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When and for whom would e-waste be a treasure trove? Insights from a network equilibrium model of e-waste flows

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Abstract

E-waste is the fastest growing waste stream. Due to its potential economic value as well as its possible negative impacts on the environment, tracing e-waste flow is a major concern for stakeholders of e-waste management. Especially, whether or not adequate amounts of electrical and electronic equipment waste (WEEE) flow into the designed recycling systems is a fundamental issue for sustainable operations. In this paper, we analyze how technical, market, and legislative factors influence the total amount of e-waste that is collected, recycled, exported and (legally and illegally) disposed of. We formulate the e-waste network flow model as a variational inequality problem. The results of the numerical examples highlight the importance of considering the interaction between the supply and the demand side for precious materials in policy-decisions. Low collection rates of e-waste lead to low profits for stakeholders and make it difficult to establish sustainable recycling operations. Increasing WEEE collection rates increases recyclers’ profits; however, it only increases smelters’ profits up to a certain limit, after which smelters cannot benefit further due to limited demand for precious materials. Furthermore, the results emphasize the importance of establishing international control regimes for WEEE flows and reveal possible negative consequences of the recent trend of dematerialization. More precisely, product dematerialization tends to decrease recyclers’ and smelters’ profits as well as to increase the outflow of e-waste from the designated recycling system.

1 Introduction

Electrical and electronic equipment waste (WEEE) is the fastest growing waste stream. In the EU alone, WEEE has been growing at a rate of 3-5% per year, about three times faster than average waste. 90% of this waste is still landfilled or incinerated, which not only damages the environment but also poses health risks to society (Savage, 2006). WEEE recycling and its management have unique associated challenges since WEEE is usually a composition of several materials, including hazardous ones, such as, for example, mercury, sulfur, and cadmium and precious ones, such as iron, copper, gold and nickel (Sodhi and Reimer, 2001).

The establishment of sound collection systems for WEEE is of high priority in many countries. Increasing the efficiency of collection and recycling
systems leading to a reduction in stockpiling by households, discarding in landfills, incineration, and illegal exports is one of the pillars of the raw material initiative of the European Union (European Commission, 2010). A similar government movement concerning precious metal recovery from WEEE can be observed in Japan (The Nikkei American Edition, 2010b) and the U.S. (The New York Times, 2010).

The Ontario Electronic Stewardship (OES), for example, gathered only one-third of the targeted WEEE of 42,000 tons in its first year although it pays recyclers $235 per ton of WEEE. OES suspects that many of the collected WEEE from end-users are exported to developing countries that lack appropriate treatment facilities for WEEE (Torstar News Service, 2010). In the EU, only 30 percent of the region’s e-waste, mostly parts without value, flowed through registered collection and recycling organizations (Shao and Lee, 2009). In China, WEEE collection programs in Qingdao and Zhejiang are finding it difficult to control WEEE flows into the informal sector that commits illegal dumping and/or inappropriate treatment (Hicks et al., 2005).

Legislative (assessing penalties for illegal dumping and minimum collection rates), market (WEEE supply and raw material demand), and technical factors (the sorting and recycling capabilities and the percentage of hazardous and precious materials in products) influence the flow of WEEE. The effects of the interaction among these factors are still largely unknown which makes it difficult to make policy recommendations. In this paper, we seek to answer the following research questions:

- How do legislative, market, and technical factors impact the total amount of waste that is collected, recycled and (legally and illegally) disposed of?
- How do legislative, market, and technical factors impact the profits of the stakeholders involved in the recycling process?
- How can policy-makers use this knowledge to design efficient and sustainable reverse logistics systems?

The results of the numerical examples emphasize the importance of an adequate supply of recycled products and demand for precious materials. Existing conflicts of interest between recyclers and smelters are shown. Furthermore, the results highlight possible negative consequences of the recent trend of dematerialization.
The remainder of this article is organized as follows. In section 2, we provide a description of factors influencing WEEE flows. In section 3, we provide an overview of the related literature. Section 4 develops the analytical framework used in the paper. In section 5, we present the numerical analysis and discuss managerial insights. Section 6 summarizes our results and presents future research questions.

2 Factors Influencing WEEE Flows

Based on a literature review of papers dealing with reverse logistics, Rahman and Subramanian (2012) identified eight factors for end-of-life computer recycling operations. Lindhqvist (2000) noted that administrative instruments (e.g., regulations related to recycling, reuse, and disposal) and economic instruments (e.g., tax and/or subsidy and disposal fee systems) are important drivers for a sustainable economy. Hosoda (2008) argued that technical conditions, legal conditions, and market conditions are key drivers for designing a material circulation society. In line with these studies, we group the factors into three groups: legislative, market, and technical factors.

In this paper, we develop a network equilibrium model which allows us to simultaneously evaluate impacts of legislative, market, and technical factors on e-waste flows, prices, and profits of decision-makers as Figure 1 shows. Furthermore, it enables us to explicitly consider effects of competition between multiple stakeholders at multiple tiers of the network on e-waste flows. A variational inequality formulation allows us to apply algorithms for solving the complex general equilibrium problem, as well as to concisely address stakeholders’ decision problems.

Legislative Factors

Many past studies (e.g., Rahman and Subramanian, 2012) determine that legislation is one of the most important driving forces for successful reverse supply chains. Legislative factors include penalties for illegal dumping, import/export duties and restrictions. The EU enacted Directive 2002/96/EC together with Directive 2002/95/EC to restrict the use of certain hazardous substances in electrical and electronic equipment (RoHS). Similar legislation or regulations are being implemented or planned in other countries including Japan, Canada, the US, and China (Toyasaki et al., 2011). Controlling
transboundary WEEE flows is a major interest to legislators for reducing environmental damages. However, a strict control of transboundary WEEE flows plays an obstructive factor for international circulation of recyclable materials, and, consequently, results in increased reliance on natural resource development. For example, Richo, Japan, tried to transship toner cartridges collected in Hong Kong to Japan for recycling; however it abandoned the program due to the enormous time required and costs incurred at customs (Kojima, 2005).

**Market Factors**

As highlighted in Rahman and Subramanian (2012), many past studies, including Teunter and van der Laan (2002), Ravi et al. (2005) and Guide Jr. and Van Wassenhove (2009), explicitly mention the importance of market factors for implementing reverse supply chain operations. Market factors that can be considered include recycled material supply and demand and costs for leachate-controlled landfills. Recently, competition for precious metals from WEEE has become more intense due to tight metal markets associated with economic growth in both India and China and tightened export controls for precious metals. A growing number of countries, including the United States, is recognizing the value of WEEE (The New York Times, 2010). In the U.S., “the House of Representatives approved a bill authorizing research to address the supply of rare earths” (The New York Times, 2010). Indeed, the high density of precious metals included in cell phones attracts the attention of relevant recycling parties.
Technical Factors

Technical factors include the percentage of precious and hazardous materials contained in products and the percentage of precious materials that smelters can extract from collected WEEE. Firms reduce or eliminate hazardous materials contained in products due to environmental concerns (e.g., Murphy and Poist (2003)) or consumer demand (e.g., Kapetanopoulou and Tagaras, 2011). Firms also seek to improve sorting precision of returned products and/or recycling technologies due to incentives (e.g., Ferguson and Toktay, 2006) or economic reasons (e.g., Ravi et al., 2005). In order to secure adequate amounts of WEEE for recycling in Japan, some smelters have developed new smelter technologies that allow them to extract rare metals from WEEE that rival smelters cannot extract. Consequently, they achieve a 99% recycling rate and can purchase WEEE for a higher price from South East Asia (The Nikkei American Edition, 2008a).

3 Literature Overview

A fast growing research stream investigates WEEE management and recycling systems. Sodhi and Reimer (2001) developed optimization models for each member of an electronics recycling network and obtained results using data from the U.S. recycling industry. In their models, prices and transportation costs are exogenously defined, and competition between members is ignored. Nagurney and Toyasaki (2005) formulated a multitiered e-cycling network model using a variational inequality formulation in a Cournot oligopoly game. The authors focused on formulating the relationship between different members of the e-cycling network. The formulation provides the endogenous equilibrium prices and material flows between tiers. Hammond and Beullens (2007) developed a closed-loop supply chain model for WEEE in a Cournot pricing game with perfect information. They numerically found that minimum recovery targets stimulated manufacturers’ reverse chain activities. Based on this work, Yang et al. (2009) developed a closed-loop supply chain network model that included raw material suppliers, manufacturers, retailers, consumers and recovery centers. Qiang et al. (2013) include demand and yield uncertainty in their closed-loop supply chain network model with competition and distribution channel investments. Rahman and Subramanian (2012) identified eight factors for end-of-life computer recycling operations.
and investigated interactions among these factors through a cognition mapping process applied at two private recycling companies in Australia.

Our model differs from previous models in the structure of the network and the behavior of the decision-makers. While Nagurney and Toyasaki (2005) assume that cost-minimizing consumers have to dispose all the waste, our research explicitly considers that consumers have the option of keeping some of the waste. Furthermore, recyclers can decide between selling the waste to local smelters or to offshore smelters. This setting allows us to study the impact of different policies on the total amount of waste that is collected and processed within a country.

In order to also represent the important issue of technology investments, our model explicitly considers impacts of technical factors, specifically, sorting precision and the percent of precious materials that can be extracted from WEEE, on the amount of collected and legally or illegally discarded WEEE. Furthermore, this paper explicitly addresses the issue of hazardous waste, illegal dumping as well as the risk of detection. Studies specifically dealing with risks related to hazardous waste reverse logistics are rare. Hu et al. (2002) presented a cost-minimization model for a multi-time-step and multi-type hazardous waste reverse logistics system. Sheu (2008) applied linear multi-objective optimization models to maximize the profit of forward flows and to minimize costs and risks in the reverse flows. In this paper, a penalty associated with disposing a unit of the electronic waste in the land-fill is imposed with a certain probability if a source is audited and found in violation of the regulation by the regulator.

4 The Model

In this section, we develop the multitiered e-cycling network model with illegal dumping and offshoring depicted in graphical form in Figure 2. The network consists of four tiers of nodes. Consider $m$ sources of electronic waste as represented by the top tier of nodes in Figure 2 ($h = 1, \ldots, m$); $n$ recyclers of electronic waste, as depicted by the second tier of nodes ($i = 1, \ldots, n$); $o$ smelters as depicted by the third tier of nodes ($j = 1, \ldots, o$), and $p$ demand markets for precious materials shown in the fourth tier ($j = 1, \ldots, p$). In addition, let node $n + 1$ at the second tier denote illegal dumping by consumers, let node $o + 1$ at the third tier denote offshoring of recycling activities by recyclers who send e-waste outside the country and let node $p + 1$
at the fourth tier denote disposal of waste in leachate-controlled landfills by smelters. Here, we follow the common network structure that can be found in Sodhi and Reimer (2001). Recyclers collect, sort and disassemble e-waste that they get from sources. We assume that recyclers disassemble e-waste into a certain number of components \( v = 1, \ldots, l \); \( v = l + 1 \) indicates components that cannot be classified (residual waste). Smelters buy these components, extract precious materials and sell them to demand markets. The remaining parts of the e-waste are categorized as hazardous and residual waste and are disposed at a legal landfill.

The endogenous and exogenous variables (Tables 1 and 2, respectively) and the functions for the model (Table 3) are presented subsequently. We assume that the functions for the aversion to product returns, the disutility of holding waste, the illegal dumping penalty, all transportation, transaction, collection, sorting, disassembly, processing and waste disposal costs are continuously differentiable. Furthermore, we require the cost functions for recyclers and smelters as well as recyclers’ offshoring costs to be continuously differentiable and convex. The demand functions for precious materials on the fourth tier of our network are linearly decreasing in prices.
### Material Flows

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_{hi}$</td>
<td>e-waste flow from source $h$ to $i$</td>
</tr>
<tr>
<td>$q_{ijv}$</td>
<td>flow of components of type $v$ from recycler $i$ to smelter $j$</td>
</tr>
<tr>
<td>$q_{i(o+1)v}$</td>
<td>e-waste flow of components of type $v$ from recycler $i$ to offshore smelter $o+1$</td>
</tr>
<tr>
<td>$q_{jk}$</td>
<td>flow of precious components from smelter $j$ to demand market $k$</td>
</tr>
</tbody>
</table>

- **$Q_h \in \mathbb{R}^{(n+1)}$** vector of e-waste flows from source $h$ to the second tier nodes, $q_{hi}$, $\forall i$
- **$Q' \in \mathbb{R}^{m}_+$** vector of e-waste flows from all sources to recycler $i$, $q_{hi}$, $\forall h$
- **$Q_i \in \mathbb{R}^{(o+1)\times(l+1)}_+$** vector of e-waste flows from recycler $i$ to the third tier nodes
- **$Q^i_j \in \mathbb{R}^{n\times(l+1)}_+$** vector of e-waste flows from all recyclers to a smelter $j$, $q_{ijv}$, $\forall i$ and $\forall v$
- **$Q_j \in \mathbb{R}^{p}_+$** vector of e-waste flows from smelter $j$ to the fourth tier nodes
- **$Q^k \in \mathbb{R}^o_+$** vector of e-waste flows from all smelters to a fourth tier node $k$, $q_{jk}$, $\forall j$

### Prices

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_h$</td>
<td>opportunity cost of source $h$ for dealing with e-waste</td>
</tr>
<tr>
<td>$\rho_{1hi}$</td>
<td>incentive (price) source $h$ receives from (pays to) recycler $i$</td>
</tr>
<tr>
<td>$\rho_{2ijv}$</td>
<td>incentive (price) recycler $i$ receives from (pays to) smelter $j$ for components of type $v$</td>
</tr>
<tr>
<td>$\rho_{3jk}$</td>
<td>price for precious materials sold to demand market $k$ by smelter $j$</td>
</tr>
<tr>
<td>$\rho_{4k}$</td>
<td>material price (of precious materials) at demand market $k$</td>
</tr>
<tr>
<td>$\rho_{4} \in \mathbb{R}^p_+$</td>
<td>vector of demand market prices</td>
</tr>
</tbody>
</table>

### Lagrange Multipliers

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma^1_{iv}$</td>
<td>Lagrange multiplier for equation (6), $\forall v$</td>
</tr>
<tr>
<td>$\gamma^2_{iv}$</td>
<td>Lagrange multiplier for equation (7), $\forall v$</td>
</tr>
<tr>
<td>(\gamma_i^{(l+1)})</td>
<td>Lagrange multiplier for equation (8)</td>
</tr>
<tr>
<td>(\gamma_i^4)</td>
<td>Lagrange multiplier for equation (9)</td>
</tr>
<tr>
<td>(\eta_j)</td>
<td>Lagrange multiplier for equation (13)</td>
</tr>
</tbody>
</table>

Table 1: Endogenous variables of the model

| \(\bar{S}_h\) | volume of e-waste possessed by source \(h\) |
| \(\pi_{1h}\) | probability that source \(h\)’s illegal dumping is discovered |
| \(\bar{\rho}_{2(o+1)}\) | unit price for e-waste at off-shore smelter \(o + 1\) |
| \(\alpha_{hv}\) | fraction of components of type \(v\) that is included in e-waste from \(h\) |
| \(\beta_{v}^{\text{prec.}}\) | fraction of precious materials included in component of type \(v\) |
| \(\beta_{v}^{\text{haz.}}\) | fraction of hazardous materials included in component of type \(v\) |
| \(\delta_i\) | sorting precision of recycler \(i\) |
| \(\delta_j\) | extraction precision of smelter \(j\); \(\delta_j = 1\) means that smelter \(j\) can extract 100\% of precious materials that are included in e-waste |
| \(M_i\) | minimum collection quantity of recycler \(i\) |

Table 2: Model parameters

| \(a_h(Q_h)\) | source \(h\)’s aversion to product return |
| \(\varepsilon_h(\bar{S}_h, Q_h)\) | disutility level of source \(h\) for holding waste |
| \(c_{hi}(q_{hi})\) | transportation costs faced by source \(h\) and option \(i\) |
| \(\tilde{p}_{1h}(q_{h(n+1)})\) | illegal dumping penalty charged to source \(h\) if detected |
| \(f_i(Q_i)\) | recycler \(i\)’s collection, sorting and disassembly costs |
| \(f_j(Q_j^j, Q_j)\) | smelter \(j\)’s transportation and processing costs |
| \(c_{kj}(q_{jk})\) | transaction costs faced by demand market \(k\) with smelter \(j\) |
| \(d_k(\rho_{4k})\) | demand at market \(k\) |
| \(\rho_{j(p+1)}(Q_j^j, Q_j)\) | total waste disposal costs for smelter \(j\) at the legal landfill |
Sources’ Behavior

The sources possess a given volume of e-waste. The assumption of a given volume of e-waste enables us to highlight the impacts of legislative, market, and technical factors on e-waste flows. The waste that each source holds differs with respect to the percentage of components of type \( v \) included, \( \alpha_{hv} \). Sources have the option of either keeping the waste or of disposing it legally or illegally. Legal and illegal disposal of waste is associated with transportation costs, which include, for example, costs associated with traveling to the collection center or landfill. Sources of waste charge or pay each recycler \( i \) a price per unit of e-waste, where the price is an endogenous variable in the model. The sources’ e-waste flow is controlled by their two incompatible feelings towards their products (aversion to returning products (Hammond and Beullens, 2007) and disutility of holding waste) and economic incentives (incentives from recyclers, penalties for illegal dumping and transaction costs). Aversion to returning products might be caused by sentimental value associated with the product. The disutility of holding, “holding costs”, represents the costs for holding on to end-of life/end-of use products. Holding costs provide an incentive to discard the product even if discarding activities are associated with costs, as it is frequently the case for the material recycling of products.

The relationship between the two contradictory feelings depends on a product’s bulkiness, the amount, and its emotional value. If an end-of-life product is relatively small (e.g. cell phones), the former might be stronger than the latter. The latter, however, might be stronger than the former if a product is very bulky (e.g. refrigerators). The specific relationship of the two feelings is unique for each consumer and each product class. For example, the amount of collected TV sets, air conditioners, washing machines, and refrigerators increased by 50% between 2001 and 2005 in Japan while the amount of collected cell phones decreased by 50% from 2000 through 2006 (The Nikkei American Edition, 2008b). Cell phones are characterized by their small size as well as high sentimental value if personal information, for example, pictures are stored. The holding cost function enables the model to consider impacts of different types of products in terms of size on sources’
decision making. Furthermore, by considering the disutility of holding prod-
ucts, the model can approximate sources’ intertemporal decision making by
accounting for the costs that accrue over several time periods. Under a series
of e-waste recycling regulations, the sources of e-waste are prohibited from
dumping waste because it contains hazardous materials. However, sources’
illegal dumping is a serious problem. For example, the amount of illegal e-
waste dumping increased by 44% in 2003 as compared to before the SHAR
law came into force in 2001. Our model reflects this reality, allows for illegal
dumping of waste by sources, and includes associated penalties.

Various structures of the audit probability were proposed by many re-
(1990) assumed that the firm’s subjective probability of being audited is a
function of the level of emissions, the level of permits, and a vector of un-
specified exogenous audit parameters set by the enforcement agency. Keeler
(1991) and van Egteren and Weber (1996) proposed an audit probability
function that was dependent on the violation level. Mrozek (1995) assumed
that the audit probability might depend on the initial allocation of permits.
In this paper, the penalty associated with disposing a unit of the electronic
waste in the landfill is monotone increasing in the amount of waste dumped.

Sources’ (End-users’) Equilibrium Conditions

The equilibrium conditions reflect sources’ behavior as described in the pre-
vious section. For all sources: \( h; h = 1, \ldots, m \), in equilibrium, one must have that:

\[
\begin{align*}
\bar{S}_h & \begin{cases}
= \sum_{i=1}^{n+1} q_{hi}^* & \text{if } \lambda_h^* > 0 \\
\geq \sum_{i=1}^{n+1} q_{hi}^* & \text{if } \lambda_h^* = 0;
\end{cases} \\
\end{align*}
\]

(1)

\[
\begin{align*}
a_h(Q_h^*) + c_{hi}(q_{hi}^*) - \varepsilon_h(\bar{S}_h, Q_h^*) + \lambda_h^* \begin{cases}
= \rho_{hi}^* & \text{if } q_{hi}^* > 0 \text{ for all } i = 1 \ldots n \\
\geq \rho_{hi}^* & \text{if } q_{hi}^* = 0 \text{ for all } i = 1 \ldots n;
\end{cases}
\end{align*}
\]

(2)

\[
\begin{align*}
a_h(Q_h^*) + c_{h(n+1)}(q_{h(n+1)}^*) - \varepsilon_h(\bar{S}_h, Q_h^*) \\
+ \lambda_h^* \begin{cases}
= -\pi_{1h} \bar{p}_{1h}(q_{h(n+1)}^*) & \text{if } q_{hi}^* > 0 \text{ for } i = n + 1 \\
\geq -\pi_{1h} \bar{p}_{1h}(q_{h(n+1)}^*) & \text{if } q_{hi}^* = 0 \text{ for } i = n + 1.
\end{cases}
\end{align*}
\]

(3)
Condition (1) states that the equilibrium shadow price $\lambda_h^*$ is zero if the total amount of product available is larger than the amount that is illegally dumped and sent to recyclers. If the amount that is available and the amount transshipped are equal, then $\lambda_h^*$ represents the opportunity cost for the different ways of dealing with the waste. Condition (2) states that source $h$ will provide her end-of-life products to recycler $i$ unless the consumer’s aversion of returning the products plus the transaction costs (from the perspective of the sources) plus the opportunity cost for treating the waste minus disutility associated with holding the waste exceeds the incentive set by the recycler for obtaining the product. Condition (3), on the other hand, states that source $h$ will illegally dump her end-of-life products unless the expected penalty of illegal dumping exceeds her disutility associated with holding the waste minus the aversion of returning the product minus transaction costs minus the opportunity cost for treating the waste.

As it is the case for the analogous conditions in the other sections, these conditions correspond to an extension of the well-known spatial price equilibrium conditions (Samuelson, 1952; Takayama and Judge, 1971; Nagurney, 1999; Hammond and Beullens, 2007, and the references therein) and have also been utilized in a variety of supply chain network equilibrium problems (Nagurney, 2006). Clearly (see e.g., Nagurney, 1999), conditions (1), (2), and (3) can be formulated as a variational inequality problem given by:

$$\text{determine } (q_{hi}^*, \lambda_h^*) \in K_1, \text{ for } h = 1, \ldots, m; i = 1, \ldots, n + 1, \text{ such that}$$

$$\sum_{h=1}^{m} \sum_{i=1}^{n} \left[ a_h(Q_h^*) - \varepsilon_h(\bar{S}_h, Q_h^*) + c_{hi}(q_{hi}^*) - \rho_{1h}^* + \lambda_h^* \right] \times [q_{hi}^* - q_{hi}]$$

$$+ \sum_{h=1}^{m} \left[ a_h(Q_h^*) - \varepsilon_h(\bar{S}_h, Q_h^*) + c_{h(n+1)}(q_{h(n+1)}^*) + \pi_{1h} \bar{p}_{1h}(q_{h(n+1)}^*) + \lambda_h^* \right] \times [q_{h(n+1)}^* - q_{h(n+1)}]$$

$$+ \sum_{h=1}^{m} \left[ \bar{S}_h - \sum_{i=1}^{n+1} q_{hi}^* \right] \times [\lambda_h - \lambda_h^*] \geq 0, \quad \forall (q_{hi}, \lambda_h) \in K_1,$$

where $K_1 \equiv \{(q_{hi}, \lambda_h) | (q_{hi}, \lambda_h) \geq 0, \quad \forall h, i\}.$

**Recyclers’ Behavior**

Following reports of companies that collect end-of-life e-wastes and previous related research (e.g., Sodhi and Reimer, 2001; Chatterjee and Kumar, 2009), recyclers in our model are entities that accept e-waste from end-users and who
segregate and disassemble the waste. According to Sodhi and Reimer (2001),
the disassembly of waste can enhance its value in three ways: 1. through
the sale of components leads to higher revenue than material recovery; 2.
through additional recoverable material from disassembled subassemblies;
and 3. through separated subassemblies being sent to different smelters for
a more profitable material recovery than the entire e-waste being sent to a
single smelter (e.g. gold content in the entire PC is 0.005%; however, 0.05%
gold content in the motherboard). For example, a cell-phone recycler in
Japan disassembles collected phones into 12-18 components and sends them
separately based on their materials.

In this model, recyclers collect e-waste from sources and pay or charge
them a price per unit of e-waste. The waste is disassembled and sorted ac-
cording to the components included. The recyclers can decide to send part or
all of the e-waste outside of the country and, thereby, offshore extraction ac-
tivities of precious materials. Waste that is offshored usually costs a uniform
price, independent of the components included (c.f., Sandner and Schilling,
2010).

Recyclers’ Optimization Problems

Recyclers’ objective is to maximize profit. Revenue thereby incurred is for
selling components to smelters and for the export to offshore markets. Costs
arise for collection, sorting and disassembly of e-waste and possibly from in-
centives recyclers have to pay to sources in order to get the e-waste. Let \( \rho^*_{2ijv} \)
denote the endogenously determined price charged by recycler \( i \) to smelter
\( j \) for component \( v \). A positive \( \rho^*_{2ijv} \) implies that recycler \( i \) receives money
from smelter \( j \) for transshipping component \( v \), a negative price implies that
recycler \( i \) needs to pay money to smelter \( j \). We can express the criterion of
profit maximization for recycler \( i \) as:

\[
\max_{Q_i, Q_j} \sum_{j=1}^{o} \sum_{v=1}^{l+1} (\rho^*_{2ijv} \cdot q_{ijv}) + \sum_{v=1}^{l+1} \bar{\rho}_{2(o+1)} \cdot q_{i(o+1)v} - \sum_{h=1}^{m} (\rho^*_{1hi} \cdot q_{hi}) - f_i(Q_i) \tag{5}
\]
subject to:

\[ \sum_{j=1}^{o+1} q_{ijv} \geq \delta_i \sum_{h=1}^{m} (\alpha_{hv} \cdot q_{hi}), \quad \forall v \in \{1, \ldots, l\}, \]  

\[ \sum_{j=1}^{o+1} q_{ijv} \leq \delta_i \sum_{h=1}^{m} (\alpha_{hv} \cdot q_{hi}), \quad \forall v \in \{1, \ldots, l\}, \]  

\[ \sum_{j=1}^{o+1} q_{ij(l+1)} \geq \sum_{h=1}^{m} q_{hi} - \sum_{v=1}^{l} \sum_{j=1}^{o+1} q_{ijv}, \]  

\[ \sum_{h=1}^{m} q_{hi} \geq M_i, \]  

\[ q_{hi} \geq 0, q_{ijv} \geq 0, \quad h = 1, \ldots, m; \quad j = 1, \ldots, o+1; \quad v = 1, \ldots, l+1. \]  

The constraints reflect recycler \( i \)'s sorting possibilities \( \delta_i \), where \( \delta_i = 0.5 \), for example, means that recycler \( i \) can correctly assign half of the disassembled e-waste to components of type \( v \). \( \delta_i \sum_{h=1}^{m} (\alpha_{hv} \cdot q_{hi}) \) is the amount of component \( v \) that recycler \( i \) received from all sources \( h \) and correctly classified. Constraints (6) and (7) indicate that recyclers need to transfer these components to the next tier but that they cannot transfer more components than they have. Constraint (8) covers all components that cannot be classified \( v = l + 1 \) (residual waste) and makes sure that recyclers transfer all residual waste to the next tier. Residual waste achieves a negative price (recyclers need to pay for its treatment), therefore, a constraint that makes sure that recyclers do not transfer more residual waste than they get is not necessary.

The WEEE legislation adopted by the EU requires a minimum collection quantity of WEEE of 4 kg per person a year. Other countries introduced similar legislation. In order to shed more light on the impact of minimum collection quantities, constraint (9) ensures a minimum amount of \( M_i \) that is transshipped from sources \( h \) to recycler \( i \).

The equilibrium conditions for all recyclers \( i, i = 1, \ldots, n \), can be formulated as a variational inequality problem as follows: determine \( (q_{hi}^*, q_{ijv}^*, \gamma_{iv}^{1*}, \gamma_{iv}^{2*}, \gamma_{ii(l+1)}^{3*}, \gamma_{i}^{4*}) \in K_2 \) for \( h = 1, \ldots, m; \quad j = 1, \ldots, o+1; \quad v = 1, \ldots, l+1 \) such that
where

\[ \mathbb{K}_2 \equiv \{(q_{hi}, q_{ijv}, \gamma_{i1}, \gamma_{i2}, \gamma_{i3}, \gamma_{i4}) \mid (q_{hi}, q_{ijv}, \gamma_{i1}, \gamma_{i2}, \gamma_{i3}, \gamma_{i4}) \geq 0, \forall h, i, j, v\} \].

**Smelters’ Behavior**

Smelters in our model are entities that recover metal and nonmetal materials from disassembled parts of products. As Chatterjee and Kumar (2009) explains, this process includes pulverization, valuation, and extraction. The recovery rate varies depending on technologies, knowledge, and economic viability. In developing countries, the recovery yield of precious metals is very poor due to unskilled operations (Chatterjee and Kumar, 2009); in developed countries, more than 90%, but less than 100%, of recovery rates are
achieved in silver, palladium, and gold (Park and Fray, 2009). However, recycling units in developed countries face shortages of collected quantities and thereby operations become economically non-viable (Chatterjee and Kumar, 2009). Furthermore, recovery technologies of rare metals and rare earths still are immature, so that most of them are still included in residue that is discarded to landfills. The Japanese Ministry of Economy, Trade and Industry therefore encourages recycling units to develop economically viable technologies for rare metal extraction (Ministry of Economy, Trade and Industry Japan and Japan Oil, Gas and Metals National Corporation, 2012). However, some smelters have successfully implemented extraction technologies and logistics systems for rare metals and rare earth, so that they establish a superior position in the market (The Nikkei American Edition, 2008a).

As noted earlier, the disposal of WEEE is a major problem due to the fact that WEEE includes hazardous materials. Due to the toxicity of these materials, WEEE has to be treated at smelters with appropriate treatment facilities, and the residuals after treatment must be disposed of in leachate-controlled landfills. In this model, smelters acquire waste from recyclers and pay/receive a price for the waste. The costs that they incur for processing the waste depend on the amount of waste that they receive and treat. Hazardous materials require specific treatment due to their toxicity and, therefore, smelters incur additional costs for dealing with hazardous materials. Smelters extract and sell precious materials from the waste. Smelters might not be able to extract precious materials completely from electronic waste. Depending on the technology used in the process, the percentage of precious materials that can be taken out of the waste varies. Smelters discard residual waste at landfills. Each smelter pays a price per unit of waste disposed of at the landfill, depending on the type of material.

**Smelters’ Optimization Problems**

Smelters’ revenue arises from selling precious materials to demand markets. They incur costs for buying e-waste from recyclers, for disposal of e-waste at legal landfills and for processing the waste. Of course, prices for components \( v \) differ due to their differing material composition, especially, the different percentages of precious materials included, \( \beta_v^{\text{prec}} \). By denoting \( \rho_{ijk}^\ast \) as the (endogenous) price that smelter \( j \) charges to demand market \( k \) for precious
materials, a smelter $j$’s optimization problem is given by:

$$\max_{Q^j, Q^j} \sum_{k=1}^{p}(\rho_{3jk}^* \cdot q_{jk}) - \sum_{i=1}^{n} \sum_{v=1}^{l+1}(\rho_{2ijv}^* \cdot q_{ijv} - \rho_{j(p+1)}(Q^j, Q^j) - f_j(Q^j, Q^j))$$  \hspace{1cm} (12)$$

subject to:

$$\sum_{k=1}^{p} q_{jk} \leq \delta_j \sum_{v=1}^{l+1}(\beta_{v}^{prec} \sum_{i=1}^{n} q_{ijv}),$$  \hspace{1cm} (13)$$

$$q_{ijv} \geq 0, q_{jk} \geq 0, \quad i = 1, \ldots, n; \quad k = 1, \ldots, p; \quad v = 1, \ldots, l + 1.$$  \hspace{1cm} (14)$$

Constraint (13) guarantees that smelters do not sell more precious materials than they extract from all components $v$ that they received from all sources $i$. The optimization conditions for all smelters $j$ can be formulated as a variational inequality problem as follows: determine $(q_{ijv}^*, q_{jk}^*, \eta_j^*) \in K_3$ for $i = 1, \ldots, n; \quad j = 1, \ldots, o; \quad k = 1, \ldots, p; \quad v = 1, \ldots, l + 1$, such that

$$\sum_{v=1}^{l+1} \sum_{i=1}^{n} \sum_{j=1}^{o} \left[ \frac{\partial \rho_{j(p+1)}(Q^j, Q^j)}{\partial q_{ijv}} + \frac{\partial f_j(Q^j, Q^j)}{\partial q_{ijv}} + \rho_{2ijv}^* - \eta_j \beta_{v}^{prec} \delta_j \right] \times [q_{ijv} - q_{ijv}^*]$$

$$+ \sum_{j=1}^{o} \sum_{k=1}^{p} \left[ -\rho_{3jk}^* + \frac{\partial \rho_{j(p+1)}(Q^j, Q^j)}{\partial q_{jk}} + \frac{\partial f_j(Q^j, Q^j)}{\partial q_{jk}} + \eta_j^* \right] \times [q_{jk} - q_{jk}^*]$$

$$+ \sum_{j=1}^{o} \left[ \delta_j \sum_{v=1}^{l+1}(\beta_{v}^{prec} \sum_{i=1}^{n} q_{ijv} - \sum_{k=1}^{p} q_{jk}^*) \times [\eta_j - \eta_j^*] \geq 0,ight.$$  \hspace{1cm} (15)$$

$$\forall (q_{ijv}, q_{jk}, \eta_j) \in K_3, \quad \text{where} \quad K_3 \equiv \{(q_{ijv}, q_{jk}, \eta_j)|(q_{ijv}, q_{jk}, \eta_j) \geq 0, \forall i, j, v, k\}.$$  \hspace{1cm} (15)$$

**Demand Market Behavior**

Consumers buy precious materials from smelters. Consumers face transaction costs for their transaction with smelters that depend on the volume of shipment/transaction between the smelter and the demand market pair.
The Demand Market Equilibrium Conditions

The equilibrium conditions associated with the precious material shipments from smelters to precious material markets take on the following form: for any smelter with associated demand market \( k \) of precious materials where \( k = 1, \ldots, p \):

\[
d_k(\rho_{4k}^*) \begin{cases} 
= \sum_{j=1}^{o} q_{jk}^*, & \text{if } \rho_{4k}^* > 0 \\
\leq \sum_{j=1}^{o} q_{jk}^*, & \text{if } \rho_{4k}^* = 0;
\end{cases}
\]

(16)

\[
\rho_{3jk}^* + c_{kj}(q_{jk}^*) \begin{cases} 
= \rho_{4k}^*, & \text{if } q_{jk}^* > 0 \\
\geq \rho_{4k}^*, & \text{if } q_{jk}^* = 0.
\end{cases}
\]

(17)

Condition (16) states that if the price the consumers are willing to pay for the precious materials at demand market \( k \), \( \rho_{4k}^* \), is positive, then the quantity consumed at the demand market is precisely equal to the demand. Otherwise, the availability of precious materials at demand market \( k \) may exceed its demand. Condition (17) states that consumers at demand market \( k \) will purchase precious materials from smelter \( j \), if the price charged by the smelter for the product, \( \rho_{3jk}^* \), plus the transaction costs (from the perspective of the consumers) do not exceed the price that consumers are willing to pay for precious materials. Clearly, (see e.g., Nagurney, 1999) conditions (16) and (17) must hold for all demand markets \( k; k = 1, \ldots, p \), and can be formulated as a variational inequality problem, given by:

\[
\text{determine } (q_{jk}^*, \rho_{4k}^*) \in \mathbb{K}_4, \text{ for } j = 1, \ldots, o; k = 1, \ldots, p \text{, such that }
\]

\[
\sum_{k=1}^{p} \left[ \sum_{j=1}^{o} q_{jk}^* - d_k(\rho_{4k}^*) \right] \times [\rho_{4k} - \rho_{4k}^*] + \sum_{j=1}^{o} \sum_{k=1}^{p} \left[ \rho_{3jk}^* + c_{kj}(q_{jk}^*) - \rho_{4k}^* \right] \times [q_{jk} - q_{jk}^*] \geq 0,
\]

(18)

\( \forall (q_{jk}, \rho_{4k}) \in \mathbb{K}_4 \), where

\[\mathbb{K}_4 \equiv \{(q_{jk}, \rho_{4k})|(q_{jk}, \rho_{4k}) \geq 0, \forall j = 1, \ldots, o; k = 1, \ldots, p\} \]
The Equilibrium Conditions of the Network:

In equilibrium, the equilibrium conditions for sources and demand markets as well as the optimality conditions for recyclers and smelters must hold simultaneously and all the constraints must be satisfied (see e.g., Nagurney, 2006).

Definition 1. Network Equilibrium

The equilibrium state of the e-waste network is one where the flows between the tiers of the network coincide and the shipments and prices satisfy the sum of the optimality conditions (11) and (15) as well as the equilibrium conditions (4) and (18).

Theorem 2. Variational Inequality Formulation

The e-waste network is in equilibrium according to Definition 1, if and only if it satisfies the variational inequality problem: Determine $q_{hi}^*$, $q_{ijv}^*$, $\lambda_i^*$, $\rho_{ik}^*$, $q_{jk}^*$, $\gamma_{iv}^*$, $\gamma_{iv}^{2*}$, $\gamma_{i(l+1)}^*$, $\gamma_i^*$, $\eta_j^* \in \mathbb{K}_5 \forall h = 1, \ldots, m; i = 1, \ldots, n+1; j = 1, \ldots, o+1; k = 1, \ldots, p; v = 1, \ldots, l + 1$, such that:
\[
\sum_{h=1}^{m} \sum_{i=1}^{n} \left[ a_h(Q_h^r) - \varepsilon_h(\tilde{S}_h, Q_h^r) + c_{hi}(q_h^i) \right] + \sum_{i=1}^{l} \left[ \gamma_i^1 \alpha_{hi} \delta_i - \gamma_i^2 \alpha_{hi} \delta_i + \gamma_i^3 (\alpha_{hi} \delta_i) \right] + \gamma_i^4 + \lambda_h^* \times [q_{hi} - q_h^i] \\
+ \sum_{h=1}^{m} \left[ a_h(Q_h^r) - \varepsilon_h(\tilde{S}_h, Q_h^r) + c_{h(n+1)}(q_h^{i(n+1)}) + \pi_{hi} \rho_{hi}(q_h^{i(n+1)}) + \lambda_h^* \right] \times [q_{h(n+1)} - q_h^{i(n+1)}] \\
+ \sum_{i=1}^{n} \sum_{j=1}^{o} \sum_{v=1}^{l} \frac{\partial \rho_j(\alpha_{ip+1})(Q_j^r, Q_i^r)}{\partial q_{jv}^{i(t+1)}} + \frac{\partial \tilde{f}_j(Q_j^r, Q_i^r)}{\partial q_{jv}^{i(t+1)}} + \frac{\partial \tilde{f}_j(Q_j^r)}{\partial q_{jv}^{i(t+1)}} - \gamma_{jv}^3 \beta_{jv} \delta_j \times [q_{jv}^{i(t+1)} - q_{jv}^{i(t+1)}] \\
+ \sum_{i=1}^{n} \sum_{v=1}^{l} \left[ \tilde{\rho}_{2(i+1)} - \tilde{\rho}_{2(i+1)} - \gamma_{jv}^2 + \gamma_{jv}^3 \delta_j \right] \times [q_{jv}^{i(1+1)} - q_{jv}^{i(1+1)}] \\
+ \sum_{i=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{p} \left[ \partial \rho_j(\alpha_{ip+1})(Q_j^r, Q_i^r) - \partial \rho_j(\alpha_{ip+1})(Q_j^r, Q_i^r) - \rho_{jk} + \eta_j^* \right] \times [q_{jk} - q_{jk}^i] \\
+ \sum_{k=1}^{p} \sum_{j=1}^{o} [q_{jk} - d_k(\rho_{jk})] \times [\rho_{jk} - \rho_{jk}] + \sum_{i=1}^{l} \sum_{j=1}^{m} \left[ \sum_{h=1}^{m} q_{jv}^{i(t+1)} - \sum_{h=1}^{m} (\alpha_{hi} \cdot q_{hi}) \right] \times [\gamma_{jv}^1 \gamma_{jv}^2 \gamma_{jv}^3] \\
+ \sum_{i=1}^{n} \sum_{v=1}^{l} \left[ \delta_i \sum_{h=1}^{m} (\alpha_{hi} \cdot q_{hi}) - \sum_{j=1}^{p} q_{jv}^{i(t+1)} \right] \times [\gamma_{jv}^3 \gamma_{jv}^3 \gamma_{jv}^3] \\
+ \sum_{i=1}^{n} \left[ \sum_{h=1}^{m} q_{hi} - M_i \right] \times [\gamma_i^4 \gamma_i^4] + \sum_{h=1}^{m} [\tilde{S}_h - \sum_{i=1}^{p} q_{hi}] \times [\lambda_h - \lambda_h^*] \\
+ \sum_{j=1}^{o} \left[ \delta_j \sum_{i=1}^{l} q_{jv}^{i(t+1)} \sum_{h=1}^{m} \beta_{jv} \sum_{i=1}^{p} q_{jv}^{i(t+1)} - \sum_{k=1}^{n} q_{jk}^i \right] \times [\eta_j - \eta_j^*] \geq 0.
\]

\( (q_{hi}, q_{ijv}, \lambda_h, \rho_{4k}, q_{jk}, \gamma_i^1, \gamma_i^2, \gamma_i^3, \gamma_i^4, \eta_j) \in \mathbb{K}_5 \), where \( \mathbb{K}_5 \equiv \{(q_{hi}, q_{ijv}, \lambda_h, \rho_{4k}, q_{jk}, \gamma_i^1, \gamma_i^2, \gamma_i^3, \gamma_i^4, \eta_j) | (q_{hi}, q_{ijv}, \lambda_h, \rho_{4k}, q_{jk}, \gamma_i^1, \gamma_i^2, \gamma_i^3, \gamma_i^4, \eta_j) \geq 0, \forall h, i, j, k, v \} \).

Qualitative properties are provided in B.
5 Numerical Simulation

This section highlights how the model developed in the previous sections can be used to explore how market, technical, and legislative factors impact the amount of waste that is collected, recycled, exported and legally or illegally disposed of. The network for the numerical simulation consists of two sources of electronic waste, four recyclers, two smelters, and one demand market (see Figure 3). Sources either illegally dump their waste, hold on to it, or send it to the recyclers. Recyclers can offshore the waste or send the components $v=1,2$ and the residual waste $v=3$ to smelters. Smelters extract precious materials and sell them to the demand market. They send the residual waste to the legal landfill. The Euler method (Dupuis and Nagurney, 1993) was implemented in Matlab and applied to compute the solutions. A description of the functions and parameters used for the numerical simulations can be found in A. We assume that each tier has an identical set of functions. By including four recyclers and two smelters, our model therefore considers competition among recyclers and smelters in the recycling market.

---

Figure 3: The 4-tiered e-cycling network assumed for the numerical simulation
5.1 Influence of Market Factors

Prices for precious materials are influenced by global changes in supply and demand for these materials and vary significantly. For example, the prices for steel refined from scrap iron varied by the generated amount of scrap iron and the demand for scrap iron of international as well as domestic blast furnace and electric furnace steelmakers (Scrap Trading News, n.d.). This section describes the impact of WEEE supply and demand for precious materials on profits, waste flows and prices in the network. Our numerical examples highlight that consumer behavior, the resulting supply for WEEE, and the potential demand market size for precious materials strongly influence the stakeholders in the WEEE recycling system.

One of the major concerns of many WEEE systems is whether or not the supply of WEEE is sufficient to ensure sustainable operations. Pilot studies and assessment for WEEE collection strategies have been conducted in some municipalities, such as Massachusetts (Massachusetts Department of Environmental Protection, 2000), Portland Metro (Cascadia Consulting Group and e4 Partners, Inc., 2002), and Tokyo (Committee of Promotion of Precious Metal Containing Product Collection, Tokyo Metropolitan Government, 2009). These studies highlighted the importance of the effective design of collection systems. Although end-users’ end-of-use/-life product return is influenced by collection strategies (Boyaci et al., 2009) and collection network designs (Aras and Aksen, 2008; Aras et al., 2008), our model pays attention to the flow of WEEE in a given downstream network, rather than to collection strategies and network design.

Our numerical example indicates that WEEE supply is strongly impacted by sources’ sentiment towards holding waste. Increasing consumer aversion to product returns \( p_1 = 0, 0.5, \ldots, 2 \) reduces WEEE flows from sources, reduces recycled material flows to offshore markets, and reduces recyclers’ profits. Increasing consumers’ holding costs for holding waste \( p_2 = 0, 1, \ldots, 4 \) has the opposite effect. These results highlight the importance of effective collection designs and reflect problems of WEEE take-back schemes caused by a low collection rate, for example, low collection rates of cell phones from end-users in Japan (The Nikkei American Edition, 2008b).

Figure 4 provides a more detailed analysis of the impact of an increase of the volume of e-waste possessed by source \( h, \bar{S}_h \), on recyclers’ and smelters’ profits. Recyclers’ profits increase steadily due to reduced prices for resources transshipped from sources of waste and increased offshoring activities. Prof-
its for smelters first increase, too. However, after a certain limit, smelters cannot benefit further from increasing supply due to limitations with respect to demand for precious materials. Hence, this example emphasizes the importance of considering the interaction between the supply and demand side for precious materials. Policies that increase WEEE supply without considering the demand side may not lead to the desired results for all market participants.

Figure 4: Impact of a variation of the demand market size on recycler $i$’s and smelter $j$’s profit

An increase of the maximum demand ($p_{11} = 1000, 2000, \ldots, 5000$) for precious materials primarily increases prices at demand markets and reduces quantities of offshored WEEE.

5.2 Technical factors

A series of WEEE laws require manufacturers of electronic equipments to bear recycling costs. These laws can encourage manufacturers to reduce material use and to improve the recyclability of products (Tojo, 2004). In developing environmentally friendly products, manufacturers change the type of materials included in products. These initiatives sometimes lead to reduced amounts of precious materials included in products as it was in the case of Hitachi that developed a motor technology without using a rare-earth magnet
Furthermore, the recent rapid development of miniaturization and lightweight technology reduces the amount of metals used in electronic equipment. In this section, we analyze the impact of technical factors on e-waste flows.

5.2.1 Extraction Possibilities

Advances in extraction technologies allow for higher quantities of precious materials to be extracted from e-waste. Figure 5 highlights the influence of these changes on offshoring. Low extraction precisions ($\delta_j < 0.3$) impede profitable smelting operations and hence cause a smelter’s output quantity to be zero. When the extraction precision is increased, the quantities that a smelter transships to demand markets increase rapidly. At first, this also leads to higher quantities being transshipped between recyclers and smelters ($0.3 < \delta < 0.6$). When the extraction precision is increased even further ($\delta > 0.6$), we observe that due to market saturation the waste stream that is brought to an offshore market rises. In our model, we observe the smallest amount of offshored waste at an extraction precision of 60%.

![Figure 5: Effects of a variation in smelter $j$’s extraction possibilities on the amount of offshoring](image-url)
5.2.2 Composition of WEEE

As Figure 6 indicates, the analysis of the impact of changes in composition of WEEE leads to interesting and partly counterintuitive results. An increase in the fraction of precious materials contained in components 1 and 2 increases quantities that are transshipped to demand markets. While this effect continues, prices that demand markets have to pay in order to get the precious materials decrease. For $\beta_{\text{prec.}} > 0.7$ decreased prices will lower the incentive for materials to remain in the network and will hence increase quantities that are brought to offshore markets. As a decrease of precious materials has a negative effect on smelters’ and recyclers’ profits, this result sheds light on the negative impact of dematerialization of electronic products on the sustainability of recycling systems.

![Figure 6: Impact of a variation of $\beta_{\text{prec.}}^1$ on transshipped quantities](image)

5.3 Legislative factors

The new directives and laws implemented in the EU, the US, Canada, and Japan that were described in the introduction, highlight the important role that legislative factors play in the development of recycling systems. Estimating the impact of these regulations on stakeholders is a difficult task due to the diversity and interrelatedness of their consequences. In Figure 7, we
highlight how the model developed in this paper can be used to analyze the impact of the probability that illegal dumping is detected. Recent attempts to raise the probability of detection of illegal dumping ($\pi_{1h}$) use geographic information systems (Tasaki et al., 2007) and satellite images (Ishihara et al., 2002). When deciding on the methods for detecting illegal dumping, policy makers need to consider the trade-offs in terms of costs and benefits.

![Figure 7: Variation of the probability that source h’s illegal dumping is detected ($\pi_{1h}$) and its impact on illegally dumped e-waste.](image)

In our numerical simulation, an increase of the detection probability $\pi_{1h}$ leads to a sharp decrease in quantities that are dumped illegally when the probability is raised from 0% to 20%, while the decrease becomes smaller when $\pi_{1h}$ is raised further. Hence, it is especially beneficial to raise the detection probability if it is currently below 20%.

6 Summary and Future Research

Reverse logistics systems for end-of-life products, especially, electrical and electronic equipment waste (WEEE), are faced with many difficulties, including illegal dumping of waste and low waste collection rates. Improving the performance of reverse logistics systems requires a good understanding of the main drivers of the logistics systems and their interaction.

In this paper, we develop a model that captures the influence of market, technical, and legislative factors on the amount of products that are
collected, recycled, and legally or illegally disposed of as well as the prices, and costs associated with these processes. The numerical examples show how this model can be used by policy makers to study the possible implications of new laws on sources of waste, smelters and demand markets, as well as on the environmental performance of the system. As the results of Sections 5.2.1 and 5.2.2 show, recyclers’ offshoring option plays a buffer for them against rapid-changing circumstances of WEEE collection and processing. This indicates the importance of establishing not only domestic but also international control regimes for WEEE flows. In this context, the e-waste recycling regime in the European Union (EU) could be a reference example. In the regime, EU Council Regulation 259/93/EEC is applied to wastes transshipped within the EU while the Basel Convention is applied to hazardous wastes into and out of the EU. Waste on the Green List of Wastes (e.g. metal scraps and waste plastics, rubber, paper, wood) defined in the Council Regulation 259/93/EEC are allowed to be transshipped within the EU without restrictions. As a result, member states of the EU enjoy the benefits of flexible trade of e-waste and its regional specialization among member states (Kojima and Yoshida, 2005). Lastly, our numerical examples also indicate the importance of adequate and secure demand for recycled materials. Easily saturated demand for recycled materials would increase the amount of offshoring. This could hinder prevalence of highly recyclable products as results of Section 5.2.2 indicate.

The model shows certain limitations and can be extended in several directions. Adding an additional tier of manufacturers would allow one to address issues associated with closed-loop supply chains. One topic of special interest in this context would be a detailed analysis of incentives and effects of new product design. In our current model, we assume that the levels of technical factors (i.e., component types, material types, sorting and smelting precision) are parameters. However, in reality, manufacturers, recyclers and smelters determine the level of sustainable product designs and recycling technologies while taking into account WEEE inflow and economic viability. In this case, the equivalence between equilibrium problems and variational inequality problems may not be achieved with a standard procedure because non-linear constraints might be included. In this case, convexity of the feasible region and conditions that satisfy a constraint qualification need to be further explored. Furthermore, our model ignores the impacts of WEEE take-back schemes. Opposing views exist concerning the desirability of different schemes (Toyasaki et al., 2011). Comparing e-waste flows and
stakeholders’ performance under different WEEE take-back schemes would be an important practical issue for recycling system designers. Our research also provides future research direction concerning the calculation method applied in the numerical examples section. The numerical experiments reveal that variations of certain parameters had a great impact on the computation time. More careful analysis of these phenomena from the perspective of algorithms and functional forms used in the numerical examples is also subject to future research. Finally, the insights gained in our numerical examples could be tested using industry data. Cross-country studies could provide insights concerning the influence of technologies while intertemporal studies could provide insights concerning the effects of policies and raw material prices.

Acknowledgements

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A Parameters and functions used for the numerical simulations

<table>
<thead>
<tr>
<th>Formula</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_h(Q_h) = p_1 \cdot \sum_{i=1}^{n+1} q_{hi}$</td>
<td>source $h$’s aversion to products returns</td>
</tr>
<tr>
<td>$\varepsilon_h(S_h, Q_h) = p_2 \cdot (S_h - \sum_{i=1}^{n+1} q_{hi})$</td>
<td>disutility level of source $h$ for holding e-waste</td>
</tr>
<tr>
<td>$c_{hi}(q_{hi}) = p_3 \cdot q_{hi}$</td>
<td>transportation cost faced by source $h$ and option $i$</td>
</tr>
<tr>
<td>$c_h(n+1)(q_{h(n+1)}) = 0$</td>
<td>transaction cost for illegal dumping (set to zero)</td>
</tr>
<tr>
<td>$\bar{p}<em>{1h}(q</em>{h(n+1)}) = p_4 \cdot q_{h(n+1)}$</td>
<td>illegal dumping penalty charged to source $h$ when detected</td>
</tr>
<tr>
<td>$f_i(Q_i) = [p_5 \cdot (\sum_{j=1}^{o+1} \sum_{v=1}^{l+1} q_{ijv})^p_k + [p_7 \cdot \sum_{j=1}^{o+1} \sum_{v=1}^{l+1} q_{ijv}] + [p_8 \cdot \sum_{j=1}^{o} \sum_{v=1}^{l+1} q_{ijv}]$</td>
<td>recycler $i$’s cost function consists of collection cost (first term), sorting cost (second term) and dismantling cost (third term); the term describing collection costs is based on Atasu et al. (2013)</td>
</tr>
<tr>
<td>$f_j(Q_j, Q_{j}) = p_3 \cdot \sum_{k=1}^{o} q_{jk}$</td>
<td>smelter $j$’s costs consist of transportation costs of processed precious materials to demand market $k$ (first term), as well as processing costs of components of type $v$ (second term); the third term reflects the fact that with an increase of $q_{jk}$ costs will increase more than proportionally due to the need of processing less attractive materials</td>
</tr>
<tr>
<td>$d_k(\rho_{4k}) = p_{11} - p_{12} \cdot \rho_{4k}$</td>
<td>demand function at demand market $k$</td>
</tr>
<tr>
<td>$c_{kj}(q_{jk}) = p_3 \cdot q_{jk}$</td>
<td>transaction costs of demand market $k$ associated with smelter $j$</td>
</tr>
<tr>
<td>$\rho_{j(p+1)}(Q_j, Q_{j}) = p_{13} \cdot \sum_{i=1}^{n+1} \sum_{v=1}^{l+1} \beta_{v}^{\text{haz}} q_{ijv} + p_{14} \cdot (\sum_{i=1}^{n} \sum_{v=1}^{l+1} q_{ijv} - \sum_{k=1}^{o} q_{jk})$</td>
<td>waste disposal cost consist of regular disposal costs (second term) and additional waste disposal costs for hazardous materials (first term)</td>
</tr>
</tbody>
</table>

Table 4: Functional forms used for the numerical simulations
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Values for the simulation</th>
<th>Base case value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_h$</td>
<td>volume of e-waste possessed by each source $h$</td>
<td>${500, 1000, 1500, 2000}$</td>
<td>$S_h = 1321.9$ (U.S. Environmental Protection Agency, Office of Solid Waste, 2008)</td>
</tr>
<tr>
<td>$\bar{p}_{2(o+1)}$</td>
<td>unit price for e-waste at off-shore smelter $o + 1$</td>
<td>$$100$ (calculated based on an argument in The Seattle Times, 2006)</td>
<td></td>
</tr>
<tr>
<td>$\alpha_{h1} = \alpha_{h2}$</td>
<td>fraction of components of type 1 (2) in e-waste that is transshipped from source $h$</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>$\beta_{1}^{prec.}$</td>
<td>percentage of precious materials contained in component of type 1 (2)</td>
<td>${0.4, 0.5, \ldots, 1}$</td>
<td>0.9</td>
</tr>
<tr>
<td>$\beta_{2}^{prec.}$</td>
<td>percentage of precious materials in residual waste (component $l + 1 = 3$)</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>$\beta_{3}^{haz.}$</td>
<td>percentage of hazardous materials contained in component of type 1 (2)</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$\beta_{3}^{haz.}$</td>
<td>percentage of hazardous materials contained in residual waste (component $l + 1 = 3$)</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>$\delta_i$</td>
<td>recycler $i$’s sorting precision</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>$\delta_j$</td>
<td>smelter $j$’s extraction precision</td>
<td>${0.1, 0.2, \ldots, 1}$</td>
<td>0.8</td>
</tr>
<tr>
<td>$M_i$</td>
<td>recycler $i$’s minimum collection quantity</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>----------------------------------------</td>
<td>----</td>
<td></td>
</tr>
<tr>
<td>$\pi_{1hi}$</td>
<td>probability that source $h$’s illegal dumping is detected</td>
<td>${0, 0.2, 0.4, 0.6, 0.8, 1}$</td>
<td>50%</td>
</tr>
<tr>
<td>$p_1$</td>
<td>parameter in the function of source $h$’s aversion to product returns</td>
<td>${0, 0.5, 1, 1.5, 2}$</td>
<td>0.1</td>
</tr>
<tr>
<td>$p_2$</td>
<td>parameter in the disutility function of source $h$</td>
<td>${0, 1, \ldots, 4}$</td>
<td>2</td>
</tr>
<tr>
<td>$p_3$</td>
<td>unit transportation costs for source $h$, smelter $j$ and demand market $k$</td>
<td>17.2 (see Boon et al., 2002)</td>
<td></td>
</tr>
<tr>
<td>$p_4$</td>
<td>fine that has to be paid when illegal dumping of waste is detected</td>
<td>1,000</td>
<td></td>
</tr>
<tr>
<td>$p_5, p_6$</td>
<td>parameters for recycler $i$’s collection cost</td>
<td>$p_5 = 0.007619, p_6 = 1.572939$ (see Atasu et al., 2013)</td>
<td></td>
</tr>
<tr>
<td>$p_7$</td>
<td>unit waste sorting costs of one ton of WEEE</td>
<td>140 (see Boon et al., 2002)</td>
<td></td>
</tr>
<tr>
<td>$p_8$</td>
<td>costs to dismantle one ton of WEEE</td>
<td>315 (see Boon et al., 2002)</td>
<td></td>
</tr>
<tr>
<td>$p_9$</td>
<td>unit costs to process one ton of WEEE</td>
<td>640 (see Ontario Electronic Stewardship, 2008)</td>
<td></td>
</tr>
<tr>
<td>$p_{10}$</td>
<td>parameter for smelter $j$’s processing cost</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>$p_{11}$</td>
<td>maximum demand at demand market $k$</td>
<td>${1000, 2000, \ldots, 5000}$</td>
<td>4000</td>
</tr>
<tr>
<td>$p_{12}$</td>
<td>slope of the demand function</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
B Supplementary Material

For easy reference, variational inequality problem (18) can be rewritten in standard variational inequality form (cf. Nagurney, 1999) as follows: determine $X^* \in \mathbb{K}_5$ such that

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \ \forall X \in \mathbb{K}_5. \quad (20)$$

where the terms of $F$ correspond to the terms preceding the multiplication signs in inequality (19), and the term $\langle \cdot, \cdot \rangle$ denotes the inner product in $N$-dimensional Euclidean space.

Since the feasible set is not compact, one cannot derive existence simply from the assumption of continuity of the function $F(X)$. One can, nevertheless, impose a rather weak condition to guarantee existence of a solution to variational inequality (19).

**Lemma 3.** Let $\mathbb{K}_b = \{(q_{hi}, q_{ijv}, \lambda_h, \rho_{4k}, q_{jk}, \gamma_{i^1v}, \gamma_{i^2v}, \gamma_{i^3(v+1)}, \gamma_{i^4v}, \eta_j)| 0 \leq \sum_{i=1}^{n+1} q_{hi} \leq S^h; 0 \leq \sum_{j=1}^{o+1} \sum_{v=1}^{3} q_{ijv} \leq q_{hi}; 0 \leq \sum_{k=1}^{p+1} q_{jk} \leq \sum_{i=1}^{n} \sum_{v=1}^{3} q_{ijv}; 0 \leq \lambda_h \leq b_1; 0 \leq \rho_{4k} \leq b_2; 0 \leq \gamma_{i^1v} \leq b_3; 0 \leq \gamma_{i^2v} \leq b_4; 0 \leq \gamma_{i^3(v+1)} \leq b_5; 0 \leq \gamma_{i^4v} \leq b_6; 0 \leq \eta_j \leq b_7\}$, where $b = (b_1, b_2, b_3, b_4, b_5, b_6, b_7) \geq 0$. Then $\mathbb{K}_b$ is a bounded closed convex subset of $\mathbb{K}_5$. Thus, the following variational inequality $\langle F(X^b)^T, X - X^b \rangle \geq 0, \ \forall X^b \in \mathbb{K}_b$, admits at least one solution $X^b \in \mathbb{K}_b$, from the standard theory of variational inequalities, since $\mathbb{K}_b$ is compact and $F$ is continuous.

The material flows $q_{hi}, q_{ijv}$ and $q_{jk}$ ($\forall h, i, j, v$) are bounded since we assume a fixed and finite amount of electronic waste $S^h$ available at each source. We assume that the demand function $d_k$ for all $k$ is monotonically decreasing in $\rho_{4k}$ for all $k$ and demand for extracted precious material will be very low when the
demand price is high. Thus, we can set an upper bound on the demand price \( \rho_{4k} \) (Nagurney, 1999). As long as \( \rho_{3jk} \) has an upper bound, the opportunity cost for dealing with WEEE (\( \lambda_h \)), the shadow prices for the transshipment of materials from the second to the third tier of our network and the shadow price for the minimum collection quantity (\( \gamma_{iv}, \gamma_{iv}^2, \gamma_{i(i+1)}, \gamma_i^4 \)) as well as the shadow price of extracted precious materials (\( \eta_j \)) will never take on an infinite value. Hence, it is possible to construct \( b_1, b_2, b_3, b_4, b_5 \) and \( b_7 \). See also the proof of existence for Proposition 1 in Nagurney and Zhao (1993).

**Theorem 4. Summation**

By solving variational inequality (19), we simultaneously solve sources’ and demand markets’ equilibrium conditions (as given in equations (4) and (18)) as well as recyclers’ and smelters’ optimization problem (given in equations (5)-(10) and (12)-(14)).

**Proof.** For specific realizations of the proof of Theorem 4 in the context of a supply chain model see for example Nagurney et al. (2002).

**Lemma 5. Strict Monotonicity**

Suppose that all \( f_i, f_j, \) and \( \rho_{j(p+1)} \) are convex in \( Q_i, Q^j \) and \( Q_j \) \( \forall i, j, v, k \) respectively, and at least one of the families of convex functions is a family of strictly convex functions in \( Q_i, Q_j \) and \( Q^j \). Furthermore, assume that \( a_h, -\varepsilon_h \) and \( c_{hi} \) are monotone increasing functions in \( Q_h \) \( \forall h \) and at least one of them is a strictly monotone increasing function in \( Q_h \). Finally assume that \( c_{kj} \) is a strictly monotone increasing and \( d_k \) is a strictly monotone decreasing function in (19). Then, the vector function \( F(X) \) that enters the variational inequality (19) is strictly monotone for any \( X', X'' \) with both \( X' \) and \( X'' \in K \), that is,

\[
\langle (F(X') - F(X''))^T, X' - X'' \rangle > 0, \quad \forall X', X'' \in K.
\]
Proof. After some simplifications, the left-hand side of inequality (19) is
\[
\sum_{h=1}^{m} \sum_{i=1}^{n} \left[ a_h(Q_h') - \varepsilon_h(S_h, Q_h') + c_{hi}(q_h') - (a_h(Q_h'') - \varepsilon_h(S_h, Q_h'') + c_{hi}(q_h'')) \right] \times [q_{hi} - q_{hi}^n] \\
+ \sum_{h=1}^{m} \left[ a_h(Q_h') - \varepsilon_h(S_h, Q_h') + c_{h(n+1)}(q_{h(n+1)}) + \pi_{1h} \rho_{1h}(q_{h(n+1)}) \right] \\
- (a_h(Q_h'') - \varepsilon_h(S_h, Q_h'') + c_{h(n+1)}(q_{h(n+1)}''') + \pi_{1h} \rho_{1h}(q_{h(n+1)}''')) \times [q_{h(n+1)}'' - q_{h(n+1)}'] \\
+ \sum_{i=1}^{n} \sum_{j=1}^{n} \left[ \frac{\partial p_{ij(p+1)}(Q_{ij}, Q_{ij}')}{\partial q_{ijv}} + \frac{\partial f_j(Q_{ij}'', Q_{ij}')}{\partial q_{ijv}} + \frac{\partial f_i(Q_{ij}')}{\partial q_{ijv}} \right] \\
- \left( \frac{\partial p_{ij(p+1)}(Q_{ij}'', Q_{ij}')}{\partial q_{ijv}} + \frac{\partial f_j(Q_{ij}'', Q_{ij}')}{\partial q_{ijv}} + \frac{\partial f_i(Q_{ij}')}{\partial q_{ijv}} \right) \times [q_{ijv} - q_{ijv}'''] \\
+ \sum_{i=1}^{n} \sum_{j=1}^{n} \left[ \frac{\partial f_i(Q_{ij}')}{\partial q_{ijv(l+1)}} - \frac{\partial f_i(Q_{ij}'')}{\partial q_{ijv(l+1)}} \right] \times [q_{ijv(l+1)} - q_{ijv(l+1)}'''] \\
+ \sum_{i=1}^{n} \left[ \frac{\partial f_i(Q_{ij}')}{\partial q_{ijv(o+1)v}} - \frac{\partial f_i(Q_{ij}'')}{\partial q_{ijv(o+1)v}} \right] \times [q_{ijv(o+1)v} - q_{ijv(o+1)v}'''] \\
+ \sum_{j=1}^{p} \sum_{k=1}^{n} \left[ \frac{\partial p_{jk(p+1)}(Q_{jk}, Q_{jk}')}{\partial q_{jk}} + \frac{\partial f_j(Q_{jk}'', Q_{jk}')}{\partial q_{jk}} + c_{jk} (q_{jk}') \right] \\
- \left( \frac{\partial p_{jk(p+1)}(Q_{jk}'', Q_{jk}')}{\partial q_{jk}} + \frac{\partial f_j(Q_{jk}'', Q_{jk}')}{\partial q_{jk}} + c_{jk} (q_{jk}') \right) \times [q_{jk} - q_{jk}'''] \\
+ \sum_{k=1}^{p} [-d_k (\rho'_{4k}) + d_k (\rho'_{4k}')] \times [\rho'_{4k} - \rho'_{4k}'] 
\]
and the term (22) is positive so the conclusion follows. \( \square \)

Convergence results for the Euler method can be found in Dupuis and Nagurney (1993) and Nagurney (2006) with specific realizations in Dhanda et al. (1999).
Theorem 6. Uniqueness
Assuming the conditions in Lemma 5, there must be a unique shipment pattern and a unique generalized price vector satisfying the equilibrium conditions of the e-waste network. In other words, if the variational inequality (19) admits a solution, that should be the only solution in $q_{hi}, q_{ijv}, \lambda_h, \rho_{4k}, q_{jk}, \gamma_{1w}^1, \gamma_{2w}^2, \gamma_{3w}^3, \gamma_{i(l+1)}^4, \gamma_i^4, \eta_h \forall h, i, j, k, v$.

Proof. Under the strict monotonicity result of Lemma 5, uniqueness follows from the standard variational inequality theory (c.f. Kinderlehrer and Stampacchia, 1980).

References


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