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Supplement to Koller, Maier, & Hatzinger: "An Empirical Power Analysis of Quasi-Exact Tests for the Rasch Model: Measurement Invariance in Small Samples"

Marco J. Maier & Ingrid Koller

This document is a supplementary text to *"An Empirical Power Analysis of Quasi-Exact Tests for the Rasch Model: Measurement Invariance in Small Samples"* by Koller, Maier, & Hatzinger (to be published in *Methodology*, ISSN-L 1614-1881), which covers all technical details regarding the simulation and its results.
First, the simulation scenarios and the introduction of *differential item functioning* (DIF) are described. Next, the different populations' distributions that were investigated are discussed, and finally, actual type-I-error rates and empirical power are displayed for all simulated scenarios.

Contents

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1 Sample Sizes, Test Lengths, DIF-Generation

The simulations were carried out with sample sizes of n = 30, 50, 100, and 200. For the investigation of differential item functioning (DIF), the first group was generated without and second with DIF in each sample (both groups were equally large, i. e., 15, 25, 50, and 100 observations per group). The number of items was determined by placing them equidistantly between −2 and +2 logits. Step sizes were 1, 0.5, and 0.25 logits which leads to test lengths of k = 5, 9, and 17.

To introduce DIF in the second group, k^{DIF} = 1, 2 and 5 items were shifted in increments of 0.1 logits up to a maximum violation of +2 logits per item. Of course, the combination of k = 5 and k^{DIF} = 5 is not possible and therefore omitted. This leads to 21 simulations for each combination of k and k^{DIF} with violations from 0.0, 0.1, . . . , to 2.0 logits. Figures 1, 2, and 3 show how items are placed in both groups and how DIF violation is introduced.

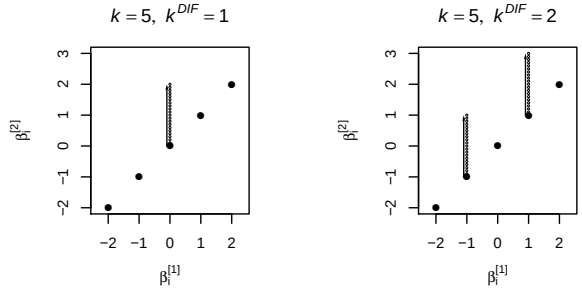


Figure 1: Scenarios with k = 5 items ($\beta_1^{[1]}$ and $\beta_1^{[2]}$ are the item difficulties for the first and second group). DIF violation increases in the second group as indicated by the arrows.

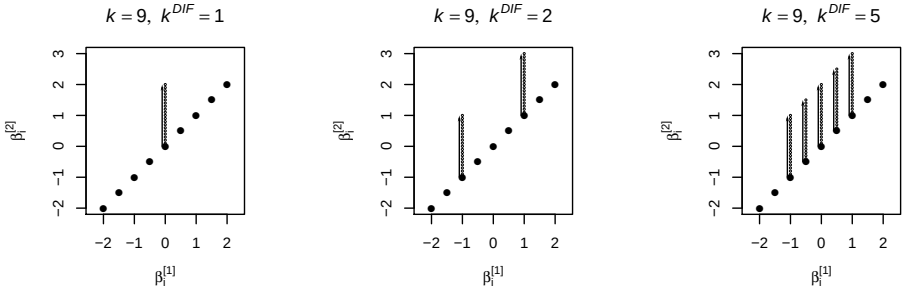


Figure 2: Scenarios with k = 9 items ($\beta_1^{[1]}$ and $\beta_1^{[2]}$ are the item difficulties for the first and second group). DIF violation increases in the second group as indicated by the arrows.

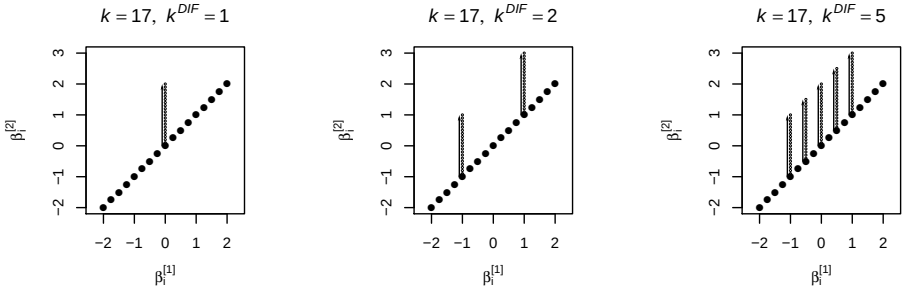


Figure 3: Scenarios with k = 17 items ($\beta_1^{[1]}$ and $\beta_1^{[2]}$ are the item difficulties for the first and second group). DIF violation increases in the second group as indicated by the arrows.

2 Distributions of θ

In addition to the simulation scenarios outlined in section 1, different distributional assumptions were assumed. The following list describes all 9 used combinations of populations (see also Fig. 4 and Tab. 1). Results of the different populations' distributions will be addressed subsequently.

A: Both Normal; Both groups are sampled from the same normal distribution with $\mu = 0$ and $\sigma = 1.5$.

B: Normal shifted +0.25; Both groups come from a normal distribution with $\sigma = 1.5$, but the group without DIF has $\mu = -0.25$ and the second group $\mu = 0.25$ (ability difference of 0.5 logits).

C: Normal shifted +0.5; Both groups come from a normal distribution with $\sigma = 1.5$, but the group without DIF has $\mu = -0.5$ and the second group $\mu = 0.5$ (ability difference of 1 logit).

D: Normal shifted −0.25; As in 2, both groups come from a normal distribution with $\sigma = 1.5$, but the group without DIF has $\mu = 0.25$ and the second group $\mu = -0.25$ (ability difference of 0.5 logits).

E: Normal shifted −0.5; As in 3, both groups come from a normal distribution with $\sigma = 1.5$, but the group without DIF has $\mu = 0.5$ and the second group $\mu = -0.5$ (ability difference of 1 logit).

F: Beta (Positive); The first group has a normal distribution ($\mu = 0$, $\sigma = 1.5$) while the group with DIF comes from a beta distribution with positive skew, $\alpha = 2$, $\beta = 3$ (note that the beta distribution is defined with a support on $x \in (0, 1)$; in order to use it for these simulations, the support was stretched to $x \in (-3, 3)$).

G: Beta (Negative); The first group has a normal distribution ($\mu = 0$, $\sigma = 1.5$) while the group with DIF comes from a beta distribution with negative skew, $\alpha = 3$, $\beta = 2$ (for notes on the support, see above).

H: Uniform P_2; The first group has a normal distribution ($\mu = 0$, $\sigma = 1.5$) while the DIF-group has a uniform distribution between −3 and +3.

I: Uniform P_1; The first group is uniformly distributed in $[-3, 3]$ while the second group follows a normal distribution ($\mu = 0$, $\sigma = 1.5$).

Table 1: Populations.

Abbreviation	P_1 (no DIF)	P_2 (with DIF)
A	$\mathcal{N}(\mu = 0, \sigma = 1.5)$	$\mathcal{N}(\mu = 0, \sigma = 1.5)$
B	$\mathcal{N}(\mu = -0.25, \sigma = 1.5)$	$\mathcal{N}(\mu = 0.25, \sigma = 1.5)$
C	$\mathcal{N}(\mu = -0.5, \sigma = 1.5)$	$\mathcal{N}(\mu = 0.5, \sigma = 1.5)$
D	$\mathcal{N}(\mu = 0.25, \sigma = 1.5)$	$\mathcal{N}(\mu = -0.25, \sigma = 1.5)$
E	$\mathcal{N}(\mu = 0.5, \sigma = 1.5)$	$\mathcal{N}(\mu = -0.5, \sigma = 1.5)$
F	$\mathcal{N}(\mu = 0, \sigma = 1.5)$	$\mathcal{B}^*(\alpha = 2, \beta = 3)$
G	$\mathcal{N}(\mu = 0, \sigma = 1.5)$	$\mathcal{B}^*(\alpha = 3, \beta = 2)$
H	$\mathcal{N}(\mu = 0, \sigma = 1.5)$	$\mathcal{U}(a = -3, b = 3)$
I	$\mathcal{U}(a = -3, b = 3)$	$\mathcal{N}(\mu = 0, \sigma = 1.5)$

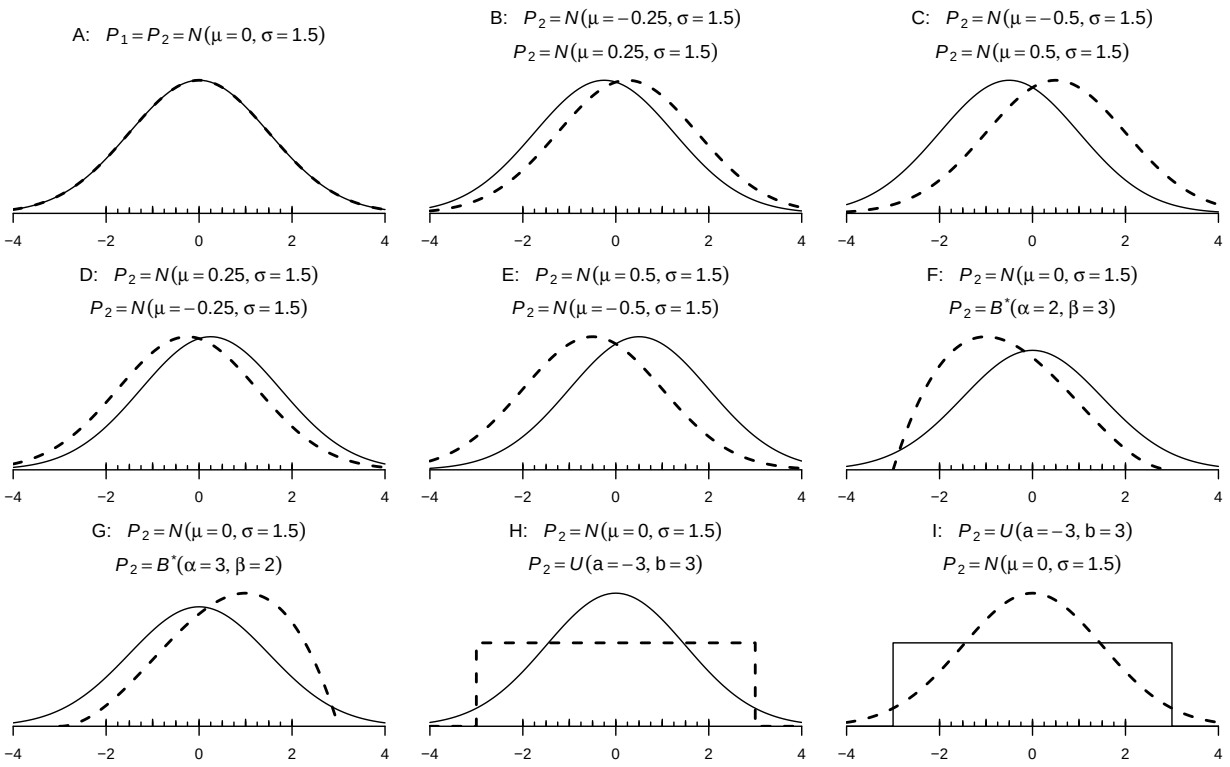


Figure 4: Used populations. Solid densities indicate the distribution of the first group (P_1, no DIF) while dashed densities are the distributions of the second group (P_2, DIF). Note that the beta distributions, which have a support $x \in (0, 1)$, are „stretched" to $x \in (-3, 3)$ and therefore denoted $B^*(\alpha, \beta)$.

3 Technical Details of the Simulation Algorithm

The simulation had four different sample sizes, eight combinations of k and k^{DIF}, 21 degrees of violation and nine distributional assumptions for both groups, which leads to a total of $4 \cdot 8 \cdot 21 \cdot 9 = 6,048$ simulations.

In each of those simulations, the target was to simulate 10,000 valid likelihood-ratio-tests. Depending on factors such as sample size, test length, DIF, etc., a lot of matrices were not appropriate for the Rasch model and/or the LRT. Non-parametric methods, however, have a much larger sample space of feasible matrices, therefore they can handle a lot of data that would not work with the LRT and there are usually more results for T_{10} and T_4.

3.1 Simulation: Hard- and Software

The simulations were carried out on a cluster of workstations running Debian GNU/Linux Squeeze in an Oracle Grid Engine (Version 6.2u5) with 44 nodes (2× Intel Xeon hexa-core CPUs with 24 GB RAM per node). For more details, see `http://www.wu.ac.at/firm/cluster_folder`.

3.2 Proportions of LRT-inappropriate Samples

Table 2 shows the median[1] proportion of samples that only worked with T_{10} and T_4, but not with Andersen's LRT. Those proportions quantify the excess number of simulated matrices that were necessary to arrive at the 10,000 desired matrices for the LRT, that is, a proportion of 2 means that roughly 20,000 matrices had to be simulated to get 10,000 that worked with the LRT.[2]

Obviously, a lot of matrices had to be discarded in the $n = 30$ scenarios. Since, for example, a single item in a subgroup with either all 0s or 1s leads to a failing LRT, but not necessarily causes the non-parametric tests to fail, the sample space of the LRT was a subset of T_{10}, respectively T_4.

This phenomenon rapidly decreases with increasing sample sizes, since the probability of having an item in a subgroup that contains only 0s or 1s falls. There were few such cases with $n = 100$ and virtually no additionally samples were needed with $n = 200$.

Table 2: Median Proportions of Sampled Matrices Compared to Targeted Frequencies.

		k^{DIF}		
	k	1	2	5
$n = 30$	5	2.866	3.096	—
	9	1.767	1.917	1.964
	17	1.720	1.945	1.985
$n = 50$	5	1.352	1.414	—
	9	1.100	1.143	1.148
	17	1.072	1.121	1.121
$n = 100$	5	1.018	1.028	—
	9	1.002	1.005	1.005
	17	1.001	1.002	1.002
$n = 200$	5	1.000	1.000	—
	9	1.000	1.000	1.000
	17	1.000	1.000	1.000

[1] The median was computed for each combination of n, k, and k^{DIF} with all population-scenarios.

[2] Due to technical issues, some combinations miss one LRT matrix, but this does not distort the results.

4 Nominal α Levels

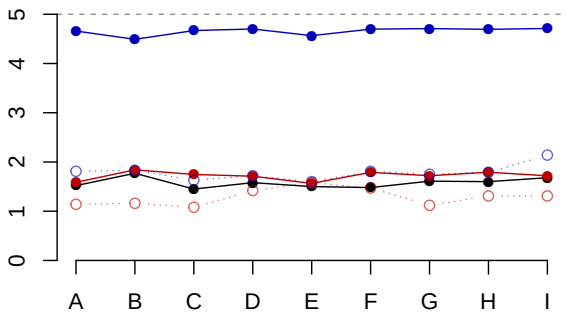
This section deals with the actual α levels that the test statistics exhibit. Actual α levels were computed $1/n \sum \mathbf{I}_{p<\alpha}$ with $\alpha = 5\%$.
There are two numbers for the non-parametric tests: the first and second columns with T_{10} and T_4 are the levels for the respective statistics using all simulated matrices. The last two columns $T_{10}^{\star}$ and $T_4^{\star}$ show the nominal levels if only those matrices are used that also worked for Andersen's LRT.

In all graphs, the nominal α levels are plotted against the used population distribution (scenarios A through I) on the x-axis.
The solid lines with filled dots are: T_{10} (blue), T_4 (red), LRT (black). The dashed lines with hollow dots are: $T_{10}^{\star}$ (blue), $T_4^{\star}$ (red).

4.1 $n = 30$

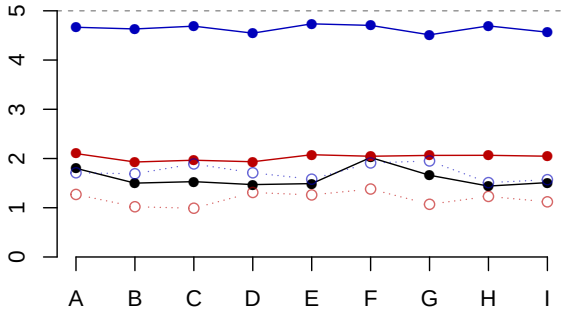
4.1.1 $k = 5$, $k^{DIF} = 1$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	4.66	1.59	1.52	1.81	1.14
B	4.49	1.84	1.77	1.83	1.16
C	4.67	1.75	1.45	1.63	1.08
D	4.70	1.71	1.58	1.72	1.42
E	4.57	1.56	1.50	1.60	1.58
F	4.70	1.79	1.48	1.81	1.47
G	4.71	1.72	1.61	1.75	1.12
H	4.69	1.79	1.60	1.79	1.31
I	4.71	1.72	1.68	2.14	1.31



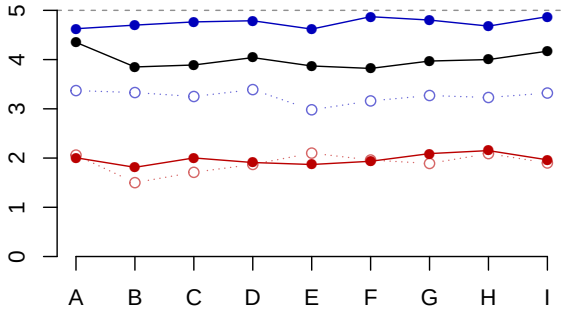
4.1.2 $k = 5$, $k^{DIF} = 2$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	4.67	2.10	1.80	1.71	1.27
B	4.63	1.93	1.50	1.69	1.02
C	4.69	1.97	1.53	1.89	0.99
D	4.55	1.93	1.47	1.71	1.31
E	4.74	2.08	1.49	1.58	1.26
F	4.70	2.05	2.02	1.91	1.38
G	4.51	2.06	1.66	1.95	1.07
H	4.69	2.07	1.44	1.51	1.23
I	4.57	2.05	1.51	1.57	1.12



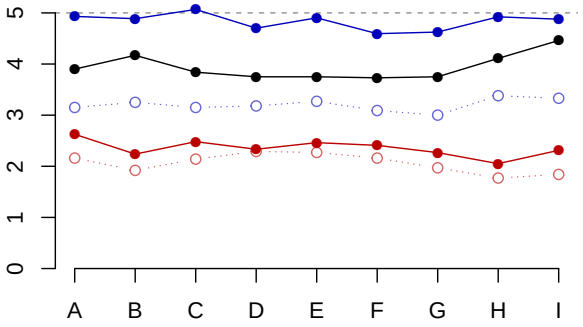
4.1.3 $k = 9$, $k^{DIF} = 1$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	4.63	2.01	4.35	3.37	2.06
B	4.70	1.81	3.85	3.33	1.50
C	4.77	2.00	3.89	3.25	1.71
D	4.79	1.91	4.04	3.39	1.87
E	4.62	1.87	3.87	2.98	2.10
F	4.87	1.93	3.82	3.16	1.96
G	4.80	2.09	3.97	3.27	1.89
H	4.68	2.15	4.00	3.23	2.09
I	4.87	1.96	4.17	3.32	1.90



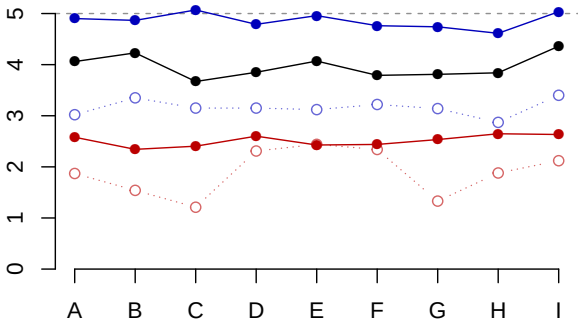
4.1.4 $k = 9$, $k^{DIF} = 2$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	4.93	2.63	3.90	3.15	2.16
B	4.89	2.24	4.17	3.25	1.92
C	5.07	2.48	3.84	3.15	2.14
D	4.70	2.33	3.75	3.18	2.29
E	4.90	2.46	3.75	3.27	2.27
F	4.59	2.41	3.73	3.09	2.16
G	4.63	2.27	3.75	3.00	1.97
H	4.92	2.05	4.11	3.38	1.77
I	4.88	2.31	4.46	3.33	1.84



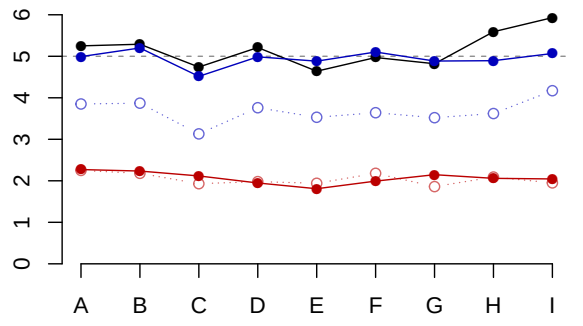
4.1.5 $k = 9$, $k^{DIF} = 5$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	4.90	2.58	4.06	3.02	1.87
B	4.87	2.34	4.23	3.35	1.54
C	5.07	2.40	3.67	3.15	1.21
D	4.79	2.60	3.85	3.15	2.31
E	4.96	2.43	4.07	3.12	2.44
F	4.76	2.44	3.79	3.22	2.34
G	4.74	2.53	3.81	3.14	1.33
H	4.61	2.65	3.84	2.87	1.88
I	5.04	2.63	4.36	3.40	2.12



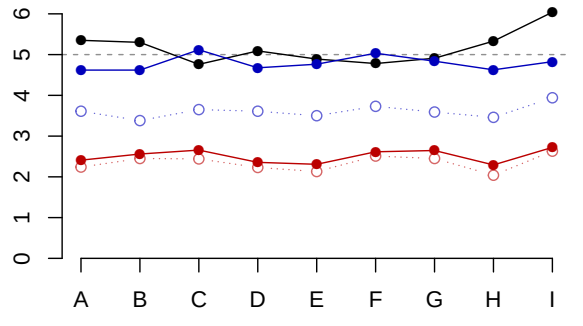
4.1.6 $k = 17$, $k^{DIF} = 1$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	4.99	2.27	5.25	3.85	2.25
B	5.20	2.24	5.29	3.87	2.18
C	4.52	2.12	4.74	3.13	1.93
D	4.99	1.95	5.21	3.76	1.98
E	4.88	1.81	4.64	3.53	1.94
F	5.10	1.99	4.97	3.64	2.18
G	4.89	2.15	4.82	3.52	1.86
H	4.89	2.06	5.59	3.62	2.09
I	5.07	2.04	5.93	4.17	1.95



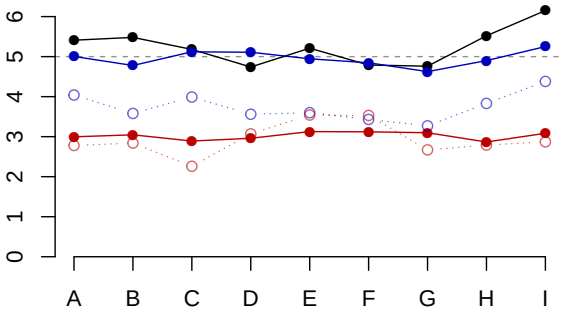
4.1.7 $k = 17$, $k^{DIF} = 2$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	4.62	2.41	5.35	3.61	2.24
B	4.62	2.56	5.30	3.38	2.45
C	5.12	2.66	4.76	3.65	2.44
D	4.68	2.36	5.09	3.61	2.23
E	4.76	2.31	4.89	3.50	2.13
F	5.03	2.61	4.78	3.73	2.51
G	4.84	2.65	4.91	3.59	2.45
H	4.63	2.29	5.33	3.46	2.04
I	4.83	2.72	6.04	3.94	2.63



4.1.8 $k = 17$, $k^{DIF} = 5$

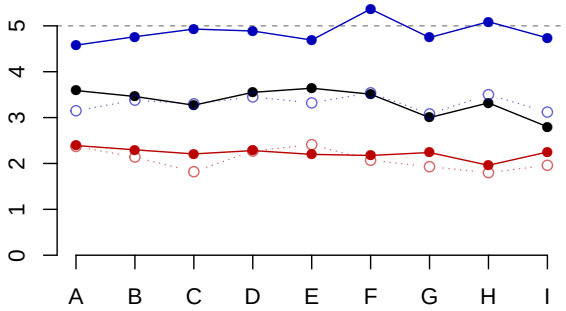
	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	5.00	3.00	5.41	4.04	2.78
B	4.78	3.05	5.48	3.58	2.84
C	5.12	2.89	5.18	3.99	2.26
D	5.11	2.96	4.74	3.56	3.07
E	4.95	3.13	5.21	3.60	3.54
F	4.85	3.12	4.79	3.43	3.53
G	4.63	3.10	4.76	3.27	2.67
H	4.90	2.87	5.52	3.83	2.79
I	5.25	3.08	6.15	4.38	2.87



4.2 $n = 50$

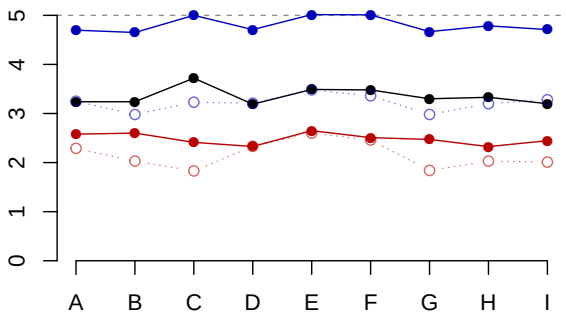
4.2.1 $k = 5$, $k^{DIF} = 1$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	4.58	2.39	3.59	3.15	2.37
B	4.76	2.30	3.46	3.38	2.14
C	4.93	2.21	3.27	3.30	1.82
D	4.89	2.28	3.55	3.45	2.27
E	4.69	2.20	3.64	3.32	2.41
F	5.37	2.17	3.51	3.54	2.07
G	4.74	2.24	3.00	3.08	1.93
H	5.09	1.96	3.32	3.50	1.80
I	4.74	2.25	2.80	3.12	1.96



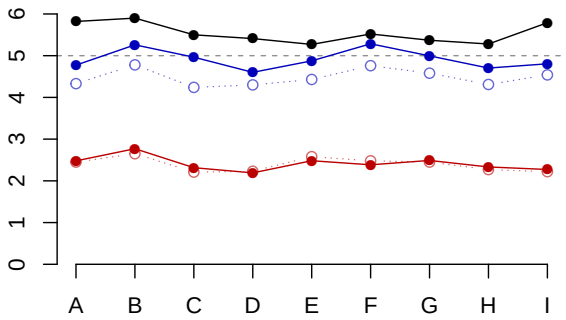
4.2.2 $k = 5$, $k^{DIF} = 2$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	4.70	2.58	3.24	3.25	2.29
B	4.65	2.60	3.24	2.98	2.03
C	5.00	2.41	3.72	3.23	1.83
D	4.71	2.33	3.19	3.21	2.33
E	5.02	2.65	3.49	3.48	2.60
F	5.01	2.51	3.48	3.36	2.46
G	4.67	2.47	3.30	2.98	1.84
H	4.79	2.33	3.33	3.20	2.03
I	4.71	2.45	3.20	3.28	2.01



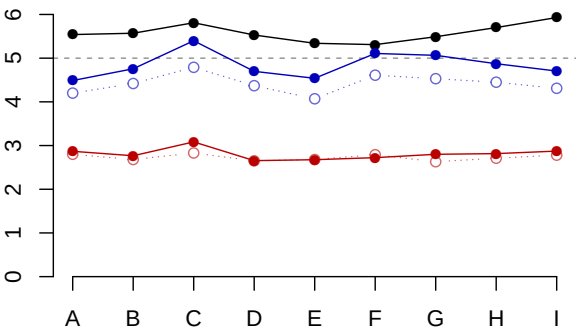
4.2.3 $k = 9$, $k^{DIF} = 1$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	4.77	2.48	5.82	4.33	2.45
B	5.26	2.77	5.90	4.78	2.65
C	4.96	2.32	5.50	4.24	2.21
D	4.60	2.19	5.41	4.30	2.23
E	4.88	2.48	5.27	4.43	2.58
F	5.28	2.38	5.52	4.76	2.48
G	5.00	2.49	5.37	4.58	2.45
H	4.70	2.33	5.28	4.31	2.27
I	4.81	2.27	5.79	4.54	2.22



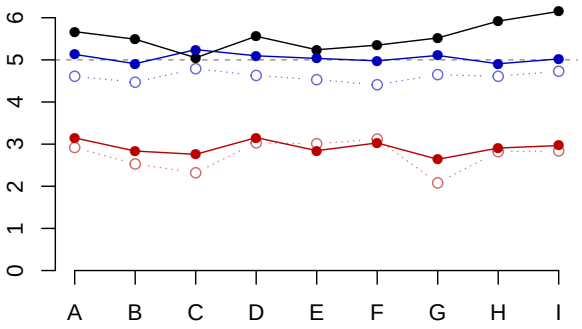
4.2.4 $k = 9$, $k^{DIF} = 2$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	4.49	2.87	5.54	4.20	2.80
B	4.76	2.77	5.57	4.42	2.68
C	5.40	3.08	5.81	4.79	2.83
D	4.70	2.65	5.53	4.37	2.65
E	4.54	2.68	5.34	4.07	2.68
F	5.11	2.72	5.31	4.61	2.79
G	5.06	2.80	5.49	4.53	2.63
H	4.87	2.82	5.70	4.45	2.71
I	4.70	2.87	5.93	4.31	2.78



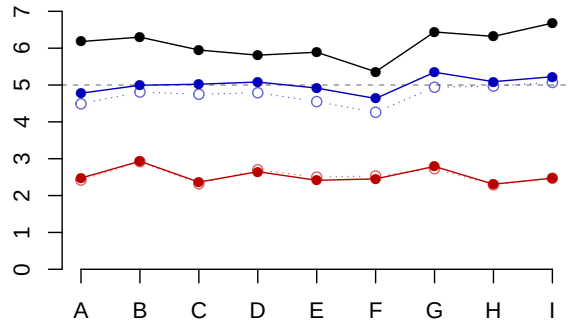
4.2.5 $k = 9$, $k^{DIF} = 5$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	5.13	3.15	5.67	4.61	2.92
B	4.91	2.84	5.49	4.47	2.53
C	5.24	2.76	5.05	4.79	2.32
D	5.10	3.15	5.56	4.63	3.03
E	5.03	2.85	5.24	4.53	3.01
F	4.98	3.03	5.35	4.41	3.12
G	5.10	2.64	5.52	4.65	2.08
H	4.91	2.91	5.92	4.61	2.82
I	5.02	2.97	6.15	4.73	2.84



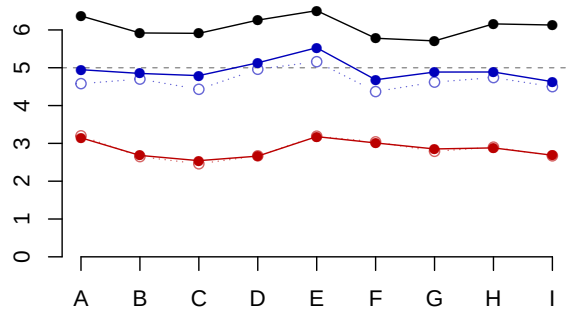
4.2.6 $k = 17$, $k^{DIF} = 1$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	4.78	2.47	6.18	4.49	2.42
B	4.99	2.93	6.30	4.81	2.92
C	5.02	2.36	5.95	4.75	2.32
D	5.08	2.65	5.81	4.79	2.70
E	4.92	2.41	5.89	4.55	2.50
F	4.63	2.45	5.36	4.26	2.53
G	5.35	2.79	6.44	4.94	2.73
H	5.09	2.31	6.32	4.97	2.29
I	5.23	2.47	6.68	5.07	2.47



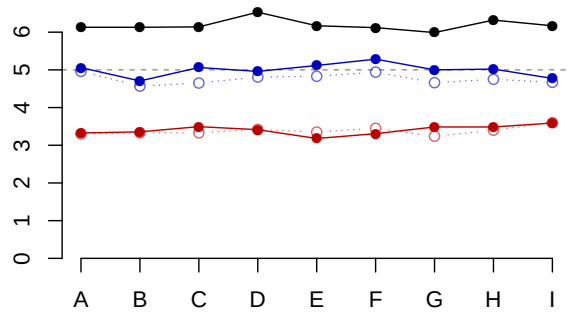
4.2.7 $k = 17$, $k^{DIF} = 2$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	4.95	3.15	6.37	4.58	3.20
B	4.85	2.68	5.92	4.70	2.65
C	4.79	2.54	5.91	4.43	2.46
D	5.13	2.66	6.26	4.96	2.67
E	5.53	3.18	6.51	5.16	3.19
F	4.68	3.01	5.78	4.37	3.04
G	4.89	2.85	5.71	4.62	2.79
H	4.89	2.88	6.16	4.74	2.90
I	4.63	2.69	6.13	4.50	2.67



4.2.8 $k = 17$, $k^{DIF} = 5$

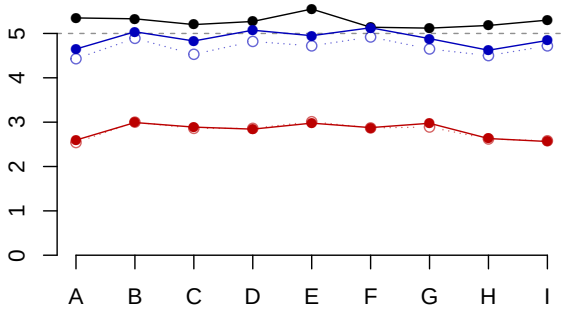
	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	5.06	3.33	6.13	4.96	3.30
B	4.71	3.36	6.13	4.57	3.33
C	5.06	3.49	6.14	4.65	3.33
D	4.96	3.41	6.53	4.81	3.42
E	5.12	3.18	6.17	4.83	3.35
F	5.28	3.31	6.12	4.94	3.45
G	5.00	3.49	5.99	4.66	3.24
H	5.02	3.48	6.32	4.75	3.40
I	4.77	3.59	6.17	4.67	3.60



4.3 $n = 100$

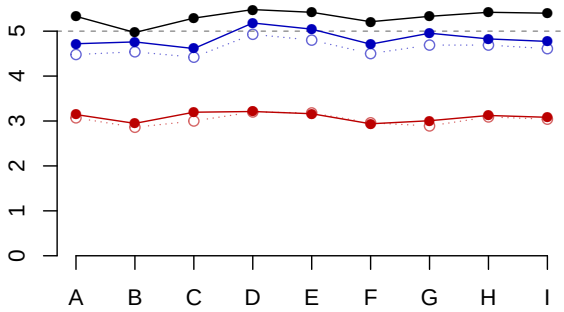
4.3.1 $k = 5$, $k^{DIF} = 1$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	4.65	2.60	5.35	4.43	2.54
B	5.04	2.99	5.33	4.89	3.00
C	4.83	2.89	5.20	4.53	2.86
D	5.07	2.84	5.27	4.82	2.86
E	4.95	2.98	5.55	4.72	3.01
F	5.13	2.88	5.14	4.92	2.87
G	4.88	2.97	5.12	4.65	2.89
H	4.62	2.63	5.18	4.50	2.62
I	4.84	2.56	5.30	4.72	2.58



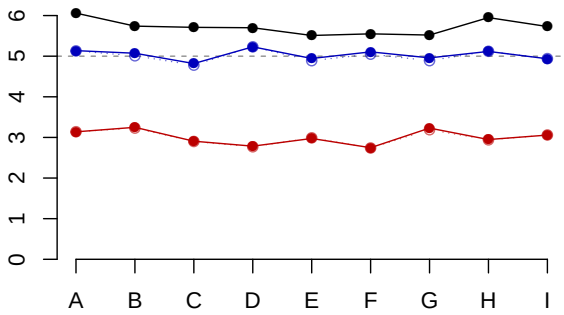
4.3.2 $k = 5$, $k^{DIF} = 2$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	4.72	3.14	5.33	4.48	3.07
B	4.76	2.95	4.97	4.54	2.86
C	4.61	3.20	5.29	4.42	3.00
D	5.18	3.21	5.48	4.93	3.20
E	5.04	3.16	5.42	4.80	3.18
F	4.71	2.94	5.21	4.50	2.96
G	4.95	3.01	5.33	4.69	2.89
H	4.83	3.12	5.42	4.69	3.09
I	4.77	3.08	5.40	4.61	3.04



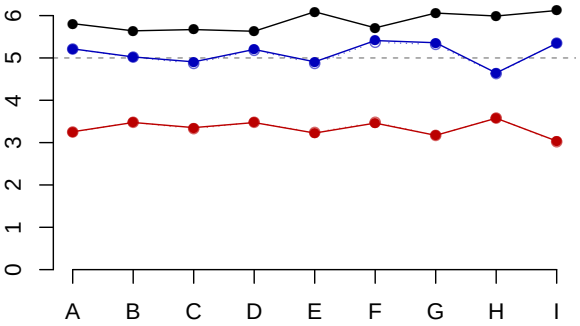
4.3.3 $k = 9$, $k^{DIF} = 1$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	5.14	3.14	6.06	5.13	3.14
B	5.07	3.25	5.74	5.01	3.23
C	4.82	2.91	5.71	4.78	2.90
D	5.23	2.78	5.70	5.23	2.77
E	4.94	2.97	5.51	4.89	2.99
F	5.11	2.75	5.55	5.05	2.74
G	4.96	3.23	5.52	4.89	3.19
H	5.12	2.95	5.95	5.12	2.94
I	4.94	3.06	5.73	4.94	3.06



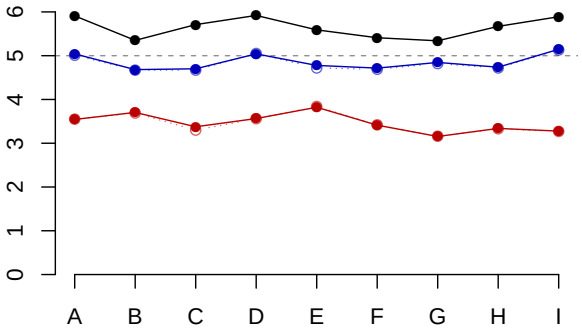
4.3.4 $k = 9$, $k^{DIF} = 2$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	5.21	3.26	5.81	5.21	3.25
B	5.02	3.48	5.64	5.02	3.48
C	4.91	3.35	5.67	4.87	3.33
D	5.20	3.47	5.63	5.18	3.48
E	4.91	3.23	6.09	4.87	3.24
F	5.41	3.46	5.71	5.37	3.48
G	5.36	3.17	6.06	5.32	3.17
H	4.64	3.58	5.99	4.63	3.58
I	5.34	3.04	6.12	5.35	3.02



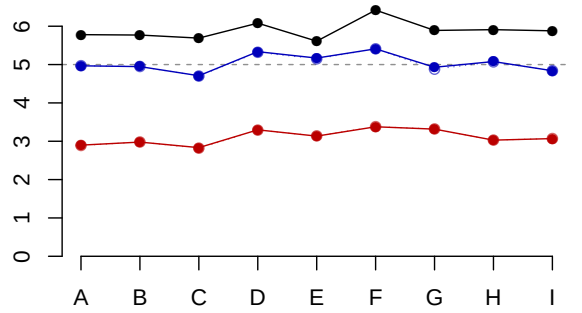
4.3.5 $k = 9$, $k^{DIF} = 5$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	5.04	3.54	5.91	5.01	3.55
B	4.68	3.70	5.35	4.67	3.69
C	4.70	3.38	5.71	4.67	3.30
D	5.04	3.56	5.92	5.05	3.56
E	4.78	3.82	5.59	4.72	3.84
F	4.72	3.41	5.41	4.69	3.42
G	4.85	3.15	5.34	4.82	3.16
H	4.74	3.34	5.67	4.72	3.33
I	5.15	3.28	5.89	5.13	3.27



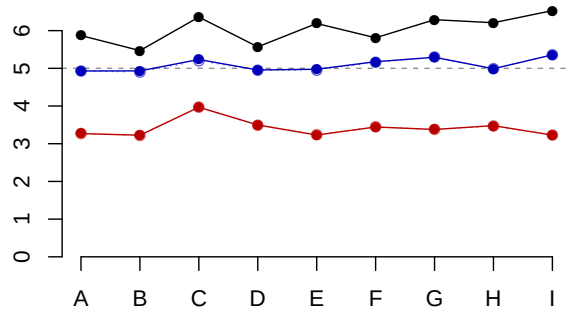
4.3.6 $k = 17$, $k^{DIF} = 1$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	4.97	2.90	5.78	4.97	2.89
B	4.95	2.98	5.77	4.95	2.98
C	4.71	2.83	5.69	4.70	2.82
D	5.34	3.30	6.08	5.32	3.29
E	5.17	3.13	5.61	5.15	3.14
F	5.41	3.38	6.42	5.41	3.38
G	4.93	3.32	5.89	4.88	3.32
H	5.08	3.03	5.91	5.07	3.03
I	4.84	3.07	5.88	4.84	3.07



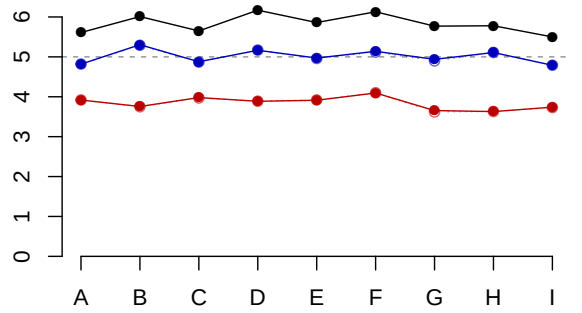
4.3.7 $k = 17$, $k^{DIF} = 2$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	4.93	3.27	5.87	4.93	3.27
B	4.93	3.23	5.47	4.90	3.22
C	5.23	3.97	6.36	5.20	3.97
D	4.96	3.50	5.57	4.95	3.49
E	4.98	3.23	6.19	4.95	3.24
F	5.18	3.44	5.81	5.16	3.45
G	5.29	3.38	6.29	5.30	3.38
H	4.99	3.48	6.21	4.99	3.47
I	5.36	3.23	6.53	5.36	3.23



4.3.8 $k = 17$, $k^{DIF} = 5$

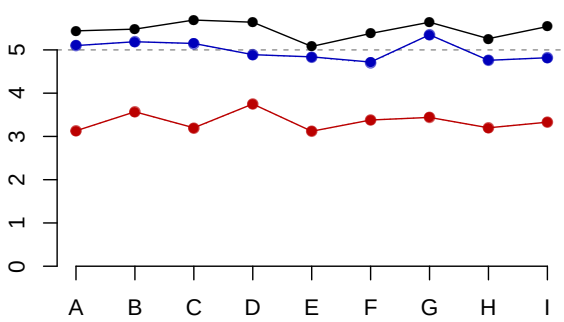
	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	4.82	3.92	5.61	4.82	3.92
B	5.30	3.76	6.01	5.29	3.75
C	4.88	3.98	5.65	4.87	3.97
D	5.17	3.89	6.17	5.17	3.89
E	4.97	3.91	5.86	4.96	3.92
F	5.14	4.10	6.13	5.12	4.10
G	4.94	3.65	5.77	4.90	3.62
H	5.11	3.63	5.78	5.11	3.63
I	4.79	3.74	5.50	4.79	3.73



4.4 $n = 200$

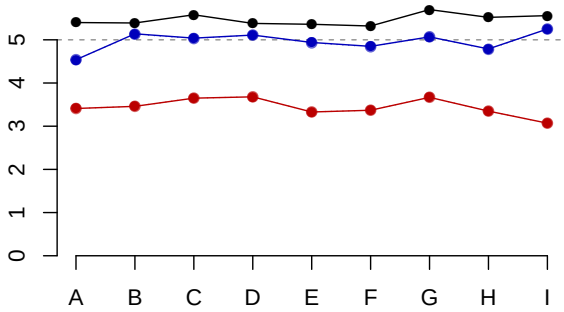
4.4.1 $k = 5$, $k^{DIF} = 1$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	5.10	3.13	5.44	5.10	3.13
B	5.19	3.57	5.48	5.19	3.57
C	5.15	3.20	5.69	5.14	3.19
D	4.89	3.75	5.64	4.89	3.75
E	4.84	3.12	5.08	4.83	3.12
F	4.72	3.38	5.38	4.70	3.38
G	5.35	3.44	5.64	5.35	3.44
H	4.76	3.20	5.26	4.76	3.20
I	4.82	3.33	5.54	4.82	3.33



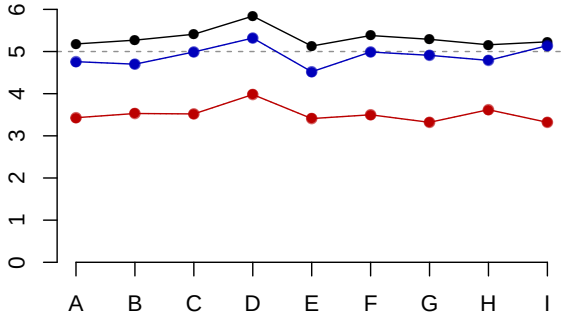
4.4.2 $k = 5$, $k^{DIF} = 2$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	4.54	3.41	5.40	4.54	3.41
B	5.14	3.46	5.39	5.13	3.46
C	5.04	3.65	5.58	5.03	3.65
D	5.11	3.68	5.38	5.11	3.68
E	4.94	3.33	5.36	4.93	3.33
F	4.85	3.37	5.32	4.84	3.37
G	5.07	3.67	5.70	5.07	3.67
H	4.79	3.35	5.52	4.78	3.35
I	5.25	3.07	5.56	5.25	3.07



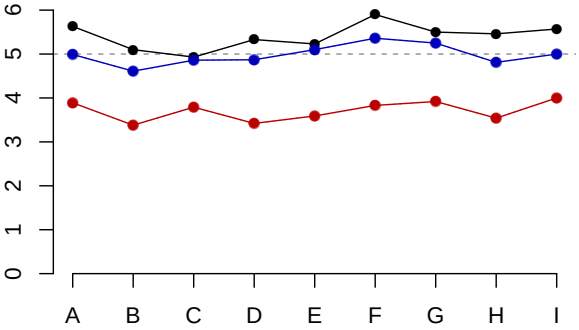
4.4.3 $k = 9$, $k^{DIF} = 1$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	4.76	3.43	5.18	4.76	3.43
B	4.70	3.53	5.27	4.70	3.53
C	4.99	3.52	5.41	4.99	3.52
D	5.32	3.98	5.84	5.32	3.98
E	4.52	3.41	5.13	4.52	3.41
F	4.99	3.50	5.38	4.99	3.50
G	4.91	3.32	5.29	4.91	3.32
H	4.79	3.62	5.16	4.79	3.62
I	5.14	3.32	5.23	5.14	3.32



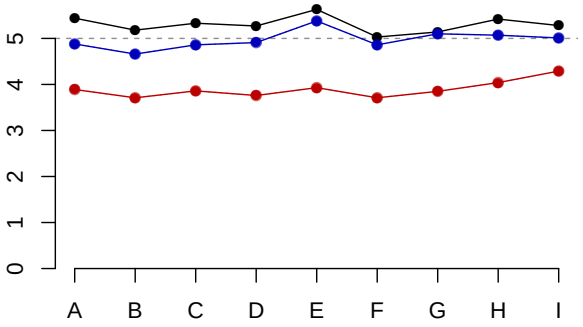
4.4.4 $k = 9$, $k^{DIF} = 2$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	4.99	3.89	5.63	4.99	3.89
B	4.61	3.38	5.10	4.61	3.38
C	4.86	3.79	4.93	4.86	3.79
D	4.87	3.42	5.33	4.87	3.42
E	5.10	3.59	5.23	5.10	3.59
F	5.36	3.83	5.90	5.36	3.83
G	5.25	3.92	5.50	5.25	3.92
H	4.81	3.54	5.46	4.81	3.54
I	5.00	4.00	5.57	5.00	4.00



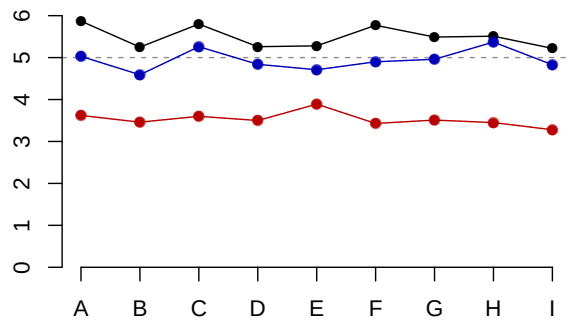
4.4.5 $k = 9$, $k^{DIF} = 5$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	4.88	3.89	5.44	4.88	3.89
B	4.66	3.71	5.18	4.66	3.71
C	4.86	3.86	5.33	4.86	3.86
D	4.91	3.76	5.27	4.91	3.76
E	5.38	3.93	5.63	5.38	3.93
F	4.86	3.71	5.03	4.86	3.71
G	5.10	3.85	5.14	5.10	3.85
H	5.07	4.04	5.42	5.07	4.04
I	5.01	4.29	5.28	5.01	4.29



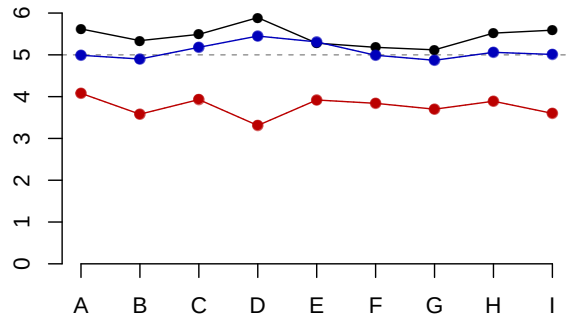
4.4.6 $k = 17$, $k^{DIF} = 1$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	5.03	3.62	5.87	5.03	3.62
B	4.59	3.46	5.25	4.59	3.46
C	5.26	3.60	5.80	5.26	3.60
D	4.84	3.50	5.26	4.84	3.50
E	4.71	3.89	5.28	4.71	3.89
F	4.90	3.43	5.77	4.90	3.43
G	4.96	3.51	5.49	4.96	3.51
H	5.37	3.45	5.51	5.37	3.45
I	4.83	3.28	5.22	4.83	3.28



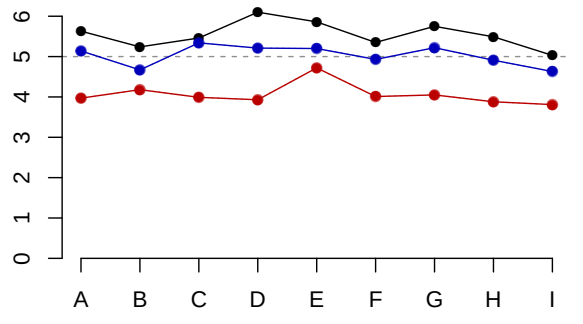
4.4.7 $k = 17$, $k^{DIF} = 2$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	4.99	4.08	5.62	4.99	4.08
B	4.90	3.58	5.34	4.90	3.58
C	5.18	3.93	5.49	5.18	3.93
D	5.45	3.31	5.89	5.45	3.31
E	5.31	3.92	5.28	5.31	3.92
F	4.99	3.84	5.18	4.99	3.84
G	4.87	3.70	5.12	4.87	3.70
H	5.06	3.89	5.52	5.06	3.89
I	5.01	3.60	5.59	5.01	3.60



4.4.8 $k = 17$, $k^{DIF} = 5$

	T_{10}	T_4	LRT	$T_{10}^{\star}$	$T_4^{\star}$
A	5.14	3.97	5.63	5.14	3.97
B	4.67	4.18	5.24	4.67	4.18
C	5.34	3.99	5.46	5.34	3.99
D	5.21	3.93	6.10	5.21	3.93
E	5.20	4.72	5.86	5.20	4.72
F	4.93	4.01	5.35	4.93	4.01
G	5.22	4.05	5.75	5.22	4.05
H	4.91	3.88	5.49	4.91	3.88
I	4.63	3.81	5.03	4.63	3.81

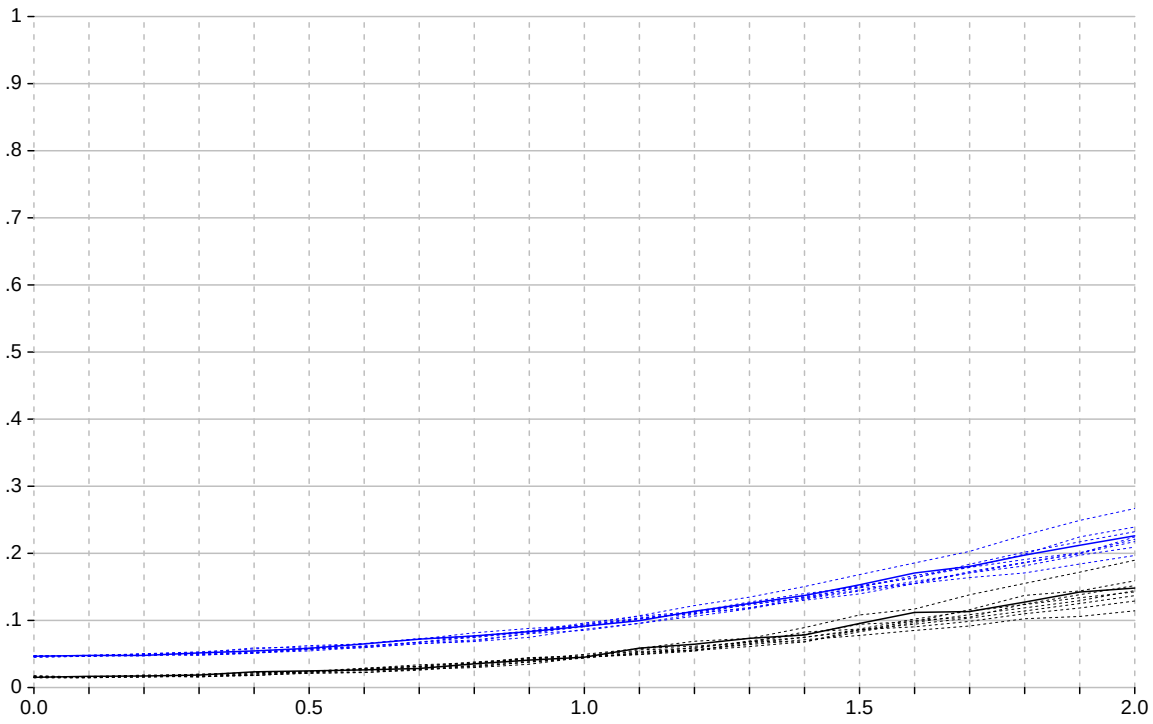


5 Observed Power: LRT vs. T_{10}

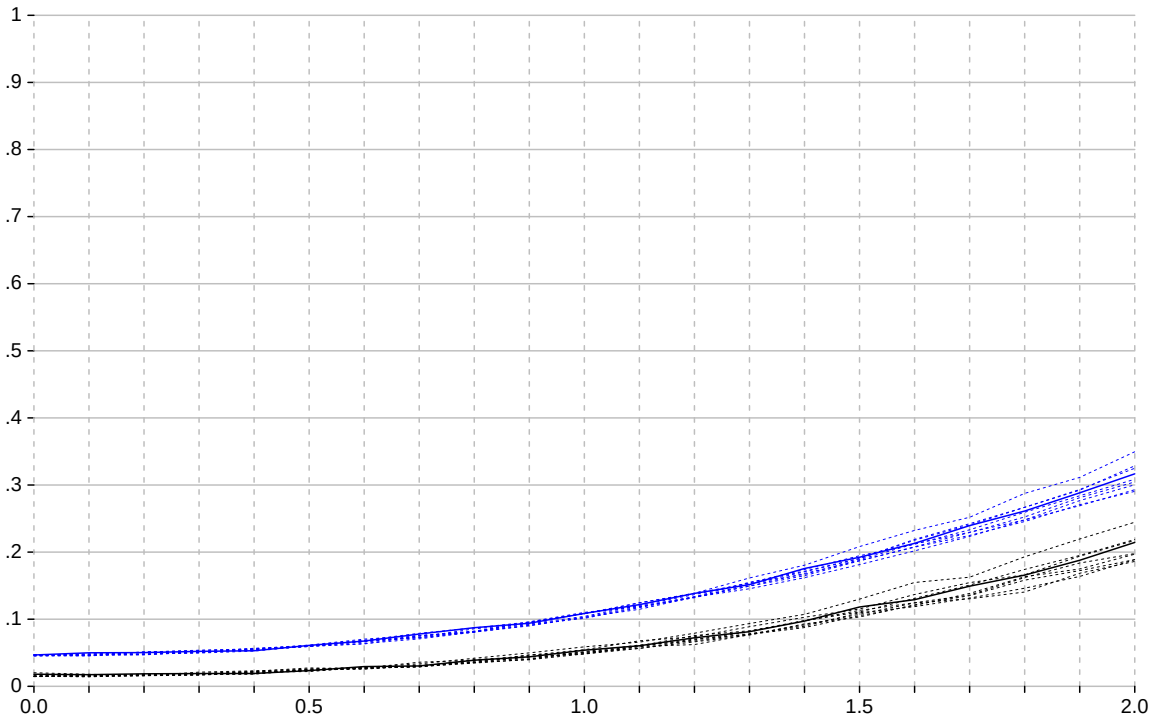
This section contains the observed power for all combinations of n, k, and k^{DIF} for the LRT and the T_{10} statistic. Although all distributional scenarios are plotted, scenario A with identical normal distributions in both groups are the solid lines, all other scenarios are displayed as dashed lines. On the x-axis, the magnitude of violation from 0 to 2 logits is plotted against the proportion of significant test results ($\alpha = .05$) on the y-axis. Black lines are the results of the LRT and blue lines stand for the performance of T_{10}.

5.1 $n = 30$

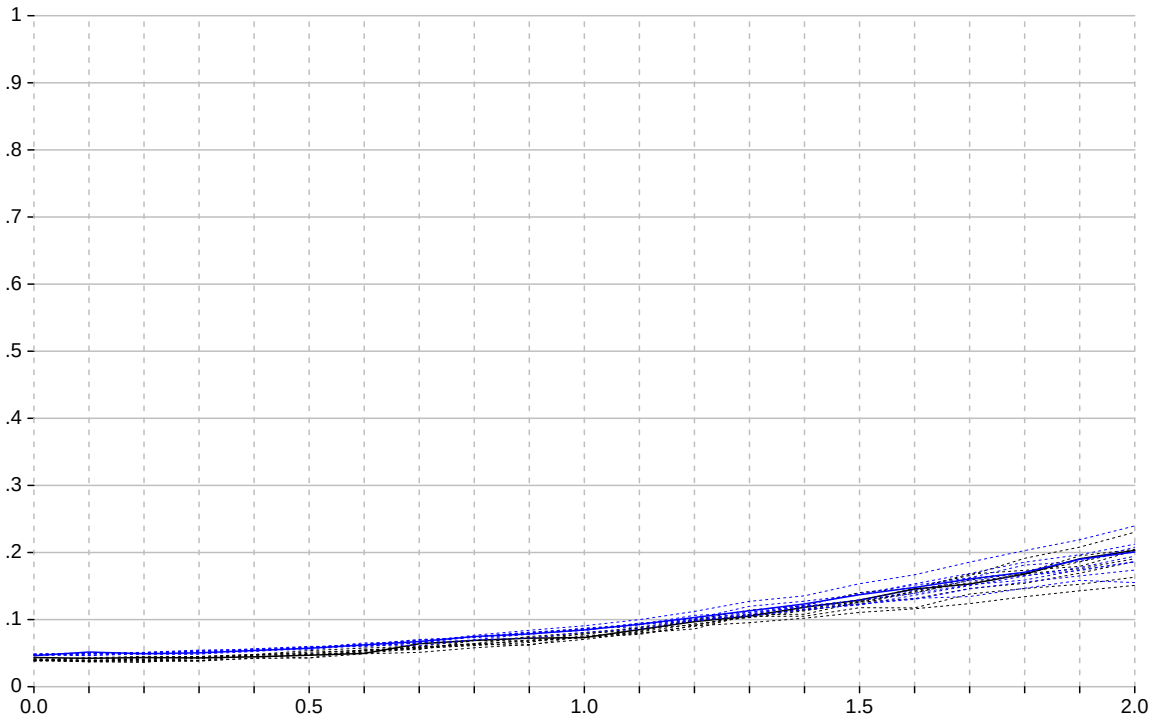
5.1.1 $k = 5$, $k^{DIF} = 1$



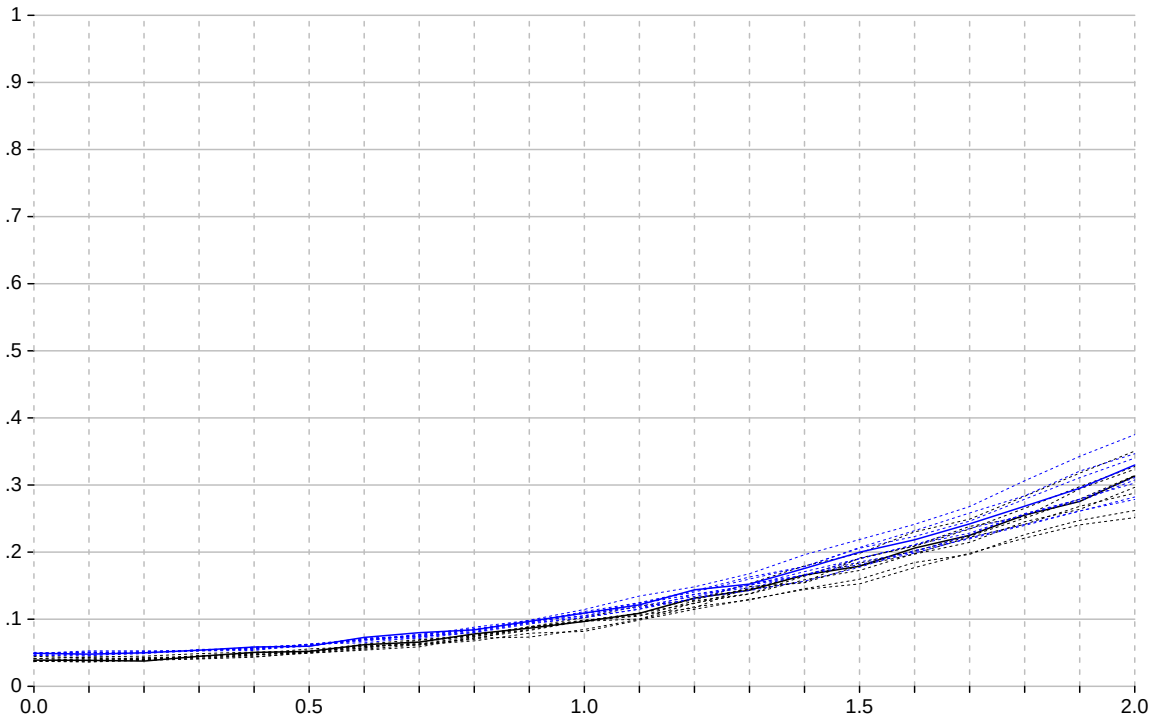
5.1.2 $k = 5,\ k^{DIF} = 2$



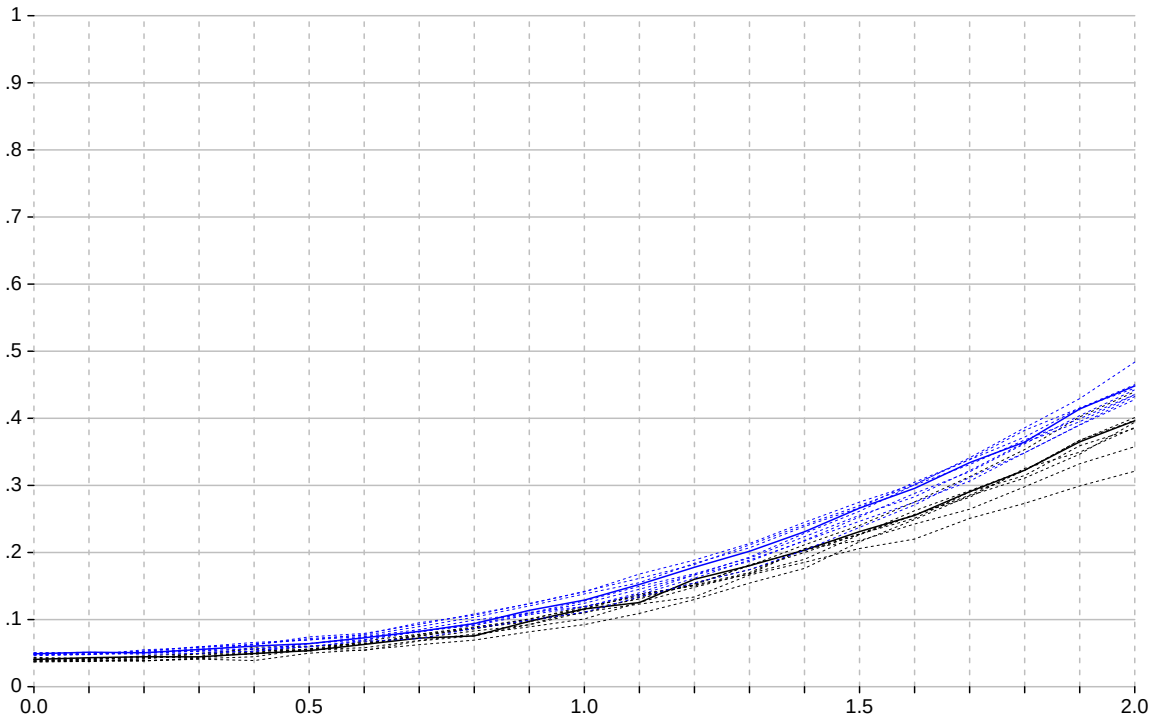
5.1.3 $k = 9,\ k^{DIF} = 1$



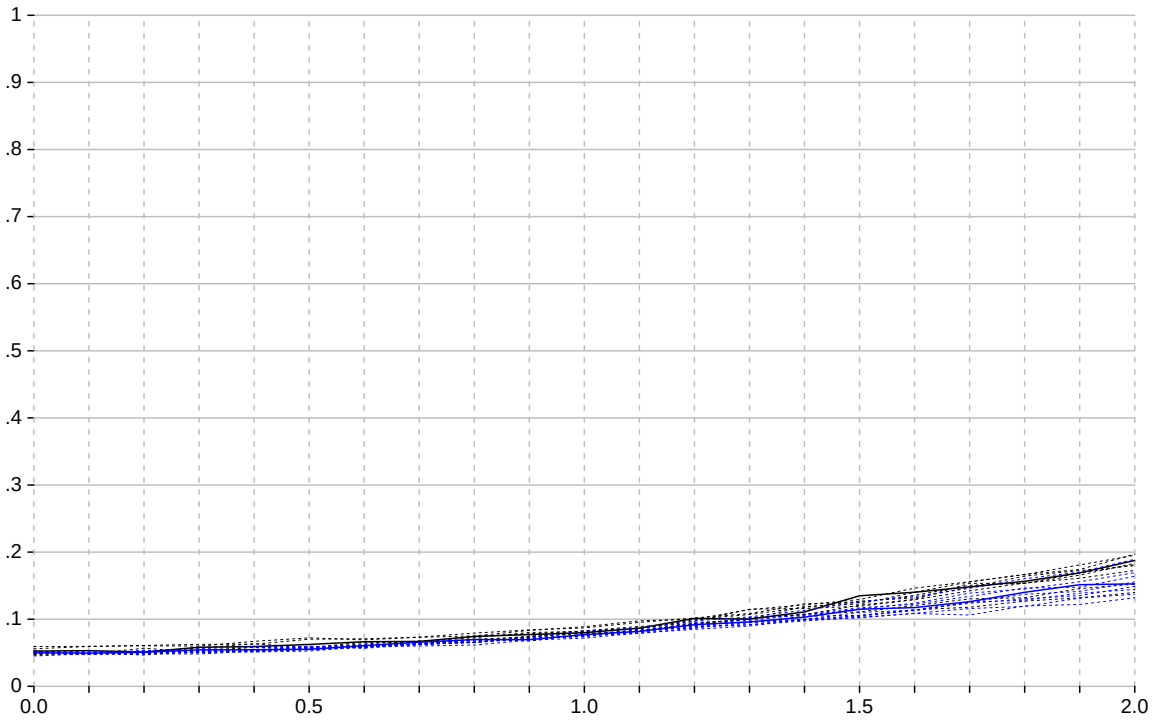
5.1.4 $k = 9,\ k^{DIF} = 2$



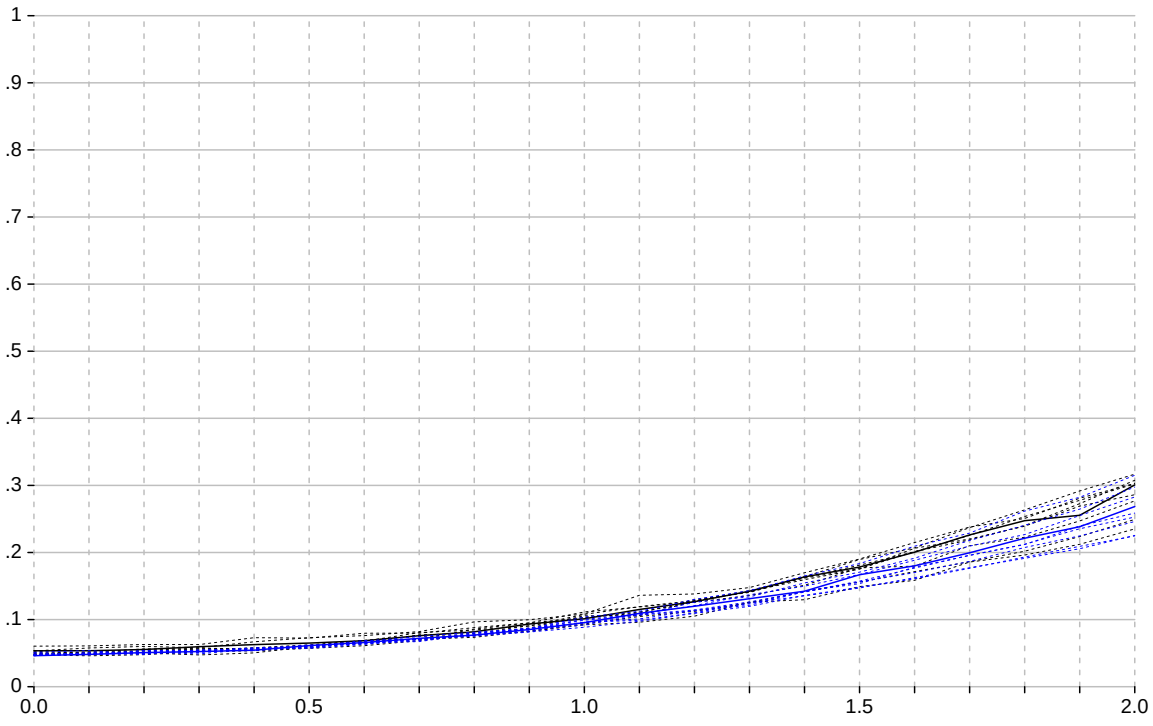
5.1.5 $k = 9,\ k^{DIF} = 5$



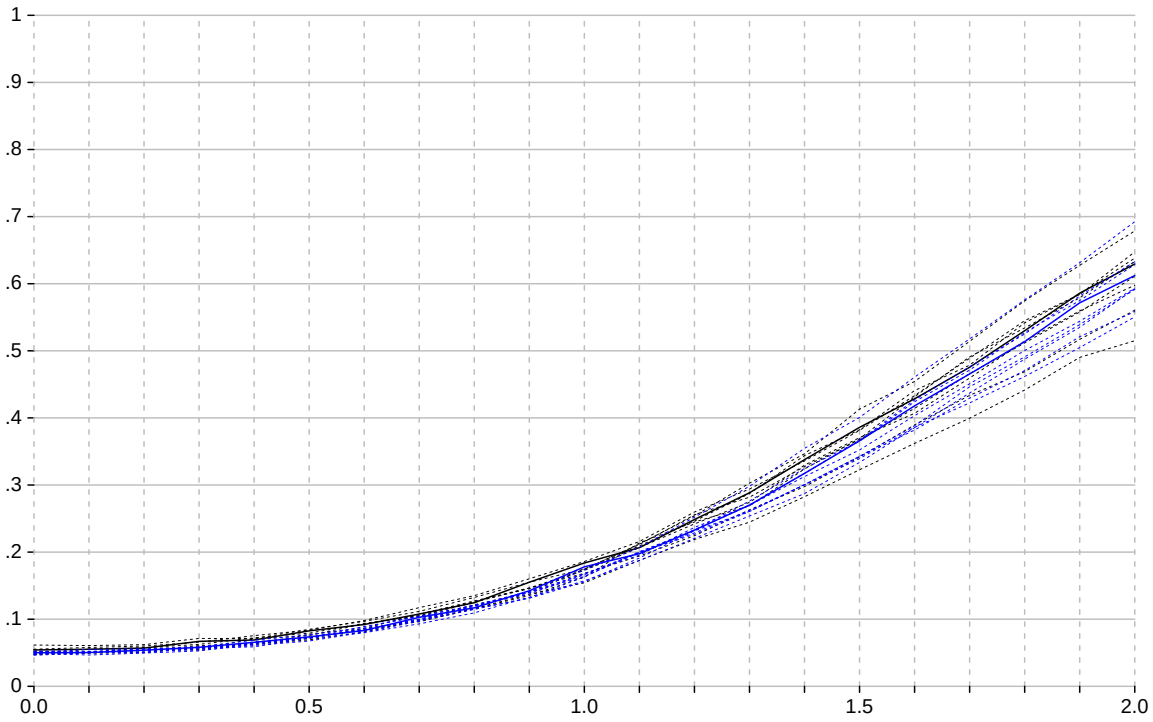
5.1.6 $k = 17,\ k^{DIF} = 1$



5.1.7 $k = 17,\ k^{DIF} = 2$

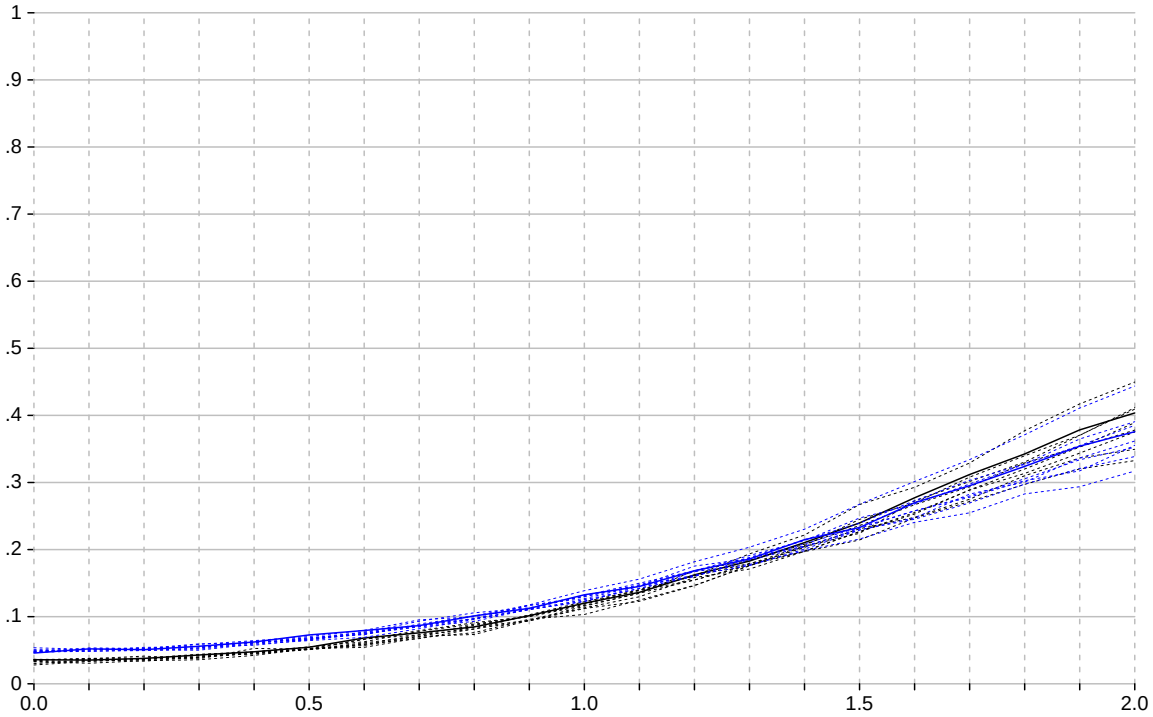


5.1.8 $k = 17,\ k^{DIF} = 5$

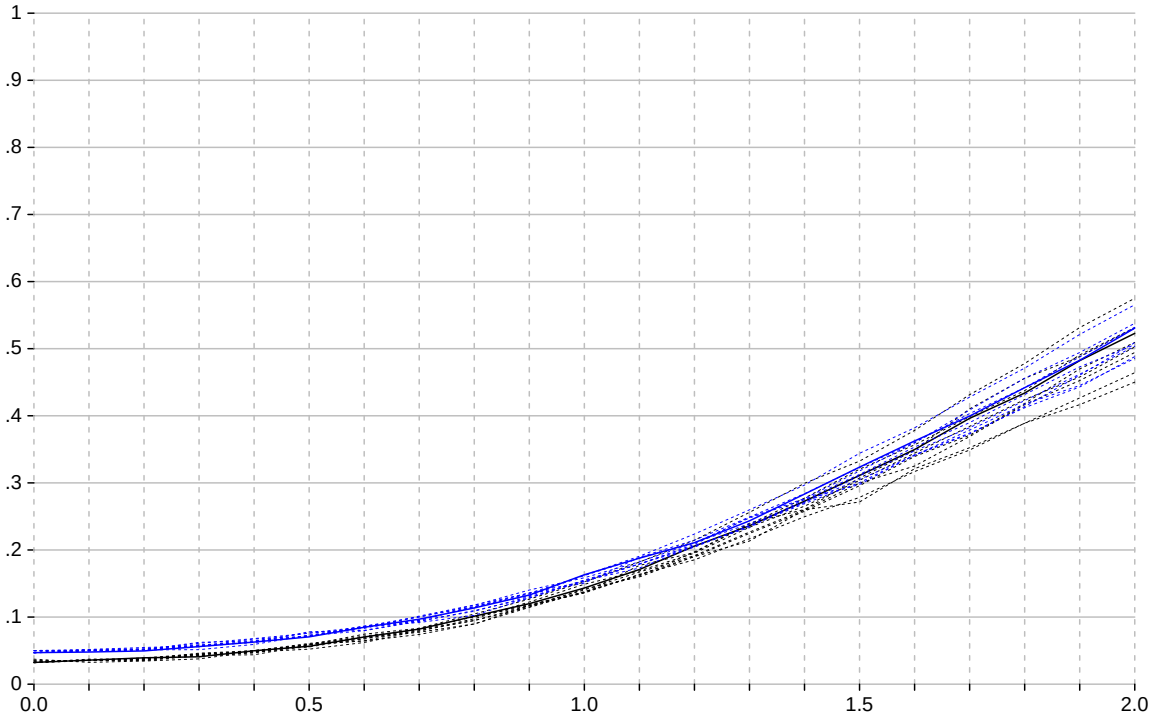


5.2 $n = 50$

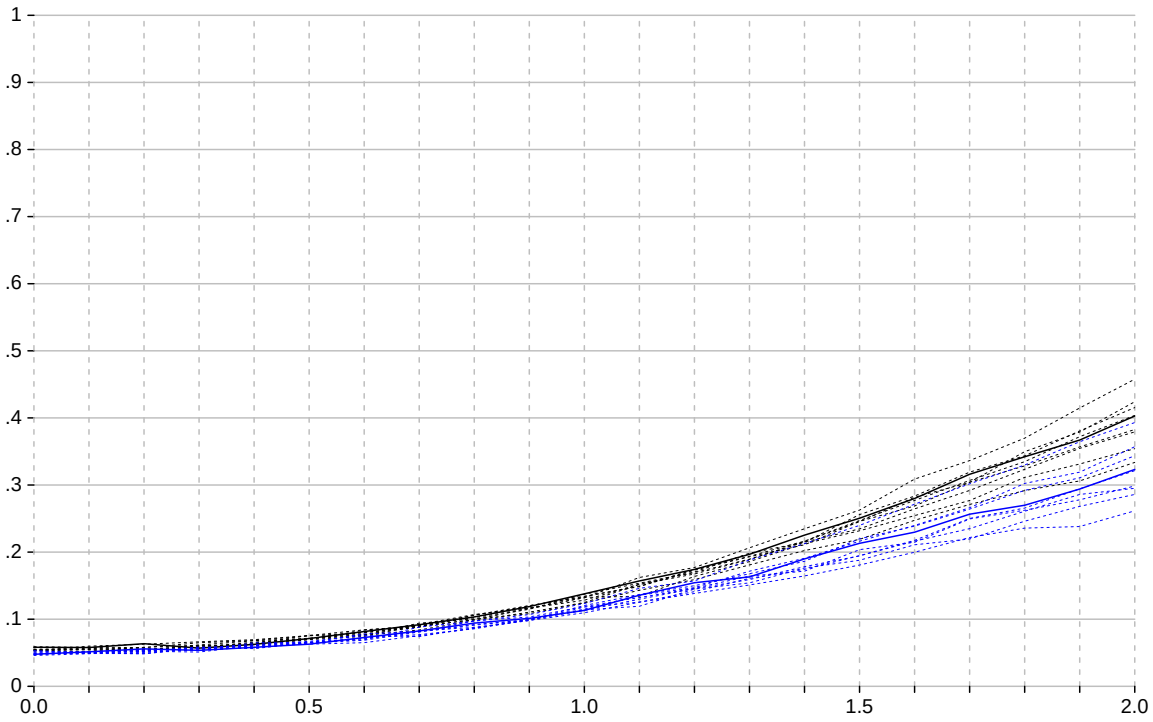
5.2.1 $k = 5,\ k^{DIF} = 1$



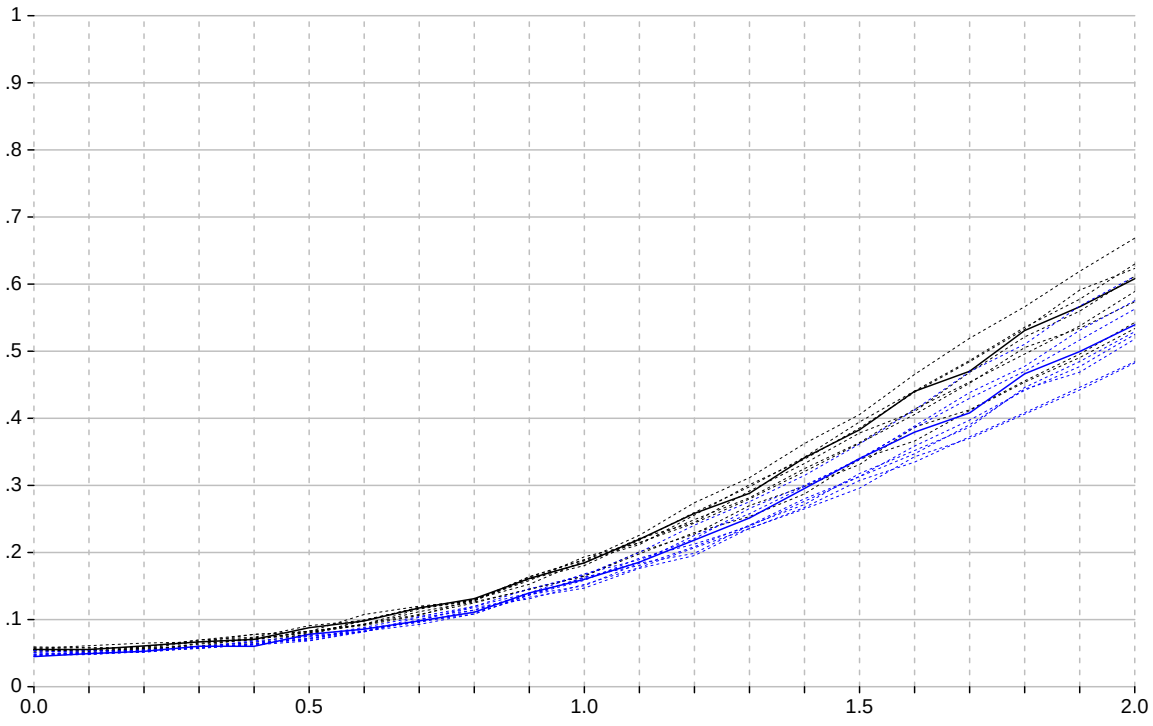
5.2.2 $k = 5,\ k^{DIF} = 2$



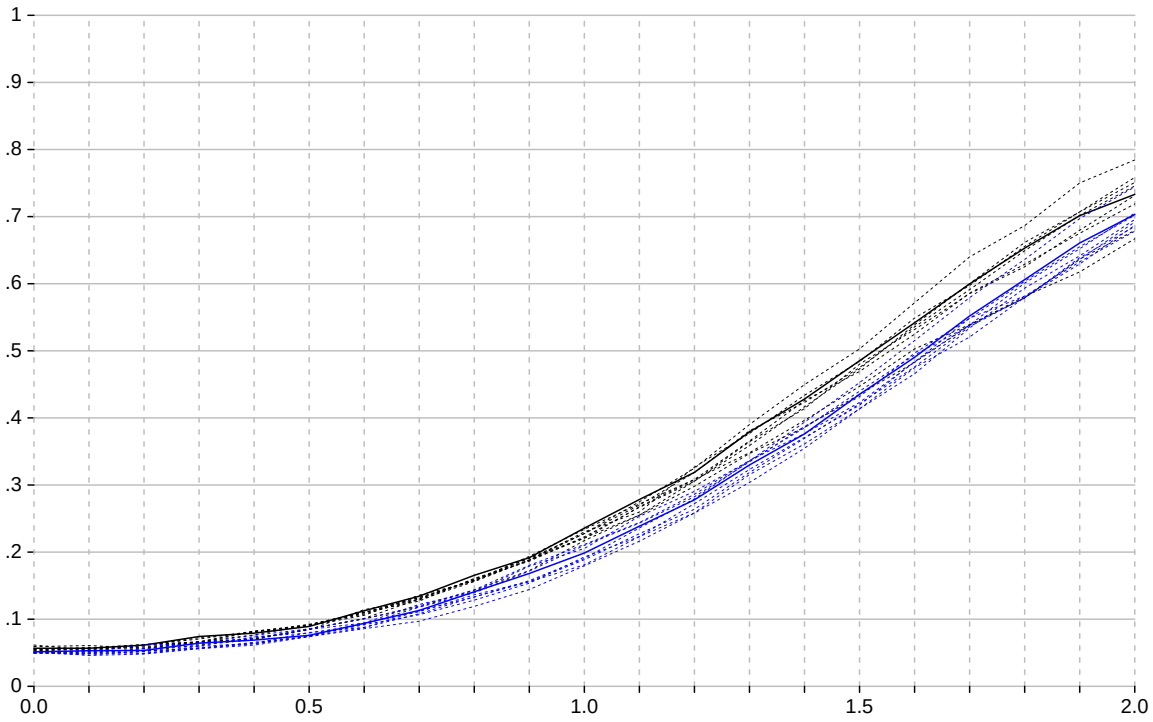
5.2.3 $k = 9,\ k^{DIF} = 1$



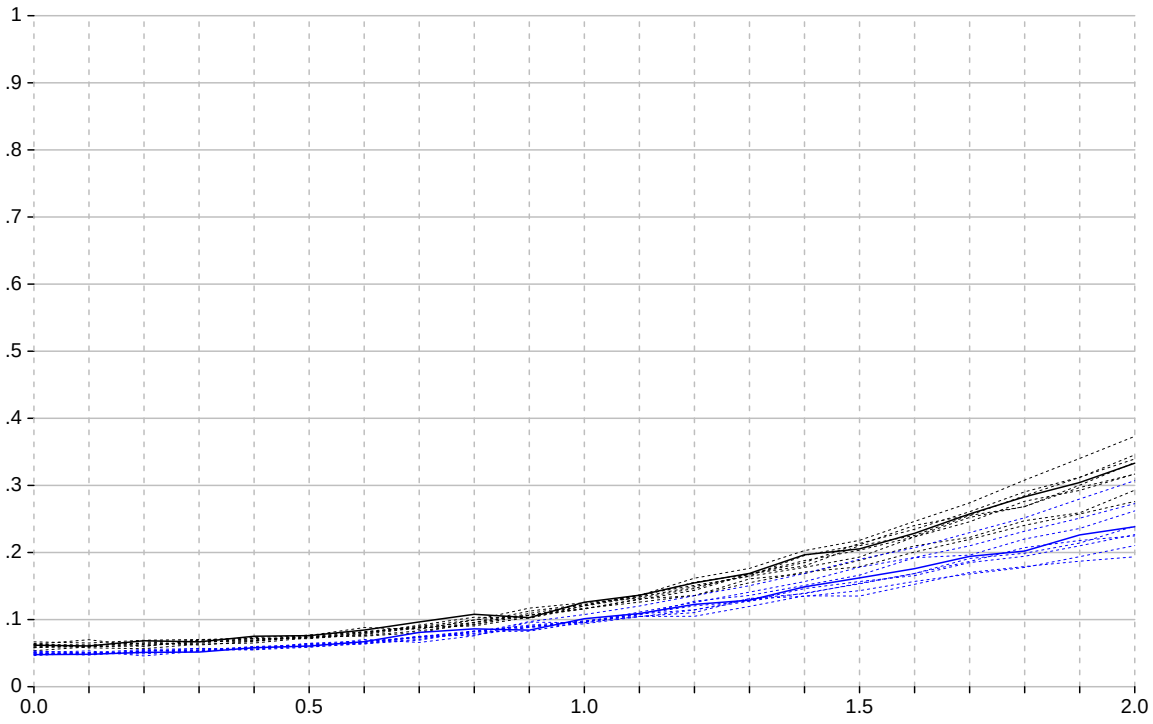
5.2.4 $k = 9,\ k^{DIF} = 2$



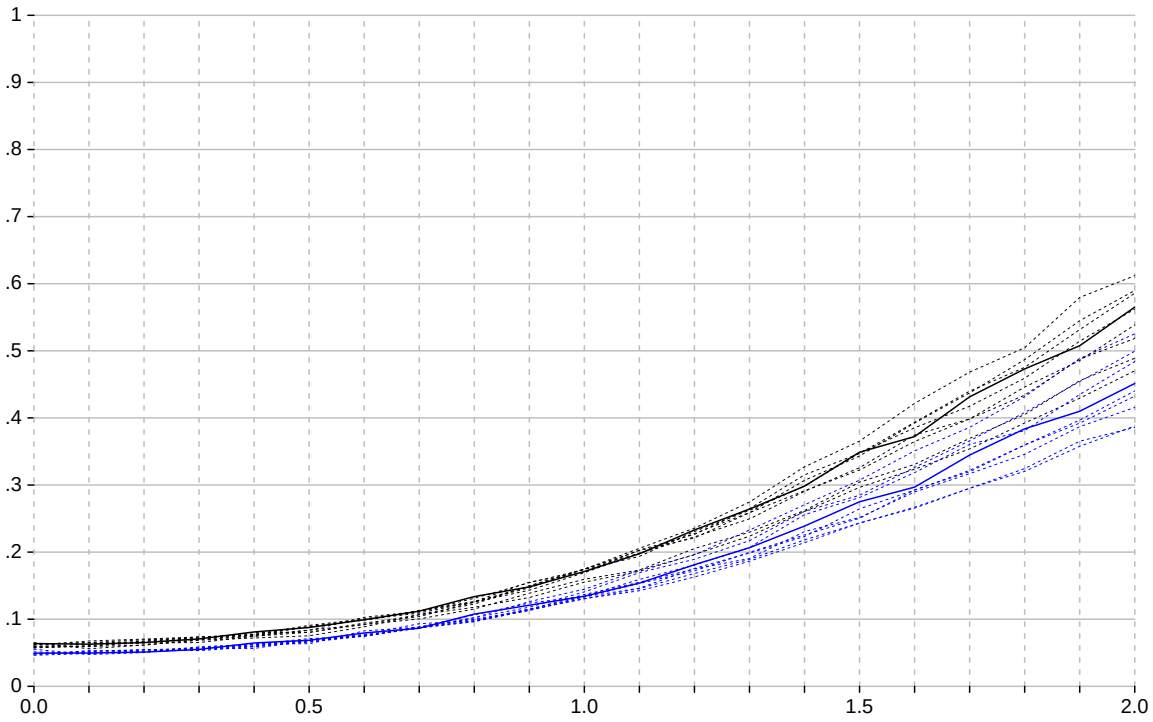
5.2.5 $k = 9,\ k^{DIF} = 5$



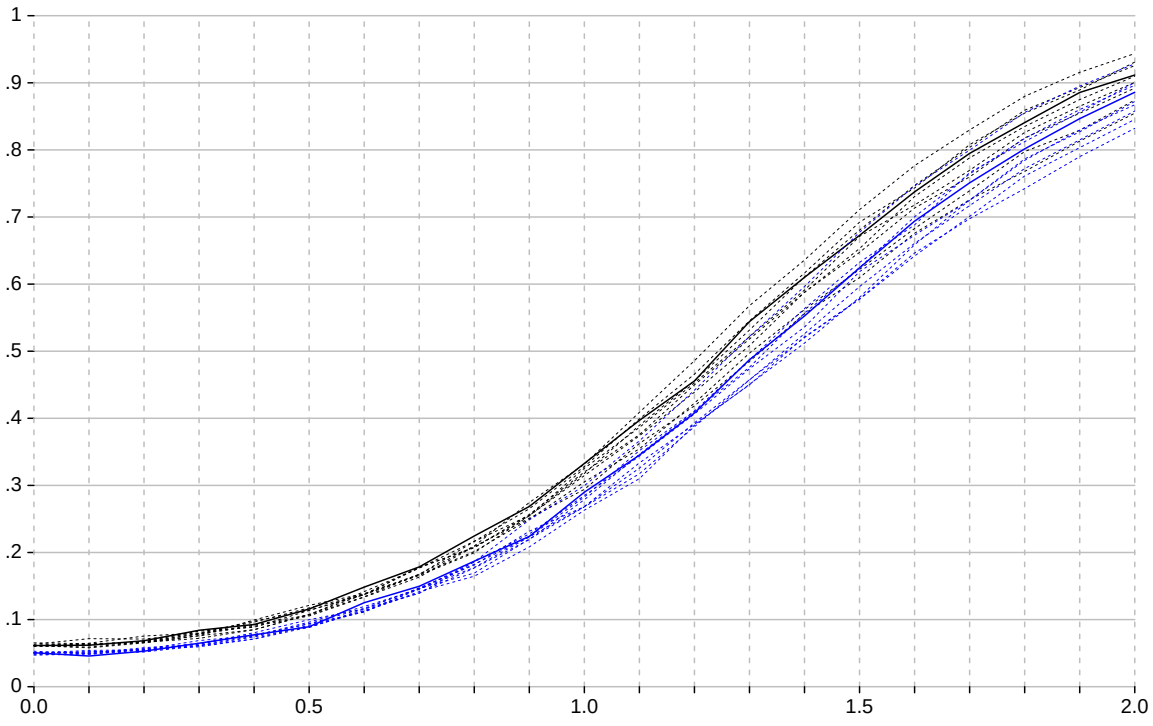
5.2.6 $k = 17,\ k^{DIF} = 1$



5.2.7 $k = 17,\ k^{DIF} = 2$

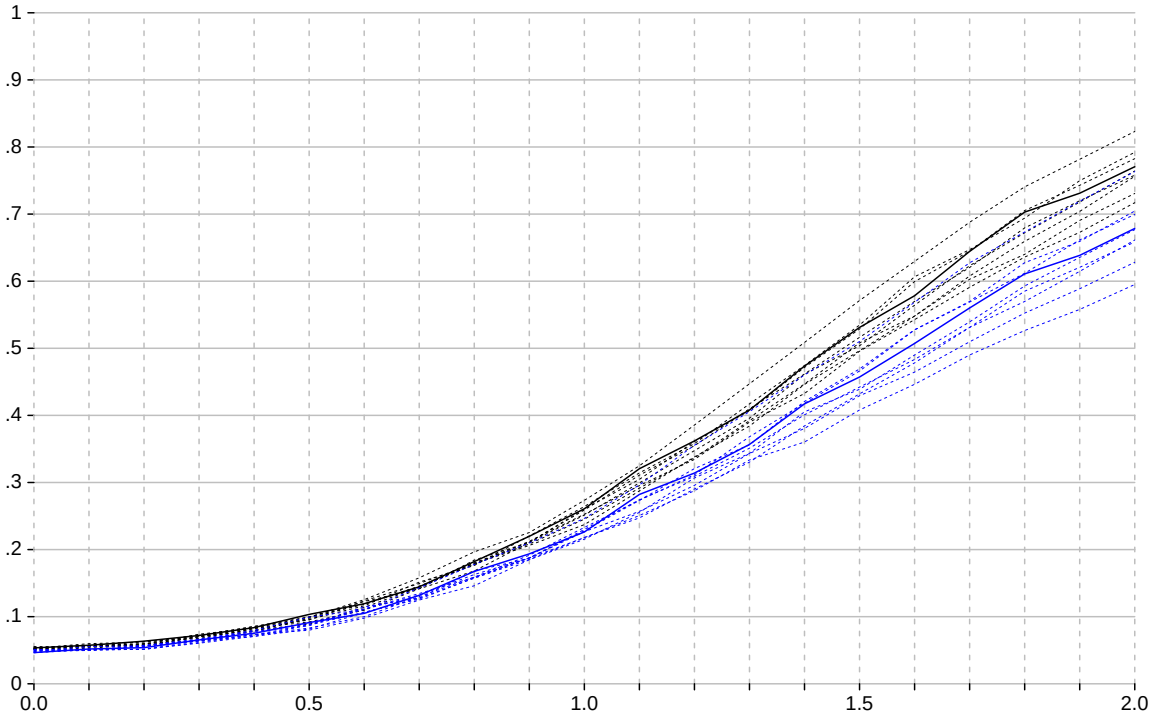


5.2.8 $k = 17,\ k^{DIF} = 5$

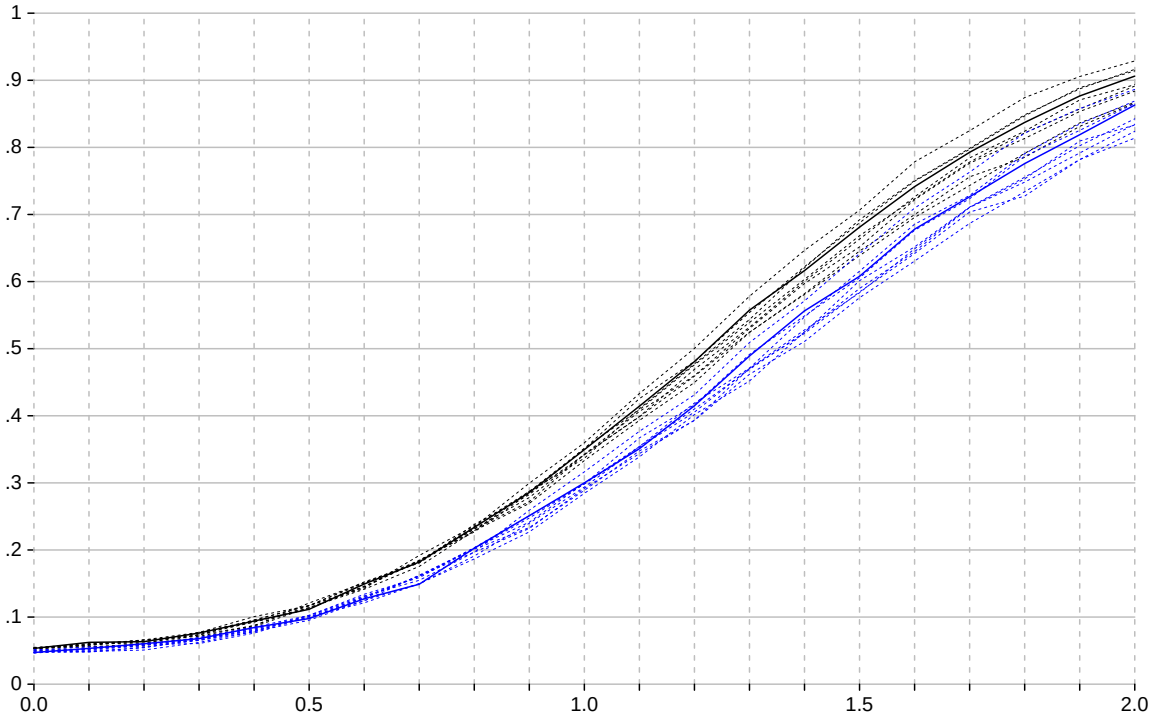


5.3 $n = 100$

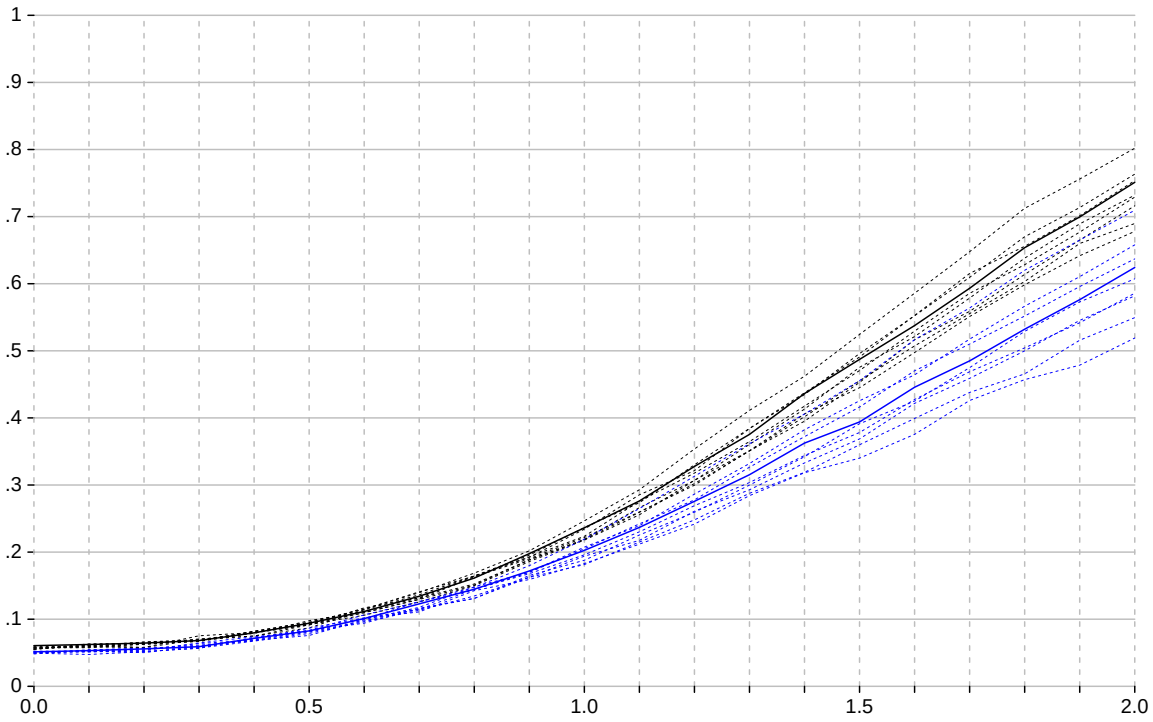
5.3.1 $k = 5,\ k^{DIF} = 1$



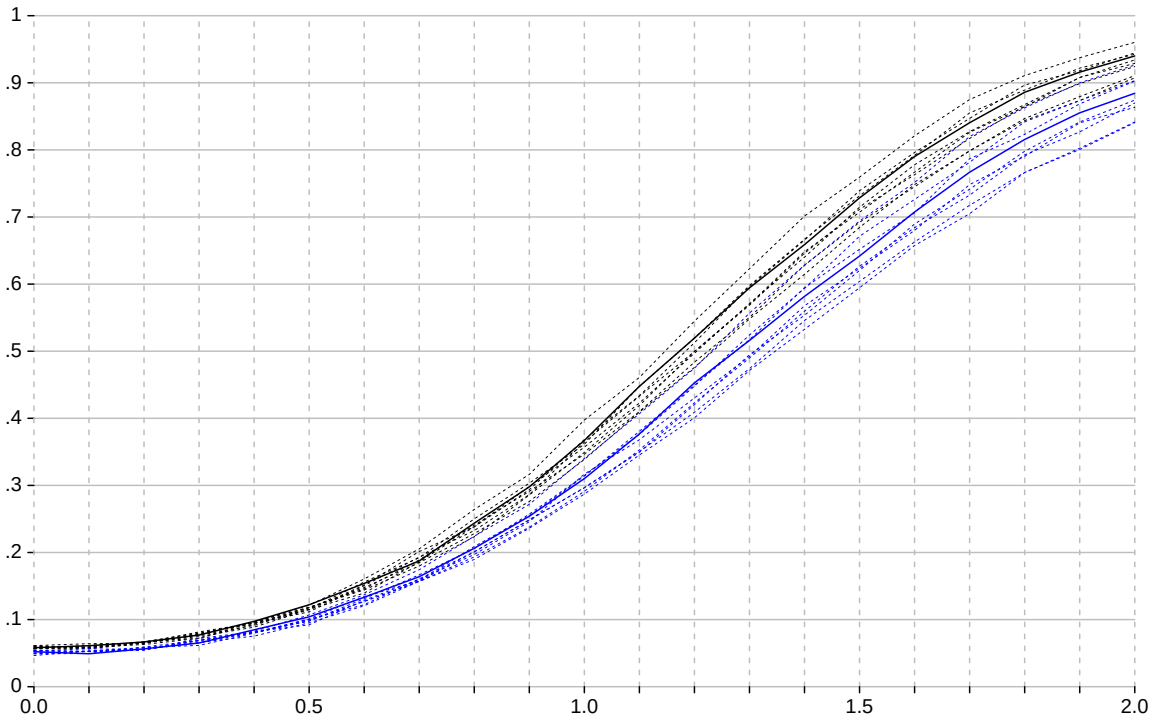
5.3.2 $k = 5,\ k^{DIF} = 2$



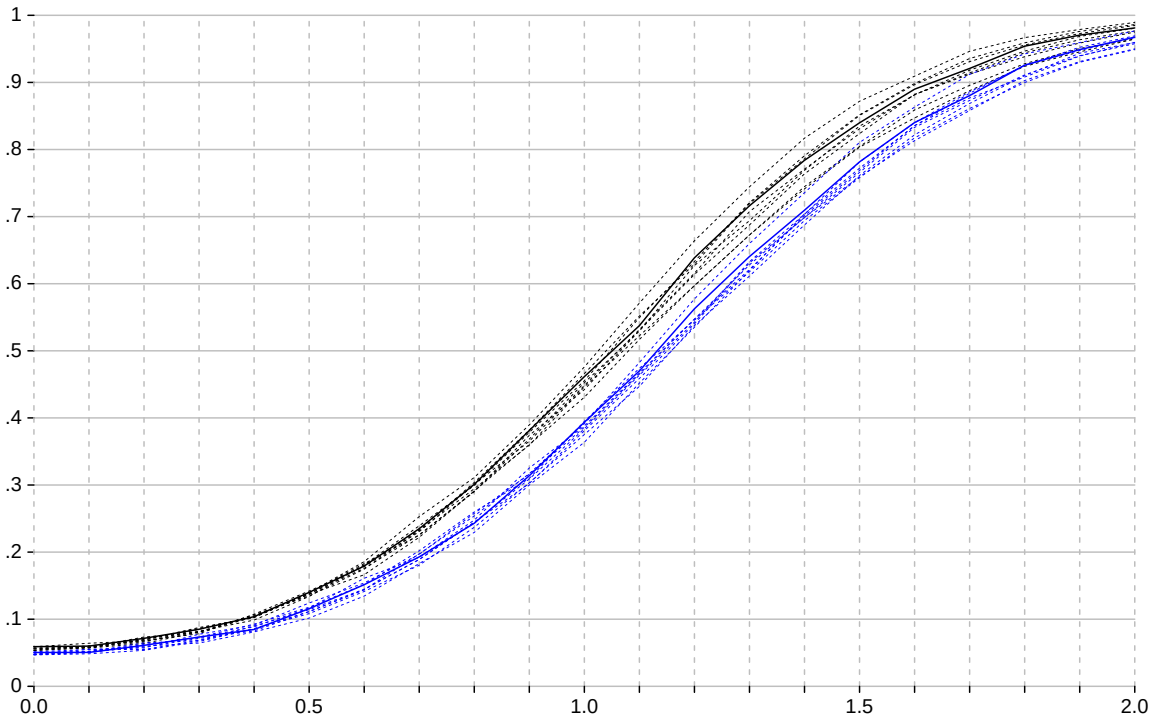
5.3.3 $k = 9,\ k^{DIF} = 1$



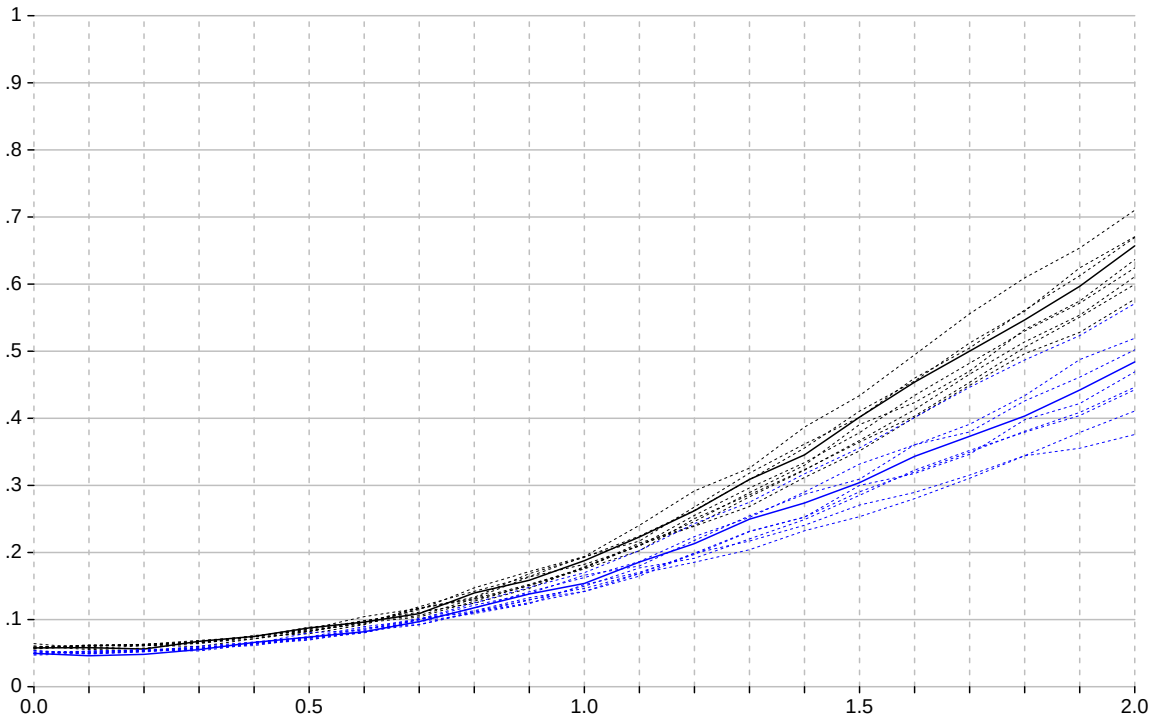
5.3.4 $k = 9,\ k^{DIF} = 2$



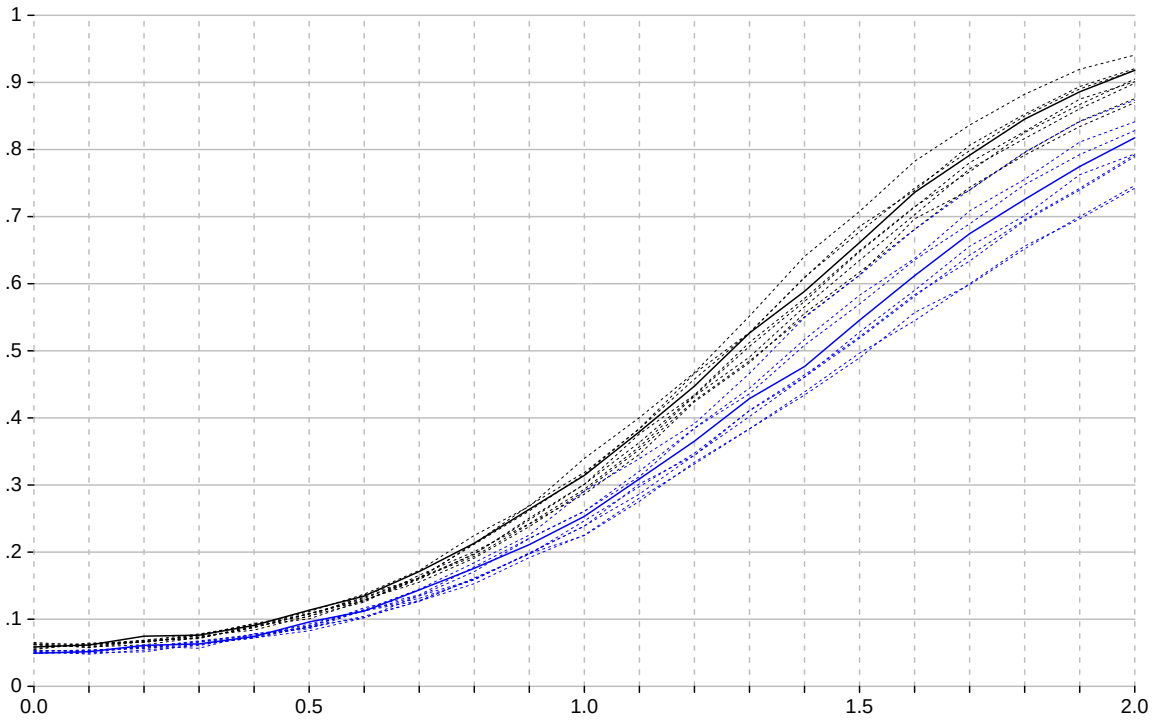
5.3.5 $k = 9,\ k^{DIF} = 5$



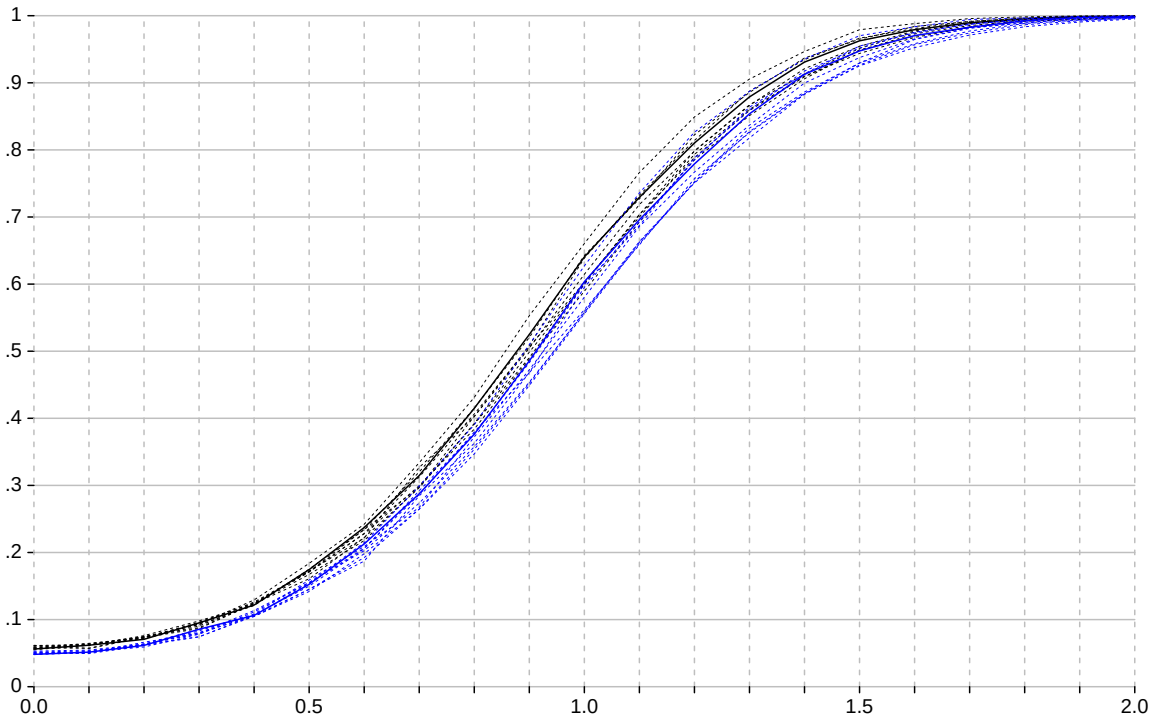
5.3.6 $k = 17,\ k^{DIF} = 1$



5.3.7 $k = 17,\ k^{DIF} = 2$

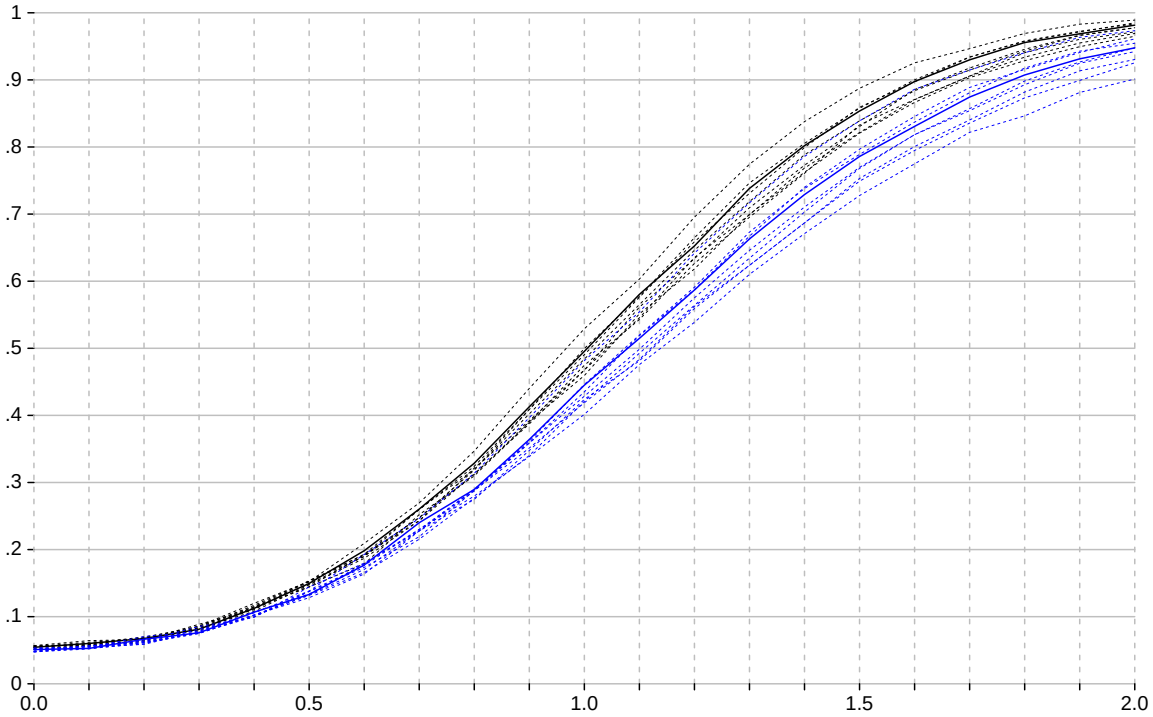


5.3.8 $k = 17,\ k^{DIF} = 5$

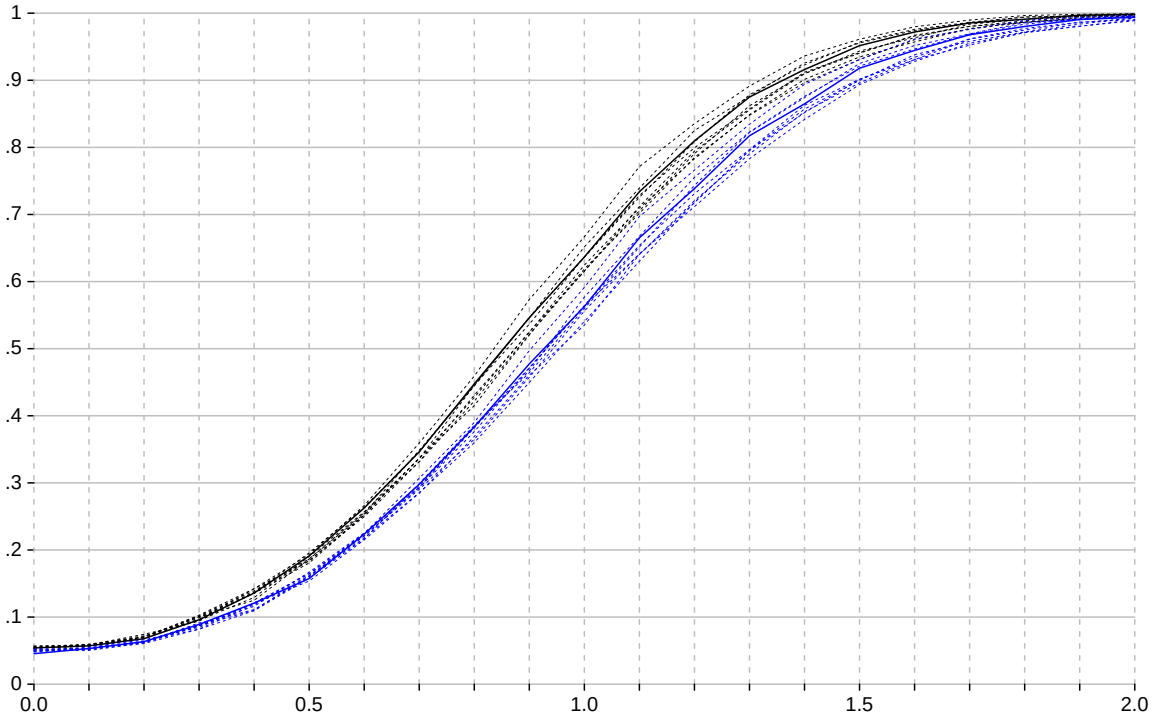


5.4 $n = 200$

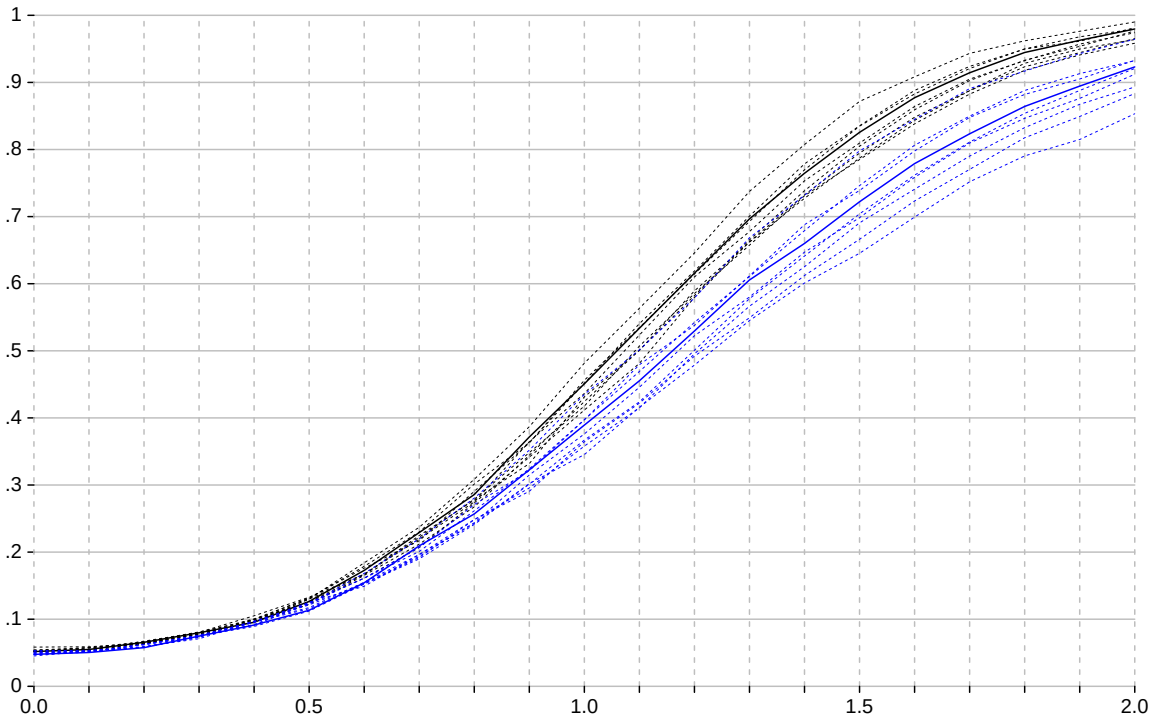
5.4.1 $k = 5,\ k^{DIF} = 1$



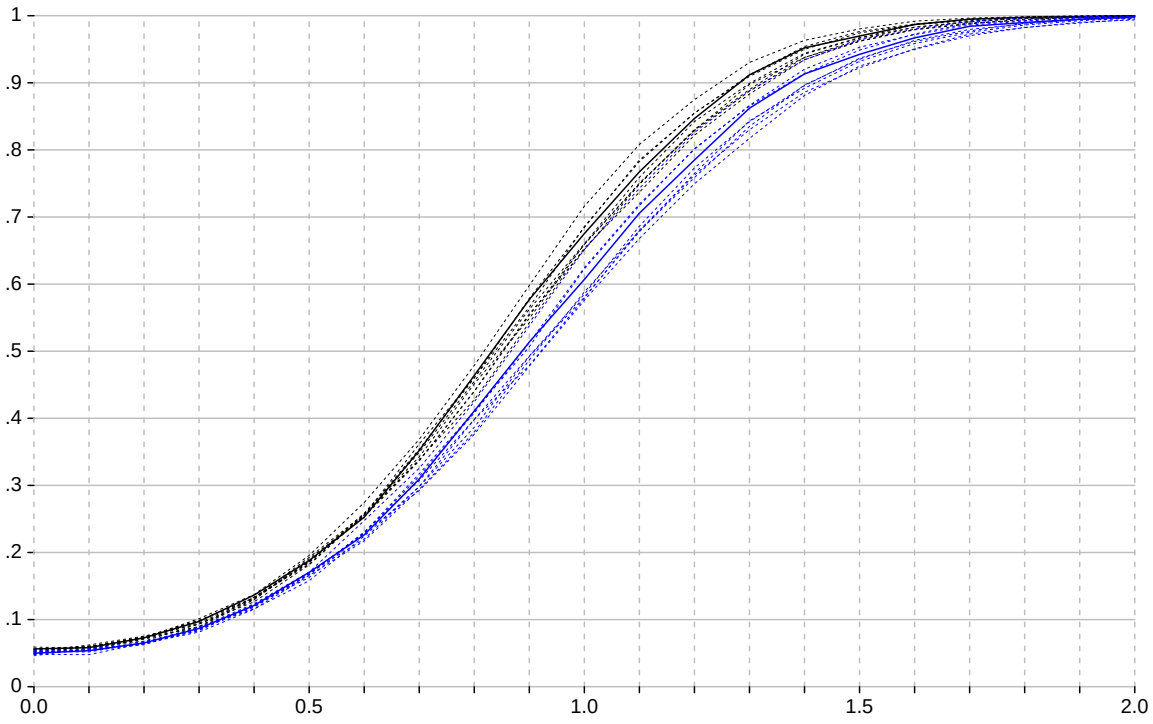
5.4.2 $k = 5,\ k^{DIF} = 2$



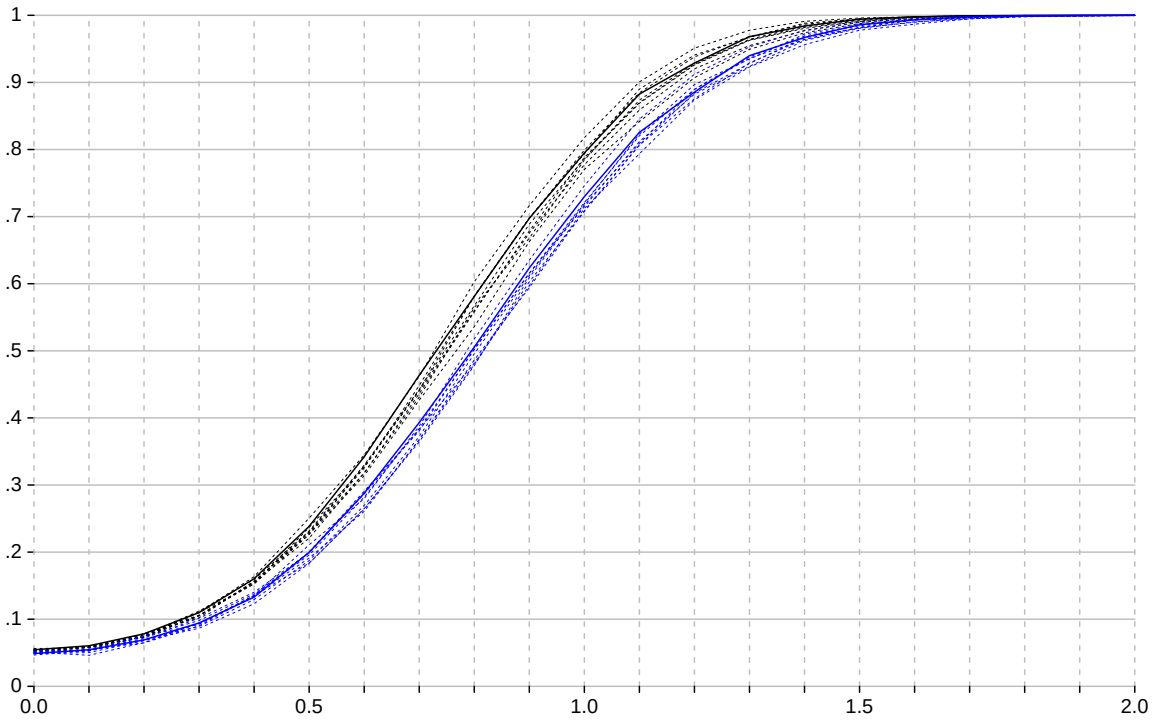
5.4.3 $k = 9,\ k^{DIF} = 1$



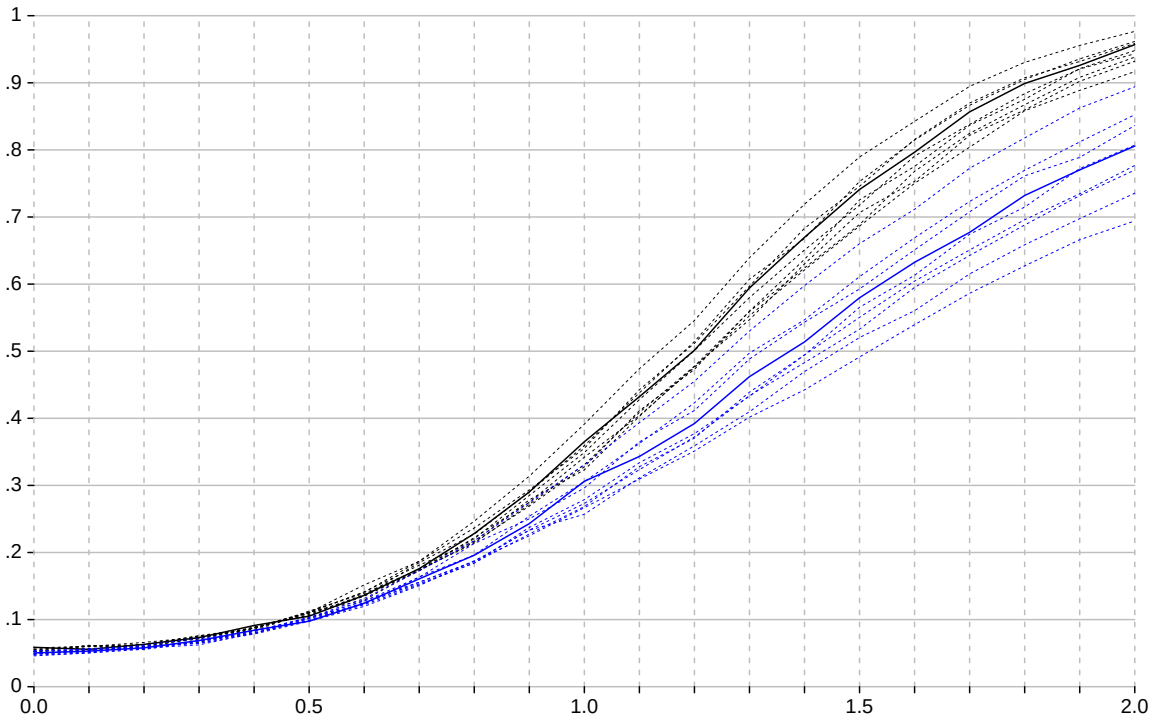
5.4.4 $k = 9,\ k^{DIF} = 2$



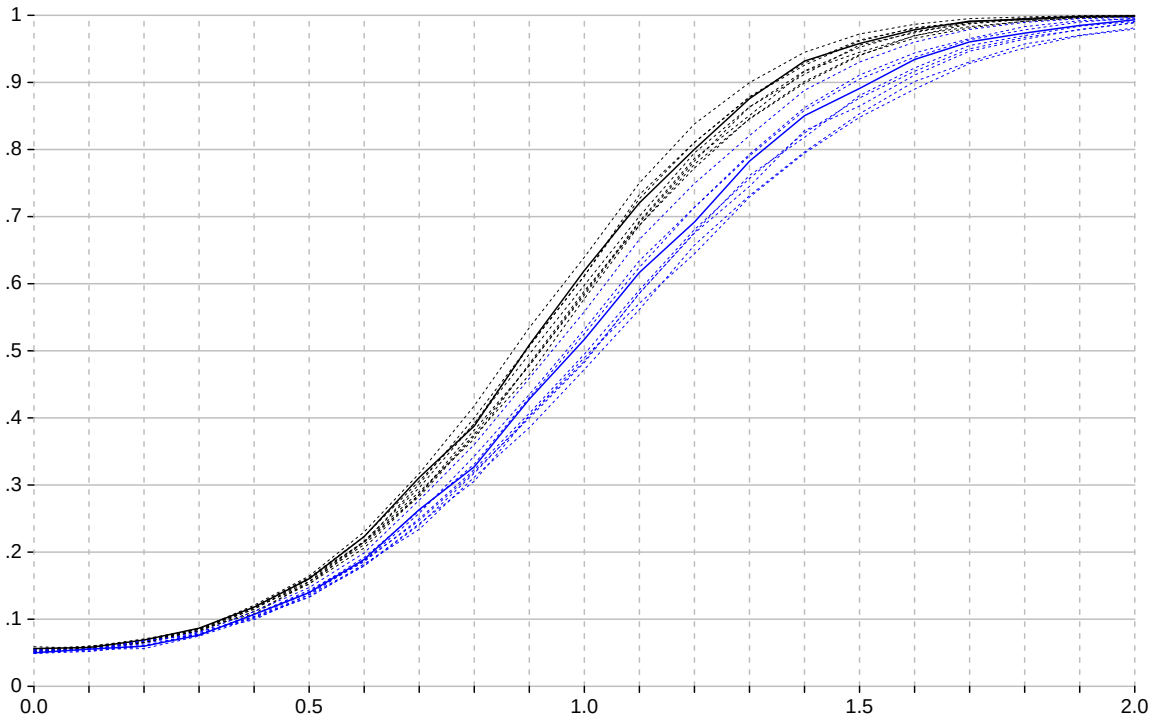
5.4.5 $k = 9,\ k^{DIF} = 5$



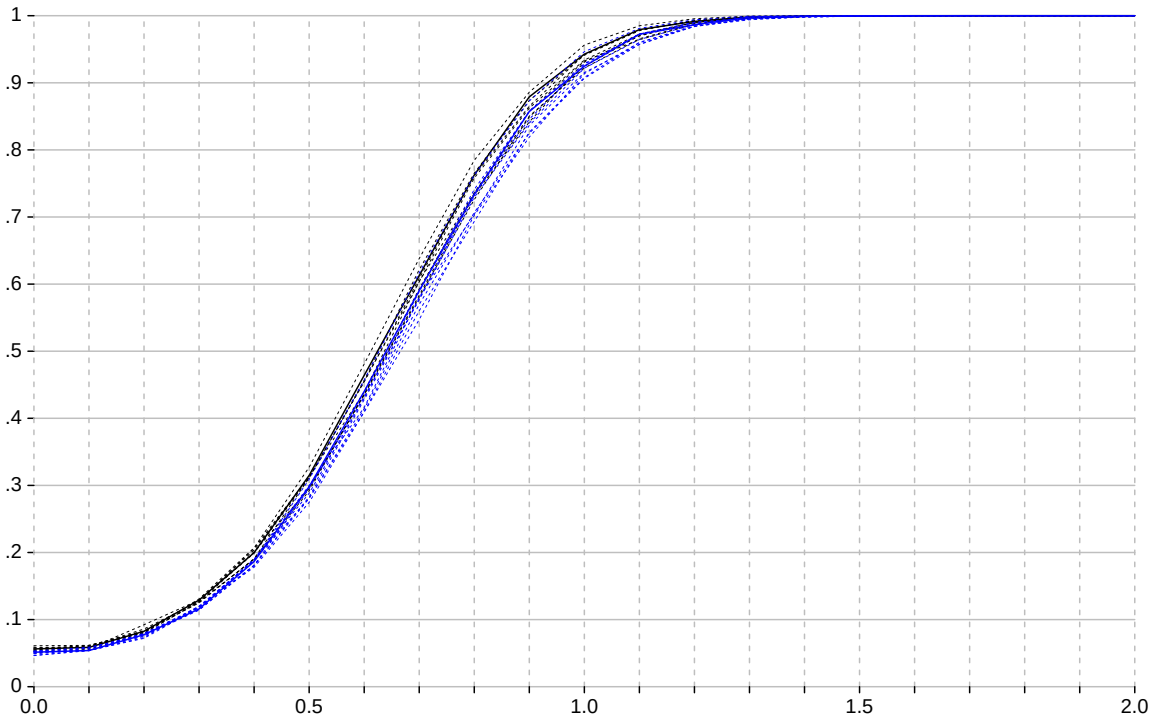
5.4.6 $k = 17,\ k^{DIF} = 1$



5.4.7 $k = 17,\ k^{DIF} = 2$



5.4.8 $k = 17,\ k^{DIF} = 5$

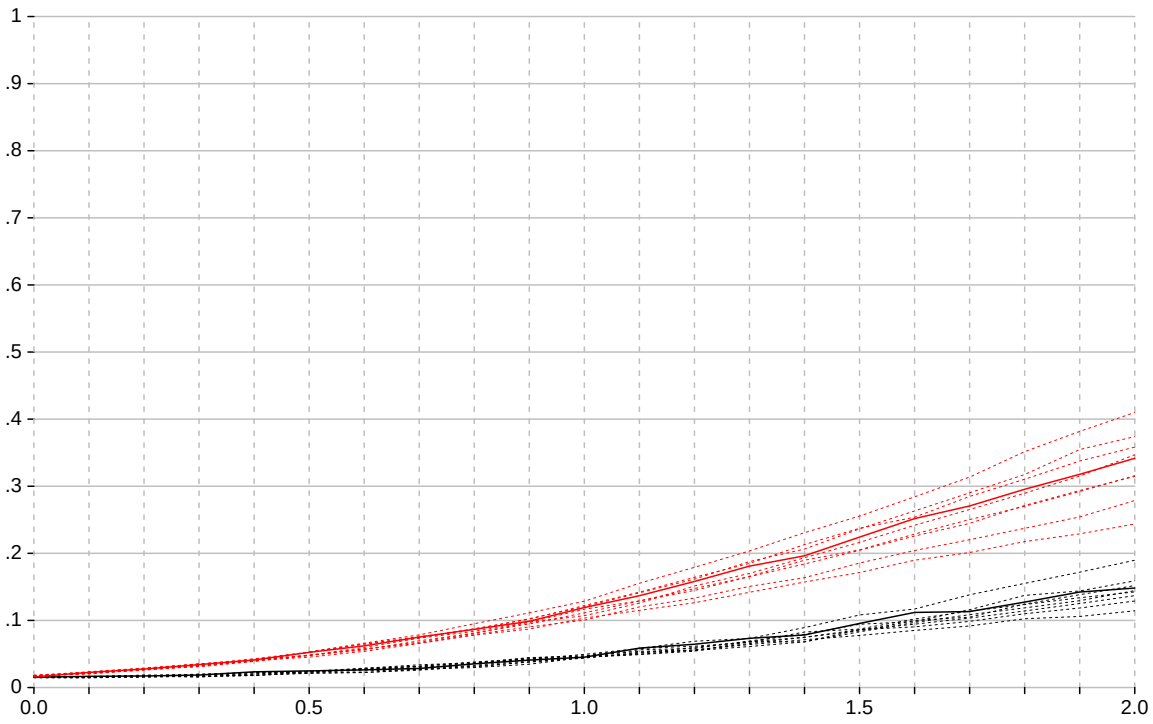


6 Observed Power: LRT vs. T_4

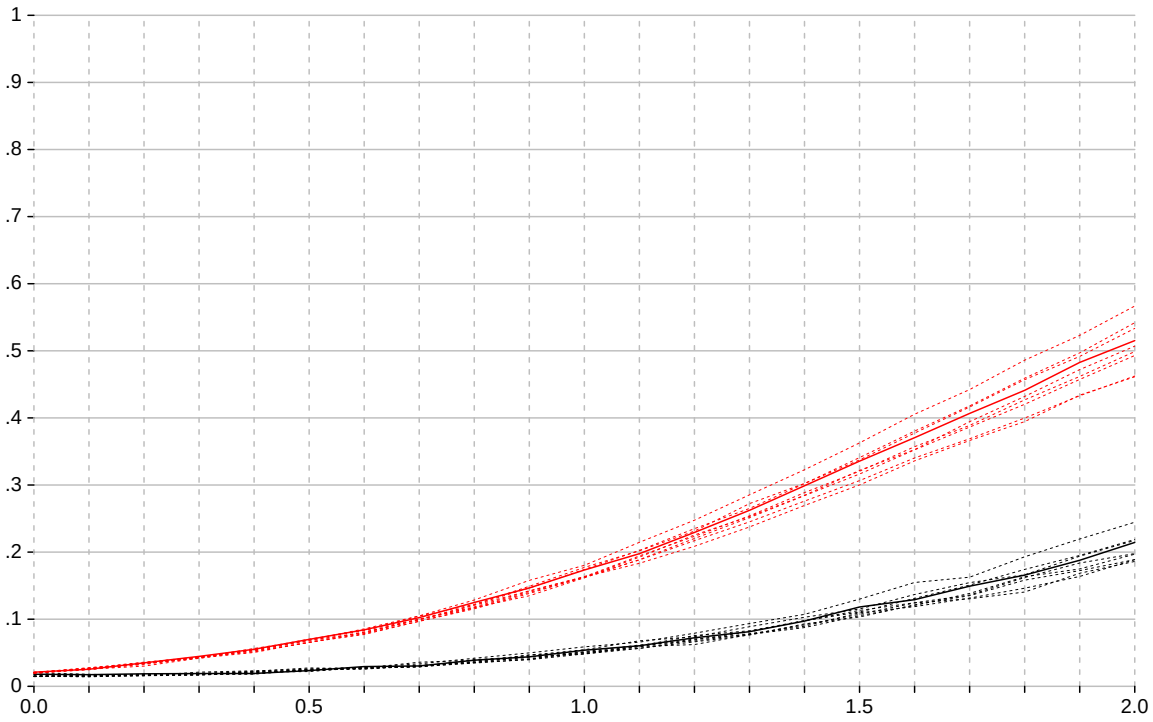
In this section, the observed power for all combinations of n, k, and k^{DIF} for the T_4 statistic (red lines) are displayed with the results of the LRT (black lines) to have a rough guideline against a global test statistic. For each test the distribution assumptions' results are plotted as separate lines where all are dashed except scenario A (both groups from an identical normal distribution). The x-axis shows the magnitude of DIF introduced from 0 to 2 logits and on the y-axis, the proportion of significant test results ($\alpha = .05$) is displayed.

6.1 $n = 30$

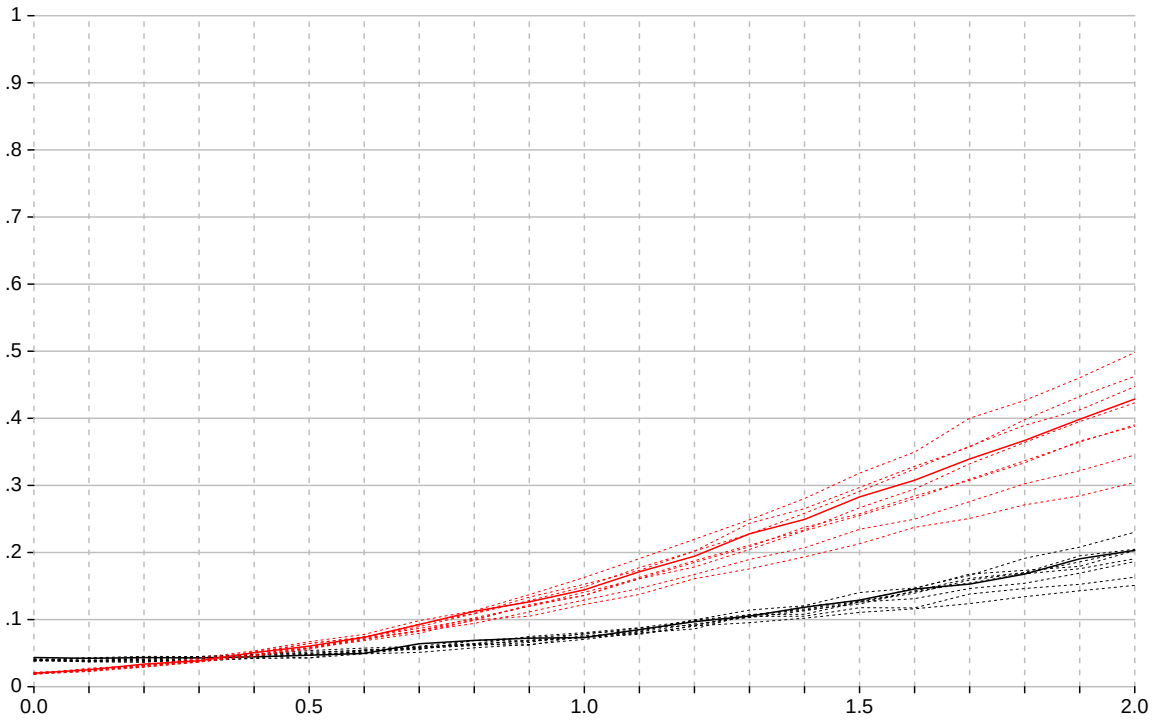
6.1.1 $k = 5$, $k^{DIF} = 1$



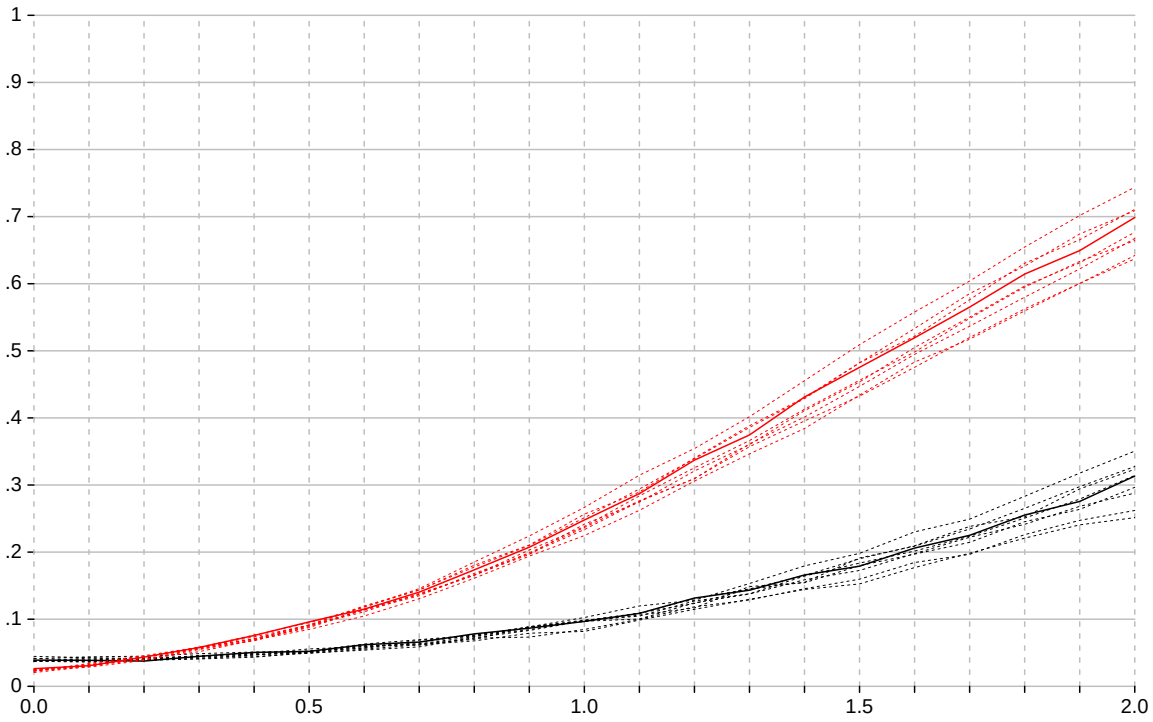
6.1.2 $k = 5,\ k^{DIF} = 2$



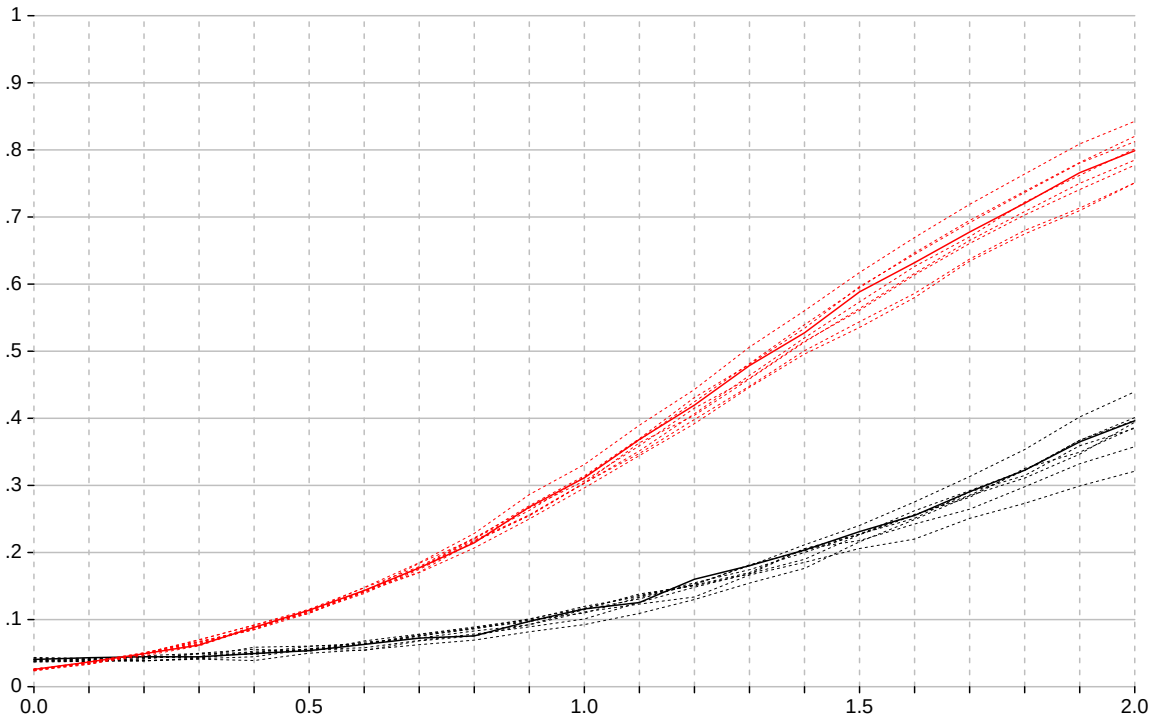
6.1.3 $k = 9,\ k^{DIF} = 1$



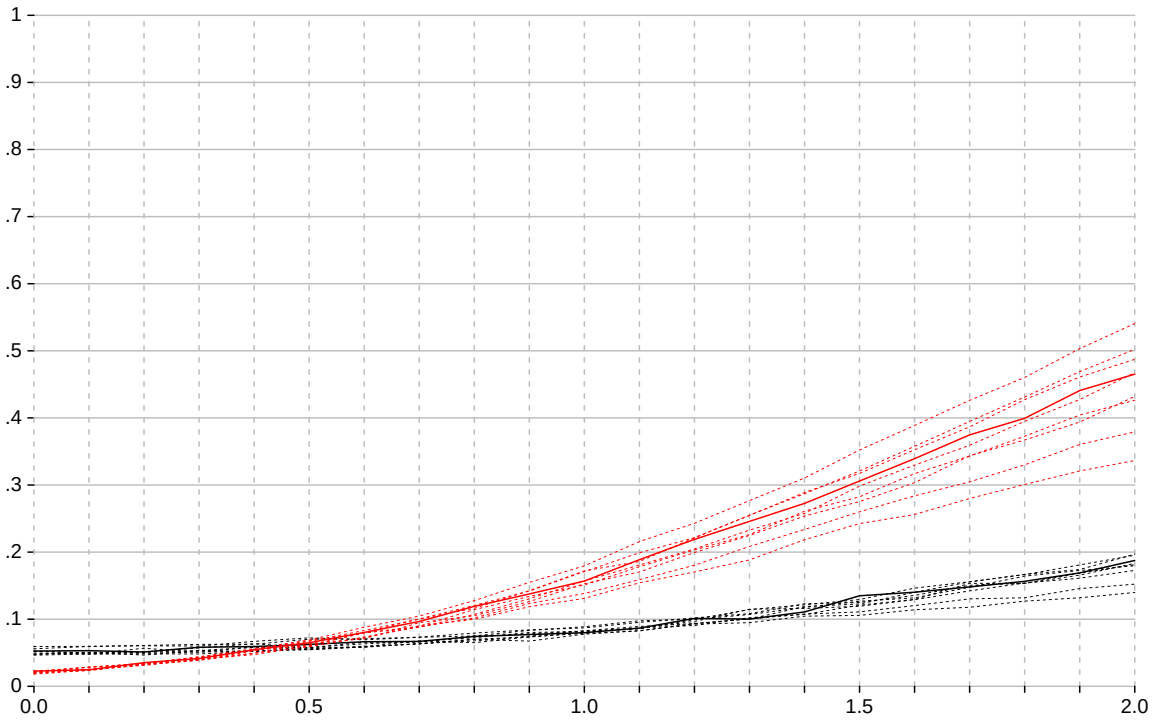
6.1.4 $k = 9,\ k^{DIF} = 2$



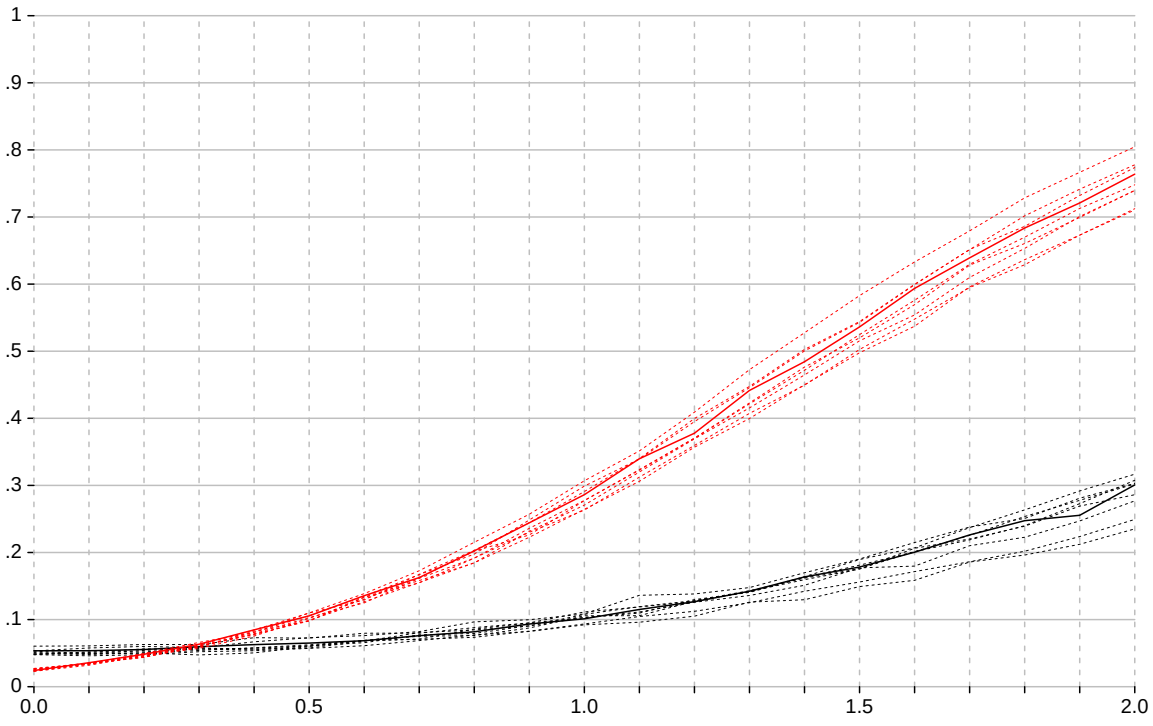
6.1.5 $k = 9,\ k^{DIF} = 5$



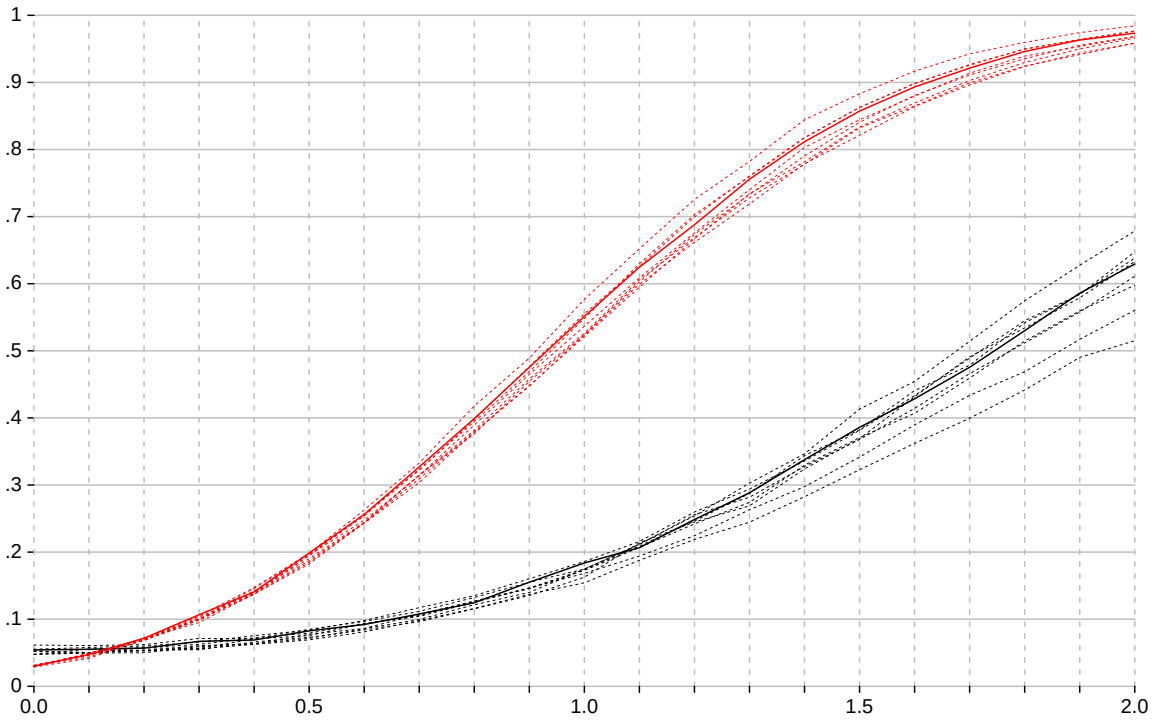
6.1.6 $k = 17,\ k^{DIF} = 1$



6.1.7 $k = 17,\ k^{DIF} = 2$

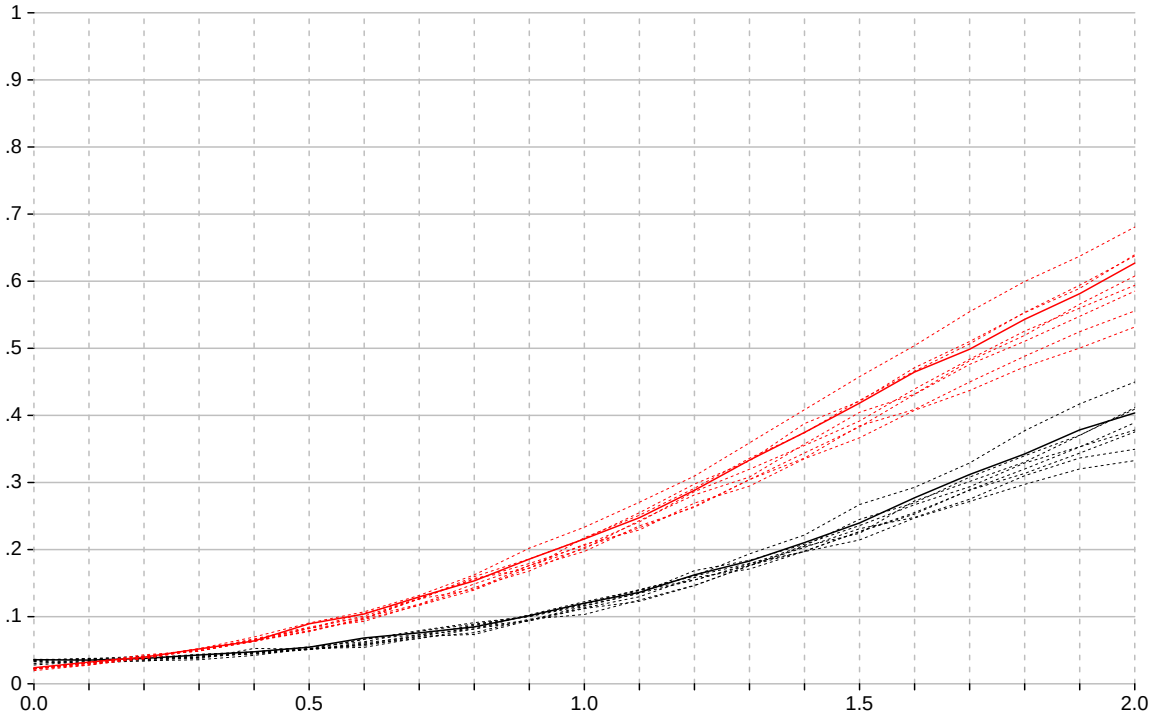


6.1.8 $k = 17$, $k^{DIF} = 5$

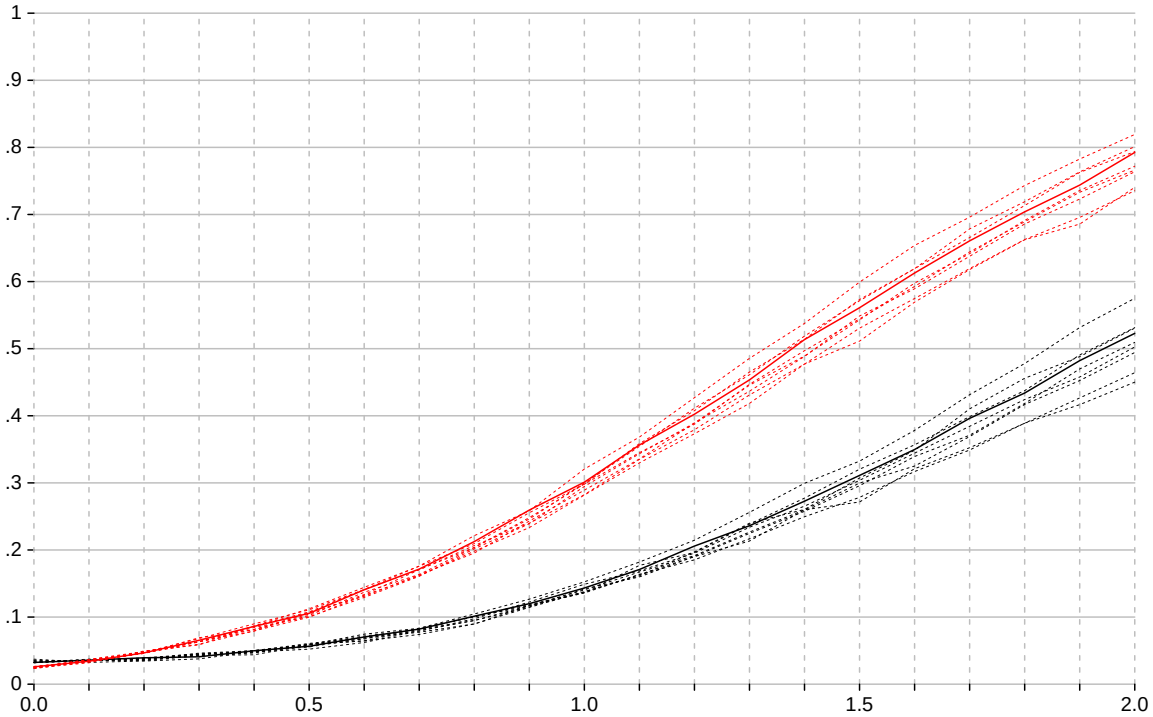


6.2 $n = 50$

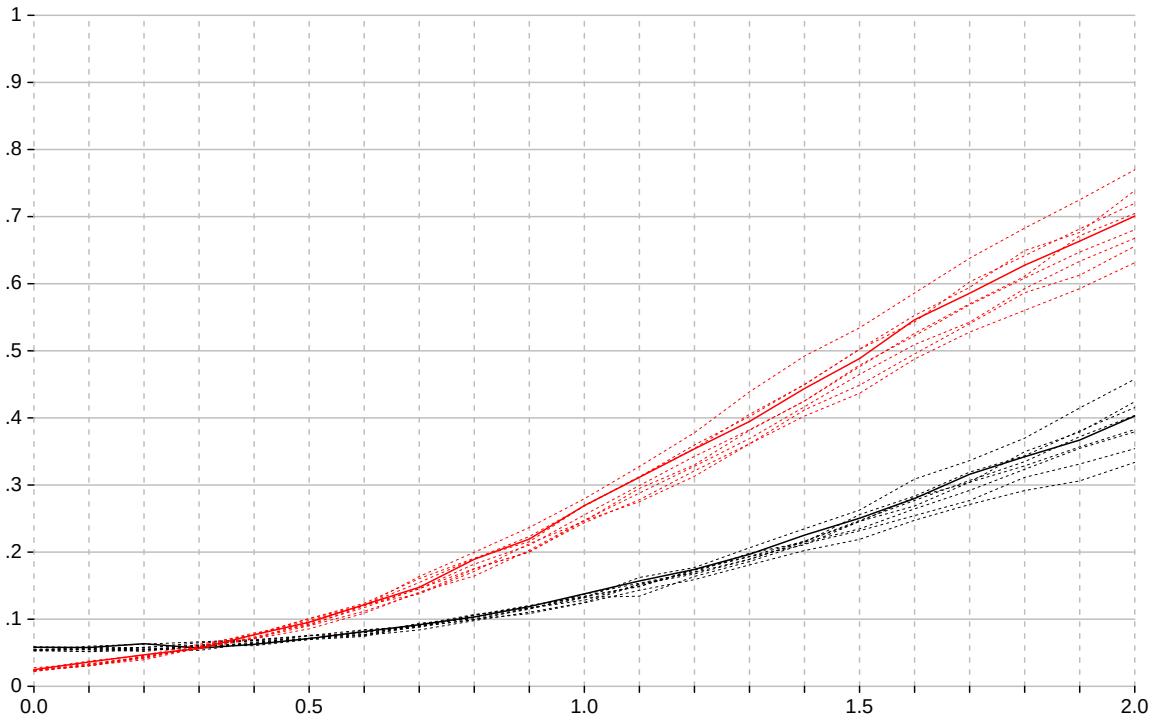
6.2.1 $k = 5,\ k^{DIF} = 1$



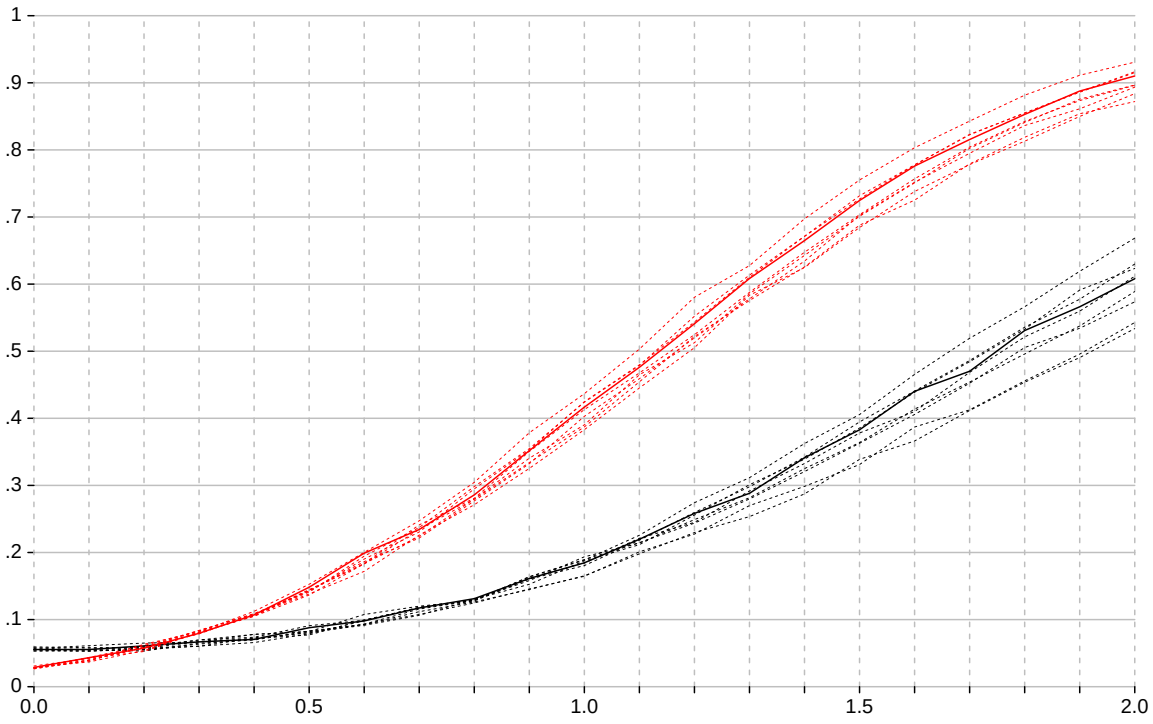
6.2.2 $k = 5,\ k^{DIF} = 2$



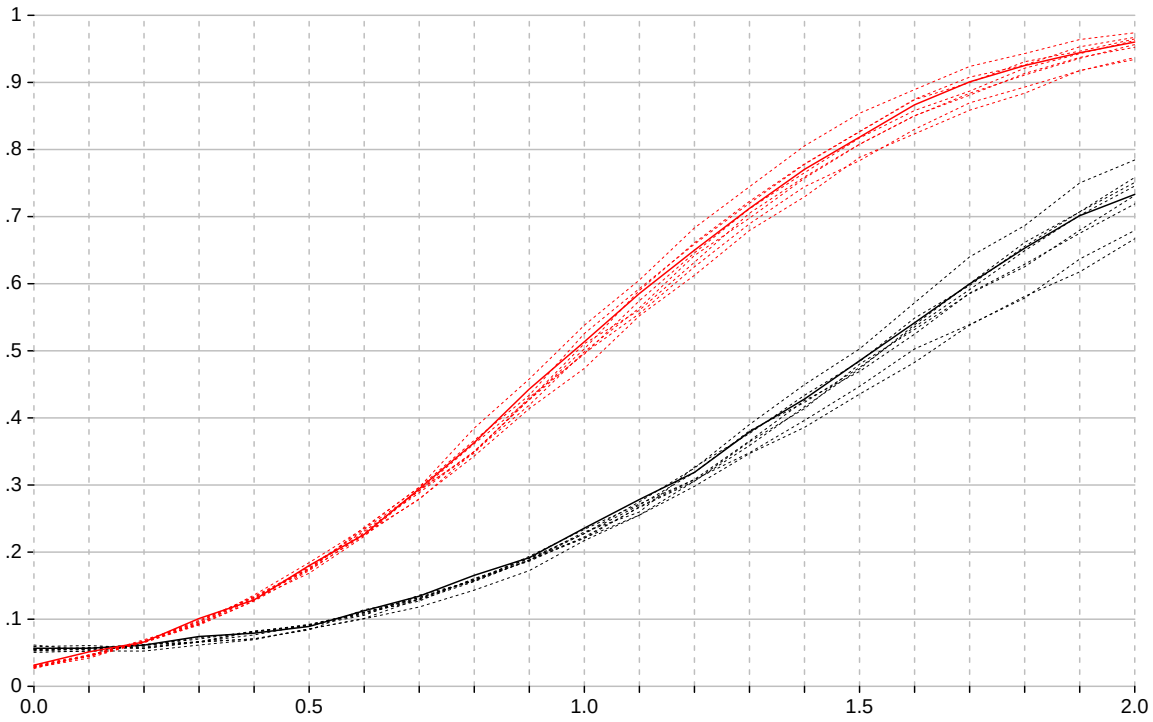
6.2.3 $k = 9,\ k^{DIF} = 1$



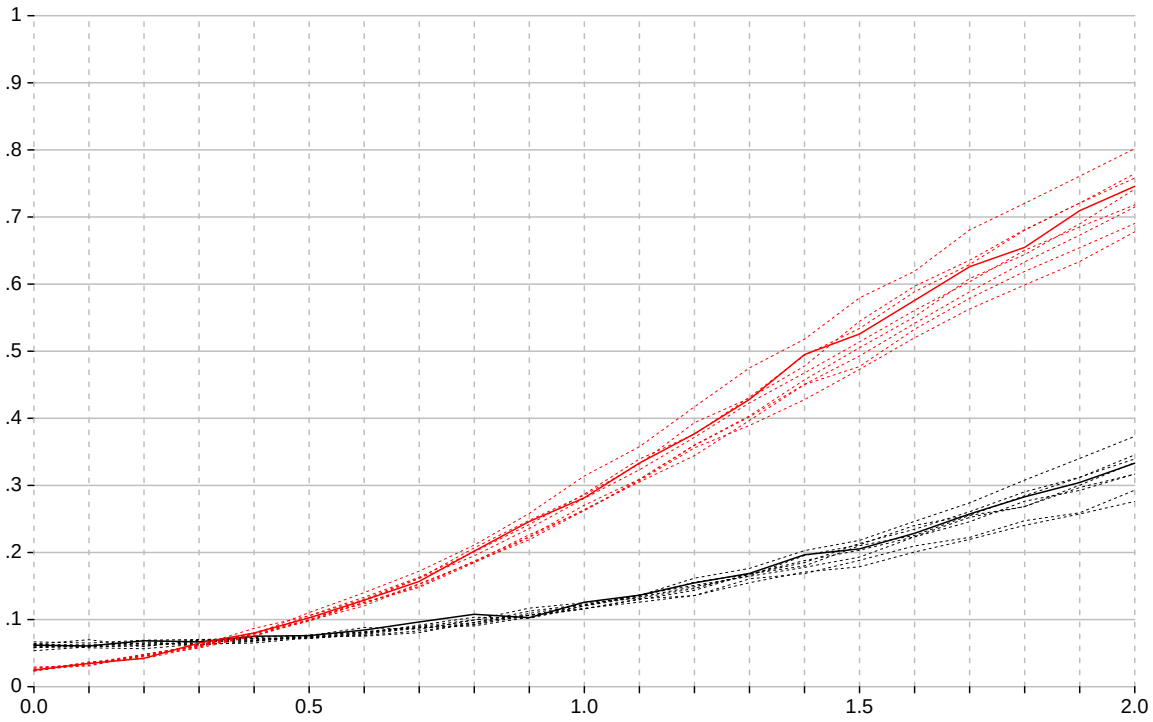
6.2.4 $k = 9,\ k^{DIF} = 2$



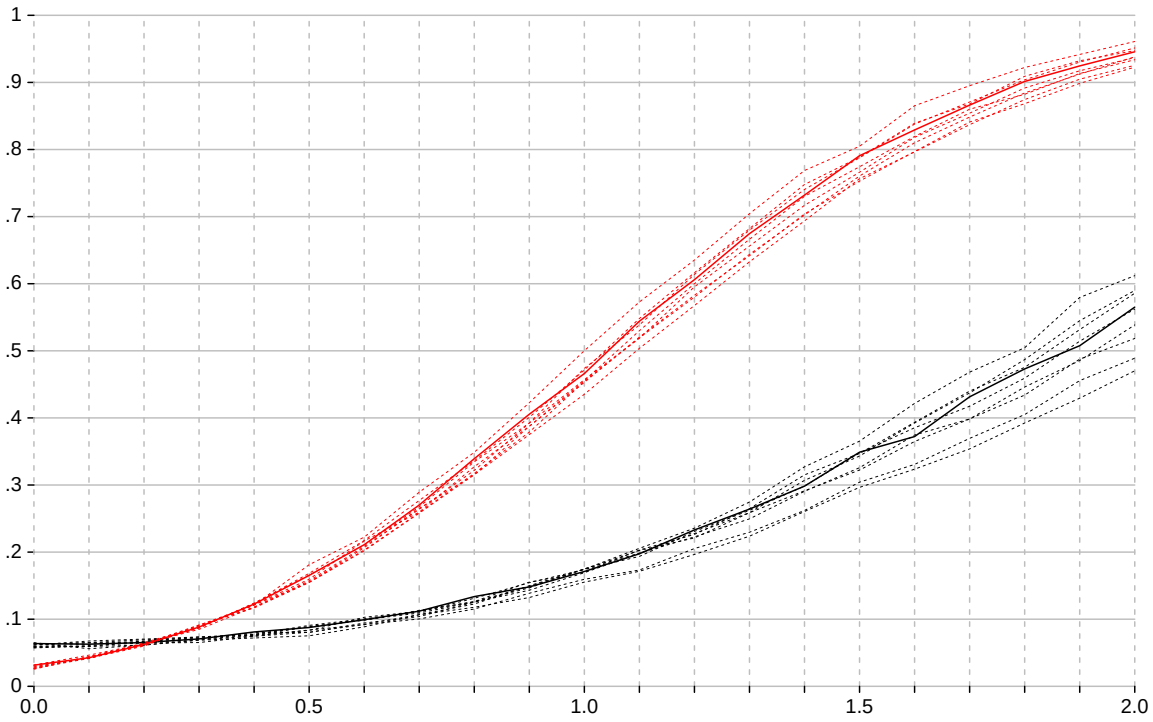
6.2.5 $k = 9,\ k^{DIF} = 5$



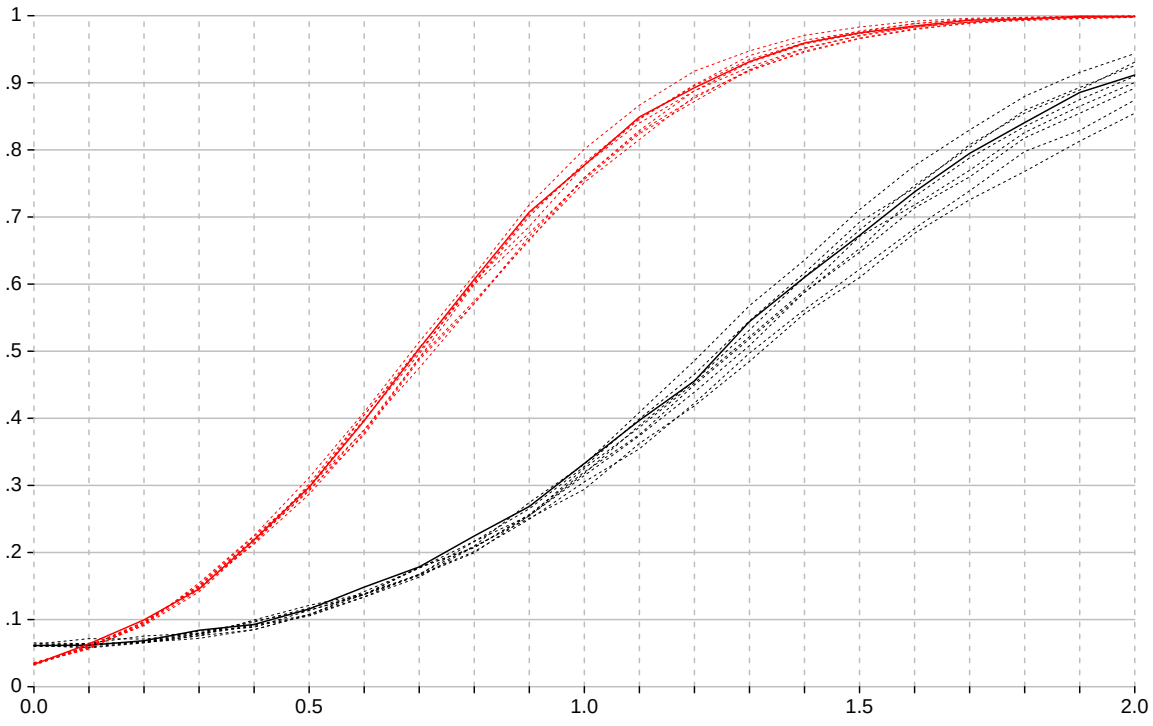
6.2.6 $k = 17,\ k^{DIF} = 1$



6.2.7 $k = 17,\ k^{DIF} = 2$

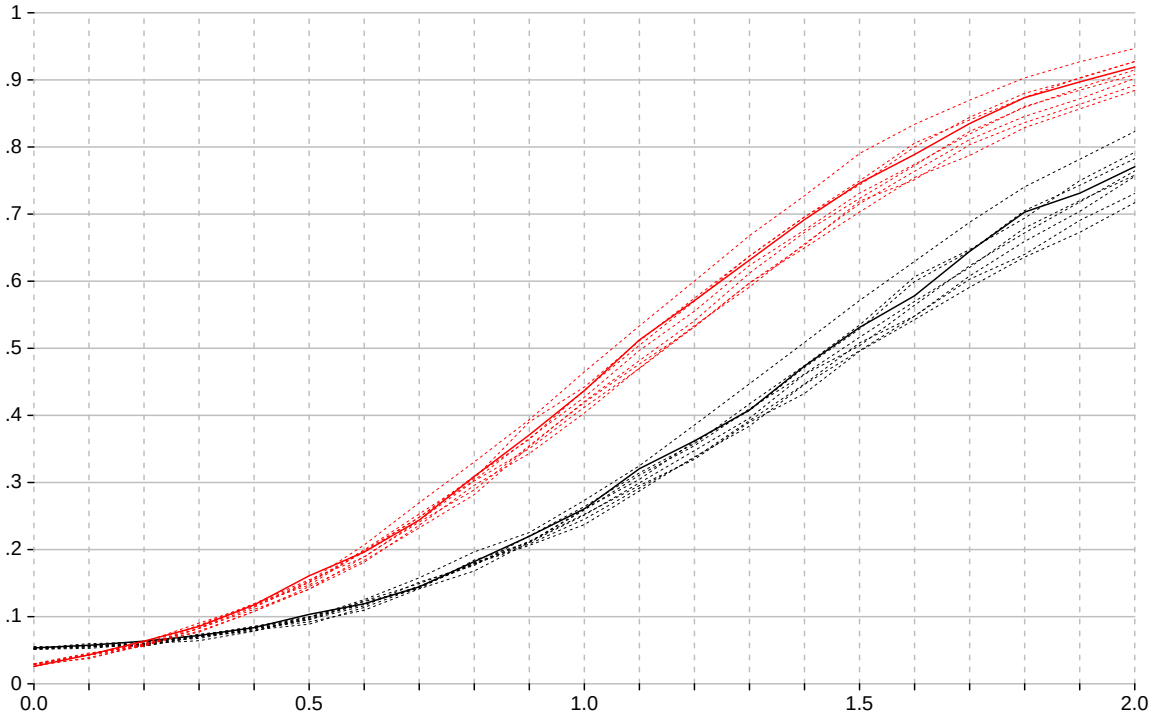


6.2.8 $k = 17,\ k^{DIF} = 5$

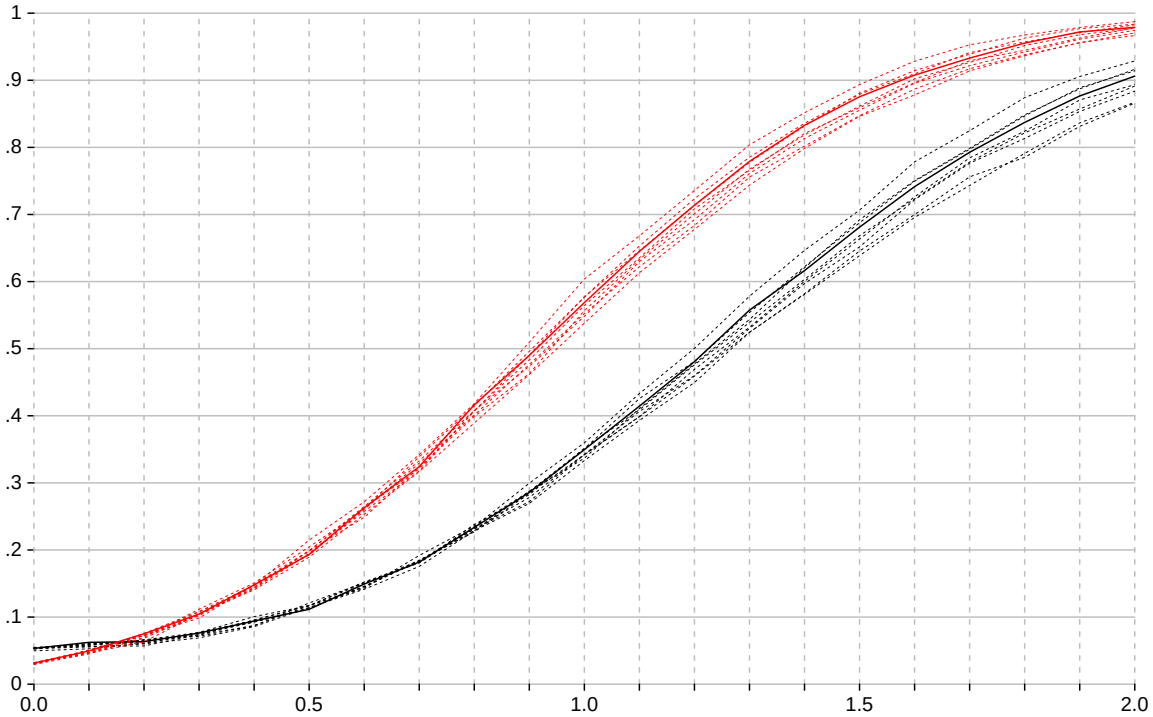


6.3 $n = 100$

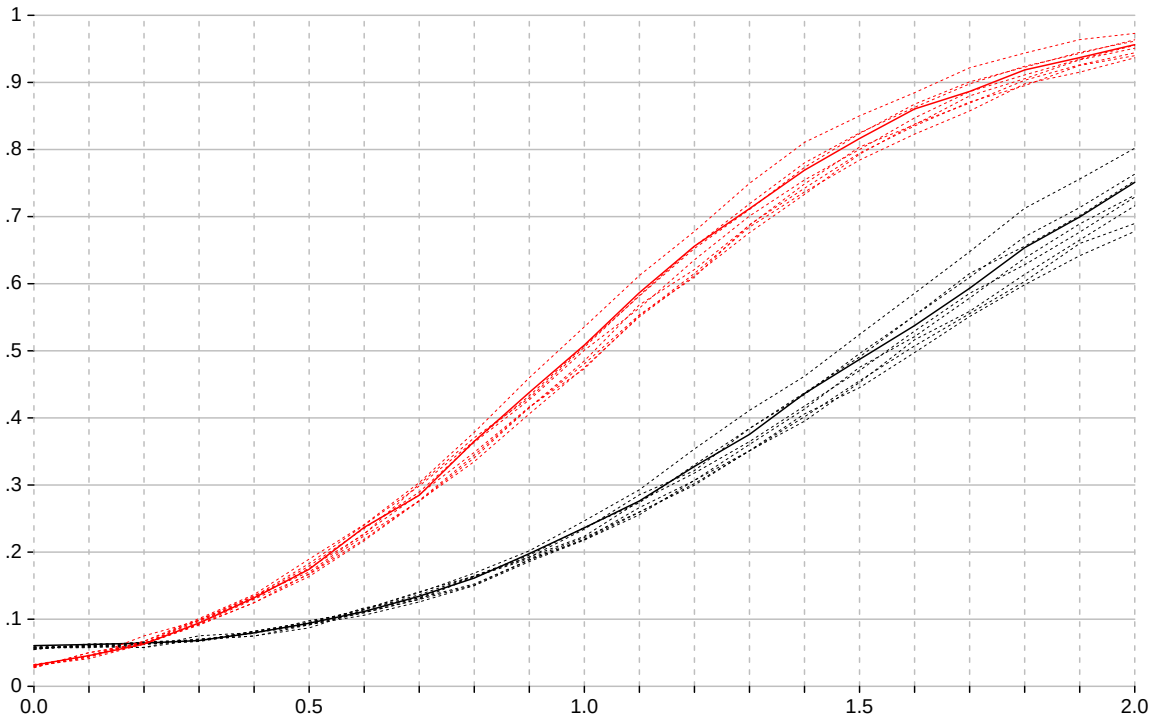
6.3.1 $k = 5,\ k^{DIF} = 1$



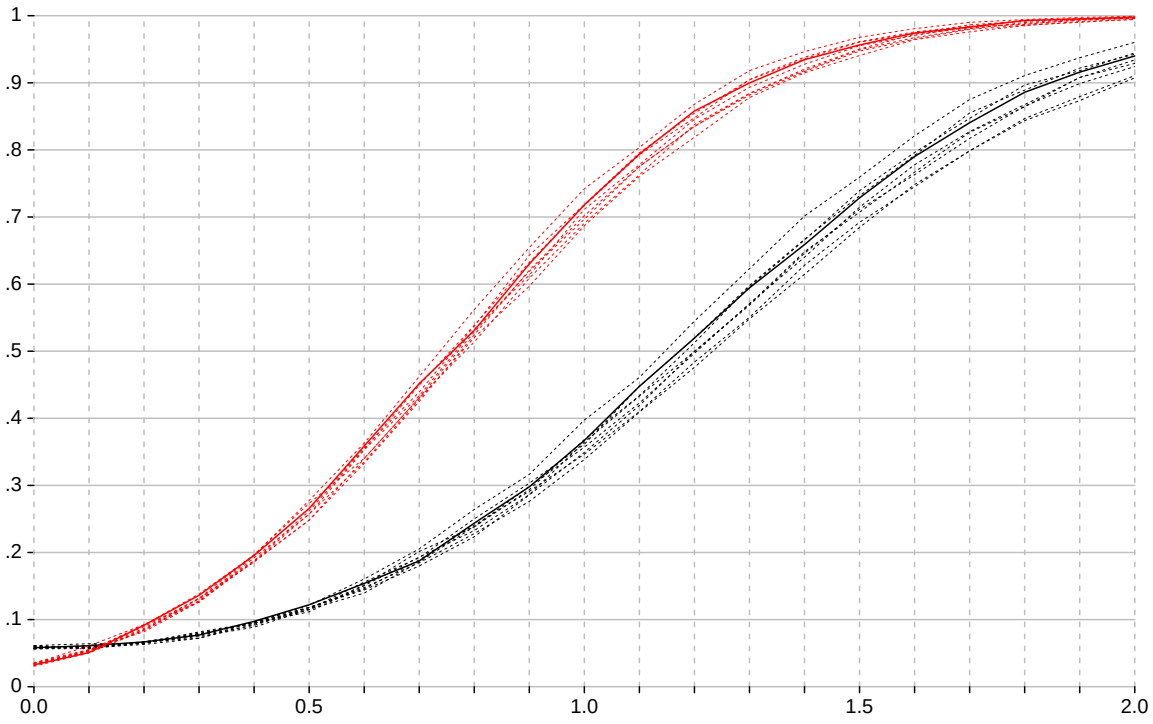
6.3.2 $k = 5,\ k^{DIF} = 2$



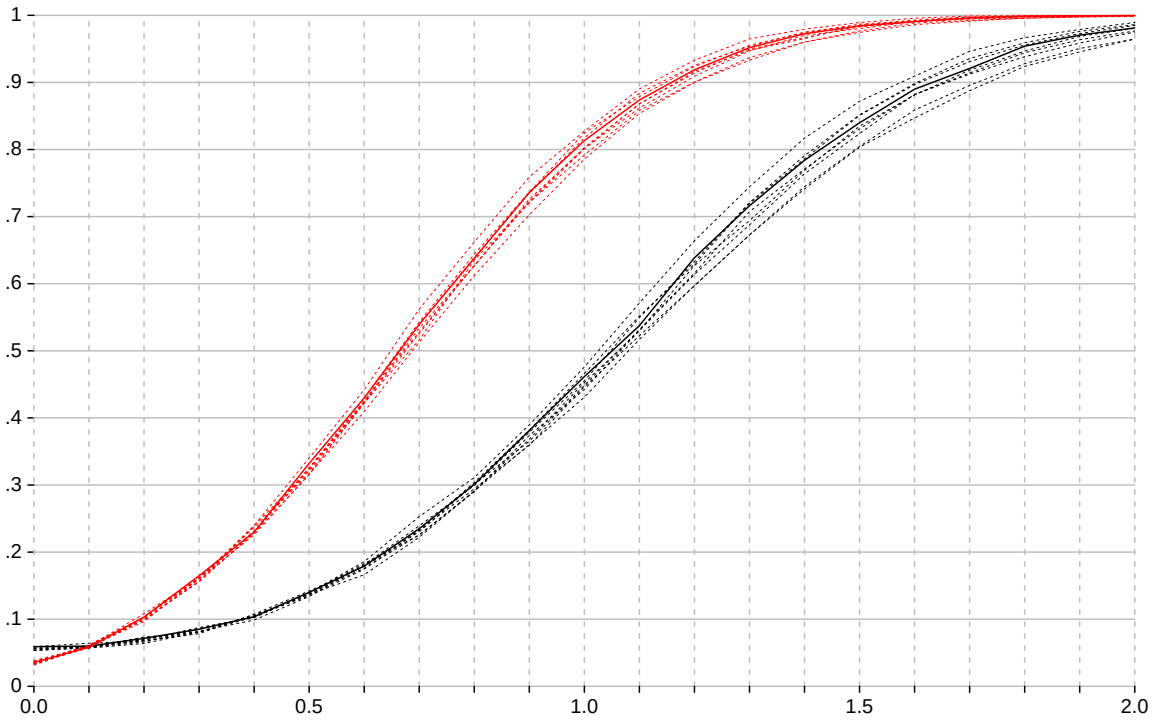
6.3.3 $k = 9,\ k^{DIF} = 1$



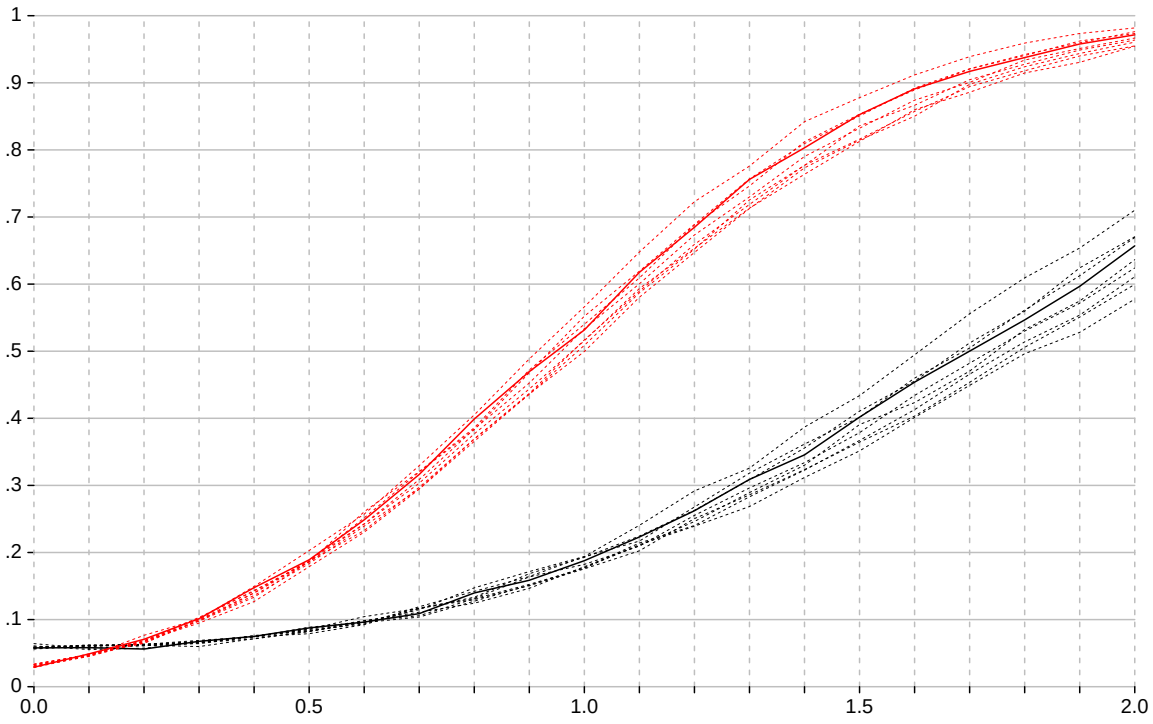
6.3.4 $k = 9,\ k^{DIF} = 2$



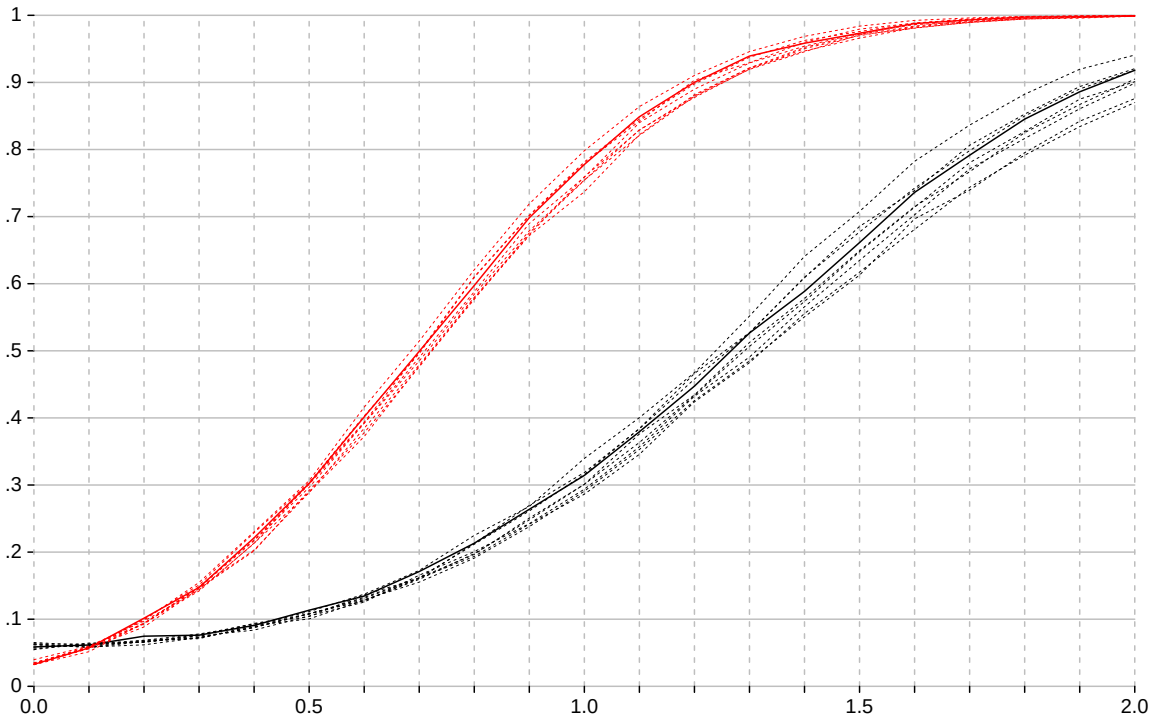
6.3.5 $k = 9,\ k^{DIF} = 5$



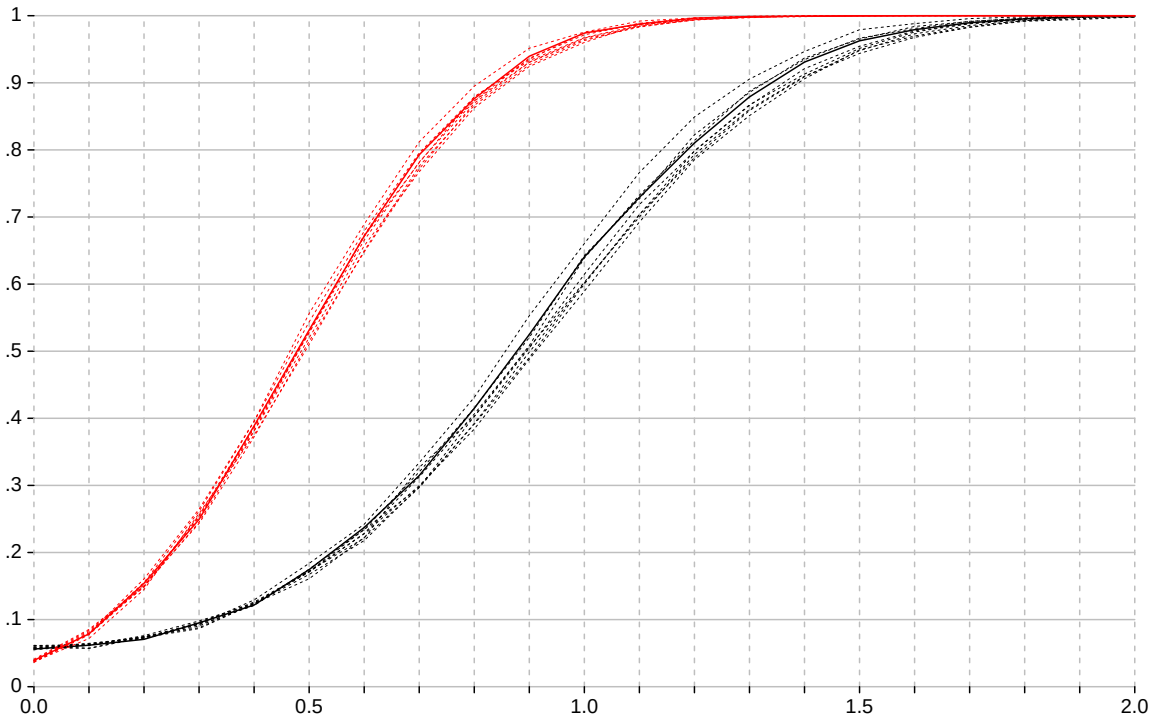
6.3.6 $k = 17,\ k^{DIF} = 1$



6.3.7 $k = 17,\ k^{DIF} = 2$

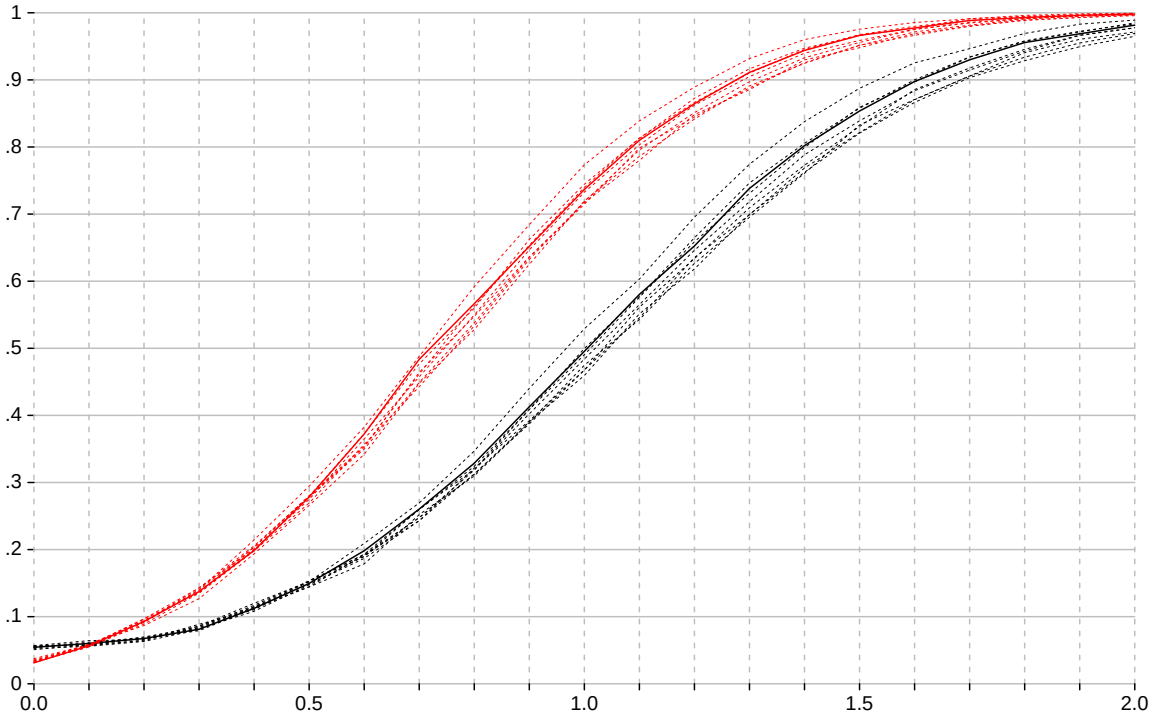


6.3.8 $k = 17,\ k^{DIF} = 5$

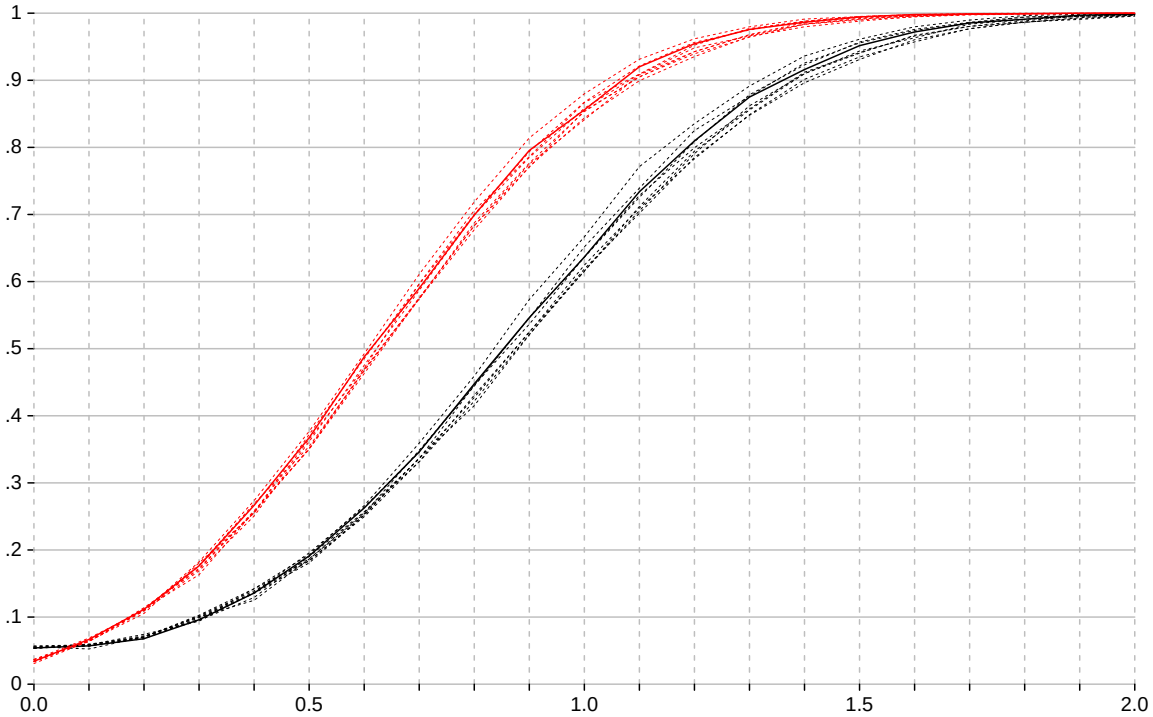


6.4 $n = 200$

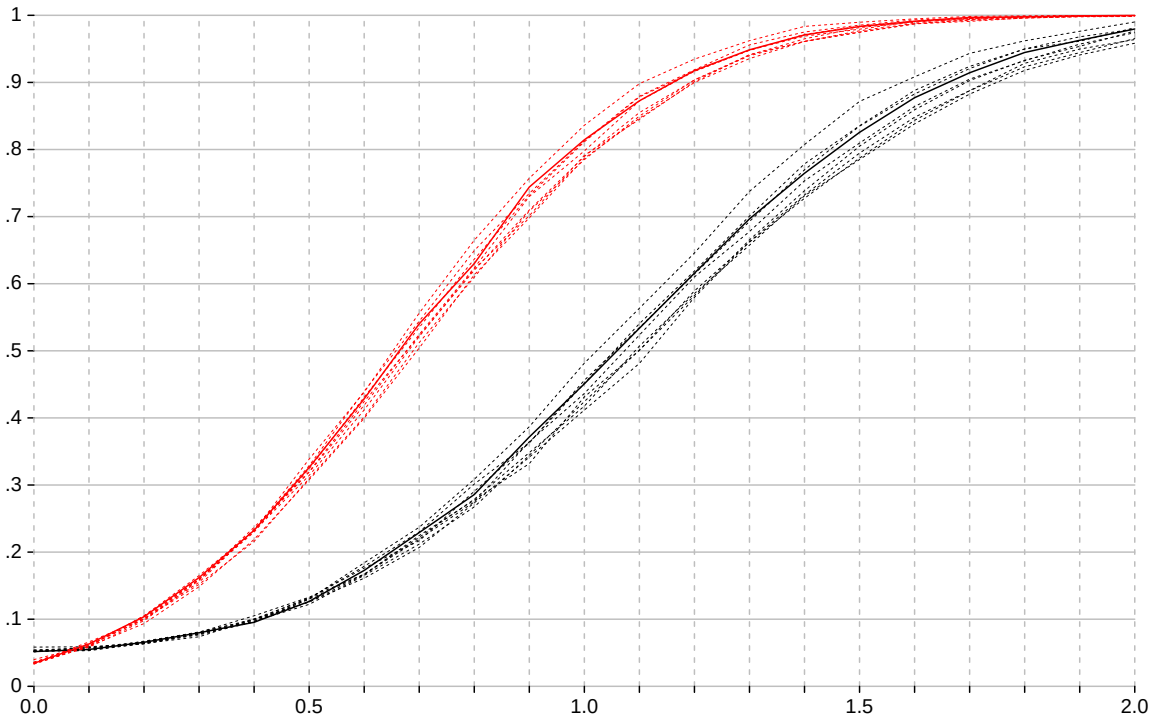
6.4.1 $k = 5,\ k^{DIF} = 1$



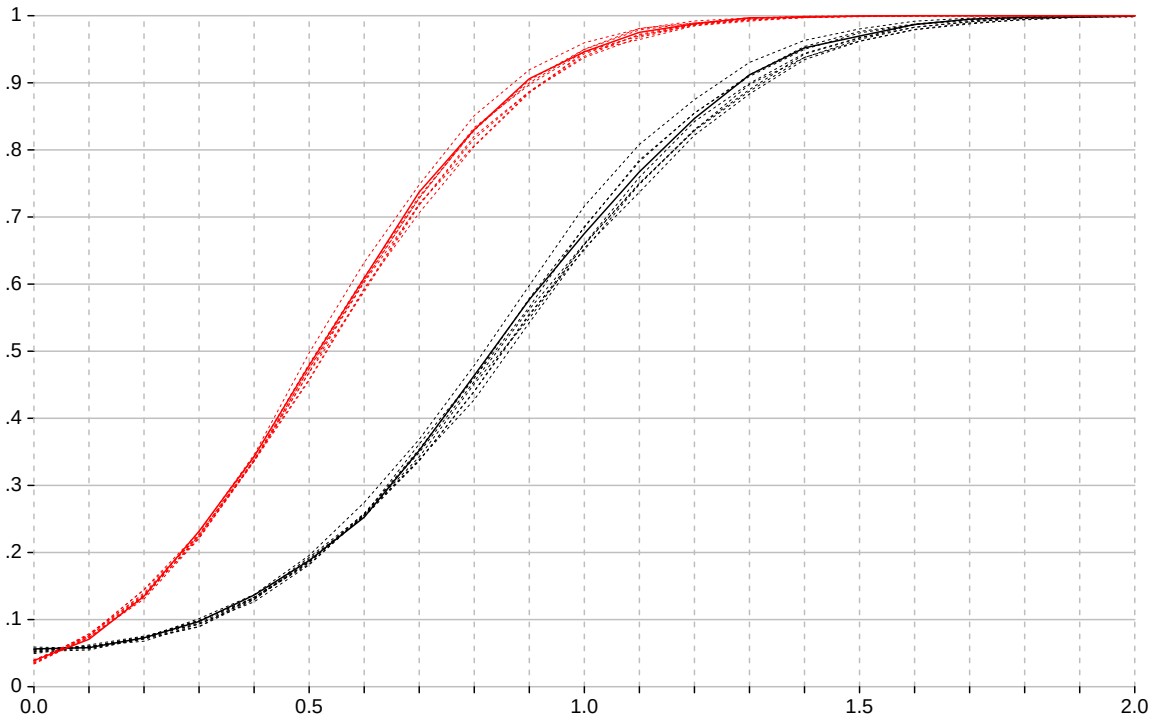
6.4.2 $k = 5,\ k^{DIF} = 2$



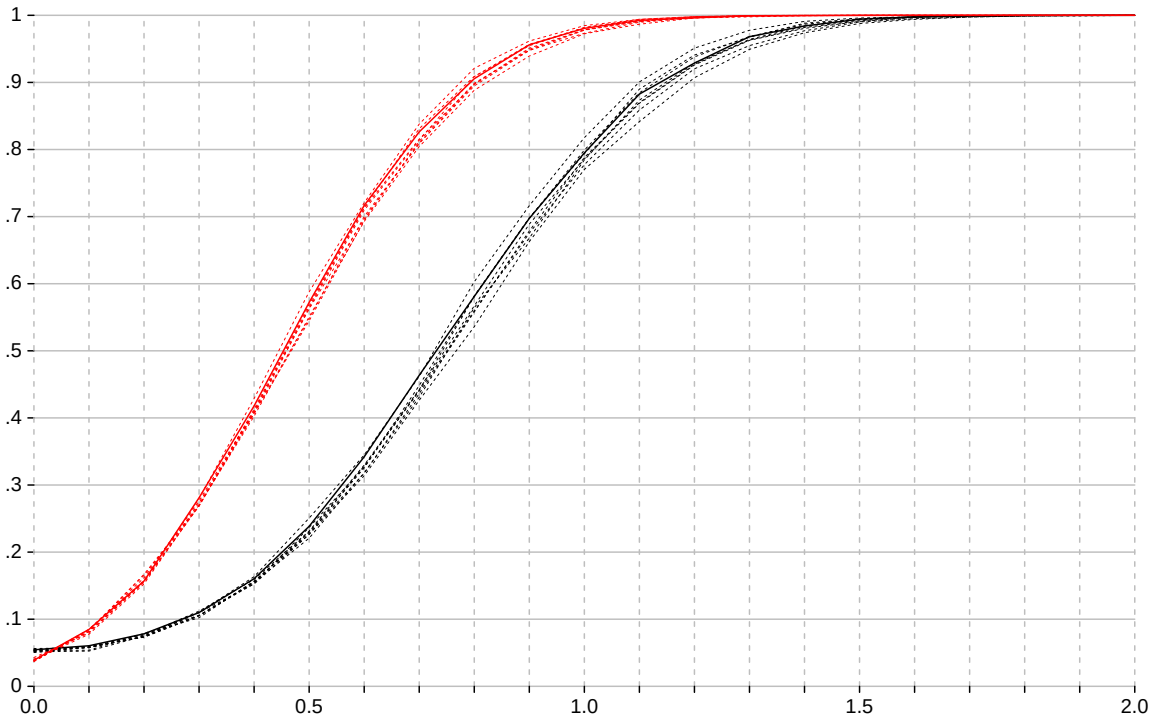
6.4.3 $k = 9,\ k^{DIF} = 1$



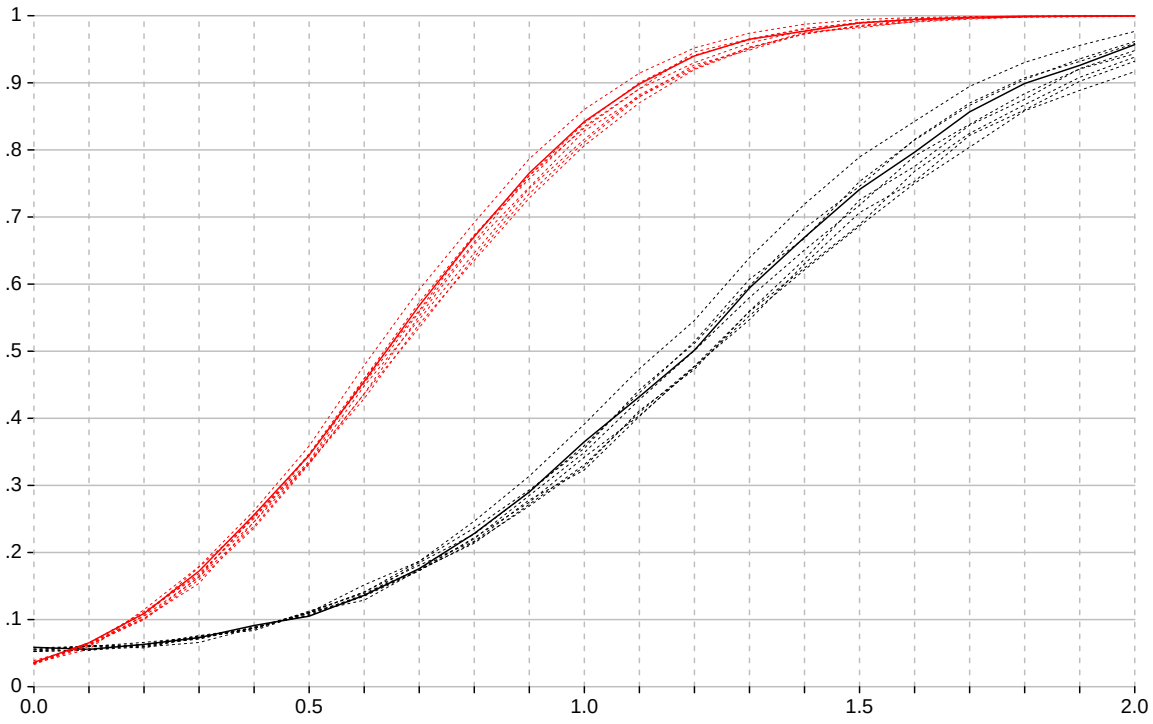
6.4.4 $k = 9,\ k^{DIF} = 2$



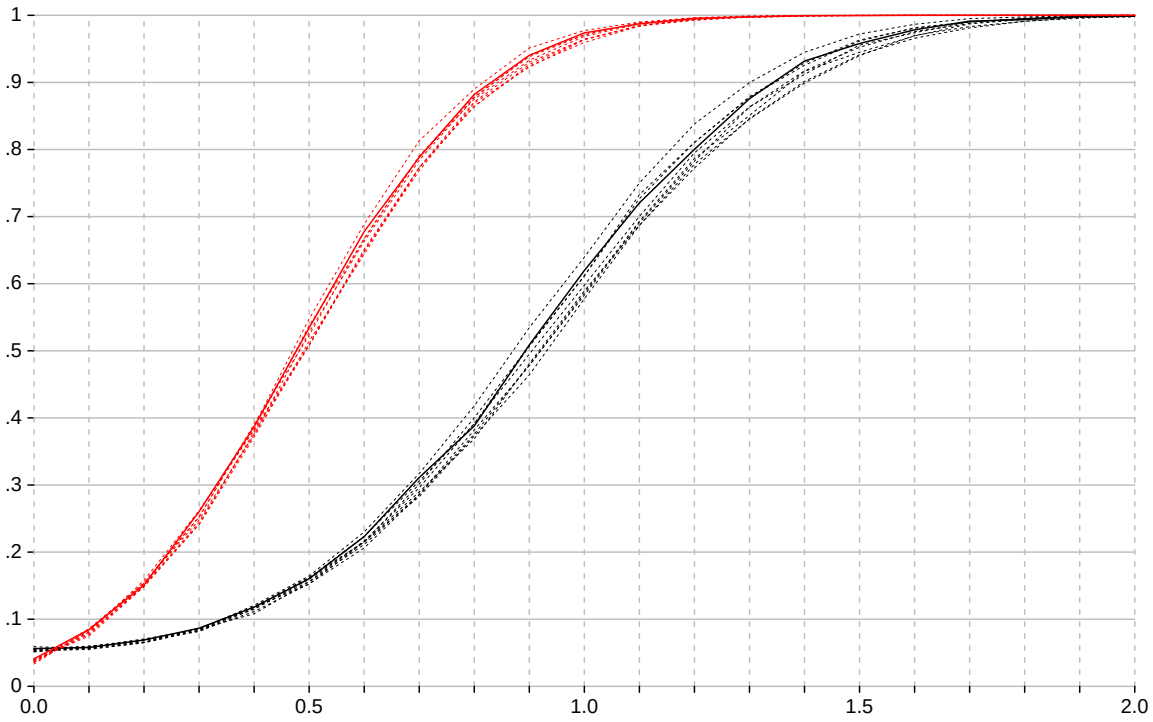
6.4.5 $k = 9,\ k^{DIF} = 5$



6.4.6 $k = 17,\ k^{DIF} = 1$



6.4.7 $k = 17,\ k^{DIF} = 2$



6.4.8 $k = 17,\ k^{DIF} = 5$