CLIMAFIN handbook: pricing forward-looking climate risks under uncertainty

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Aligning finance to sustainability requires methodologies to price forward-looking climate risks and opportunities in financial contracts and in investors’ portfolios. Traditional approaches to financial pricing models cannot incorporate the nature of climate risk (i.e. deep uncertainty, non-linearity and endogeneity), and of financial risks (interconnectedness and complexity). To fill this gap, we developed a transparent, science-based framework to assess and price climate financial risks under uncertainty, the CLIMAFIN tool. It embeds climate scenarios adjusted financial pricing models (for equity holdings, sovereign and corporate bonds), climate scenarios conditioned risk metrics (such as the Climate Spread and the Climate Value-at-Risk). These allow us to introduce forward-looking climate risk scenarios in the valuation of counterparty risk, in the probability of default and largest losses on investors’ portfolios. This handbook is intended to support investors in the assessment of forward-looking climate risks in their portfolios and in the identification of portfolios’ risk management strategies, and financial supervisors in the analysis of risk exposures that could have implications for systemic risk and in the design of prudential measures to mitigate such risk.

Keywords: CLIMAFIN, forward-looking climate transition risk, climate deep uncertainty, financial contracts, financial pricing models, Climate Spread, Climate Value at Risk
1. Introduction

There is growing awareness among academics, practitioners and financial supervisors of the fact that unmitigated climate change and a disorderly transition to a low-carbon economy could affect the profitability of several economic activities and cause relevant losses for investors’ portfolios (Carney 2015, NGSF 2018, Battiston et al. 2017).

Nevertheless, recent research highlighted that investors are not pricing yet climate-related risks in their portfolios (Monasterolo and de Angelis 2019, Morana and Sbrana 2019, Ramelli et al. 2018).

Since financial investors take decisions based on what they can measure, and their decisions do influence (and are influenced by) the benchmark in their respective markets, evaluating climate risks in financial contracts is crucial from an investors’ risk management perspective, and for financial supervisors whose mandate is about preserving financial stability.

Main barriers that investors face in pricing climate-related financial risks are represented by (i) the nature of climate risks (physical, transition), (ii) the poor understanding of existing classifications to assess financial exposures to climate risks, (iii) the need to move from the backward-looking nature of traditional financial risk assessment and of investors’ benchmarks to the forward looking nature of climate risks, and (iv) the integration of forward looking climate shocks in financial risk metrics and management approaches.

In this handbook, we show how the CLIMAFIN tool can guide a risk averse investor in integrating climate risks considerations in her counterparty credit and financial risk valuation and probability of default, including new climate scenarios adjusted risk metrics (Climate Value-at-Risk, Climate Spread). CLIMAFIN provides a transparent, science-based framework to assess investors’ exposure to forward-looking climate risks and to price climate risks in the value of their financial contracts and portfolios. This allows investors to align to the recommendations of the Network for Greening the Financial System (2019) on climate

⋆This is a first version of a work in progress. The aim is report and discuss in a single document the results of a stream of scientific works on the pricing of climate risk across financial instruments and markets.
financial risk assessment and climate stress-testing, and financial regulators to identify the
drivers of climate-related financial instability and to design prudential measure to mitigate
it.

In this first part of the handbook we focus on the following CLIMAFIN’s characteristics:

- The information set that a rational risk averse investor should use to assess financial
  risk under climate transition scenarios;

- The forward-looking climate transition risk scenarios and shocks and the transmission
  channels through which they hit economic activities (low-carbon and carbon-intensive)
  and firms’ profitability;

- The climate financial pricing models for climate scenarios adjusted counterparty risk
  valuation for individual contracts (equity, corporate and sovereign bonds, loans);

- Climate scenarios conditional financial risk metrics such as the Climate Value at Risk
  (Battiston et al. 2017) and the Climate Spread (Battiston and Monasterolo 2019);

- Climate Stress-testing models (see Battiston et al. 2017, Roncoroni et al. 2019). The
  presentation of the Climate Stress-test and its applications to investors’ portfolios is
  included in the second part of this Handbook.

The CLIMAFIN Handbook is organized as follows. Section 2.3 describes the information
set of a risk averse investor that aims to minimize climate risks in her portfolio. Section 3
describes the climate scenarios adjusted financial risk evaluation model for equity holdings.
Section 4 and Section 5 present the climate scenarios adjusted credit risk evaluation models
for corporate and sovereign bonds respectively. Section 6 introduces the Climate Spread
while Section 7 introduces the Climate Value-at-Risk.

2. Model component, investors’ information set and risk management strategy

2.1. Climate risk: not a Normal type of risk for financial actors

In this section we introduce the concepts of climate physical and transition risks, and
we discuss the main differences between the properties of climate risks and of the risks
usually considered in finance.

2.1.1. *Climate physical risks*

Climate change physical risk refers to risk of damages to physical assets, natural capital and/or human lives resulting into output losses, as a result of climate induced weather events. Based on the available scientific information, the Greenhouse Gases (GHG) emissions trajectory currently followed by UN countries would lead to severe socio-economic consequences, resulting in particular from sea level rise, icesheet and permafrost melting, and the increased frequency of extreme weather events such as drought, floods and heat-waves. These events will have economic consequences both at the firm and macroeconomic level, and include:

- The destruction of immobilized productive capital, with negative implications on firms’ profitability, investments, employment and eventually on Gross Domestic Product (GDP) (Burke et al. 2015, 2018, Hsiang et al. 2017);

- Drops in properties’ values (see e.g. the example of luxury coastal properties in Florida and South Carolina, that would eventually become not insurable anymore, US 4th Climate Assessment Report), with implications for banks and insurance companies;

- Loss of arable land productivity, with implications on food commodities’ production and prices, and thus on famine and social unrest, and eventually the relocation of millions of people currently living in areas particularly exposed to climate physical risks, even within developed countries (FAO SOFA 2018, IPCC 2014).

2.1.2. *Climate transition risks*

Climate change transition risk refers instead to the risk arising from sudden assets’ values adjustments and repricing as a result of coordination of expectations of market participants about the implementation of climate policies (e.g. a carbon tax, or the revision of the Emissions Trading Scheme (ETS) scheme in Europe). These adjustments are expected to negatively impact the value of fossil fuels related assets (the so-called carbon stranded
assets, see e.g. Leaton et al. 2012). They are also expected to impact indirectly the value of assets in other sectors that use fossil fuel energy and electricity as a production input, or that are involved in the value chain of companies that do it, thus generating cascading losses. In addition, in today’s interconnected business and financial sectors, a shock generated from an economic activity could cascade on the investor who is exposed to the financial contracts issued by that activity. However, the sign of the impact can be positive or negative, depending on whether firms are able to anticipate the policy and adapt their business to alternative sources of energy (e.g. in certain scenarios, renewable-based utilities or energy-intensive processes that manage to diversify their energy sources away from fossil-fuels are expected to grow in market share).

Complexity of climate risks and limits of traditional financial pricing models

Climate risks are characterised by deep uncertainty, non-linearity, fat tailed distributions, path-dependency and endogeneity. These characteristics, that we briefly outline below, cannot be easily embedded in traditional financial pricing models that stand on assumptions of Normally distributed shocks, perfect information, complete markets, absence of arbitrage and short term valuation.

**Non-linearity.** Climate shocks probability distribution can’t be inferred from historical data being forward-looking in nature. In addition, recent studies showed that past temperature data are not normally distributed. For instance, Western European summer of 2003 was 5.4 above mean temperature for 1864-2000. Within a Normal distribution, 5.4 summer would occur once every 30 million years. But Eastern Europe had similar heat wave in 2010. Thus, if such events happen every 7 years, temperatures are not normally distributed (Ackerman 2017).

**Deep uncertainty.** The forecasts of climate change and its impact on humans and ecosystems contain irreducible uncertainties because of the nature of the earth system, including the presence of tail events (Weitzman 2009) and tipping points (Solomon ea. 2009), which cannot be overcome by model consensus (Knutti 2010). This means that largest shocks expected to occur in mid-to long-term but their exact localization and magnitude is unknown.
In addition, uncertainty characterises the costs and benefits estimates in each scenario that vary substantially with the assumptions on agents’ utility function, future productivity growth rate, and intertemporal discount rate. These assumptions, sometimes implicit or given for granted in the mathematical treatment of economic agents’ behavior, ultimately imply fundamental philosophical and ethical considerations (Nordhaus 2007; Stern 2008, Ackerman et al. 2009, Stern 2013, Pyndick 2013).

**Complexity.** Even if costs and benefits could be predicted precisely, the likelihood of the realization of a given pathway depends on the assumptions on agents’ rationality and on the ability of countries to coordinate on international policies. The political economy of the actors involved is complex and plays a fundamental role. However, this is not accounted for by the literature on the social cost of carbon nor by the literature on Integrated Assessment Models (IAMs). **Endogeneity of risk.** On the one hand, the likelihood of achieving climate targets and the mitigation of climate risks in financial markets and investors’ portfolios depends from the orderly introduction of climate policies and the scaling up of financial investments in low-carbon sectors. However, the endogeneity between uncertainty of policy decisions and announcements and investors’ expectations on the financial risk deriving from the policies generates the possibility of multiple equilibria. In this context, a rational agent cannot identify a preferred investment strategy.

In this context, the standard approach to financial risk analysis, consisting of: identifying the most likely scenario, computing expected values, and estimating financial risk based on backward looking metrics and historical values of market prices, is not an adequate approach (Battiston 2019).

### 2.2. Model components

We define and implement a model that is composed of the following:

- Definition of the investor’s portfolio of risky financial contracts;
- A discussion on the nature of climate risks considered;
- Macro-economic trajectories and climate transition risk at issuer/counterparty level;
• A valuation model to price equity risk;
• A structural model to price credit risk;
• A model of forward-looking climate transition risk using the Climate Policy Shock Scenarios from the Investor Information Set
• The definition of Climate Spread and Climate VaR
• The assessment of impact of Climate Policy Shocks on bonds default probability, Climate Spread, Portfolio Climate Value at Risk (VaR).

2.3. Investors’ information set

Building on Battiston and Monasterolo (2019) we consider a risk averse investor that aims to assess the exposure of her portfolio to forward-looking climate transition risk in a context of incomplete information and deep uncertainty (Keynes 1973, Knight 1921, Greenwald and Stiglitz 1986, Nalebuff and Stiglitz 1983).

We identify an Information Set relevant to climate transition risk, suitable for investor that does not necessarily have a greening mandate but who does need to implement a financial valuation (risk) of its portfolio. We want to identify the properties of portfolio’s risk management strategy accounting for investor’s risk aversion, counterparty risk, Probability of Default (PD), Spread and Value-at-Risk (VaR) adjusted for forward-looking climate transition risk scenarios. In this context, implementing the strategy requires to adjust the traditional Probability of Default (PD), the Spread and VaR, conditional to forward-looking Climate Policy Shock Scenarios (i.e. happening in the future).

The information set of the risk averse investor is composed of:

• A set of Climate Policy Scenarios \( P_i \) corresponding to GHG emission reduction target across regions \( (B = \text{Business-as-Usual}) \):

\[
\text{ClimPolScen} = \{B, P_1, ..., P_l, ..., P_{n\text{Scen}}\}
\]
• A set of economic output trajectories for each country $j$, sector $k$ under each scenario $P_l$, estimated with each climate economic model $M_m$:

$$\text{EconScen} = \{Y_{1,1,1,1}, ..., Y_{j,k,P_l,M_m}, ...\}$$

• A set of forward-looking Climate Policy Shock Scenarios (disorderly transition $B \rightarrow P_l$):

$$\text{TranScen} = \{B \rightarrow P_1, ..., B \rightarrow P_l, ..., B \rightarrow P_{n\text{Scen}}\}$$

• A set of Climate Policy Shocks on economic output for $j$, $k$ under transition scenario $B \rightarrow P_l$, estimated with model $M_m$

$$\text{EconShock} = \{..., \frac{Y_{j,k,P_l,M_m} - Y_{j,k,B,M_m}}{Y_{j,k,B,M_m}}, ...\}$$

By defining the information set we want to:

• Include the current available knowledge about transition risk factors related to climate change and climate change mitigation that can affect the investment value. We consider the climate policy scenarios developed by the International Scientific Community and reviewed by the Intergovernmental Panel on Climate Change (IPCC). Then, we translate the economic trajectories for both low-carbon and carbon-intensive economic activities obtained from climate economics models (e.g. Integrated Assessment Models (IAM) as well as other models) into climate policy shocks on the Gross Value Added (GVA) of those activities and firms.

• Cover a time horizons that is relevant both for investment strategies and for the low-carbon transition, and ideally covers several decades, from 2020 to 2050 and possibly beyond that.

• Include varying investor’s risk aversion preferences. We consider multiple scenarios that account for different risk aversion and allow to go beyond the inadequate notion of “most likely scenario” and include the notion of ”worst case scenarios”.

• Compatible with the hypothesis of the possibly incomplete information and incomplete markets (Greenwald and Stiglitz 1986), the economic shocks led by a disorderly low-carbon transition allow to model to be temporary out-of-equilibrium.

• Be relevant for institutions with a focus on financial risk valuation and financial stability mandate (thus, we do not assume financial actor’s mandate beyond risk).

2.4. Investors’ Climate Risk Management Strategy

The investor risk management strategy is based on the VaR and aims to minimize climate risk in its portfolio by:

• Accounting for investor-specific risk aversion level (i.e. varying subsets of investor information set $\text{InfoSetClimRisk}$).

• Accounting for counterparty risk adjusted for climate policy shock scenarios (e.g. probability of default, spread)

• Accounting for metrics relevant for financial regulation e.g. risk measure such as VaR.

In this context, the risk averse investor aims to minimize her Climate Value-at-Risk ($\text{Climate VaR}$) under the investor information set $\text{InfoSetClimRisk}$ i.e. the forward-looking climate policy shocks, the scenarios of economic trajectories for low-carbon and carbon-intensive economic activities’ GVA, and the climate models (e.g. the IAM) used to estimate the economic shock on GVAs.

The Climate VaR Management Strategy that aims to minimize the worst-case losses of the portfolio across the forward-looking Climate Policy Shock Scenarios can be written as:

$$\text{ClimVaRStr} = \min_{\text{Portfolios}} \{ \max_{\text{Shocks}} \{ \text{VaR}(\text{Portfolio},\text{Adj.PD} | \text{Policy Shock}) \} \}$$

In this context, future asset prices are subject to shocks that depend on the issuer’s future economic performance, the risk premia demanded by the market, as well as the timing and magnitude of the climate policy introduced and the outcome of the energy transition of individual firms and countries. The investor considers different feasible climate policy
scenarios (but has no information on the probability associated) for which she can calculate the impacts (negative or positive) on the market share of carbon-intensive or low-carbon economic activities and firms.

The investor is subject to incomplete information on her (and competitors’) exposure to risk stemming from a disordered transition from a climate policy scenario to another one, uncertainty on the outcome of the firms and country’s energy transition, and no information on the probability distribution. Thus, her risk management strategy is to consider a set of feasible climate transition scenarios that her portfolio should withstand, and then compute the VaR conditional to those scenarios.

2.5. Climate policy scenarios

With the aim to assess the impact of a disorderly low-carbon transition, i.e. forward-looking climate policy shocks on the value of contracts of the investor’s portfolio, we consider the climate policy scenarios of the IPCC 2014 report, described in .

<table>
<thead>
<tr>
<th>Scenario Name</th>
<th>Scenario Type</th>
<th>Near-term Target / Fragmented Action</th>
<th>Fragmented Action until</th>
<th>Long-term Target, in 2100</th>
<th>Barrier-sharing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>Baseline</td>
<td>None</td>
<td>N/A</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>RefPol</td>
<td>Reference</td>
<td>Weak</td>
<td>2100</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>StrPol</td>
<td>Reference</td>
<td>Stringent</td>
<td>2100</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>450</td>
<td>Benchmark</td>
<td>None</td>
<td>N/A</td>
<td>450 ppm CO2e (2.8 W/m²)</td>
<td>None</td>
</tr>
<tr>
<td>500</td>
<td>Benchmark</td>
<td>None</td>
<td>N/A</td>
<td>500 ppm CO2e (3.2 W/m²)</td>
<td>None</td>
</tr>
<tr>
<td>RefPol-450</td>
<td>Climate Policy</td>
<td>Weak</td>
<td>2020</td>
<td>450 ppm CO2e (2.8 W/m²)</td>
<td>None</td>
</tr>
<tr>
<td>StrPol-450</td>
<td>Climate Policy</td>
<td>Stringent</td>
<td>2020</td>
<td>450 ppm CO2e (2.8 W/m²)</td>
<td>None</td>
</tr>
<tr>
<td>RefPol-500</td>
<td>Climate Policy</td>
<td>Weak</td>
<td>2020</td>
<td>500 ppm CO2e (3.2 W/m²)</td>
<td>None</td>
</tr>
<tr>
<td>StrPol-500</td>
<td>Climate Policy</td>
<td>Stringent</td>
<td>2020</td>
<td>500 ppm CO2e (3.2 W/m²)</td>
<td>None</td>
</tr>
</tbody>
</table>

(Source: IIASA, Kriegler et al. 2013) Characteristics of the mild and tight climate policy scenarios considered in the LIMITS project

In particular, we select four climate policy scenarios aligned to the 2°C target from the LIMITS database of IAM and a baseline of no climate policy, described in Table 1. We
use the LIMITS project database (Kriegler et al. 2013) to compute the trajectories of the market shares for several variables including the output of primary energy from fossil fuel and the output of secondary energy in the form of electricity both from fossil fuel sources and renewable energy sources. Then, we estimate the effect of the introduction of market-based climate policies (i.e. a carbon tax). The two emissions concentration targets chosen under milder and tighter climate policy scenarios (i.e. 500 and the 450 ppm), determine the amount of CO2 to be emitted in the atmosphere by 2100 consistently with the 2°C aligned IPCC scenarios (IPCC 2014). The 500 and 450 ppm scenarios are associated to a probability of exceeding the 2°C target by 35-59% and 20-41% respectively (Menishausen et al. 2009). Thus, the choice of specific emissions concentration targets could be considered as a proxy for the stringency of the global emission cap imposed by potential climate treaty.

A change in climate policy (i.e. in the value of the carbon tax every 5-years time step) implies a change in the sectors’ macroeconomic trajectory, and thus a change in the market share of primary and secondary energy sources. The shock in the market share could differ in sign and magnitude depending on the scenario \( S \), the region \( R \), the model \( M \) used and the sector \( S \). We consider a shock occurring in 2030, affecting the market shares of the economic activities and firms (low-carbon and carbon-intensive, see Figure 2.1) to which the investor’s portfolio is exposed via financial contracts (equity, corporate and sovereign bonds, loans).

2.6. Climate Policy Shocks

In the model, the climate and energy targets of each countries are assumed to be known by the investor. These targets translate in a share of energy and electricity produced by renewable energy sources.

However, for each country, the investor does not known if and when the country will introduce climate policies to foster the alignment of the economy to its targets. She also does not know along which economic trajectory, which means, the change in energy mix of the economy that leads to a change in the market share of different renewable/fossil sub-sectors of the economy and thus the revenues of the firms in those sectors.
Table 1: Selected climate policy scenarios from the LIMITS database. The table shows the four climate policy scenarios considered (plus the Base scenario), i.e. RefPol-450, RefPol-500, StrPol-450, StrPol-500.

<table>
<thead>
<tr>
<th>Climate policy shock scenario</th>
<th>Climate policy scenario</th>
<th>Scenario Class</th>
<th>Target by 2020</th>
<th>Target between 2020 and 2100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not applicable</td>
<td>Base</td>
<td>No climate policy</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>Disorderly switch from Base to RefPol-450</td>
<td>RefPol-450</td>
<td>Countries</td>
<td>Lament</td>
<td>450 ppm: 2.8 W/m² in 2100, overshoot allowed</td>
</tr>
<tr>
<td>Disorderly switch from Base to StrPol-450</td>
<td>StrPol-450</td>
<td>Countries</td>
<td>Strengthened</td>
<td>450 ppm: 2.8 W/m² in 2100, overshoot allowed</td>
</tr>
<tr>
<td>Disorderly switch from Base to RefPol-500</td>
<td>RefPol-500</td>
<td>Countries</td>
<td>Lament</td>
<td>500 ppm: 3.2 W/m² in 2100, overshoot allowed</td>
</tr>
<tr>
<td>Disorderly switch from Base to StrPol-500</td>
<td>StrPol-500</td>
<td>Countries</td>
<td>Strengthened</td>
<td>500 ppm: 3.2 W/m² in 2100, overshoot allowed</td>
</tr>
</tbody>
</table>
The investor does not have priors on the probability of these events and assumes that if a country implements the low-carbon transition, then it does so by switching from its BAU scenario to one of the climate policy scenarios described by the scientific community (i.e. the energy and economic scenarios based on IEA roadmap and IPCC climate scenarios, see Kriegler et al. 2013, IPCC 2014). This assumption is motivated by the fact that there is policy and scientific consensus on these climate policy scenarios and their trajectories.

The transition of a country from the Business as Usual $B$ to a climate policy scenario $P$ can occur orderly or disorderly.

**Orderly**, means here that the introduction of a climate policy is carried out timely enough for the country to achieve its renewable energy targets and with a public and predictable schedule. In this scenario, investors can anticipate it and discount the effects on asset prices of the economic activities affected. For instance, the phasing out of coal-based electricity plants is announced to happen with a certain schedule, which is maintained and the market players know that it will be maintained. Thus, they can discount the future value of investments in assets that have these plants as underlying, accordingly, and they can price the risk associated to their exposure to financial contracts related to those plants.

In contrast, **disorderly** means that the transition is carried out at a schedule that is not predictable by markets and investors, e.g. the government introduces the climate policy in a late and sudden way, or retroactively revise its policies. In this case, we assume that the climate policy shock stemming from a disordered transition is not anticipated (despite potentially expected) by the investor. This is due to the backward looking nature of the benchmark considered by asset managers and on which asset managers’ performance (and thus remuneration) is assessed. It is common knowledge that asset managers take investment decisions based on the benchmark in their respective markets (Greenwald and Stiglitz 1986). Recent research shows that the market benchmark is carbon intensive (see e.g. Battiston and Monasterolo 2019 for the case of corporate bonds market benchmark against which the European Central Bank’s corporate bonds purchase (CSPP) has been assessed).

If the investor cannot anticipate the policy shock, then we can assume that she cannot discount correctly the effect of a climate policy on the change in asset prices of the economic
activities affected by the transition. A failure to anticipate the climate policy shock leads to a failure in pricing it correctly. In turn, this has potentially severe implications on price volatility, on portfolio’s performance and financial stability.

It is important to notice that the assessment of the policy shock could be incorrect even on average across market participants. The motivation for considering this possibility is due to the fact that several recent policy events (achievement of Paris Agreement, outcome of US elections, the US withdrawal from Paris Agreement, Brexit, the outcome of 2018 Italian elections) have been incorrectly forecast by most observers and investors. Nevertheless, these events and their incorrect pricing are having long-lasting economic effects (see e.g. the spread on Italy’s sovereign bonds). This implies that these effects could not be priced in by market participants, and this possibility should be considered in financial pricing models of sovereign bonds. Since the experience shows that the possibility that markets do not anticipate correctly policy events and their economic impact is material, we assume that the investor wants to include this possibility among her scenarios. For instance, the phasing out of coal based electricity plants could occur late on the policy agenda, behind the initially announced schedule (e.g. in Poland), in a situation where market players are thinking that it won’t happen any longer. This implies that they do not discount correctly the future value of investments in the assets that have these plants as underlying.

Today, the information available to policy makers and market players on the trajectories of future values of economic sectors’ market share comes mostly from Integrated Assessment Models (IAM). These are (partial or general) equilibrium models, calibrated on the recent state of the economy and climate targets, and provide trajectories in which the economy remains in equilibrium along any given trajectory. Thus, moving from a BAU to a climate policy scenario implies jumping from an equilibrium condition to another one. Moreover, the levels of output of the sectors of the economy must be consistent one with each other to reach again equilibrium conditions. The latter feature means that, for instance, a decrease in electricity generation based on coal has to be compensated by an increase in generation based on other sources to be consistent with the internal demand. This, in turn, affects the
relative prices. Each trajectory is also consistent with a specific target in terms of GHG by 2050, and with a specific scenario on the status of international coordination on climate efforts. The trajectories integrate also the estimates of climate change damages to physical assets in the economy by means of a climate module. There exists only a limited number (less than 10) of established IAM in the world, run by independent and internationally recognized scientific institutions. The models consider a common set of internationally agreed climate policies and emissions scenarios but differ in the way they define certain output variables and in the data used for the calibration (e.g. Kriegler et al. 2013). There is a consensus in considering the IAMs’ set of trajectories as the information set available today about the future economic impact of climate change. Nevertheless, it is increasingly recognized that such models have some limitations (e.g. in the computation of the trajectories and outputs) that relate to the model structure and behaviour, and can affect the policy relevance of the outcomes (see e.g. Battiston and Monasterolo 2018).

2.7. Composition of the economy

We consider $n$ countries $j$ whose economy is composed of $m$ economic sectors $S$. Economic activities included in $S$ are based on a refined classification of the Climate Policy Relevant Sectors (CPRS), which was originally introduced in Battiston et al. (2017). NACE codes (4 digits) are mapped to CPRS (2017), which identifies the main sectors that are relevant for climate transition risk (fossil-fuel, electricity, energy-intensive, transportation, buildings). CPRS classification departs from the NACE classification of economic sectors (at 4 digit level) in so far, it catches the energy and electricity technology of the economic activity. Its refinement (i.e. CPRS Rev2 2019) provides a more granular classification of the economic activities in terms of technologies (utility—electricity—wind, solar, gas).

Within $S$, we focus on the fossil fuel and renewable energy primary and secondary sectors and subsectors, due to the main role they play in the low-carbon transition via the energy and electricity supply along the value chain. Firms that compose economic sectors $S$ are considered as a portfolio of cash flows from fossil fuel and renewable energy activities. The classification of countries and regions affected by the climate shock is based on the
LIMITS/CD-LINKS aggregation, see Kriegler et al. (2013), McCollum et al. (2018).

Figure 2.1: Climate Policy Relevant Sectors. The figure shows the classification of economic activities by different degrees of granularity by technology

<table>
<thead>
<tr>
<th>CPRS 2017</th>
<th>CPRS 2019</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-fossil</td>
<td>1-fossil</td>
</tr>
<tr>
<td>1-fossil</td>
<td>oil</td>
</tr>
<tr>
<td>2-utility</td>
<td>electricity</td>
</tr>
<tr>
<td>2-utility</td>
<td>electricity</td>
</tr>
<tr>
<td>2-utility</td>
<td>electricity</td>
</tr>
<tr>
<td>2-utility</td>
<td>electricity</td>
</tr>
<tr>
<td>2-utility</td>
<td>other</td>
</tr>
<tr>
<td>2-utility</td>
<td>waste</td>
</tr>
</tbody>
</table>

In particular, we can define a set of issuers \( \{1, ..., j, ..., n\} \) from economic sectors \( \{1, ..., s, ..., n^{\text{Sect}}\} \), where the issuers’ GVA in a country is the sum of sectors’ contributions: \( \text{GVA}_j = \sum_s \text{GVA}_{j,s} \).

2.8. Impact of climate policy shock on economic activities’ GVA and profitability

We consider the contribution of issuer \( j \)'s to the sector \( S \)'s GVA and fiscal assets and how this can be affected by changes in its economic performance, either negatively or positively. We then relate the performance of the economic activity to the change in its market share as a result of a disorderly climate policy transition scenario.

In a disorderly transition, a climate policy shock affects the performance of issuers in sectors \( S \) via a change in economic activities’ market share, cash flows and profitability, eventually affecting the GVA of the sector. The climate policy shock is calculated at the sector, country and regional level. The country’s GVA composition is available at NACE 2 digit level from official statistics (e.g. Eurostat). Negative shocks result from the policy impact on the GVA of sectors based on carbon-intensive (i.e. fossil fuels) technologies, while
positive shocks result from the impact on the GVA of sectors based on low-carbon (i.e. renewable energy) technologies.

We consider macroeconomic trajectories of output over time for sector \( s \) consistent with climate policy scenario \( P \in \{\ldots, P_{\text{RefPol}}, P^{450}, \ldots\} \) The **Climate Policy Shock Scenario** consists in the transition from a trajectory Business-as-usual (\( B \)) to trajectory with climate policy \( P \). Forward-looking climate policy shock arises from investors that are not fully anticipating the introduction and impact of the climate policy (as an analogy, we can consider the introduction/impact of Brexit). We focus on shocks on the GVA of 3 Climate Policy Relevant Sectors (CPRS):

- primary energy fossil (\( \text{PrFos} \))
- electricity fossil (\( \text{ElFos} \)) / renewable (\( \text{ElRen} \))

\[
\begin{align*}
GVA_j(P) &= u_{GVA,_j}(P) = u_{GVA,_j}(P) w_{GVA,_j}(P) + u_{GVA,_j}(P) w_{GVA,_j}(B) + u_{GVA,_j}(P) w_{GVA,_j}(B) \\
GVA_j(B) &= \sum_s (GVA_{j,s}(P) - GVA_{j,s}(B))
\end{align*}
\]

We assume that a \( \% \) shock on output \( \approx \% \) shock on GVA, \( u_{GVA,j} \), for each sector of \( j \)

\[
\begin{align*}
u_{GVA,j}(P) &= \frac{GVA_j(P) - GVA_j(B)}{GVA_j(B)} = \sum_s \frac{GVA_{j,s}(P) - GVA_{j,s}(B)}{GVA_{j,s}(B)}
\end{align*}
\]

where then \( u_{j,s}(P) \): GVA shock on sector \( s \); \( w_{j,s}(B) \): share of GVA of sector \( s \)

From an accounting perspective, at the level of an individual firm, it holds true that a decrease (increase) \( x \) in the market share translates in a relative decrease (increase) \( x \) in its sales, as long as market conditions are the same\(^1\). Indeed, a body of empirical literature has found a strong and positive relation between firms’ market-share and profitability (Szyman-\( \kappa \)ski et al. 1993; Venkatraman et al. 1990). At similar argument can be made at the level\(^{17}\)

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\(^1\)More precisely, it holds under the conditions that total demand and prices remain unchanged in the period considered, and that returns to scale are constant.
of countries’ economic sectors, such as their utility sectors. A decrease (increase) $x$ in the market share in a given region of countries competing on the energy market translates in a relative decrease (increase) $x$ in its sales. As a result, there is a decrease (increase) in the tax revenues that the sovereign issuer $j$ collects from the firms operating in that sector in its country.\footnote{Notice that while the tax rate may vary in principle with firms’ size (e.g. total level of pre-tax profits), in many cases large firms are subject to similar tax rates than smaller firms. Hence, agents assume that an $x\%$ drop in firm’s profits implies the same $x\%$ drop in revenues.} In the case of the energy and utility sectors, this argument is corroborated by the fact that ownership is very concentrated in both fossil and renewable business. Indeed in most EU countries there is just a major energy firm (e.g. OMV in Austria, ENI in Italy) and one major utility firm.

The net effect of the change in energy mix on the profit of a given sector depends on the pre-shock energy mix and the post-shock energy mix. For instance, sector $S_{j_1}$ will have a larger post-shock profit compared to $S_{j_2}$, denoted as $\pi(S_{j_1}, P) > \pi(S_{j_2}, P)$, because it starts from a larger pre-shock share of renewable-based power (everything else being equal). Moreover, $S_{j_2}$’s profit (summed over the two business lines) could decrease after the policy shock, denoted as $\pi(S_{j_2}, P) < \pi(S_{j_2}, B)$, if it is not possible for $S_{j_2}$ to more than compensate on the renewable business line the losses on the fossil business line.

The final impact of the climate policy shock on the net fiscal assets of an issuer $j$ depends not only on the tax revenues from sector $S_j$ and thus on its profit $\pi(S_j, P)$, but also on the expenses that the issuer incurs. If we consider $j$ as a sovereign issuer, the consideration discussed earlier in this section lead us to make the assumption that a relative change in the market share of sector $S$ within the country $j$, implies a proportional relative change in the net fiscal assets of issuer $j$ from sector $S$.

In the case of a sovereign issuer, we define the net fiscal assets related to sector $S$, denoted as $A_j(S)$, as the difference between accrued fiscal revenues from sector $S$ and public investments and subsidies granted by $j$ to the same sector.

The impact of the market share shock (resulting from the policy shock $P$) on net fiscal assets of sector $S$ is thus assumed to imply a change $\Delta A_j(S, P, M)$, estimated under model
\[ \frac{\Delta A_j(S, P, M)}{A_j(S)} = \chi_S u_j(S, P, M), \]

where \( \chi \) denotes the elasticity of profitability with respect to the market share.

The forward-looking trajectories of sectors’ market shares are taken from the LIMITS IAM scenario database (Kriegler et al. 2013), considering combinations of IAM \( M \) and four climate policy scenarios \( P \), characterized by different Greenhouse Gases (GHG) emissions targets and way to achieve them.\(^3\)

Because, in general, the policy shock affects at the same time several sectors in the economy of the issuer \( j \), we have to consider the total net effect on the issuer’s net fiscal assets as follows:

\[ \frac{\Delta A_j(P, M)}{A_j} = \sum_S \frac{\Delta A_j(S, P, M)}{A_j(S)} A_j(S) = \sum_S \chi_S u_j(S, P, M) \frac{A_j(S)}{A_j}, \]

In principle, in our approach, the elasticity coefficient could be estimated empirically for the specific sectors of the sovereign issuers in the portfolio. In this work, the data to carry out this estimation was not available. Being our goal to provide an estimation of the upper bounds of the magnitude of the shocks due to a given climate policy scenarios \( P \) (see section 5), where the shock is transmitted to the value of the sovereign bond via the change in sectors’ market share, GDP and fiscal assets, we have assumed a value of \( \chi \) constant and equal to 1 (typical empirical values range between 0.2 and 0.6).

### 3. Pricing climate risk in equity holdings

In this section, we focus on the risk-neutral valuation of equity holdings in sectors subject to potential forward-looking climate policy shocks. We first derive the valuation formula in the case where the timing and the characteristics of climate policy shock are known. Then, we discuss how to extend the valuation model in the case in which the timing and magnitude of the climate policy shock are subject to further uncertainty.

\(^3\)See the LIMITS database documentation for more details [https://tntcat.iiasa.ac.at/LIMITSDB/static/download/LIMITS_overview_SOM_Study_Protocol_Final.pdf](https://tntcat.iiasa.ac.at/LIMITSDB/static/download/LIMITS_overview_SOM_Study_Protocol_Final.pdf)
In the valuation model, \( t_0 = 0 \) denotes the time at which valuation is carried out and \( E \) denotes a generic equity contract. In absence of climate policy, we assume that all relevant information is captured by expected future flow of dividends \( (\text{div}(t))_{t \geq t_0} \) and, following Gordon’s formulation (Gordon 1959), we further consider that dividends grow at a constant rate \( g(B) \) so that for all \( t \geq t_0 \), \( \text{div}(t+1) = (1 + g(B))\text{div}(t) \). Denoting by \( r \) the cost of risky capital, the value of equity is then determined as the net present value of future dividends, that is:

\[
V_{E, t_0}^B = \sum_{t=1}^{+\infty} \frac{(1 + g(B))^t \text{div}(t_0)}{(1 + r)^t} = \frac{\text{div}(B)(1 + g(B))}{r - g(B)}
\]

(1)

where \( \text{div}(B) = \text{div}(t_0) \).

We then consider a situation where a climate policy shock is assumed to occur at time \( t^* \) following which the dividend is assumed to shift to \( \text{div}(P) \) and the growth rate of dividends to \( g(P) \) where \( P \) identifies a specific climate policy scenario. The value of equity is then determined as

\[
V_{E, t^*}^P = \sum_{t=1}^{t^*} \frac{(1 + g_0)^t \text{div}(B)}{(1 + r)^t} + \sum_{t=t^*+1}^{+\infty} \frac{(1 + g(P))^{t-t^*} \text{div}(P)}{(1 + r)^t}
\]

(2)

or equivalently

\[
V_{E, t^*}^{P, t^*} = (1 - \frac{1 + g_0}{1 + r})^{t^* - t_0} \frac{\text{div}(B)(1 + g(B))}{r - g(B)} + \frac{1}{(1 + r)^{t^*}} \frac{\text{div}(P)(1 + g(P))}{r - g(P)}
\]

(3)

In particular, if the climate policy shock occurs at valuation time, i.e. \( t^* = t_0 \), we obtain

\[
V_{E, t_0}^P = \frac{\text{div}(P)(1 + g(P))}{r - g(P)}
\]

(4)

In a climate policy scenario \( P \), it is expected that \( \text{div}(P) \) and \( g(P) \) decrease for carbon-intensive economic activities and increase for low-carbon economic activities. In sectors such as energy production, where climate policy shocks induce substitution from high-carbon to low-carbon sources, these impacts can be directly inferred from market shares under the assumption that (i) the growth rate of total revenues in the sector (high-carbon plus low-carbon) remain constant, (ii) the dividend to revenue ratio is similar across subsectors and
(iii) dividends are proportional to market share. Indeed, one then has \( g(P) = g(B) \), and using the notations of the preceding section, one has (up to a discount factor if \( t^* > t_0 \)):

\[
div(P) = \frac{m_E(S, P, M)}{m_E(S, B, M)} \cdot div(B).
\]

We further highlight two basic applications of our equity valuation methodology:

- The discontinuous change of valuation in the case of a disorderly transition occurring at time \( t^* \) is given by \( V_{E}^{B,t^*} - V_{E}^{P,t^*} \).

- Given a probability distribution \( \mathbb{P} \) on the time of occurrence and/or the impact of the policy scenarios, one can compute the expected value and the value-at-risk or order \( \alpha \) associated to an equity contract respectively as \( \int V_{E}^{P,t_0} d\mathbb{P}(P, t_0) \) and \( X \) such that \( \mathbb{P}(V_{E}^{P,t_0} \geq X) = 1 - \alpha \).

4. Pricing climate transition risk in corporate bonds

We define here a model for counterparty valuation in the case of a corporate bonds issuer and we define the default conditions and default probability.

4.1. Model for corporate bonds valuation

We consider a risky (defaultable) bond of corporate issuer \( j \), issued at \( t_0 \) with maturity \( T \). The bond value at \( T \), with \( R \) bond Recovery Rate (i.e. % of notional recovered upon default), and LGD Loss-Given-Default (i.e. % loss) can be defined as:

\[
v_j(T) = \begin{cases} 
R_j = (1 - \text{LGD}_j) & \text{if } j \text{ defaults (with prob. } q_j) \\
1 & \text{else (with prob. } 1 - q_j) 
\end{cases}
\]

The expected value of bond’s payoff can then be written as:

\[
\mathbb{E}[v_j] = (1 - q_j) + q_j R_j = 1 - q_j (1 - R_j) = 1 - q_j \text{LGD}_j
\]
The bond price $v^*_j$ is equal to the bond discounted expected value, with $y_f$ risk-free rate. The price defines implicitly the yield $y_j$ of bond $j$ (under risk neutral measure) as follows:

$$v^*_j = e^{-y_f T} \mathbb{E}[v_j] = e^{-y_f T} (1 - q_j \text{LGD}_j) = e^{-y_j T}$$

Finally, the bond spread can be defined as: $s_j = y_j - y_f$, with $e^{-s_j T} = 1 - q_j \text{LGD}_j$

An useful fact about spread is that:

$$s_j \approx \frac{1}{T} q_j (1 - R_j) = \frac{1}{T} q_j \text{LGD}_j \text{ (for small } s_j \text{)}$$

4.2. Corporate bond default conditions

We consider the corporate bond issuer $i$ balance sheet: $A_j(t_0), A_j(T) \text{ asset, with } t_0 \text{ issue time and } T \text{ maturity; } L_j(T) \text{ liability.}$

The default condition (e.g. following Merton 1974) reads as

$$A_j(T) = A_j(t_0)(1 + \eta_j(T)) < L_j(T)$$

with $\eta_j(T) \in \mathbb{R}$: idiosyncratic shock (e.g. firm $j$ productivity), $\phi(\eta_1, ..., \eta_j, \eta_n)$ joint probability distribution (possibly correlated)

We add the climate policy shock $\xi_j$ on $j$'s assets (as a “jump” up/down in the probability of default), assuming that the idiosyncratic shock $\eta_j$ and the policy shock $\xi_j$ are independent.

We can then define the new default condition as:

$$A_j(T) = A_j(t_0)(1 + \eta_j(T) + \xi_j(P)) < L_j(T)$$

$$\iff \eta_j(T) \leq \theta_j(P) = L_j(T)/A_j(t_0) - 1 - \xi_j(T, P)$$

with $\theta_j(P)$ default threshold under scenario $P$ and $\xi_j(P)$ the climate policy shock can be either positive or negative (given the composition of $j$: $\xi_j(P) > -1$), and possibly correlated across $j$. 

22
4.3. Corporate default probability

We can define the default probability (PD) \( q_j \) of issuer \( j \) under Climate Policy Scenario \( P \), with \( \phi_P(\eta_j) \) being the probability distribution of the idiosyncratic shock \( \eta_j \), \( \eta_{inf} \) lower bound of distribution support:

\[
q_j(P) = \mathcal{P}(\eta_j < \theta_j(P)) = \int_{\eta_{inf}}^{\theta_j(P)} \phi_P(\eta_j) \, d\eta_j.
\]

We introduce now a proposition of the PD adjustment \( \Delta \) under the climate policy shock following the intuition that frequent small productivity shocks across time and firms occur in a similar way with/without climate policy shock. Then, the policy shock shifts the probability distribution of the small productivity shocks and thus the default probability of \( j \).

We introduce the following assumption: the idiosyncratic shocks are independent from policy shock, i.e. conditional to occurrence of \( \xi_j \).

We obtain that the PD adjustment under policy shock scenario is:

\[
\Delta q_j(P) = q_j(P) - q_j(B) = \int_{\theta_j(B)}^{\theta_j(P)} \phi(\eta_j) \, d\eta_j, \text{ with } \theta_j(P) = \theta_j(B) - \xi_j(P)
\]

Then, assuming that the idiosynchronic shocks are independent from the policy shock, and that the policy shock on assets is proportional to shock on GVA via elasticity \( \xi_j = \chi_j u^{GVA}_j(P) \), we obtain that the adjustment \( \Delta q_j(P) \) in default probability of \( j \) under Climate Policy Shock Scenario:

- Increases with GVA shock magnitude \( |u^{GVA}_j(P)| \) if \( u^{GVA}_j(P) < 0 \), and decreases vice versa (under mild condition on \( \phi \));

- Is proportional to the GVA shocks on climate relevant sectors (in the limit of small Climate Policy Shock):

\[
\Delta q_j(P) \approx -\chi_j \left( u^{GVA}_{j,PrFos} w^{GVA}_{j,PrFos} + u^{GVA}_{j,ElFos} w^{GVA}_{j,ElFos} + u^{GVA}_{j,ElRen} w^{GVA}_{j,ElRen} \right).
\]

Climate policy shock corporate bond value adjustment

Being \( \Delta v^*_j \) defined as the change in the discounted expected value of the corporate bond, \( v^*_j \), conditional to a Climate Policy Shock Scenario \( B \rightarrow P \)

\[
\Delta v^*_j = v^*_j(q_j(P)) - v^*_j(q_j(B)) = -e^{-y_j T} \Delta q_j(P) \text{LGD}_j
\]
Proposition: conditional to policy shock scenario $B \rightarrow P$, and assuming everything else the same regarding the issuer’s balance sheet, then the corporate bond value adjustment $\Delta v^*_j(P)$:

- Is negative and increases with magnitude of policy shock $|\xi_j(P)|$ if $\xi_j(P) < 0$;
- Is positive and increases with magnitude of policy shock if $\xi_j(P) > 0$, with the constraint $v^*_j \leq 1$;

5. Pricing climate transition risk in sovereign bonds

We define here a model for counterparty valuation in the case of a sovereign bond issuer and we define the default conditions and default probability.

5.1. Model for sovereign bonds valuation

We consider a risky (defaultable) bond of sovereign $j$, issued at $t_0$ with maturity $T$. The sovereign bond value at $T$, with $R$ bond Recovery Rate (i.e. % of notional recovered upon default), and LGD Loss-Given-Default (i.e. % loss) can be defined as:

$$v_j(T) = \begin{cases} R_j = (1 - \text{LGD}_j) & \text{if } j \text{ defaults (with prob. } q_j) \\ 1 & \text{else (with prob. } 1 - q_j) \end{cases}$$

The expected value of bond’s payoff can be defined then as:

$$E[v_j] = (1 - q_j) + q_j R_j = 1 - q_j (1 - R_j) = 1 - q_j \text{LGD}_j$$

The sovereign bond price $v^*_j$ can be defined as the bond discounted expected value, with $y_f$ risk-free rate.

The price defines implicitly the yield $y_j$ of sovereign bond $j$ (under risk neutral measure) as follows:

24
\[ v_j^* = e^{-y_j T} \mathbb{E}[v_j] = e^{-y_j T} (1 - q_j \text{LGD}_j) = e^{-y_j T} \]

Finally, the bond spread can be defined as: \( s_j = y_j - y_f \), with \( e^{-s_j T} = 1 - q_j \text{LGD}_j \)

An useful fact about spread is that:

\[ s_j \approx \frac{1}{T} q_j (1 - R_j) = \frac{1}{T} q_j \text{LGD}_j \text{(for small } s_j) \]

5.2. Sovereign default conditions

Following a stream of literature (Gray et al. 2007), we model the payoff of the defaultable sovereign bond as dependent on the ability of the sovereign to repay the debt out of its fiscal revenues accrued until the maturity. More in detail, the balance sheet of the sovereign entity is modelled as follows:

- Assets: net fiscal assets, i.e. the accrued value over time of tax revenues minus expenditures such as investments and subsides;

- Liabilities: debt securities issued as sovereign bonds with the same maturity.

We have a sovereign \( i \) balance sheet defined as: \( A_j(t_0), A_j(T) \textbf{ net fiscal asset at } t_0 \text{ and } \text{maturity; } L_j(T) \text{ liability.} \)

The default condition (e.g. Gray-Merton-Bodie 2007) reads as:

\[ A_j(T) = A_j(t_0)(1 + \eta_j(T)) < L_j(T) \]

We add then a climate policy shock \( \xi_j \) on \( j \)'s net fiscal assets ("jump" up/down), assuming idiosyncratic shock \( \eta_j \) and policy shock \( \xi_j \) are \textbf{independent}.

The new sovereign default condition reads as:

\[ A_j(T) = A_j(t_0)(1 + \eta_j(T) + \xi_j(P)) < L_j(T) \]

\[ \iff \eta_j(T) \leq \theta_j(P) = L_j(T)/A_j(t_0) - 1 - \xi_j(T,P) \]
where \( \theta_j(P) \) is the default threshold under scenario \( P \), \( \xi_j(P) \) is the \textbf{climate policy shock} from \( B \) to \( P \) (can be positive or negative), \( \xi_j(P) > -1 \), possibly correlated across \( j \).

Differently from Gray et al. 2007, we do not consider whether debt is issued in local or foreign currency, and we do not consider exchange rate risk.

In the context of climate change, there is a consensus among scholars and practitioners on the fact that markets and investors are not yet pricing in all the information available about climate-related financial risks. Therefore, we relax the classic assumptions of efficient and frictionless markets that is needed in the Merton model (Merton 1974) to solve the pricing in closed form. Our goal here is to model the mechanism of the shock transmission channel from fiscal revenue to the value of the sovereign bond, in a market that is non necessarily efficient.

### 5.3. Sovereign default probability

We can define the \textbf{Default probability} \( PD \) of issuer \( j \) under Climate Policy Scenario \( P \), with \( \phi_P(\eta_j) \) probability distribution of idiosyncratic shock \( \eta_j \), \( \eta_{inf} \) lower bound of distribution support:

\[
q_j(P) = \mathcal{P}(\eta_j < \theta_j(P)) = \int_{\eta_{inf}}^{\theta_j(P)} \phi_P(\eta_j) \, d\eta_j,
\]

We introduce now a proposition of the PD adjustment \( \Delta \) under the climate policy shock following the intuition that frequent small productivity shocks across time and firms occur in a similar way with/without climate policy shock. Then, the policy shock shifts the probability distribution of the small productivity shocks and thus the default probability of issuer \( j \).

We introduce the assumption that the idiosyncratic shocks are independent from policy shock, i.e. conditional to occurrence of \( \xi_j \).

And we obtain that the PD adjustment under policy shock scenario is:

\[
\Delta q_j(P) = q_j(P) - q_j(B) = \int_{\theta_j(B)}^{\theta_j(P)} \phi_P(\eta_j) \, d\eta_j, \text{ with } \theta_j(P) = \theta_j(B) - \xi_j(P)
\]

Then, assuming that:

- The idiosyncratic shocks are independent from the policy shock;
• The policy shock on fiscal asset is proportional to shock on GVA via elasticity
  \( \xi_j = \chi_j u_j^{GVA}(P) \)

The adjustment \( \Delta q_j(P) \) in default probability of sovereign \( j \) under Climate Policy Shock Scenario:

• Increases with GVA shock magnitude \( |u_j^{GVA}(P)| \) if \( u_j^{GVA}(P) < 0 \), and decreases viceversa (under mild condition on \( \phi \));

• Is proportional to the GVA shocks on climate relevant sectors (in the limit of small Climate Policy Shock):
  \[
  \Delta q_j(P) \approx -\chi_j (u_j^{GVA} w_j^{GVA} + u_j^{GVA} w_j^{GVA} + u_j^{GVA} w_j^{GVA} + u_j^{GVA} w_j^{GVA}).
  \]

Climate policy shock sovereign bond value adjustment

Being \( \Delta v_j^* \) defined as the change in the discounted expected value of the bond, \( v_j^* \), conditional to a Climate Policy Shock Scenario \( B \rightarrow P \)

\[
\Delta v_j^* = v_j^*(q_j(P)) - v_j^*(q_j(B)) = -e^{-y_j^T \Delta q_j(P)} \text{LGD}_j
\]

Proposition: conditional to policy shock scenario \( B \rightarrow P \), and assuming everything else the same regarding the issuer’s balance sheet, then the bond value adjustment \( \Delta v_j^*(P) \):

• Is negative and increases with magnitude of policy shock \( |\xi_j(P)| \) if \( \xi_j(P) < 0 \);

• Is positive and increases with magnitude of policy shock if \( \xi_j(P) > 0 \), with the constraint \( v_j^* \leq 1 \);

6. Climate Spread

The Climate spread \( \Delta s_j \) is defined as the change in the spread \( s_j \), conditional to Climate Policy Shock Scenario

\[
\Delta s_j = s_j(q_j(P)) - s_j(q_j(B))
\]

Conditional to the climate policy shock scenario, the climate spread \( s_j(P) \):
• Increases with magnitude of policy shock $|\xi_j(P)|$ if $\xi_j(P) < 0$;

• Decreases with magnitude of policy shock if $\xi_j(P) > 0$;

• For small GVA shocks $u_j^{GVA}(P)$ it holds:

$$\Delta s_j \approx -\frac{1}{T}\chi_j \times$$

$$\times (u_{j,PrFos}^{GVA} w_{j,PrFos}^{GVA} + u_{j,ElFos}^{GVA} w_{j,ElFos}^{GVA} + u_{j,ElRen}^{GVA} w_{j,ElRen}^{GVA})$$

7. Investor and Portfolio Value-at-Risk and Climate Value-at-Risk

We can define an investor $i$’s portfolio value $z_i$ and portfolio rate of return $\pi_i$ at $T$, with $W_{ij}$ amount (numeraire) of $j$’s bond purchased by $i$ as:

$$z_i(T) = \sum_j W_{ij}v_j(T), \quad \pi_i = \frac{z_i(T) - z_i(t_0)}{z_i(t_0)}.$$

The **Value-at-Risk** (VaR) on investor’s rate of return is the “worst case loss” at confidence level $C^{VaR}$. Given the probability distribution $\psi(\pi_i(T))$, the VaR = value of return $\pi_i$ (e.g. left tail) such that:

$$\mathcal{P}\{\pi_i < \text{VaR}\} = \int_{\inf(\pi_i)}^{\text{VaR}} \pi_i \psi_i(\pi_i) d\pi_i = C^{VaR}$$

The **Climate VaR** is defined as the Value-at-Risk of the portfolio of the investor, conditional to Climate Policy Shock Scenario with $\pi$ portfolio return, $\psi_P(\pi)$ distribution of returns conditional to the climate policy shock:

$$\text{ClimateVaR}(P) = \int_{\inf(\pi)}^{\text{ClimateVaR}} \pi \psi_P(\pi) d\pi = C^{VaR}$$

Conditional to the policy shock scenario $B \rightarrow P$, the ClimateVaR($P$):

• Increases with magnitude of policy shock $|\xi_j(P)|$ if $\xi_j(P) < 0$;

• Decreases with magnitude of policy shock if $\xi_j(P) > 0$;

• Increases with marginal default probability adjustment $\Delta q_j(P)$ of bond $j$. 

28
8. References


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