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A Periodic Location Routing Problem for Collaborative Recycling

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Motivated by collaborative recycling efforts for non-profit agencies, we study a variant of the periodic location routing problem, in which one decides the set of open depots from the customer set, the capacity of open depots, and the visit frequency to nodes, in an effort to design networks for collaborative pickup activities. We formulate this problem, highlighting the challenges introduced by these decisions. We examine the relative difficulty introduced with each decision through exact solutions and a heuristic approach which can incorporate extensions of model constraints and solve larger instances. The work is motivated by a project with a network of hunger relief agencies (e.g., food pantries, soup kitchens and shelters) focusing on collaborative approaches to address their cardboard recycling challenges collectively. We present a case study based on data from the network. In this novel setting, we evaluate collaboration in terms of participation levels and cost impact. These insights can be generalized to other networks of organizations that may consider pooling resources.

Key words: periodic location routing problems, non-profit operations research

1. Introduction

This paper considers a variant of the periodic location routing problem (PLRP), which combines elements of the periodic vehicle routing problem and the location routing problem. In the PLRP, a set of customers requires regular service from a set of potential depots. Each depot has a maximum capacity and a fixed cost. Each customer has a given visit frequency and a set of possible visit-day combinations specifying the days of service. One visit-day combination is selected for each customer. For each day, routes from the open depots are built to supply customers. The vehicle fleet is homogeneous and a fixed cost is charged for vehicle use.

In this paper, we study a variant of the PLRP, motivated by a project with an organization of hunger relief agencies operating food pantries and soup kitchens. The organization was founded with the goals of advocating for local hunger relief agencies and their clients, providing education and training, and sharing resources to foster a healthier community. Member agencies have identified cardboard recycling as a costly expense given the quantity of donations provided in cardboard boxes and the lack of resources to dispose of the cardboard. When cardboard volume is sufficiently high to warrant the purchase of a cardboard baler (a machine to compress cardboard), the resulting cardboard bales can generate profits from recycling companies. Per-ton prices for baled cardboard ranged from \$60 to \$160 between 2012 and 2013 (Forest2Market (2013)). While most of the organization's member agencies do not generate sufficient cardboard volume individually to purchase a baler, aggregating the volume across the network may be a profitable, or at least cost-saving, venture. Even without baling cardboard, economies of scale can be achieved through aggregation. Working with the organization, our research group has examined ways in which the organization can leverage their network structure to develop a recycling plan.

Multiple options exist for cardboard disposal. We focus on two strategies presented in Figure 1. In Strategy 1, agencies independently dispose of cardboard, either by sending cardboard to a recycling center or by using bins serviced by a waste collector. Strategy 1(0) represents the status quo in which no agency has a baler. In Strategy 1(P), an agency may purchase a baler for cardboard accumulated at that agency only. Strategy 2 involves coordinated removal for subsets of agencies, with one agency in each subset acting as a depot (with or without a baler) to accumulate and collectively dispose of material. Transporting cardboard to the depot is coordinated by the organization.

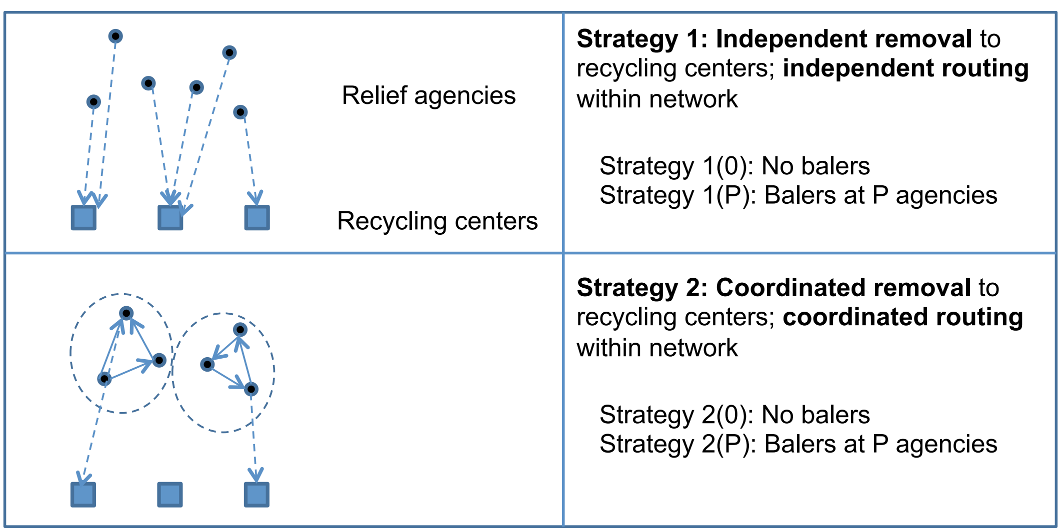

Figure 1 **Network recycling strategies**

Strategy 1 can be evaluated with simple cost models: for each agency, choose the least cost disposal option. Strategy 2 adds periodic routing and location decisions, which can be modeled as a PLRP variant. We formulate this PLRP variant and develop solution approaches for a case study. In the problem, (i) the set of eligible depots are selected from the set of customers; (ii) depot capacity is a variable; and (iii) the frequency with which customers are visited is a decision variable. This PLRP variation is the first to integrate these three elements together. Our computational study suggests that the combination of these three elements leads to greater cost savings.

We present a mixed-integer programming (MIP) model of this PLRP variant, highlighting the challenges introduced by the above decisions. We examine the relative difficulty with an exact solution approach and a heuristic approach which allows us to study more relaxed versions of the problem. Based on data provided by hunger relief organizations, we present a case study to develop insights for collaboration. Importantly, we examine two goals of participation and cost impact, and discuss the inherent trade-offs. These insights can be generalized to other networks of organizations that may consider pooling resources, such as schools and libraries.

The paper is organized as follows. Section 2 reviews literature on related problems. Section 3 describes the problem setting and mathematical formulation. Section 4 introduces the ALNS heuristic approach. Section 5 evaluates the model and solution approaches, and presents a case study to compare the strategies in Figure 1. Section 6 concludes with final remarks and extensions.

2. Literature Review

We review relevant literature in terms of modeling (Section 2.1) and application (Section 2.2).

2.1. Related modeling literature

The PLRP combines the periodic vehicle routing problem (PVRP) with the location-routing problem (LRP); requiring simultaneous facility location and multi-day routing decisions. The PVRP is a multi-day VRP extension in which each customer must be visited a specified number of times over a time horizon. The PVRP is first proposed by Beltrami and Bodin (1974) to route and schedule periodic collection of municipal waste in one of the earliest papers applying vehicle routing to waste collection. Reviews of the PVRP literature can be found in Francis et al. (2008) and Campbell and Wilson (2014). The LRP is the single-period version of the PLRP. Early work in location-routing includes Christofides and Eilon (1969), which uses approximations of the routing costs to estimate combined location and routing decisions, and Laporte and Nobert (1981), which first models and solves the LRP with a MIP model. There are several extensive surveys of the LRP literature, including the PLRP. Nagy and Salhi (2007) covers the literature through 2007, Prodhon and Prins (2014) and Drexl and Schneider (2015) cover the literature from 2007 to 2013, and Lopes et al. (2013) categorizes LRP problems based on modeling and solution approaches.

Alvim and Taillard (2013) solve large-scale LRP instances with a POPMUSIC (partial optimization metaheuristic under special intensification conditions) framework. In the problem considered, the set of eligible depots is also selected from the set of customers. Unlike the problem studied in this paper, they consider a single-period problem and uncapacitated depots.

The PLRP is introduced by Prodhon (2008), which develops MIP models and heuristics for the problem. Several PVRP algorithms have been developed including hybrid local search and evolutionary metaheuristics (Prodhon and Prins (2008), Prodhon (2009), Prodhon (2011)); matheuristics (Pirkwieser and Raidl (2010)); and large neighborhood search-based metaheuristics (Hemmelmayr (2015)). The heuristic we use is based on that of Hemmelmayr (2015) (this is the best performing PLRP algorithm to date), with modifications discussed in Section 4. Tunalıoğlu et al. (2016) introduces a multiperiod LRP motivated by the collection problem of Olive Oil Mill Wastewater, which determines the location of treatment facilities and the capacity level of each facility. The visit days are selected in the problem, but not from a set of predefined visit-day combinations as in our problem. The paper proposes an adaptive large neighbourhood search metaheuristic.

Other related multi-period location-routing problems have been proposed. Liu and Lee (2003) is the earliest work to combine the LRP with the inventory routing problem - multiple depots are located and goods are routed to customers considering both transportation and inventory costs. Albareda-Sambola et al. (2012) combines location and routing on separate time scales (unlike the LRP), with visit frequency defined by customers.

In summary, our paper extends recent advances in related work by modeling a PLRP in which (i) the set of eligible depots are selected from the set of customers; (ii) depot capacity is a variable; and (iii) the frequency with which customers are visited is a decision variable. Section 3 shows how these elements can be incorporated into PLRP formulations and Section 5.1.2 shows the impact of these elements on solution objectives.

2.2. Related application literature

This work also contributes to the literature on location and routing applied to waste collection. Beltrami and Bodin (1974) models the PVRP for waste collection for residential and commercial customers, routing of barges to collect large containers of garbage, and arc routing for street sweeping. Beliën et al. (2014) surveys the literature on routing for municipal waste collection, and categorize papers based on the type of collection: (1) curbside collection, (2) collection at a consolidation site such as a neighborhood recycling center, and (3) collection of large quantities of goods from industrial customers. Our application has some resemblance with categories (1) and (2). The setting posed here is concerned with collecting recycling from hunger relief agencies (similar to curbside collection) and we model the collection of goods from intermediate sites (our depots) as an out-of-network collection with a variable collection cost.

Angelelli and Speranza (2002) models waste collection in which vehicles stop at waste treatment centers with the PVRP with intermediate facilities. Archetti and Speranza (2004) models a waste collection problem with high-volume industrial customers, in which full containers are collected from customers, delivered to treatment plants, and empty containers are returned to customers. Hemmelmayr et al. (2013) models the waste bin allocation and routing problem, combining a PVRP with service choice (visit frequency as a decision variable) and intermediate facilities with a capacity allocation problem. The quantity and type of bins are chosen for removal sites.

Operations research literature on hunger relief operations has been growing, focusing on food distribution: Bartholdi et al. (1983), Gunes et al. (2010), Yildiz et al. (2012), Mahadevan et al. (2013), Balcik et al. (2014) and Lien et al. (2014). Notably, Solak et al. (2014) models a single period LRP for food distribution, with decisions including locating food delivery sites, assigning agencies to those delivery sites, and routing vehicles to those sites from a central food bank.

Our paper contributes to this literature by focusing on the collection of the materials used to distribute food and, importantly, analyzing opportunities for organizations to jointly plan operations, leveraging their network structure in new ways. The case study shows how logistics operations can be a source of interagency cohesion.

3. Problem setting and model formulation

Section 3.1 describes the general problem setting and Section 3.2 presents the formulation.

3.1. Problem setting

Given are a set of customers, $\mathcal{V}$, each with an amount of goods to be collected, which may not be uniformly distributed over the planning horizon, and a subset of customers $\mathcal{J} \subseteq \mathcal{V}$ which, given their size and geographic location, are potential depots. Customers are connected by arcs in the set, $\mathcal{A}$. The planning horizon is defined by the set of periods $\mathcal{T}$ (typically days in a week).

At each potential depot $j \in \mathcal{J}$, there is a set of feasible depot sizes, denoted $\mathcal{P}_j$. For each $p \in \mathcal{P}_j$, Q_p denotes the maximum capacity and F_p denotes the cost of opening a depot of that size.

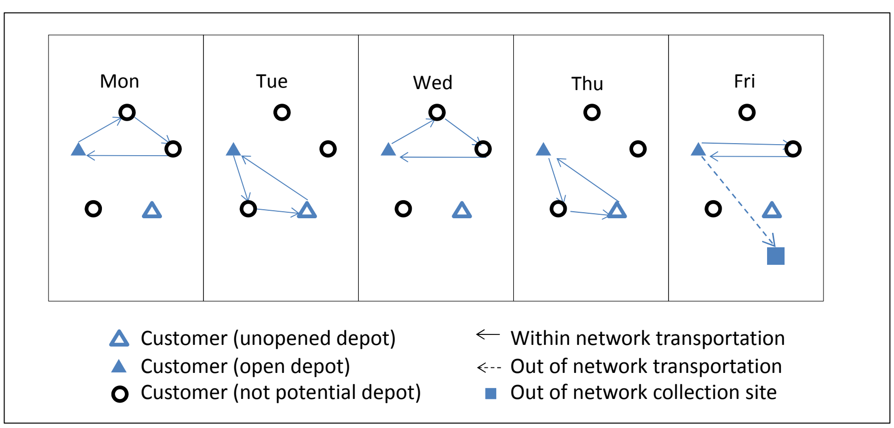

Figure 2 Example solution with one open depot for an instance with $|\mathcal{V}| = 5$ and $|\mathcal{J}| = 2$

At most one vehicle from a set of feasible vehicles $\mathcal{K}_j$ can be used at a depot $j \in \mathcal{J}$. The set $\mathcal{K}$ is the set of all vehicles: $\mathcal{K} = \bigcup_{j \in \mathcal{J}} \mathcal{K}_j$. For vehicle $k \in \mathcal{K}$, Q_k denotes its capacity and F_k denotes its fixed cost. Vehicle size impacts the per-distance transportation cost: C_{ijk} is the travel cost on arc $(i,j) \in \mathcal{A}$ by vehicle $k \in \mathcal{K}$. For each day $t \in \mathcal{T}$, each customer $i \in \mathcal{V}$ is visited by at most one vehicle. These vehicles are used within a subset of customers, collecting material from customers and delivering to the depot. Transport of collected material from the depot is considered exogenous. Parameter $\mathcal{R}_i$ represents the set of feasible service schedules for customer $i \in \mathcal{V}$. The amount collected from customer i on day t with visit schedule r is represented by q_{irt}. Figure 2 shows routes over a five-day planning horizon with two potential depots, one of which is open. Material is transported to the depot on each day of the planning horizon. Transport out of the network occurs at the end of the period. This impacts depot capacity, which must be sufficient to accommodate the volume collected over the planning horizon, while vehicle capacity must meet only the volume on an individual route.

We define a set of cost-revenue levels, $\mathcal{G}$, with associated cost-revenue values, F_g, for $g \in \mathcal{G}$. The value F_g is positive if there is a cost for collection and negative if there is a revenue. Table 1 presents an example of a disposal cost profile from our motivating example. For lower volumes, there is a cost to dispose material. Recyclers usually charge a fee based on the size of the recycling container, regardless of the quantity collected. At higher volumes, material can yield revenue that is dependent on the size and number of cardboard bales.

Cardboard Accumulated (M_g)	**Weekly Cost or Revenue (F_g)**
0 - 50 lbs	\$6 cost for roll cart recycling bin
50 - 400 lbs	\$13.50 cost for 2 cubic yard dumpster
400 - 600 lbs	\$23.75 cost for 4 cubic yard dumpster
Over 600 lbs	\$21 revenue for each 600 lb bale

Table 1 **Sample cost-revenue profile for recycling [Waste Management (2014), Larimer County (2014)].**

3.2. Model formulation

We model this problem with four sets of decision variables, defined below.

$$z_{jpk} = \begin{cases} 1 & \text{if customer } j \in \mathcal{J} \text{ opens a depot of size } p \in \mathcal{P}_j \text{ and uses vehicle } k \in \mathcal{K}_j \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ijr} = \begin{cases} 1 & \text{if customer } i \in \mathcal{V} \text{ is assigned to depot } j \in \mathcal{J} \text{ with schedule } r \in \mathcal{R}_i \\ 0 & \text{otherwise} \end{cases}$$

$$w_{jg} = \begin{cases} 1 & \text{if collected volume at depot } j \in \mathcal{J} \text{ is within the range of cost-revenue level } g \in \mathcal{G} \\ 0 & \text{otherwise} \end{cases}$$

$$x_{ijkt} = \begin{cases} 1 & \text{if arc } (i,j) \in \mathcal{A} \text{ is traversed by vehicle } k \in \mathcal{K} \text{ on day } t \in \mathcal{T} \\ 0 & \text{otherwise} \end{cases}$$

The model is as follows:

$$\min \sum_{j\in\mathcal{J}} \sum_{p\in\mathcal{P}_j} \sum_{k\in\mathcal{K}_j} (F_p + F_k) z_{jpk} + \sum_{(i,j)\in\mathcal{A}} \sum_{k\in\mathcal{K}} \sum_{t\in\mathcal{T}} C_{ijk} x_{ijkt} + \sum_{j\in\mathcal{J}} \sum_{g\in\mathcal{G}} F_g w_{jg} \tag{1a}$$

$$\sum_{i\in\mathcal{V}} \sum_{r\in\mathcal{R}_i} \sum_{t\in\mathcal{T}} q_{irt} y_{ijr} \geq \sum_{g\in\mathcal{G}} M_g w_{jg} \quad \forall j\in\mathcal{J} \tag{1b}$$

$$\sum_{p\in\mathcal{P}_j} \sum_{k\in\mathcal{K}_j} z_{jpk} = \sum_{g\in\mathcal{G}} w_{jg} \quad \forall j\in\mathcal{J} \tag{1c}$$

$$\sum_{i\in\mathcal{V}} \sum_{r\in\mathcal{R}_i} \sum_{t\in\mathcal{T}} q_{irt} y_{ijr} \leq \sum_{p\in\mathcal{P}_j} \sum_{k\in\mathcal{K}_j} Q_p z_{jpk} \quad \forall j\in\mathcal{J} \tag{1d}$$

$$\sum_{j\in\mathcal{J}} \sum_{r\in\mathcal{R}_i} y_{ijr} = 1 \quad \forall i\in\mathcal{V} \tag{1e}$$

$$\sum_{p\in\mathcal{P}_j} \sum_{k\in\mathcal{K}_j} z_{jpk} = \sum_{r\in\mathcal{R}_j} y_{jjr} \quad \forall j\in\mathcal{J} \tag{1f}$$

$$\sum_{h\in\mathcal{V}} \sum_{k\in\mathcal{K}} x_{ihkt} - \sum_{j\in\mathcal{J}} \sum_{r\in\mathcal{R}_i} a_{rt} y_{ijr} \geq 0 \quad \forall i\in\mathcal{V}\backslash\mathcal{J},\, \forall t\in\mathcal{T} \tag{1g}$$

$$\sum_{h\in\mathcal{V}} \sum_{k\in\mathcal{K}} x_{ihkt} + \sum_{p\in\mathcal{P}_i} \sum_{k\in\mathcal{K}_i} z_{ipk} - \sum_{j\in\mathcal{J}} \sum_{r\in\mathcal{R}_i} a_{rt} y_{ijr} \geq 0 \quad \forall i\in\mathcal{J},\, \forall t\in\mathcal{T} \tag{1h}$$

$$\sum_{r\in\mathcal{R}_i} y_{ijr} + \sum_{h\in\mathcal{V}} \sum_{k\notin\mathcal{K}_j} x_{ihkt} \leq 1 \quad \forall i\in\mathcal{V},\, \forall j\in\mathcal{J}, \forall t\in\mathcal{T} \tag{1i}$$

$$\sum_{i\in\mathcal{V}:i\neq j} \sum_{r\in\mathcal{R}_i} q_{irt} y_{ijr} \leq \sum_{p\in\mathcal{P}_j} \sum_{k\in\mathcal{K}_j} Q^k z_{jpk} \quad \forall j\in\mathcal{J},\, \forall t\in\mathcal{T} \tag{1j}$$

$$\sum_{i\in\mathcal{V}} x_{ijkt} - \sum_{h\in\mathcal{V}} x_{jhkt} = 0 \quad \forall j\in\mathcal{V},\, \forall k\in\mathcal{K},\, \forall t\in\mathcal{T} \tag{1k}$$

$$\sum_{i,j\in\mathcal{S}} x_{ijkt} \leq |\mathcal{S}| - 1 + \sum_{j\in\mathcal{S}\cap\mathcal{V}} \sum_{p\in\mathcal{P}_j} z_{jpk} \quad \forall \mathcal{S}\subseteq\mathcal{V},\, \forall k\in\mathcal{K},\, \forall t\in\mathcal{T} \tag{1l}$$

$$y_{ijr} \in \{0,1\} \quad \forall i\in\mathcal{V},\, \forall r\in\mathcal{R}_i,\, \forall j\in\mathcal{J} \tag{1m}$$

$$z_{jpk} \in \{0,1\} \quad \forall j\in\mathcal{J},\, \forall p\in\mathcal{P}_j,\, \forall k\in\mathcal{K}_j, \tag{1n}$$

$$x_{ijkt} \in \{0,1\} \quad \forall i\in\mathcal{V},\, \forall j\in\mathcal{V},\, \forall k\in\mathcal{K},\, \forall t\in\mathcal{T} \tag{1o}$$

$$w_{jg} \in \{0,1\} \quad \forall j\in\mathcal{J},\, \forall g\in\mathcal{G} \tag{1p}$$

The first term of the objective function represents the fixed depot and vehicle costs. The second term represents the vehicle-dependent travel cost. The final term is the cost or revenue realized for the volume collected. At low volumes, the last term is a positive fee; at high volumes, it is a negative revenue.

The first two sets of constraints model the piecewise-constant cost-revenue function. To minimize (1a), the best feasible cost-revenue level is chosen. Constraints (1b) ensure that w_{jg} values correspond to the cost-revenue level for the amount collected at depot $j \in \mathcal{J}$. A depot must collect sufficient volume to qualify for a lower cost or higher revenue. Constraints (1c) enforce the choice of exactly one cost-revenue level for each open depot (i.e., when $\sum_{p\in\mathcal{P}_j}\sum_{k\in\mathcal{K}_j} z_{jpk} = 1$). For $F_g > 0$ (cost), this constraint forces the cost to be charged. For $F_g < 0$ (revenue), this constraint prevents revenue over-estimation with multiple revenue levels. Constraints (1d) ensure that customers are assigned to open depots with ample capacity. Constraints (1e) guarantee each customer is assigned to one schedule and one depot.

If transportation costs are high relative to fixed depot costs and/or the objective function favors highly aggregated collection, there is an incentive to open a depot at a customer j_1, but assign the volume at j_1 to another depot at j_2. This may result in a higher revenue, while neglecting transportation costs from j_1 to j_2 because j_1 is a depot. To prevent this, Constraints (1f) require a depot to be assigned to itself. Constraints (1f) also ensure that at most one vehicle serves a depot.

The first two sets of routing constraints guarantee that material is collected from customers when needed. Parameter a_{rt} indicates whether schedule r includes a visit on day t. The volume at each customer $i \in \mathcal{V}$ that is not an open depot must be transported to the customer's assigned depot. For customers that are not candidates to become depots ($i \in \mathcal{V}\backslash\mathcal{J}$), these constraints follow standard PLRP constraints, as in Constraints (1g). If a customer is a potential depot ($i \in \mathcal{J}$), visiting constraints are needed only if i is not an open depot. The depot choice variable is added in Constraints (1h): if $\sum_{p\in\mathcal{P}_i}\sum_{k\in\mathcal{K}_i} z_{ipk} = 1$, then $\sum_{h\in\mathcal{V}}\sum_{k\in\mathcal{K}} x_{ihkt}$ may equal 0. This is the case on a day in which no other customers assigned to depot i are visited. Constraints (1i) prohibit assignment of a customer to vehicles that are not stationed at its assigned depot. If the first term of a constraint of type (1i) is zero (i.e., i is not assigned to j), then customer i cannot be visited by a vehicle from j.

Routing variables x_{ijkt} do not contain customer schedules; thus, vehicle capacity constraints are written with assignment variables, in constraints (1j). This simplification is possible due to the single vehicle route assumption for each depot. Constraints (1k) are standard vehicle routing flow conservation constraints. Traditional subtour elimination constraints are adjusted to include the depot choice among customers. Constraints (1l) ensure that if no depot is located within the customer subset $\mathcal{S}$, the arcs cannot form a tour. A tour can only be formed if one of the customers in the subset is a depot.

In our computational tests, instances with five to eight customers can be solved with CPLEX; Section 4 describes a heuristic for solving larger instances.

4. Adaptive Large Neighborhood Search for the PLRP

We develop an adaptive large neighborhood search (ALNS) heuristic solution approach that allows us to solve larger instances and to study relaxations of some operating assumptions of Model (1);

see Ropke and Pisinger (2006) for the first application of an ALNS heuristic. Our heuristic is based on one sequential LNS heuristic for the PLRP from Hemmelmayr (2015), with changes to account for the PLRP extensions.

In each iteration of ALNS, the current solution is partly destroyed by a destroy operator and reconstructed by a repair operator. The destroy operators remove customers from their current position on each day of visit; some destroy operators also open or close depots. Customers that are removed are put to the temporary customer pool and then reinserted by means of repair operators that select service frequency, service combinations, depot assignment and insertion position in the partial routes. Since the set of eligible depots are selected from the set of customers, opening a depot means that a customer can then serve itself. Closing a depot means that the node must be assigned to an open depot.

4.1. Operators

We use five destroy operators. In the operator *open-depot*, a node is randomly selected as a depot. The η closest customers in terms of distance are removed and put to the customer pool. Each time this operator is used, η is chosen randomly between 1 and $\left\lceil \frac{|\mathcal{V}|}{4} \right\rceil$. The operator *close-depot* closes a random depot and moves assigned customers (including the closed depot node) to the customer pool. *Swap-depot* performs a step of *close-depot* and opens a new depot. *Change-combination* removes η customers and assigns them a new random visit combination. For this operator, η is chosen randomly between 1 and $\left\lfloor \frac{|\mathcal{V}|}{2} \right\rfloor$. *Random-removal* removes η random customers, with η randomly chosen between 1 and $\left\lceil \frac{|\mathcal{V}|}{4} \right\rceil$.

Repair operators *greedy-random* and *greedy-best* insert customers in the partial solution, determining a service frequency and service combination, depot assignment, and insertion position in the routes for each customer in the customer pool. Repair operator *greedy-random* keeps the current visit combinations. The depot is assigned randomly and the insertion position is chosen that minimizes insertion cost. The operator *greedy-best* iterates for each customer insertion over all possible depots, visit schedules and insertion positions and selects the depot, visit schedule, and visit frequency that minimize total insertion cost. If this operator is used in combination with the destroy operator *change-combination*, these new visit combinations are used instead of the random ones so that *change-combination* corresponds to *random-removal.* For both insertion operators, depot capacity and vehicle capacity are chosen such that the smallest feasible capacity level is used. If demand exceeds the largest capacity level, penalty costs are added to the objective function.

4.2. Iterations

In the initial solution, a random number of depots are opened and every customer is assigned a random visit combination and a random open depot. Routes for each day are built using the

Savings Algorithm (Clarke and Wright 1964). The operators are chosen in a roulette wheel selection (Jong 1975) based on their scores. An operator's score increases when it finds a new best solution.

After each destroy and repair iteration, local search is performed for solutions that are within 5% of the objective function value of the best found solution. Local search is performed for each route separately with a 2-opt operator in a first improvement fashion. Following local search, solutions are accepted or rejected, using simulated annealing (SA) in the acceptance decision to include non-improving solutions. The starting temperature is set such that there is a 50% probability that solutions which are 5% worse than the initial solution are accepted. The temperature is decreased in every iteration. Constraints on vehicle and depot capacity may be violated during the search. Any violation is penalized and added to the objective function value. We terminate the heuristic after an iteration or time limit.

4.3. Modifications for model relaxations

The assumptions in Model (1) are consistent with operational constraints in our motivating application. Nevertheless, additional operational flexibility in our motivating setting and in other settings may be potentially cost-saving despite added complexity. The ALNS heuristic allows us to explore the following relaxations.

No node-depot assignment. We assume that the assignment of customers to depots is consistent over the planning period. This assumption facilitates the decomposition of operations by depot. With ALNS, we can relax this assumption and allow assignments to multiple depots over the planning horizon. In our motivating application, allowing a customer to be visited by vehicles originating from different depots may be possible, for example if recycling is picked up from unattended containers. In cases where collection must be scheduled, this relaxation may have a benefit of creating interaction and collaboration across more agencies, which is a goal of the network's leadership.

In the *no node-depot assignment* version, the insertion operators are adapted to allow customers to be served from multiple depots. The new version of operator *greedy-random* randomly chooses a depot separately for each visit day, instead of a single depot for the planning horizon. The operator *greedy-best* assigns a customer on each visit day to the depot that minimizes total insertion cost.

Multiple trips. We assume a vehicle is used at most once per day at an open depot, motivated by limits on vehicle usage and volunteer time, as volunteers donate their time and, potentially, usage of a personal vehicle. We relax this assumption to allow multiple trips per day at each open depot; this may be especially beneficial with a large volume of recycling and tight vehicle capacity constraints.

For the *multiple trips* version, we also adapt the insertion operators. When the insertion of a customer node in a route would result in a violation of the vehicle capacity constraint, we also consider that the vehicle goes back to the depot to unload and then starts a new trip. The position in this new trip is also considered as a potential insertion position in the insertion operators.

Multiple vehicle types. We assume that the assignment of vehicle type $k \in \mathcal{K}$ to depot $j \in \mathcal{J}$ is fixed over the planning horizon. We relax this assumption and allow different types of vehicles to serve a depot over the planning horizon. With the increasing availability of flexible and short-term rentals and ride sharing programs, using the most appropriate vehicle for each day may be helpful.

To account for the *multiple vehicle type* extension, we examine at each insertion and removal if a larger or smaller vehicle can be used, which also results in a change of the vehicle-dependent routing and fixed cost.

5. Computational Study

In the first study, we examine the PLRP variant by solving Model (1) with CPLEX and the ALNS heuristic. For this study, we develop a comprehensive set of instances, designed to highlight the impact of key parameters. In the second study, we evaluate the potential of collaborative recycling in our motivating setting. The study is based on data and observations from a local hunger relief organization. As these instances are based on our recycling application, we refer to customers as agencies throughout Section 5.

5.1. Test Case Descriptions

In this section, we describe the instances in both studies. In total, we consider 1,080 instances encompassing each combination of parameters at levels shown in Table 2, described next.

5.1.1. Instances for Problem Analysis We test small (5 and 8 agencies) and larger (10, 15 and 50 agencies) instances. We consider demand parameters that are homogeneous (*hmg*) and mixed (*mix*) across agencies. Demand satisfied per day depends on visit schedule. We consider standard schedule sets (*std*) and flexible schedule sets (*flex*) with more visit options.

The set of potential depots comprises two agencies, half of all agencies, and all agencies. The set of potential depots is chosen randomly, but is fixed for each of the three cases. That is, for each instance with half of all agencies as potential depots, the same set of agencies are the potential depots. For vehicle and depot capacity, we consider instances with either a single type (small or large) or with a choice of small and large types. We test two disposal cost-revenue functions:

Two-level function **Three-level function**

$$= \begin{cases} +\$30 & \text{if volume } < 500 \\ -\$25 & \text{if volume } \geq 500 \end{cases} \qquad = \begin{cases} +\$30 & \text{if volume } < 500 \\ -\$25 & \text{if volume } \geq 500, < 1{,}000 \\ -\$100 & \text{if volume } \geq 1{,}000 \end{cases}$$

Parameter	**Problem Analysis** (Section 5.2)	**Collaborative recycling case study** (Section 5.3)
Agency-related		
Number of agencies ($\|\mathcal{V}\|$)	5, 8, 10, 15, 50	6-34
Demand types (q)	*hmg* - all with 70 units *mix* - $\frac{1}{2}$ at 40 units, $\frac{1}{2}$ at 105 units	Estimated from agency data
Set of service schedules ($\mathcal{R}$)	*std* - {MTWThF, MWF, TTh} *flex* - {MWF, MW, TTh, MTF, MTh}	{MWF, MW, M, W} {MWF, MW, WF, M, W, F} {MWF, MW, TTh WF, M, T, W, Th, F}
Depot-related		
Number of potential depots ($\|\mathcal{J}\|$)	Two agencies Half of all agencies All agencies	Specified by agency capacity
Set of vehicle types ($\mathcal{K}_j, \forall j \in \mathcal{J}$)	*small* - 120 units *large* - 600 units *both* - 120 units and 600 units	120 units and 600 units
Set of depot types ($\mathcal{J}_j, \forall j \in \mathcal{J}$)	*small* - 750 units *large* - 3750 units *both* - 750 units and 3750 units (no-baler depot not considered)	6 sizes, including no-baler depot
Set of cost-revenue levels ($\mathcal{G}$)	2 levels 3 levels	3 levels

Table 2 Parameter values for PLRP analysis and collaborative recycling case study.

5.1.2. Experiment Design for Hunger Relief Case Study Membership in the hunger relief organization has changed throughout its history, in terms of number of locations and composition. The organization's leadership views collaborative recycling as a potential opportunity to strengthen and expand membership. The case study is designed to evaluate the feasibility of collaborative recycling at different membership levels, from a core of the most active current members to an expansion including agencies in the region that are not currently members. Figure 3 maps the 34 agencies under consideration. The six *core agencies* are those most likely to participate in a pilot program, based network activity and public visibility.

Demand, in terms of volume of cardboard produced, is known for a subset of agencies (including all core agencies) who provided data on need for recycling, and/or monthly waste removal. Demand at other agencies is estimated based on agency activity level. Most agencies publish the number of meals or pounds of food distributed and we assume that agencies with similar characteristics require a similar amount of waste removal. For agencies that did not provide demand estimates, we use known demand of agencies that are similar in the quantity of food distributed, current quantity of waste removal, and/or number of hours per week in which food is distributed. The marker size in Figure 3 represents estimated recycling demand. As demand is both a difficult parameter to estimate and a critical factor in system design, we perform sensitivity analysis on demand estimates. We vary the total quantity of cardboard collected by ±15% of baseline estimates consistently across agencies. In addition, we vary the cardboard salvage value to either \$50, \$100,

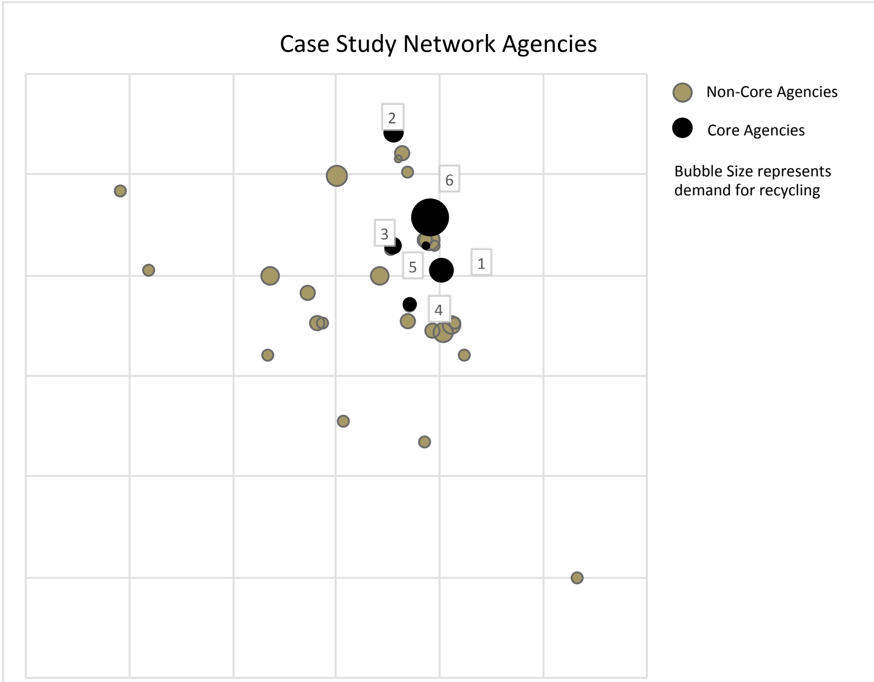

Figure 3 Case study relief agencies and relative recycling demand values

\$150, or \$200 per ton. The first and last values are outside the range of cardboard prices in the recent years but within 90% of cardboard price in the last ten years (Larimer County 2014). The value is fixed at \$100 per ton when varying other parameters.

Every agency can act as a depot without a baler, collecting only loose cardboard. The option to collect loose cardboard is modeled as a depot with no fixed cost and a recycling cost for waste collection. In order to operate a baler, additional space may be needed to load and unload cardboard; larger balers may require additional space for a forklift to move heavy bales. We identify agencies that can operate balers by physical size and activity level. Using a baler has additional requirements, such as requiring trained individuals to operate the baler; larger agencies with more activity are more likely to be able to accommodate a baler in practice. We consider two options for baler size: mini-balers and full size balers. The baler sizes differ in capacity, cost to purchase, operating cost, and ability to generate revenue. For the purchase (fixed) cost of a baler, we use prices of balers from several providers and assume that balers are financed over a year. Mini-balers create less compact bales, which sell for less per pound than full bales. The case study uses the cost-revenue profile from Table 1, in which cost-revenue levels are discretized to reflect the volume (i.e., number of bales) collected.

We consider two vehicle sizes: a personal vehicle belonging to either an agency or volunteers (with no fixed cost) and a larger vehicle leased or rented for daily use. We assume that personal vehicles are identical in capacity and travel cost, and that the larger vehicle has a travel cost and capacity five times larger than the personal vehicle. Travel cost between agencies is proportional to the travel time between agencies given by Google Maps.

5.2. Model and solution approach analysis

We solve Model (1) with CPLEX 12.6 on a quad-core AMD processor with 8 GB of RAM. The model is solved iteratively without subtour elimination constraints (1l); a set of subtour elimination constraints is added for each day and each vehicle if they are violated in a solution for any combination of day and vehicle. Identifying violated constraints requires less than 0.1% of the total computational time; the majority of time is spent iteratively re-solving the MIP model with additional constraints. For larger instances, CPLEX can not find a feasible solution in three hours.

The ALNS heuristic runs until a number of iterations or a time limit is completed. The time limit is n minutes for an instance with n agencies. The number of iterations is 1,000,000 for the 5 and 8 node instances and 5,000,000 for the 10, 15 and 50 node instances and the case study instances. The heuristic is repeated 5 times for each instance, and we report the best solution over all repetitions. The heuristic is implemented in C++, compiled with Intel compiler v12.0 and run on a cluster of Intel Xeon X5670 at 2.93Ghz.

Section 5.2.1 evaluates the heuristic performance, using exact solutions as a benchmark for smaller instances. Section 5.2.2 evaluates the impact of decisions on solution quality and solution speed. Section 5.2.3 studies the potential impact of relaxing model assumptions.

5.2.1. Analysis of the ALNS heuristic To evaluate the ALNS heuristic performance, we solve the Model (1) for the five and eight agency instances described in Table 2 and compare the results of the ALNS heuristic with these solutions. Table 3 presents the performance of the two solution approaches for the small instances, additional details are provided in the Appendix.

	Number of	CPLEX Performance		ALNS Performance	
Agencies	Instances	Gap threshold	Average final gap	Average opt gap	Max opt gap
5	176	0%	0%	0%	0%
8	178	$\leq 1\%$	0%	0%	0%
8	24	$> 1\%$	1489%	91%	535%
8	14	no feasible	—	—	—

Table 3 Solution approach performance on small instances

All five agency instances are solved to optimality by CPLEX within four minutes. The eight agency instances are divided according to the ability of CPLEX to find a feasible solution within the time limit: 178 instances for which CPLEX found a feasible solution with an optimality gap of 1% or less; 24 instances with a gap larger than 1%; and 14 instances with no feasible solution.

The final two columns of Table 3 present the gap between the ALNS solution and the CPLEX lower bound, averaged over all instances (*average opt gap*) and the maximum gap (*max opt gap*), respectively. The ALNS heuristic yields an optimality gap of at most 0.01% in instances for which CPLEX finds a feasible solution within 1% of the lower bound. For the 24 remaining eight agency instances, the average optimality gap is 91%, likely due to weak lower bounds from CPLEX. The average time to find the best solution is 3 seconds. Thus, in the following subsections, we use ALNS to solve larger instances.

5.2.2. Analysis of basic PLRP model We explore the impact of PLRP decision options on solution quality and the challenges of solving larger instances with wider ranges of choice. For each instance size, we group instances according to characteristics: vehicle capacity (large, small, choice of large and small); demand distribution (homogeneous demand or two levels of demand); and cost-revenue function (two levels or three levels). This results in 12 subsets of instances. For each subset, we introduce PLRP choices along three dimensions: depot types (small, large, or both); schedule choice (standard or flex); and potential depot sets (two, half, or all agencies $\mathcal{V} = \mathcal{J}$).

Table 4 presents the impact on the objective function for each dimension of choice, for the 12 instance subsets and averaged over the subsets. For each instance size, the rows correspond to different values of PLRP choice. The values are as defined in the second column of Table 2. (1) Depot capacity from large only to both small and large (indicated by Large $\rightarrow$ Both) and from small only to both small and large (Small $\rightarrow$ Both); (2) schedule choice from standard to flexible (Std $\rightarrow$ Flex); and (3) number of potential depots from 2 nodes to half of all nodes (2 $\rightarrow$ Half) and half of all nodes to all nodes (Half $\rightarrow$ All). The impact of increased flexibility is calculated as the average change in the objective function for instances in the subset with the PLRP choice available relative to those instances without the added PLRP choice. For example, to calculate the impact of introducing both large and small depot types over large depots only, we compare instances of the same characteristics (both subset characteristics and levels of the other PLRP choice dimensions) that have only large depots and those with both large and small, and average the improvement in the objective function when multiple depot types are available.

From the results of Table 4 and analysis of the objective function components, we make the following observations.

Depot capacity. The most significant impact on the objective function is ensuring appropriate levels of depot capacity. Particularly in smaller instances, when only the large depot types are available, costs increase dramatically with the unnecessary expense of large depots. As the number of agencies increases, solutions make use of mixed depot types, and we see the increased value in introducing larger depot types. Further, with larger vehicle capacity, using large depots only has a smaller impact on cost, as larger vehicles can make more efficient use of consolidated material.

Schedule choice. For instances with small vehicles, flexibility in schedule choice can reduce transportation costs. For most of these instances, the number of open depots does not change, but

Agencies	Vehicle Capacity / Demand / Cost-Rev Levels		*Large*				*Both*				*Small*			
	Demand		*mix*		*hmg*		*mix*		*hmg*		*mix*		*hmg*	
	Cost-Rev Levels		*2*	*3*	*2*	*3*	*2*	*3*	*2*	*3*	*2*	*3*	*2*	*3*
		Average	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
5	Large → Both	**795%**	92%	165%	50%	77%	386%	3014%	188%	566%	447%	3491%	265%	798%
	Small → Both	**0%**	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	Std → Flex	**3%**	0%	0%	0%	0%	0%	0%	4%	12%	0%	0%	4%	12%
	2 → All	**0%**	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
8	Large → Both	**518%**	57%	83%	57%	49%	315%	1839%	329%	335%	343%	2004%	357%	453%
	Small → Both	**3%**	0%	0%	0%	3%	1%	6%	3%	6%	1%	6%	4%	8%
	Std → Flex	**3%**	0%	0%	0%	0%	1%	6%	3%	4%	2%	9%	6%	7%
	2 → Half	**1%**	0%	0%	0%	0%	0%	1%	0%	3%	0%	2%	3%	6%
	Half→All	**0%**	0%	0%	0%	3%	0%	0%	0%	2%	0%	-5%	1%	3%
10	Large→Both	**415%**	49%	56%	49%	42%	317%	1336%	342%	357%	318%	1347%	343%	427%
	Small→Both	**5%**	0%	0%	0%	2%	3%	14%	6%	3%	3%	14%	6%	3%
	Std→Flex	**3%**	0%	0%	0%	0%	2%	7%	6%	0%	2%	8%	7%	0%
	2→Half	**7%**	0%	0%	0%	5%	3%	20%	4%	4%	4%	29%	4%	6%
	Half→All	**0%**	0%	0%	0%	0%	0%	0%	2%	-1%	0%	0%	2%	-2%
15	Large→Both	**216%**	44%	29%	24%	28%	292%	338%	191%	359%	292%	382%	212%	404%
	Small→Both	**3%**	0%	0%	0%	0%	3%	3%	5%	7%	3%	3%	5%	7%
	Std→Flex	**3%**	0%	0%	0%	0%	6%	3%	4%	3%	5%	5%	6%	4%
	2→Half	**1%**	0%	0%	0%	0%	0%	2%	1%	3%	0%	3%	1%	4%
	Half→All	**1%**	0%	0%	0%	0%	0%	0%	1%	2%	0%	0%	2%	3%
50	Large→Both	**162%**	27%	18%	27%	18%	231%	240%	245%	250%	220%	222%	222%	227%
	Small→Both	**7%**	1%	0%	1%	1%	8%	10%	13%	13%	8%	8%	11%	14%
	Std→Flex	**7%**	0%	0%	0%	0%	3%	10%	6%	9%	12%	13%	16%	17%
	2→Half	**2%**	2%	1%	3%	1%	3%	5%	0%	2%	2%	2%	1%	-1%
	Half→All	**1%**	0%	0%	0%	0%	0%	-2%	0%	1%	2%	2%	3%	5%

Table 4 Objective function improvement from PLRP choice

the routing is more efficient. For instances with large vehicles only, schedule choice flexibility has no impact on routing decisions, and thus no impact on overall objective functions.

Potential depot set. When the potential depot set expands, the number of depots often stays constant, but the choice of open depots changes to reduce routing costs. Particularly with small vehicles, more flexibility in depot choice yields larger improvements. Combined with schedule choice flexibility, this can have a larger impact on the objective function.

5.2.3. Impact of relaxing model assumptions Table 5 shows the impact on the objective function of relaxing model assumptions. The values in the table are the percentage improvement in total cost from the same instances with the base assumptions.

Multiple trips. In general, multiple trips are beneficial when more demand is assigned to a depot and no large vehicle is available, or it is cheaper to use the smaller vehicle and perform multiple trips. We observe large improvements with respect to the base case with only small or both vehicle types. With only large depot capacity, the possibility of multiple trips can yield improvements given the larger demand aggregation. Larger improvements can be achieved with three cost-revenue levels. Vehicle capacity limits the ability to consolidate enough to reach the largest revenue level; multiple trips increase this capacity at an increased travel cost.

Parameter Type	Parameter Value	**Multiple Trips**	**No Vehicle-Depot Assignment**	**No Agency-Depot Assignment**
Potential Depots	All	13%	2%	5%
	Half	13%	2%	4%
	Two	13%	2%	3%
Depot Capacities	Large	29%	5%	1%
	Both	4%	0%	6%
	Small	5%	0%	6%
Cost/Rev Levels	2	11%	1%	6%
	3	15%	2%	3%
Vehicle Capacity	Large	1%	0%	6%
	Both	18%	5%	3%
	Small	20%	1%	3%
Schedules	Std	12%	2%	4%
	Flex	14%	2%	4%

Table 5 Objective function improvement from relaxation of basic assumptions

No vehicle-depot assignment. The flexibility of multiple vehicles from a depot can only be exploited for instances with more than one vehicle type available (small and large vehicle types). Improvements can be observed for instances where only a large depot capacity is available, since for a small capacity depot it suffices to use the small vehicle type.

No agency-depot assignment. In this version the assumption that each agency is assigned to a depot for the entire planning period is relaxed. For instances with a large depot capacity available, the improvement is not as large as for instances with small or both depot capacities. When only small depot capacities used, depot capacity is more restrictive, so that the additional flexibility provides more room for improvement. As expected, for instances with more possible depots, we observe a greater improvement.

5.3. Collaborative Recycling Case Study

The case study of recycling for hunger relief agencies provides a novel setting for collaboration: as noted earlier, the potential benefits of recycling collaboration comes both from the ability to lower costs – or even make a profit – and from the increased participation of agencies in a joint venture. Therefore, we assess participation with two goals that may conflict: cost savings and agency participation. The goal of the case study is to address the following key question: *under what conditions is it beneficial for a network of agencies to collaboratively recycle, in terms of cost impact and participation levels?* Additionally, we evaluate the stability of resulting solutions, given the uncertainty in parameters (most critically, estimation of cardboard usage by agency and fluctuations in cardboard prices).

5.3.1. Core Agency Collaboration We begin with an analysis of core agency collaboration. We analyze the robustness of collaboration decisions for the six core agencies, relative to changes in demand and cardboard salvage value. An agency can either act as a depot for other agencies, be assigned to another agency, or act as a depot for itself without other agencies assigned to it. With six agencies, we can solve Model (1) to optimality for each combination of low, baseline, and high demand, and cardboard salvage values at 50%, 100%, 150%, and 200% of baseline values.

We evaluate solutions in terms of the optimal network configuration, the potential for revenue or cost savings, and the distribution of savings across agencies. We find, across all 12 demand and salvage cost scenarios, that cost savings are possible, but revenue generation is not given the demand ranges. Importantly, while most agencies realize cost savings across all scenarios, this is not always the case for core agency 2.

Two preferred network configurations emerged. We refer to these as the *1-depot* configuration and the *2-depot* configuration. In the *1-depot* configuration, one agency acts as a depot and all others are assigned to it, with a route visiting each agency twice per week. All *1-depot* configurations use the same depot capacity. Since the fixed cost of operating a depot is identical across agencies and there is only a single route visiting all agencies, the agency which operates the depot is arbitrary. In the *2-depot* configuration, one agency acts as a depot serving all agencies except for core agency 2. Core agency 2 acts as a small (non-baler) depot and serves itself alone.

Given that the network structure differs in whether core agency 2 is part of a route or serves itself alone, we also evaluate a 1-depot collaborative network with only 5 participants, removing core agency 2 from the network entirely. In some instances, core agency 2 does not save costs from participating in the network, and thus it may not be willing to participate in the network and share costs. Thus, we add to the configurations an additional argument denoting the number of participants (i.e., *1-depot (6)*, *2-depot (6)*, and *1-depot (5)*).

We present results for the 12 instances in Table 6 evaluating the *1-depot (6)*, *2-depot (6)* and *1-depot (5)* configurations. Each entry reports the cost per pound for all three strategies. The minimal cost per pound for each instance among configurations is shaded in each cell. Cost per pound is calculated as the total cost of recycling for the participating agencies divided by the total demand of the participating agencies. For *1-depot (5)*, cost per pound includes only the demand of the five agencies included. The value in parentheses to the right is the average savings per pound when compared with agencies serving themselves for recycling with no network. An asterisk indicates that in this solution, core agency 2 does not save from using the network configuration. In the scenarios where demand exceeds the baseline, core agency 2 is served by both itself and the depot in the *2-depot(6)* configurations.

From Table 6, we make the following observations regarding collaboration.

Solution stability. The solution value is relatively stable across network configurations. This is important, as capital decisions, such as depot location and baler acquisition, are difficult to change.

Demand multiplier	Salvage value multiplier 0.5			Salvage value multiplier 1			Salvage value multiplier 1.5			Salvage value multiplier 2		
1.15	1-depot (6) $/lb	0.13	(0.19)	1-depot (6) $/lb	0.13	(0.19)	1-depot (6) $/lb	0.13	(0.18)	1-depot (6) $/lb	0.13	(0.18)
	2-depot (6) $/lb	0.14	(0.18)	2-depot (6) $/lb	0.14	(0.18)	2-depot (6) $/lb	0.14	(0.17)	2-depot (6) $/lb	0.13	(0.17)
	1-depot (5) $/lb	0.12	(0.16)	1-depot (5) $/lb	0.12	(0.16)	1-depot (5) $/lb	0.12	(0.16)	1-depot (5) $/lb	0.11	(0.16)
1	1-depot (6) $/lb	0.15	(0.14)	1-depot (6) $/lb	0.15	(0.14*)	1-depot (6) $/lb	0.15	(0.14*)	1-depot (6) $/lb	0.14	(0.13*)
	2-depot (6) $/lb	0.14	(0.16)	2-depot (6) $/lb	0.14	(0.15*)	2-depot (6) $/lb	0.13	(0.15*)	2-depot (6) $/lb	0.12	(0.15*)
	1-depot (5) $/lb	0.14	(0.19)	1-depot (5) $/lb	0.14	(0.19)	1-depot (5) $/lb	0.13	(0.18)	1-depot (5) $/lb	0.13	(0.18)
0.85	1-depot (6) $/lb	0.18	(0.16)	1-depot (6) $/lb	0.18	(0.16)	1-depot (6) $/lb	0.17	(0.16)	1-depot (6) $/lb	0.17	(0.16)
	2-depot (6) $/lb	0.18	(0.17)	2-depot (6) $/lb	0.17	(0.17)	2-depot (6) $/lb	0.17	(0.17)	2-depot (6) $/lb	0.16	(0.17)
	1-depot (5) $/lb	0.17	(0.20)	1-depot (5) $/lb	0.16	(0.18)	1-depot (5) $/lb	0.16	(0.18)	1-depot (5) $/lb	0.15	(0.20)

Table 6 Core agency per pound costs and average savings from collaboration with varying parameter values

The solution value difference among the three configurations is minimal at less than two cents per pound per week. The preferred network structure in the solution differs across the instances, yet with the small cost difference, the choice of network configuration may be made with criteria outside of cost per pound, as discussed next.

Value of collaboration. The average savings across the agencies for each of the parameter settings is always positive, ranging from $0.13 per pound to $0.20 per pound. Over all core agencies, this translates to savings of approximately $4500 per year, with additional savings if a baler is eventually owned outright and baler lease costs do not need to be paid. With the exception of core agency 2, all agencies realize a cost savings in each scenario, with individual savings as high as $0.50 per pound for centrally located agencies with low demand. This indicates that, for most agencies, the decision to participate is robust to changes in demand and cardboard salvage value.

For core agency 2, which is not located close to the other agencies, the decision to participate is less straightforward. In 9 of the 12 instances, core agency 2 realizes a savings by participating, although for 5 of these instances, core agency 2 is essentially acting independently since it operates a depot serving only itself. However, if cost savings are shared across all agencies, this still represents a savings for core agency 2. For 3 of the 12 instances, the cost per pound increases for core agency 2 (between 1 and 4 cents per pound). As shown by the *1-depot (5)* costs per pound, the costs to the other five core agencies which participate is almost always lower with only five agencies than with all six. For core agency 2, in some instances, the costs will increase, but this increase is minimal.

These results pose a challenging dilemma for the network that is dependent on the goal of collaboration. If the goal is to create interactions across agencies, then requiring all core agencies to participate and sharing savings across agencies is a fairly stable solution. If the goal is to ensure savings for all participating agencies, then it may be desirable to omit core agency 2.

5.3.2. Network Expansion Beyond Core Agencies In this section, we explore strategies for expanding the network beyond the core agencies, with the goal of evaluating whether expanding the network can be beneficial for recycling operations. The network always contains the core agencies, with additional agencies added sequentially. We test different strategies for expansion: (1) *proximity*: adding the agency that is closest to the center of the current agencies; (2) *volume*: adding the agency with largest demand, using proximity as a tiebreaker; (3) *furthest*: adding the agency that is furthest from its closest current agencies, as a proxy for a worst case scenario; and (4) *random*: adding agencies in a random order (five different random orders are tested). In practical settings, network expansion may not be related to agency features such as demand or proximity. Agencies may participate if they are dissatisfied with their current recycling options or wish to engage more with the network. Random expansion mimics such considerations.

We find feasible solutions to the model with additional agencies, ranging from seven to 34 (all agencies in the recycling network). All instances are solved with the ALNS heuristic. We present results for the baseline demand and cardboard value parameter assumptions.

When an additional agency is added to the network, we do not add restrictions that the solution is consistent with solutions from smaller networks. That is, the depots may change when an additional agency is added, and agencies need not be assigned to the same depot as in a smaller instance. Further, agency participation may change as the network size changes. As in the core agency analysis, an agency may serve itself alone as a depot or be served by another agency, and this may change as the network size changes.

Figure 4 shows the cost per pound of demand as the network is expanded from the core agencies, using the different strategies. The network locations are the same at the left (core agencies only) and right (all 34 agencies) endpoints of graph, with costs varying in between as the network configuration varies. Cost per pound is calculated as the total cost to serve the current set of agencies divided by the total demand of the current set of agencies. Each line shows cost per pound for a specific network expansion strategy. The flat line at \$0.137 is the cost per pound with the core agencies only, as a baseline cost for comparison as the network expands.

From Figure 4, we make the following observations regarding collaboration.

Impact of expansion strategy. As expected, the choice of expansion strategy has a significant impact on the cost per pound with proximity being the best choice. With this strategy, the minimum cost is in the middle-range network size (15-25 agencies); costs increase as the network size increases. With more than 25 agencies, there is additional cost to the system from adding agencies to the network which are less centrally located.

The cost differences across the expansion strategies are mainly driven by routing cost differences. Compared to the proximity strategy, the agencies added to the network in other expansion strategies are further away from current agencies, and the cost of serving them is either the cost of adding them to existing depots' routes or the cost for them to serve themselves alone. Even with network expansion to all 34 agencies, the quantity of demand across agencies does not reach the

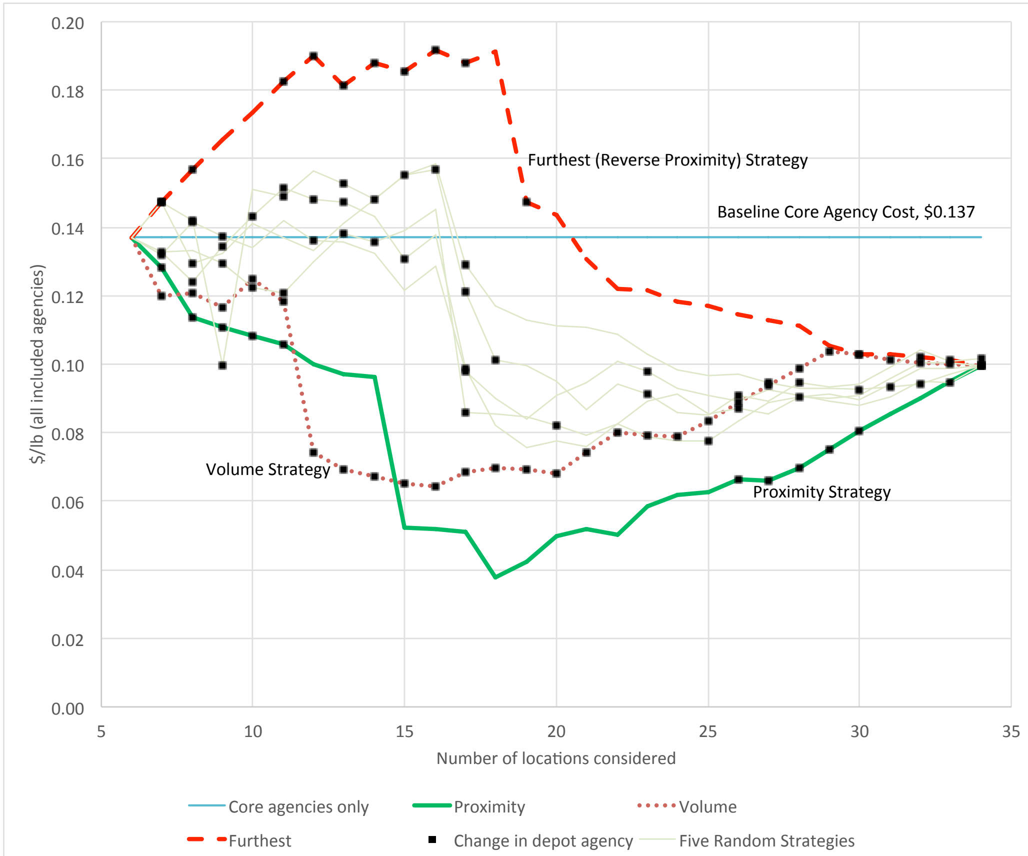

Figure 4 **Cost per pound by expansion strategy**

level to add a second large depot. As a result, distance to current agencies is a major factor in the total cost. The volume strategy performs well both because larger agencies in this setting tend to be centrally located and because a critical level of demand is reached with fewer agencies, creating revenue to offset increased distance in the objective function.

Expansion strategies beyond proximity and volume may pose greater implementation challenges; costs do not begin to decrease until a significant number of agencies participate. With fewer agencies, it may be beneficial to carefully choose an expansion strategy to decrease or maintain costs, although even for the worst expansion strategy costs are still relatively low.

Value of expansion. For all expansion strategies, expanding the network eventually saves costs beyond the initial set of agencies, with per pound costs lower than that with the core agencies alone when all agencies are included. Given the values in Figure 4, it is cost saving to expand, especially beyond sixteen to eighteen agencies, regardless of the expansion strategy. With the expanded network, each individual agency saves over their cost without the network: the cost per pound for an agency to serve itself alone as a depot is at least \$0.13 per pound.

Per unit costs assume that all agencies share the costs of recycling for the network, although in practice their participation level may vary, and costs may be shared in different ways, affecting the value of expansion. As the size of the network increases towards all 34 agencies, the last agencies added share the costs, but act alone, serving themselves as depots without being part of another depot's route.

Stability of solutions. Another goal of this study is to explore the stability of solutions (e.g., how different would the network configuration be for n agencies in the network versus $n+1$ agencies). To illustrate the stability of the expansion strategies, a dot on a line in Figure 4 indicates an instance in which the depots change from the previous instance with one fewer agency. For example, the dot on the proximity line at 25 agencies indicates that the solution found by the heuristic with 25 agencies has a different set of depots than the solution with 24 agencies. As shown in the figure, the location of the main depot changes multiple times as the network size increases. The frequency of change varies depending on the expansion strategy. The proximity strategy is more stable because new agencies are likely to be close to existing routes and can be added to routes without other changes.

Some depot changes cause large changes to the structure of routes, in terms of service frequency and the total length and number of routes. Other depot changes result in small changes, with agencies inserted into existing routes, and the new depot changing the last leg of routes. In addition to changes in depot location, we also observe changes in agency participation. The number and set of agencies that serve themselves alone (i.e., act as a depot without serving other agencies) also change as the network size increases. These changes represent smaller capital investment decisions than changing the depot, since these agencies are not served by a depot when serving themselves. Nevertheless, this may still be a complex change to manage organizationally.

To compare the impact of the changes in network design and participation, we impose two additional restrictions: (1) the large-capacity depot is fixed to one of five core agencies (excluding the distant core agency 2); and (2) all agencies are required to be part of a vehicle route as a depot or served by a depot (i.e., agencies cannot serve themselves alone). We impose (1) and (2) separately. Core agencies 1,3, 4, 5, and 6 are selected frequently as large-capacity depots, regardless of expansion strategy.

Table 7 presents the average and maximum increases in cost per pound resulting from these restrictions, across networks sizes from 7 to 34 agencies with the proximity strategy. Additionally, the table shows the percentage of solutions within 1%, 5%, or 10% of the best solution found without either (1) or (2) imposed.

From Table 7, we see that with a centrally located depot (core agencies 1, 3, 5 or 6), the objective function value is within 13% of the best solution found, and within 4% of the best solution found for core agency 5. Agency 4 performs worse as a large-capacity depot because of the greater distance

Table 7 Proximity expansion strategy, stability imposed: Fixed depot locations; All agencies on routes

	Large-Capacity Depot Location					All Participating
Large-Capacity Depot Location	**1**	**3**	**4**	**5**	**6**	
Average % Diff	3%	4%	14%	0%	4%	12%
Maximum % Diff	13%	13%	27%	4%	13%	40%
Within 1% of Best	42%	31%	8%	77%	35%	32%
Within 5% of Best	62%	54%	19%	100%	54%	36%
Within 10% of Best	81%	92%	23%	100%	92%	39%

from other agencies on average. Crucially, collaborating with any of these five fixed depots results in savings over independent operations.

Looking at the final column of Table 7, we observe that required participation is more costly. As the network size increases, the number of agencies that serve themselves alone as a depot increases, and with all 34 agencies the cost per pound without allowing this flexibility is larger than the cost per pound with only the core agencies. The participation restriction raises similar questions as in the analysis of the core agencies. The increased participation may have other benefits, such as giving agencies a larger stake in the recycling network. Thus, the goals of participation and cost savings appear to conflict in this setting and the model provides a valuable tool to evaluate such conflicts, which may be necessary in practice.

5.3.3. Guidelines and Insights The model provides decision-makers with a framework to consider collaboration network options. The solutions for this case study suggest that a recycling network can be cost saving over a range of demand and cardboard salvage values. Choosing network participants based on distance can be especially beneficial, but with a sufficiently large network, recycling revenue saves cost across a variety of network compositions. With other networks, choosing agencies based on quantity of demand may be more beneficial, especially if transportation costs are high and smaller vehicle routes with more depots could yield better solutions. It can be useful to evaluate the costs of a variety of expansion strategies, since the set of agencies that participate may not be related to costs.

When evaluating network expansion strategies, costs can be weighed against factors such as the minimum network size needed to save costs, or how the network configuration changes as the network expands. The solutions found raise challenges about how participation in the network should be defined. Better overall cost savings can be achieved if some of the agencies act alone, and it is not clear how these agencies should share costs with others, or if they should be part of another agency's collection route at the loss of some cost savings. The case study demonstrates the instability of solutions with respect to depot locations and agency participation as the network expands. Although the model is static, the case study gives insight into good potential depot locations, and can be used to evaluate the cost with a fixed set of depot locations.

6. Concluding remarks

In this paper, we study an extension to the periodic location routing problem with various types of operational flexibility, examining their individual and combined effects on managerial practices and operating costs. This problem is motivated by an application of recycling for non-profits, in which corrugated cardboard from non-profit agencies is pooled, compressed, and salvaged to generate revenue. In the proposed system, the collaborating non-profits manage the collection and operate cardboard compressors at their sites.

We propose a mixed-integer programming model for a new PLRP variant and an adaptive large neighborhood search heuristic. We find that increasing flexibility in delivery schedules can be especially beneficial with tightly capacity-constrained vehicles, which is frequently the case in our motivating non-profit application. We also find that a large set of potential depot locations can reduce total costs when combined with flexible schedules without increasing the number of, and thus fixed cost of, depot locations.

Through a case study, we present a framework for analyzing network recycling strategies in this novel setting. We find that both the level of savings and the recycling network structure are sensitive to the expansion strategy of the network. With sufficient volume of recycling, the network strategy is cost saving for all members of the network across a variety of input parameters and expansion strategies. We show that the model can give insight into the value of expanding a recycling network based on cost-related parameters. We also analyze the stability of the solution structure. The locations of depots and the structures of vehicle routes can change significantly as the recycling network expands; the cost increase from fixing some of these choices may be worthwhile for the increase in stability to the network.

This work could be extended by considering separate time scales for different decisions. In this paper, we consider the decision of the capacity and location of cardboard balers simultaneously with routing decisions over a short time horizon. The decision to lease or purchase a cardboard baler may be a much larger capital investment with the agency chosen to explicitly consider future network expansion. For example, in our case study, the most probable locations for expansion of the food pantry network are southwest of the current agencies in the network, and locating depots closer to newer members may be crucial in increasing participation and network expansion.

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7. Appendix

Table 8 Five node instances: solved with CPLEX

Depots	Vehicles	Cost-Revenue Levels	Schedules	Solution time Average	Solution time Max
2	Small Only; Large Only	2	Std	1	3
			Flex	2	8
		3	Std	1	2
			Flex	1	3
	Small and Large	2	Std	5	10
			Flex	12	33
		3	Std	6	14
			Flex	13	36
All	Small Only; Large Only	2	Std	6	12
			Flex	10	21
		3	Std	5	12
			Flex	9	25
	Small and Large	2	Std	26	46
			Flex	88	195
		3	Std	19	28
			Flex	79	178
All instances				**13**	**195**

Table 9 Eight node instances: solved with CPLEX

Depots	Vehicles	Cost-Revenue Levels	Schedules	Solution time Average	Solution time Max	Instances not solved	Opt. Gap Average
2	Small Only; Large Only	2	Std	1	3	0	0%
			Flex	3	7	0	0%
		3	Std	2	5	0	0%
			Flex	4	10	0	0%
	Small and Large	2	Std	3,607	10,801	0	34%
			Flex	19	26	2	0%
		3	Std	45	82	2	0%
			Flex	430	901	2	0%
Half	Small Only; Large Only	2	Std	12	21	0	0%
			Flex	107	160	0	0%
		3	Std	43	123	0	0%
			Flex	158	283	0	0%
	Small and Large	2	Std	425	514	3	0%
			Flex	3,873	10,801	0	63%
		3	Std	3,012	10,802	1	27%
			Flex	6,522	10,801	2	198%
All	Small Only; Large Only	2	Std	71	127	0	0%
			Flex	654	1,210	0	0%
		3	Std	266	609	0	0%
			Flex	1,587	5,239	0	0%
	Small and Large	2	Std	5,714	10,802	0	98%
			Flex	10,802	10,803	1	666%
		3	Std	8,141	10,802	0	684%
			Flex	10,801	10,804	0	2500%
All instances				**1,590**	**10,804**	**13**	**121%**