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Abstract

In this paper I propose a two-step theoretical extension of the baseline model by Diamond and Rajan (2011) and examine the amplification mechanisms when collateralized funding shocks are endogenously affected by liquidity shocks. Based on high returns on illiquid assets that are potentially available conditional on future fire sales, liquid banks increase their cash holdings by limiting term lending – a speculative motive of liquidity hoarding directly aggravated by a cash reduction due to increased haircuts on collateralized borrowing. As a result, funding liquidity shrinks steadily and credit freezes are more likely. On the other hand, illiquid banks refuse to sell more illiquid assets than necessary to meet depositors' claims – a speculative motive of illiquidity seeking indirectly amplified as fire sale prices are endogenously depressed via increased collateral requirements. Illiquid banks are forced to sell more assets, the problem of insolvency becomes more severe and market freezes are thus even more likely.

Keywords: Fire Sales; Insolvency; Market & Credit Freezes; Collateralized Borrowing

JEL Classification: G01; G12; G21

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1 Introduction

As the complexity in the financial system increased steadily over the last decades, the effects of traditional bank runs as introduced in Diamond and Dybvig (1983) fall short of explaining current financial crises. The liquidity risk arising from maturity mismatches between short-term deposits and long-term assets captures only part of the striking features of modern financial crises. External financing in short-term debt markets using long-term assets as collateral in order to keep complex investment strategies attractive, exposes financial institutions to funding risks and makes them vulnerable to collateral runs via increased haircuts by potential lenders (Brunnermeier and Oehmke (2013)). In such situations, even very small initial liquidity shocks can be amplified as market and funding liquidity are mutually reinforcing[1] (Brunnermeier and Pedersen (2009)). This liquidity spirals can cause dramatic consequences for the whole financial system leading to severe and persistent economic downturns as drastically seen in the recent financial crisis: Huge deterioration of various types of assets, dramatic rises in haircuts, forced reductions in leverage and an increased preference for liquidity hoarding accompanied by a significant drop in credit supply culminating in sudden freezes in specific asset markets[2].

In this paper I develop a theoretical extension of the baseline model by Diamond and Rajan (2011), generalizing it by introducing endogenous collateral runs affecting deep-pocketed potential buyers of illiquid assets. New amplification mechanisms on asset liquidity and potential market and credit freezes as well as the aggravated impact on leverage, liquidity hoarding and term lending are shown theoretically. Thereby I add an essential feature of deep liquidity crises to the model such that one can better understand the behaviour of a modern financial system under extreme economic stress.

The model assumes two types of financial institutions henceforth called the seller and the buyer bank. The seller bank is endowed with a quantity of assets which can be purchased only by a limited set of natural buyers with limited resources. Given the bank has to raise cash in order to meet liquidity needs before the assets mature, they can recall a fraction of their loans and/or sell assets to less informed buyer banks in the secondary market at fire sale prices[3]. The reason why assets have to be sold at prices far below fundamentals is because natural buyers of these assets are likely to be restricted by the same liquidity shock as well (Shleifer and Vishny (1992) and Allen and Gale (1994)).

As the future return on such illiquid assets can be potentially very high, the seller bank would rather hold on to those assets than selling at fire sale prices and losing the sure return if the bank

[1] There is a growing literature that finds empirical evidence for the joint interdependence between market and funding liquidity. To show some recent contributions, Ilerisoy et al. (2017) find evidence for the interaction between hedge funds' performance and market liquidity during 1994-2012. Moinas et al. (2017) provide evidence for the European Treasury Bond Market. For another very recent contribution see Chung et al. (2017).

[2] There is substantial empirical evidence for these striking features. Krishnamurthy (2010) and Gorton and Metrick (2012) provide evidence regarding asset deterioration and haircut increases. Adrian and Shin (2010a) find evidence for the procyclicality of leverage. Ivashina and Scharfstein (2010), Acharya and Merrouche (2012) and de Haan and van den End (2013) show very interesting results on lending, liquidity hoarding and fire sales.

[3] For empirical evidence see, e.g., Coval and Stafford (2007).

remains solvent conditional on liquidity shocks. This is because of the debt overhang implication of Myers (1977) and the risk-shifting[4] motive of Jensen and Meckling (1976). In case of such high returns potentially available in the future, buyer banks with sufficiently deep pockets and enough expertise in trading such illiquid assets will reduce their current term lending in order to have more cash for buying such assets cheaply in the future. According to Gale and Yorulmazer (2013), such behaviour embodies a speculative motive for liquidity hoarding. If this is the case for a significant part of the term lending market, even healthy financial institutions will be affected by reduced term lending[5], which is absent in standard dynamic asset pricing models with financially constrained investors as in Kondor (2009).

But why should buyer banks always have deep pockets and not be liquidity restricted in times of financial stress as well? What would be the overall impacts if the model incorporated collateral funding constraints on the supply side of liquidity? In order to investigate such implications, the basic model in this paper introduces a new investment strategy for buyer banks. They can use their assets as collateral to finance long-term lending or asset purchases depending on the relation between expected returns and debt capacity[6]. In contrast to Diamond and Rajan (2011), buyer banks will leverage up in economically good times to finance term lending which makes them more vulnerable to financial shocks (see, e.g., Shleifer and Vishny (2010) or Acharya and Viswanathan (2011)). Simultaneously, the funding constraint is binding in case of seller banks' premature liquidity needs as asset illiquidity and debt capacity are directly linked by discounted fire sale prices induced by the limited set of natural buyers of the underlying asset used as collateral. More precisely, as the collateral potentially has to be liquidated in the secondary market at fire sale prices as well, the lender incorporates this liquidation risk in the underlying collateral by increasing the haircut which reduces the maximum amount that can be borrowed against it – a collateral run.

Thus, the key ingredients of the extended model are illiquid asset fire sales and their endogenous impact on buyer bank's funding constraints. The paper points out two main results: First, the seller bank is forced to sell more illiquid assets and to recall more loans in order to meet premature liquidity demand if buyer banks are exposed to collateral runs via increased haircuts. Insolvency of the seller bank is more probable and market freezes become more likely. Second, the fear of future fire sales increases the return on buying illiquid assets and increases current haircuts endogenously. Hence, in order to satisfy buyer banks' speculative motive for liquidity hoarding, they will reduce their current lending even more. Buyer banks' funding liquidity shrinks endogenously and credit freezes are more likely.

The paper adds to the existing literature by providing an extended theoretical framework to model joint amplification mechanisms of market and funding liquidity on the one hand, and by

[4] The seller bank will risk-shift in the sense that they refuse to sell illiquid assets to potentially liquid buyer banks. This interpretation is analogous to that in Diamond and Rajan (2011).

[5] This implies that counterparty risk alone falls short of explaining such reductions in term lending in times of financial crises. For empirical evidence see, e.g., Schwarz (2017) who attributes most of the interest spreads between 2007-2009 to liquidity risk.

[6] According to Acharya et al. (2011), the debt capacity is defined as the maximum amount that can be borrowed using the asset as collateral and is inversely proportional to the haircut.

explaining aggravated impacts on term lending on the other hand. Additionally, this novel model explains simultaneous leverage changes endogenously as it incorporates endogenous funding constraints for both types of banks by explicitly modeling their balance sheets. This allows to explain endogenous impacts of simultaneous market and funding frictions on asset fire sales, term lending and leverage by combining traditional bank runs with endogenously induced collateral runs. The model developed is also more appropriate for examining policy interventions and regulations as well as for efficiency analyses.

The rest of this paper is organized as follows. Section 2 specifies the model setup considering the key fundamentals of the baseline model according to Diamond and Rajan (2011). In Section 3, I present the main implications in case of exogenous collateral constraints before explaining the overall impacts conditional on endogenous haircuts. Section 4 discusses the related literature and highlights the main differences before Section 5 concludes. All derivations and proofs are presented in the Appendix.

2 The Model

2.1 Agents and Basic Endowments

In the theoretical framework of the three period discrete time model there exist two types of agents called seller and buyer bank. Each of these types has a continuum of identical banks normalized to one. All agents are risk neutral and pay an interest rate of zero – discounting future cash flows is ignored. The objective of both types is to maximize the date-2 expected return.

At date 0, the seller bank is endowed with financial assets that will be worth Z at date 2 with certainty. This implies that the assets are riskless when held until maturity. Let these assets be composed of a fraction β of potentially illiquid[7] assets which can be sold to the buyer bank at either date 0 or date 1. The remaining fraction $1-\beta$ of seller banks' financial assets are loans with face value Z maturing at date 2. It is assumed that these loans can be recalled at date 1 by liquidating borrowers' projects but cannot be sold. Focusing on the liability side, the seller bank is financed with demand deposits of face value D at date 2. Moreover, it is assumed that the bank is solvent in the long run and that the difference between total assets and liabilities is equity financed.

On the asset side, buyer bank's date-0 balance sheet is composed of riskless assets that will be worth C at date 2 and of an amount θ of cash endowment that can be stored intertemporally at an interest rate of zero. Term loans to industrials are provided at date 0 and illiquid assets can be purchased at either date 0 or date 1 paying with cash. This basic endowment structure is assumed to be equity financed.

The key event happens at the intermediate date 1: Seller banks are hit by a common liquidity shock with probability q where a fraction f withdraw their deposits prematurely. To meet such future liquidity needs, the seller bank can either sell enough illiquid assets at date 0 in anticipation

[7]Assets are illiquid if they have to be sold for prices below their fundamentals. I will be more specific about the source of illiquidity in Section 2.3.

of the date-1 liquidity shock or postpone any sale until after the shock has hit. Recalling loans at the intermediate date 1 gives an additional source of liquidity for the seller bank. The buyer bank has to decide whether to invest into term lending to industrials at date 0 or into buying illiquid assets at either date 0 or date 1.

2.2 Buyer Banks' Investment Opportunities

Let us now be more specific about the dynamics of the buyer banks' two dimensional set of investment opportunities: Making term loans to industrial firms and buying illiquid assets from seller banks.

Given the sure return at date 2 per unit of cash lent at date 0 is R, the available volume of loans is defined as $I(R)$ for all $R \geq 1$, where $\partial I(R)/\partial R < 0$ and $\partial^2 I(R)/\partial R^2 > 0$ for $R > 1$ and $I(1) = \bar{I}$. This implies that an increase in the return on holding cash for buying illiquid assets reduces the amount of lending to industrials at date 0 and increases the amount of disposable cash $[\theta - I(R)]$ at both dates[8] 0 and 1. I will explain the intertemporal impact of $I(R)$ in more detail at the end of this section when prices are defined.

In contrast to Diamond and Rajan (2011), the buyer bank has the opportunity of external financing in secured debt markets by using its assets as collateral. To capture potential risks, the secured creditor (henceforth briefly called creditor) maintains a haircut $h \in [0, 1]$ on the received collateral. As the interest rate on collateralized lending is assumed to be zero[9] the amount lent to the buyer bank is $(1 - h)C$. In this simple setting, collateralization takes place if the date-2 expected return $\tilde{R}$ per unit borrowed at date 0 or at date 1 exceeds the repayment at date 2, that is, $[(1 - h)C]\,\tilde{R} \geq (1 - h)C$.

Combining both investment opportunities, the date-2 expected return on buying illiquid assets can be formulated as

$$[\theta - I(R) + (1 - h)C]\,\tilde{R}, \tag{2.1}$$

where the term in square brackets represents the disposable amount of cash for asset trading considering the liquidity constraint $\theta + (1 - h)C \geq I(R)$. As the buyer bank's investment portfolio is financed additionally via collateralized borrowing, the expected return $\tilde{R}$ spans all investment decisions entirely. This implies that the bank leverages up to the maximum permitted level constrained by the haircut, in order to finance term loans to industrials or illiquid asset purchases conditional on the yet to be determined date-2 expected returns R and $\tilde{R}$ respectively. Following this, buyer bank's (b) leverage ratio L_b can be derived as

$$L_b = 1 + \frac{(1 - h)C}{\theta + C}, \tag{2.2}$$

where the numerator represents buyer banks' outstanding debt from collateralized borrowing and

[8] Note that the interest rate of liquidity storing is assumed to be zero.

[9] This is done for clarity reasons only. Incorporating such an interest rate would only strengthen the results.

the denominator corresponds to their equity endowment. It is important to note, however, that a change in the amount of term lending corresponds to an asset swap, that is, buyer bank's leverage ratio is not affected as the total amount of assets remains directly unchanged.

Before formulating buyer bank's optimization problem to derive R and $\tilde{R}$ it is necessary to determine prices for which buyer banks are willing to purchase illiquid assets. Given the return on cash is lower than on buying illiquid assets, banks are indifferent between buying such illiquid assets at date 0 or at date 1 as long as they receive the same date-2 expected return. Let the price per unit of date-2 face value be P_0 at date 0 and P_1 at date 1. Given $P_0 \leq 1$ and $P_1 \leq 1$, the return on buying is $1/P_0$ and $1/P_1$ at date 0 and date 1 respectively. Note that assets are not traded at date 1 for less than unity in the absence of the shock. Hence, $1/P_0$ solves

$$\frac{1}{P_0} = q\frac{1}{P_1} + (1-q), \tag{2.3}$$

where the left hand side represents the return on buying illiquid assets at date 0 and the right hand side shows the expected return on buying those at date 1 given the fire sale price P_1 and the probability of the seller bank's liquidity shock q. Based on (2.3), collateralization gives the same date-2 expected return independent of the date on which collateral lending is realised. Together with the fact that the interest rate on liquidity storing is zero, collateral lending is intertemporally convertible. Up to Section 3.2, I assume that collateral lending takes place at date 0 only.

The date-2 expected return on buying illiquid assets at date 0 or at date 1 is $\tilde{R} = R = 1/P_0$ or $\tilde{R} = 1/P_1$ respectively. Hence, (2.1) indicates that the date-2 expected return on buying illiquid assets[10] is maximized by

$$\frac{\theta - I\left(\frac{1}{P_0}\right) + (1-h)C}{P_0}. \tag{2.4}$$

Based on (2.4) it should be obvious that the amount of industrial lending is reduced as long as the date-2 expected return on buying illiquid assets exceeds the sure return of lending. Thus, the minimum required rate for date-0 lending to industrials equals $1/P_0$. In addition, conditional on $[(1-h)C]\,\tilde{R} \geq (1-h)C$ and the fact that assets trade below their fundamental values in case of liquidity shocks, collateral borrowing is always profitable. As a result, the buyer bank is left with an amount $[\theta - I\,(1/P_0) + (1-h)C]$ in cash for asset trading with the seller bank.

2.3 Seller Banks' Liquidity Needs

Thus far it was not explicitly determined why assets trade at a discount to face value, i.e., why $P_0 \leq 1$ and $P_1 \leq 1$. Let us now clarify things. The key event in this framework happens at date 1 at which the seller banks face a common liquidity shock with probability q where a fraction f withdraw their deposits D prematurely. Assets are traded at a discount to face value if and only if the generated amount of cash from selling these assets to the buyer bank falls short of premature

[10] Note that the expected return on asset trading is the same for date 0 and date 1. For notational convenience (2.4) will be written as if assets are traded at date 0 only.

liquidations, that is, $\theta + (1-h)C - \bar{I} < fD$. Note that for exogenous haircuts it is assumed that $h = 0$ if $P_0 = P_1 = 1$ and hence $\theta + C - \bar{I} < fD$.

Conditional on the liquidity shock at date 1, the seller bank has the additional possibility to liquidate loans[11] prematurely. As the liquidation value of the seller bank's loan portfolio is assumed to be uniformly distributed ranging between full face value Z and zero, they will liquidate all loans up to $P_1 Z$ before selling any illiquid assets to the buyer bank at depressed P_1. The liquidation value of the seller bank's loan portfolio X is a continuous random variable with pdf

$$f_X(x) = \begin{cases} \frac{1-\beta}{Z} & \text{for } P_1 Z \leq x \leq Z \\ 0 & \text{otherwhise} \end{cases}$$

where $x \in [0, Z]$ as $P_1 \to 0$ conditional on the shock at date 1. Thus, the total value of cash generated from recalled loans can be calculated as

$$\frac{1-\beta}{Z} \int_{P_1 Z}^{Z} x dx = (1-\beta) \frac{Z}{2} \left(1 - P_1^2\right).$$

Conditional on the liquidity shock at date 1, the seller bank has to raise cash in order to meet premature withdrawals by a combination of recalling loans and selling illiquid assets for cash to the buyer bank. Hence, the seller bank's liquidity constraint is defined as

$$(1-\beta) \frac{Z}{2} \left(1 - P_1^2\right) + \left[\theta + (1-h)C - I\left(\frac{1}{P_0}\right)\right] = fD, \tag{2.5}$$

where the first term represents the cash generated by recalled loans and the second term is the amount of cash received from selling an amount $\eta_{0,1}$ of illiquid assets to the buyer bank.

Assumption 1: Subsequently it is assumed that not all of the illiquid assets have to be sold to attract all the available cash from the buyer bank, that is, $P_1 \beta Z > [\theta + (1-h)C - I(1/P_0)]$. This implies that no loans with liquidation values less than $P_1 Z$ have to be liquidated to meet the liquidity demand.

Conditional on this assumption the seller banks' cash equivalent in order to satisfy the liquidity constraint (2.5) from selling an amount $\eta_1 < 1$ of their assets to the buyer bank is $P_1 \eta_1 \beta Z = [\theta + (1-h)C - I(1/P_0)]$. Following this, it is obvious that the seller bank will never use their assets as collateral to pay out premature withdrawals, even if they had the possibility to act in this market as well. But why is this so clear? As the return on illiquid assets increases, all who have expertise in trading such assets will hold them instead of cash, that is, the seller bank will not sell more assets than necessary to satisfy (2.5). If they use the same amount of assets, the cash received from collateralized lending is lower than from selling to buyer banks, that is, $P_1 \eta_1 \beta Z(1-h) \leq \eta_1 \beta Z P_1$, where h is the same as for the buyer bank. As opposed to the buyer bank, seller banks have no

[11] Loans cannot be sold to the buyer bank or to the market as there is a lack of information technology to monitor the borrowers.

cash endowment and would therefore have to use their proceeds at date 2 to repay debt[12] to the creditor additionally. This forces the creditor to ascribe an even higher haircut h^C than to buyer banks satisfying $P_1\eta_1\beta Z(1-h^C) < P_1\eta_1\beta Z(1-h) \leq \eta_1\beta Z P_1$. In sum, the seller bank will never use their assets as collateral.

The crucial condition for the liquidity constraint (2.5) to determine the fire sale price P_1 is that the bank stays solvent conditional on the liquidity shock. To derive the solvency condition it is essential to determine seller banks' equity first[13]. I am starting with the case that asset sales occur at date 0 only. Conditional on the shock the date-2 equity will be

$$E_S^{2,0} = \left\{ \left[\beta Z - \frac{\theta + (1-h)C - I\left(\frac{1}{P_0}\right)}{P_0} \right] + (1-\beta)P_1 Z - (1-f)D \right\},$$

where the first term in square brackets is the illiquid asset endowment left after selling for P_0, the second term is the endowment left after loan liquidation and the last term shows the amount of deposits remaining[14]. Note that the first term in square brackets is obviously positive conditional on Assumption 1. In the absence of the shock (NS) the date-2 equity will be

$$E_{NS}^{2,0} = \left\{ Z - D + \left[\theta + (1-h)C - I\left(\frac{1}{P_0}\right) \right] \left(1 - \frac{1}{P_0}\right) \right\},$$

where the last term is the implicit return that the bank will forego with certainty by selling at date 0. By simple rearrangements the forgone return is $[(1/P_0) - 1]$ on $\left[(1-\beta)Z/2(1-P_1^2) - fD\right]$ units of assets.

Based on previous analyses, the date-2 equity value if assets are sold at date 1 equals

$$E_S^{2,1} = \left\{ \left[\beta Z - \frac{\theta + (1-h)C - I\left(\frac{1}{P_0}\right)}{P_1} \right] + (1-\beta)P_1 Z - (1-f)D \right\}$$

conditional on the shock and $E_{NS}^{2,1} = Z - D$ in the absence of the liquidity shock. As long as there is enough cash after date-1 withdrawals to pay out depositors who stay until date 2 the bank is solvent, that is, the discounted date-1 equity conditional on the shock $P_1 E_S^{2,1}$ is positive. Hence, the solvency condition will be

$$P_1\beta Z - \left[\theta + (1-h)C - I\left(\frac{1}{P_0}\right) \right] + (1-\beta)P_1^2 Z - (1-f)DP_1 \geq 0. \tag{2.6}$$

The solvency condition implicitly relies on the fact that the price the buyer bank is willing to pay is no less than the price the seller bank is willing to offer. As long as condition (2.6) holds, there will be active trading at date 0. In contrast, date-0 asset trading will cease if the bank becomes

[12] Note that the proceeds obviously equalize debt at date 2.

[13] Detailed formal analyses of the subsequent derivations are presented in Appendix A. The notational form of the derivations is inspired by Song (2011).

[14] $E_S^{2,0}$ denotes the date-2 equity (E) if assets are traded at date 0 conditional on the shock (S).

insolvent conditional on the shock at date 1, as the seller bank asks a higher price than the buyer bank is willing to pay. This is equivalent to a market freeze.

In addition, previous analyses allow for an explicit formalization of the seller bank's (s) leverage at date 0 or at date 1. Given that illiquid assets are sold at date 1 only, the date-1 leverage ratio $L_s^{1,1}$ will be

$$\frac{P_1\beta Z - \left[\theta + (1-h)C - I\left(\frac{1}{P_0}\right)\right] + (1-\beta)P_1^2 Z}{P_1\beta Z - \left[\theta + (1-h)C - I\left(\frac{1}{P_0}\right)\right] + (1-\beta)P_1^2 Z - (1-f)DP_1}, \tag{2.7}$$

where $L_s^{1,1} \geq 1$ as long as the bank is solvent. Note that (2.7) directly implies the date-0 leverage ratio $L_s^{0,0}$ conditional on equation (2.3) and date-0 asset trading.

2.4 Model Timeline

Figure 1 illustrates the sequence of events. In order to meet premature liquidity needs conditional on the date-1 liquidity shock, the seller bank can sell an amount of its illiquid assets to the buyer bank at date 0 or postpone any sale until after the shock has hit at date 1. As an additional source of liquidity, the seller bank can recall loans at date 1.

Buyer banks leverage up to the maximum permitted level by the haircut at date 0 and have to decide on whether to invest into loans to industrials at date 0 or into buying illiquid assets at date 0 or at date 1, depending on the date-2 expected return conditional on the liquidity shock.

The creditor covers himself against the liquidation risk in the underlying collateral by reducing the maximum amount of permitted leverage of the buyer bank conditional on the date-1 liquidity shock.

At the final date 2, seller banks' assets pay off and equity holders consume their proceeds after paying depositors. Buyer banks' investments pay off and equity holders consume their proceeds after debt to creditors is repaid.

3 Analyses and Results

In this section I analyse the overall effects of premature liquidity needs by decomposing them into direct and indirect impacts. Starting with the case of exogenous haircuts, I derive basic analytical results by focusing on the amplification mechanisms and segmented market freezes. To generalize these results and to demonstrate the impact of endogenous collateral funding constraints, haircuts have to be endogenized.

3.1 Exogenous Haircuts

Conditional on the liquidity shock at date 1 assets are traded at a discount to face value in order to meet premature withdrawals fD. As the fire sale price P_1 drops, the seller bank will liquidate all loans with liquidation values greater than $P_1 Z$ before selling any securities to the buyer bank

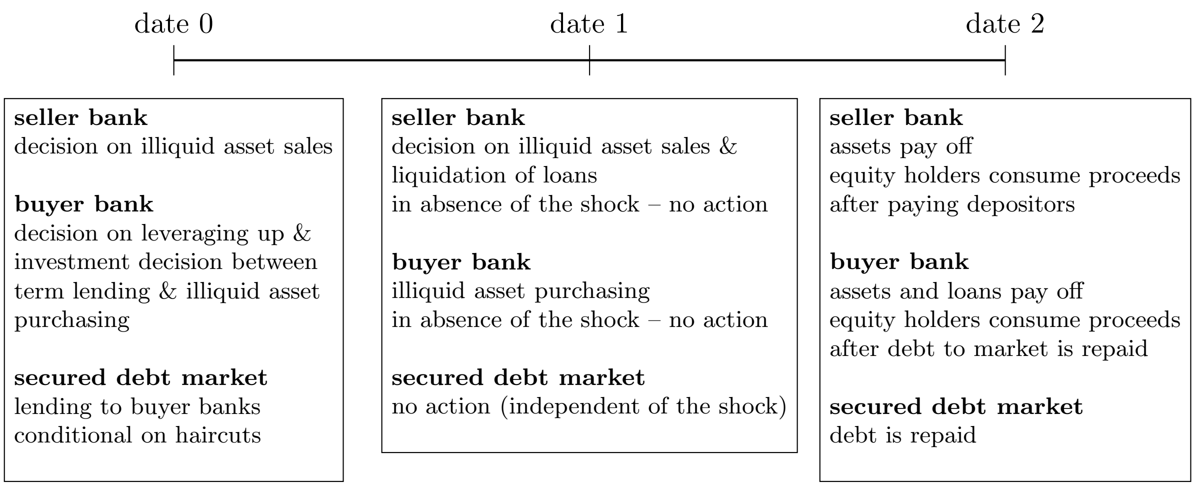

Figure 1: Sequence of Events

at depressed fire sale prices P_1. The higher returns on illiquid assets that are potentially available in the future will affect buyer banks' cash holdings as current lending to industrials shrinks - the speculative motive of liquidity hoarding. Simultaneously, however, the disposable amount of cash is reduced as haircuts are assumed to move countercyclically. Subsequently I refer to this as a funding shock[15]. This liquidity reduction due to an increased haircut depresses fire sale prices further and forces the bank to sell even more illiquid assets and to recall more loans as the lower boundary $P_1 Z$ shifts downwards. This price reduction increases the return on holding cash again and induces current lending to industrials to shrink further.

Assumption 2: To satisfy the liquidity constraint (2.5), the implied reduction of buyer banks' cash holdings due to increased haircuts is assumed to be less than the joint effect of the contemporaneous fall in lending to industrials and the increased liquidation value of loans recalled by the seller bank conditional on the shock.[16]

This implies that the seller bank must generate more cash from loan liquidation and illiquid asset sales in comparison to the reduction of buyer banks' cash holdings stemming from an increase in the haircut. Moreover, note that even if an increase of buyer banks' cash holdings in anticipation of buying illiquid assets cheaply in the future is in their own interest, it is possible that the effect on lending to industrials is lower than the effect of the funding shock. To meet premature withdrawals in such cases, the shortfall must be balanced from the seller bank by recalling more loans as long as Assumption 1 is satisfied. Hence, a funding shock affects buyer banks' investment decisions directly and seller banks' necessary reactions in order to satisfy their liquidity constraint indirectly.

Turning to solvency issues, the reduction of date-1 fire sale prices affects the solvency constraint

[15]For empirical evidence and theoretical foundations of countercyclical haircuts see, e.g., Adrian and Shin (2010b) and Adrian and Shin (2014).

[16]In case of endogenous haircuts I will show that this assumption is a necessary condition for the liquidity constraint to be satisfied.

(2.6) by a direct loss in asset valuation. The quantity of seller banks' assets as well as the volume of buyer banks' loans to industrials left after date-1 fire sales is reduced in response to the asset depreciation. A simultaneous increase in the haircut amplifies the deterioration by a reduction of buyer banks' disposable cash holdings which induces a deeper date-1 fire sale and a further reduction in date-0 lending to industrials. Following Assumption 2, buyer banks try to push their current lending to industrials down until the amount of disposable cash increases in order to satisfy their speculative motive of liquidity hoarding. This is equivalent to the implication that more illiquid assets have to be traded to meet premature liquidity demand. Overall, this makes it even harder to meet the solvency constraint (2.6) and date-0 asset trading becomes more unlikely. In the worst case, the amplified date-1 fire sale price falls below the limit for the solvency condition (2.6), that is, the seller bank is insolvent conditional on the shock, date-0 asset trading stops completely and hence a market freeze occurs.

3.1.1 Direct Impacts

Based on previous analyses it can be distinguished between two main channels induced by the liquidity shock to which I refer as direct and indirect effects. To be more specific, illiquid assets must be sold and loans have to be recalled up to date-1 fire sale prices conditional on the liquidity shock, that is, the seller bank has to delever positions. Additionally, a simultaneous funding shock reduces buyer banks' disposable cash holdings and induces a further reduction of date-1 fire sale prices. This forces the seller bank to recall more loans and to sell even more illiquid assets causing a more pronounced need for deleveraging.

Proposition 1 *(i) An increase in the fraction of premature withdrawals f, of seller bank's debt D or of the haircut h, as well as a reduction of the collateral value C or of buyer bank's cash endowment θ leads to a decrease in the date-1 fire sale price P_1 and to a reduction of seller bank's leverage $L_s^{1,1}$.*
(ii) The effect on date-1 fire sale prices P_1 as well as on seller bank's leverage $L_s^{1,1}$ triggered by an increase in the fraction of premature withdrawals f or by seller bank's debt D is amplified conditional on a simultaneous funding shock. Similarly, the effect on future fire sale prices P_1 caused by a reduced amount of buyer bank's cash endowment θ is amplified by a simultaneous funding shock.
(iii) Buyer bank's leverage L_b is reduced by an increase in the haircut h or by a reduction of the collateral value C.

Proof. See Appendix B. ■

The intuition behind the effects on date-1 fire sale prices in Proposition 1 (i) and (ii) was already discussed in detail in Section 3.1. Focusing on the change in banks' leverage ratios, two different concepts have to be considered. In contrast to the seller bank's need to reduce their leverage by a combination of illiquid asset sales and loan liquidations in order to meet premature withdrawals conditional on the date-1 liquidity shock, the buyer bank is restricted in leveraging up as the maximum permitted leverage is reduced by an increase in the haircut or by a collateral depreciation. Considering the source of the liquidity shock in more detail, it is important to note that unlike an increase in the fraction of premature withdrawals, seller bank's leverage initially increases if the amount of outstanding debt increases. Amplified by a simultaneous funding shock, illiquid asset sales and loan liquidations are forced to return to the permitted targeted leverage to avoid being cut off from external financing. Similarly, buyer banks have to reduce their leverage to counter potential collateral depreciations. This means that even if the leverage ratios of both types of banks are finally decreasing, the reasons differ fundamentally.

Finally, part (iii) immediately follows from equation (2.2) and deliberately neglects the implicit direct impacts on buyer banks' disposable amount of cash triggered by the liquidity shock, as such a balance sheet transaction represents a straightforward accounting exchange on the asset side leaving buyer bank's leverage ratio unchanged.

3.1.2 Indirect Impacts and Market Freeze

Let us now focus on the indirect effects on date-0 lending to industrials, date-0 asset trading and the seller banks' leverage conditional on the direct impacts derived in previous Section 3.1.1.

Proposition 2 *A drop in date-1 fire sale prices* P_1
(i) leads to a decrease in date-0 lending to industrials, makes date-0 asset trading more unlikely and
(ii) forces the seller bank to delever,
where the impacts in (i) and (ii) are amplified by a simultaneous funding shock.

Proof. See Appendix B. ■

The intuition behind (i) is straightforward. Date-0 lending to industrials is reduced as the return on buying illiquid assets increases which, in turn, leads to an increase in the amount of disposable cash holdings of the buyer bank. Simultaneously, the drop in date-1 fire sale prices makes it even harder to meet the solvency constraint (2.6) and date-0 asset trading becomes more unlikely, as endogenous price movements affect the likelihood of seller banks' insolvency.

In addition, an increase in the haircut due to the funding shock leads to a reduction in buyer banks' available amount of cash. In order to meet premature withdrawals, the seller bank is forced to sell more illiquid assets and to recall more loans, that is, the date-1 fire sale price decreases further. In comparison to constant haircuts, date-0 lending will be reduced more, date-0 asset trading becomes even more unlikely and hence insolvency is more likely as well. This implies that conditional on a simultaneous funding shock, a smaller liquidity shock is sufficient to jointly freeze date-0 asset trading and date-0 lending to industrials.

Focusing on the implications of (ii), this downward price spiral leads to a direct loss of the seller bank's asset net worth and hence to an increase in their leverage ratio. In order to keep the leverage constant the bank is forced to delever through illiquid asset sales and loan liquidations. However, if an additional funding shock increases the haircut simultaneously, more illiquid assets have to be sold and more loans must be recalled in order to meet premature withdrawals, as the maximum permitted leverage is reduced. Thus, the buyer bank's funding constraint directly affects the seller bank's leverage, as the cash received from illiquid asset sales shrinks. This, in turn, leads to the indirect price adjustment effect as the assets net worth is depressed and hence more illiquid assets have to be sold and more loans have to be recalled, i.e., a further need to deleverage.

3.2 Endogenous Haircuts

In this section I will generalize the results from previous sections and derive the impact of endogenous funding constraints by making haircuts endogenous. One of the key risk factors driving the size of haircuts for collateral lending is the risk of liquidation in the underlying assets, which is determined in this framework by the date-1 fire sale price. The intuition is similar to Acharya et al. (2011) who incorporate the risk of asset liquidation as one factor driving their market freeze results by using the fire-sale discount incurred in the event of liquidation of the assets by the lender/creditor. The expectation of a lower future price for the collateral will reduce the debt capacity, as the creditor incorporates the risk of selling in a falling market by a precautionary increase of the haircut.

Following Bindseil (2013) the haircut function will be

$$\begin{aligned} H(P_0) &= (1-P_0)^{(1-\delta)} \\ &= \left(1 - \frac{1}{\frac{q}{P_1} + (1-q)}\right)^{(1-\delta)} \end{aligned} \tag{3.1}$$

conditional on equation (2.3). Using (3.1), the haircut for the most liquid assets will be close to zero (as the date-1 fire sale price is close to one) and approach one for the least liquid assets (as the date-1 fire sale price will approach zero). The parameter $\delta \in [0,1]$ defines the speed of convergence over the interval of asset liquidity. For very low values of δ the haircut function converges slowly to one when asset liquidity shrinks. In contrast to that, as δ is close to one, even for very liquid assets the haircut approaches one[17]. Finally, as the date-1 liquidity shock becomes more likely, the creditor asks for a higher haircut in order to hedge against potential collateral illiquidity. Thus, the haircut function (3.1) is decreasing in P_1 and increasing in δ and q.

As the date-2 expected return on buying illiquid assets is the same for trading at date 0 or at date 1, the date-2 expected return of collateralization depends on the haircut only.[18] Hence, as long as the haircut at date 0 is lower compared to date 1, collateral lending takes place at date 0 only. To show this, haircuts on both dates have to be compared. Assuming $\delta = 0$,

$$\begin{aligned} 1-P_0 &\leq 1-P_1 \\ 0 &\leq \frac{1}{\frac{q}{P_1} + (1-q)} - P_1, \end{aligned} \tag{3.2}$$

which is obviously true for all $0 < P_1 \leq 1$ and $0 \leq q \leq 1$. Thus, the timing of events is the same as

[17] We can may think of it as the relative costs of monitoring the buyer bank.

[18] Recall that up to now it was assumed that collateral lending is realised at date 0.

in the case of exogenous haircuts (see Figure 1). Based on the haircut function (3.1) and condition (3.2) the liquidity constraint (2.5) becomes

$$(1-\beta)\,\frac{Z}{2}\left(1-P_1^2\right)+\left[\theta+(1-H(P_0))C-I\left(\frac{1}{P_0}\right)\right]=fD, \tag{3.3}$$

the solvency condition (2.6) will be

$$P_1\beta Z-\left[\theta+(1-H(P_0))C-I\left(\frac{1}{P_0}\right)\right]+(1-\beta)P_1^2 Z-(1-f)DP_1\geq 0 \tag{3.4}$$

and equations (2.2) and (2.7) are modified as

$$1+\frac{(1-H(P_0))C}{\theta+C} \tag{3.5}$$

and

$$\frac{P_1\beta Z-\left[\theta+(1-H(P_0))C-I\left(\frac{1}{P_0}\right)\right]+(1-\beta)P_1^2 Z}{P_1\beta Z-\left[\theta+(1-H(P_0))C-I\left(\frac{1}{P_0}\right)\right]+(1-\beta)P_1^2 Z-(1-f)DP_1} \tag{3.6}$$

in order to derive the buyer and seller bank's leverage ratio, respectively.

Let us now discuss the key impacts of endogenous haircuts. Conditional on the liquidity shock at date 1, the assets will trade at a discount to face value and the seller bank is forced to sell an amount of their illiquid assets and to recall enough loans in order to meet premature withdrawals. The drop in date-1 fire sale prices has a double effect on the buyer bank: On the one hand, the amount of date-0 lending to industrials is reduced as the return on buying illiquid assets increases and on the other hand, the haircut increases endogenously as the liquidity risk of the underlying collateral rises[19]. This endogenous decrease in the amount of disposable cash depresses date-1 fire sale prices further and leads to an amplified reduction in date-0 lending to industrials.

Lemma 3 *To satisfy the liquidity constraint (3.3), endogenous reductions of buyer banks' cash holdings due to increased haircuts necessarily have to be less than the joint effect of the contemporaneous fall in lending to industrials and the increased liquidation value of recalled loans conditional on the shock.*

Proof. See Appendix B. ■

As assumed in Assumption 2, the cash reduction triggered by a rise in haircuts has to be less than the joint reduction in date-0 lending to industrials and the increase in the liquidation value of recalled loans in order to pay out premature withdrawals. Lemma 3 states that this is a necessary condition for (3.3) to be satisfied in case of endogenous haircuts. As a result of a more pronounced

[19] Please note that the assets C are exposed to the same liquidity risk as those of the buyer bank. To avoid additional complexity I model this risk as a change in haircuts by leaving the net worth C constant. Formulating $(1-H(P_0))P_0C$ instead of $(1-H(P_0))C$ would only strengthen the results.

decline in buyer banks' cash holdings, more illiquid assets have to be sold and more loans have to be recalled. Hence, the seller bank is endogenously forced to reduce its leverage ratio further and the buyer banks' borrowing capacity falls as the maximum amount of permitted leverage is reduced endogenously as well. As seller banks' equity is simultaneously reduced by the joint endogenous impacts of date-1 fire sale prices and haircuts, the solvency constraint (3.4) is harder to meet and date-0 asset trading becomes even more unlikely.

Proposition 4 *Conditional on the date-1 liquidity shock*
(i) the drop in date-1 fire sale prices P_1 *and in banks' leverage* L *as well as*
(ii) the decline of date-0 lending to industrials and date-0 asset trading
are amplified by an endogenous increase of the haircut $H(P_0)$*.*

Proof. See Appendix B. ■

Proposition 4 compactly modifies Proposition 1 and Proposition 2 and emphasizes the amplifying impacts of the buyer bank's binding endogenous funding constraint conditional on the seller bank's liquidity shock on the one hand, and explains why funding liquidity steadily shrinks[20] based on the endogenous reinforcement between date-1 fire sale prices and haircuts on the other hand.

4 Related Literature

Beside Diamond and Rajan (2011), probably the closest work to the theoretical framework presented in this paper is Acharya and Viswanathan (2011). The funding problem in their model arises by the great maturity mismatch of financial institutions as the ability to roll over short-term debt is limited in case of economic downturns. The authors' main result shows that adverse asset shocks will lead to greater deleveraging and asset price deterioration when firms are highly leveraged ex ante. To highlight the differences to the model derived in this paper, I have to be more specific about the source of the funding constraint. Lemma 1 in Acharya and Viswanathan (2011) suggests that significantly highly leveraged financial institutions have to reduce their credit supply as they cannot raise new external financing conditional on the inability of debt rollovers. Thus, the funding liquidity constraint arises from credit rationing induced by risk-shifting moral hazard. In my model, however, funding liquidity is limited due to a reduction in the collateral' debt capacity triggered by fire sales. This, in turn, forces a reduction in credit supply conditional on the speculative motive for liquidity hoarding. To put it simple, credit rationing is the result of and not the condition for limited funding liquidity.

There is a substantial related literature that focuses on funding constraints via collateralized borrowing. Acharya et al. (2011) derive the debt capacity of a structured investment vehicle based on a stationary, finite, Markov chain information structure that switches randomly between pessimistic and optimistic regimes. In the worst case scenario the debt capacity can drop dramatically

[20]Since $H(P_0(P_1)) \to 1$ as $P_1 \to 0$ the occurrence of a complete endogenous funding freeze is theoretically possible. Considering Assumption 1, the funding freeze would be a limiting case though.

when the market becomes suddenly pessimistic about the collateral value and the market may freeze completely. Contrary to Acharya et al. (2011), the debt capacity in this paper is affected by fears about the value of the collateral triggered by potential future fire sales without considering rollover risks. Hence, the fear of a potential future liquidation of the collateral in a falling market determines the debt capacity. As my paper investigates the amplification mechanism between market and funding liquidity, the fundamental work of Brunnermeier and Pedersen (2009) has to be considered. The authors derive the expected change in margin requirements based on the estimated volatility of price changes in the collateral conditional on investors' information sets. Contrary to uninformed investors, informed investors observe whether the source of liquidity need is caused by a change in fundamentals or by a liquidity shock. They link the margin requirement to a (Value-at-Risk) VaR-rule, where the upper bound of the probability of default is exogenously given. The two main differences compared to my model are determined by the source of asset illiquidity and credit freezes, however. On the one hand, asset prices are derived endogenously conditional on premature liquidity needs considering the assumption of a limited set of natural buyers, whereas price shocks in Brunnermeier and Pedersen (2009) are essentially exogenous. On the other hand, credit supply in my model is jointly affected by the speculative motive for liquidity hoarding and the fear about the future value of collaterals as opposed to rollover risks. In a similar vein Gromb and Vayanos (2002) and more recently Garleanu and Pedersen (2011) investigate the impacts of exogenous funding constraints and asset liquidity, but do not explicitly consider why such collateral requirements actually are imposed (Acharya and Viswanathan (2011)).

Several interesting, methodically yet somewhat different, contributions on endogenous funding constraints one may find for example in Adrian and Shin (2010b) who derive endogenous leverage changes by a VaR-constraint in a contracting framework and demonstrate the procyclical nature of the financial system. Geanakoplos (2003) investigates endogenous collateral constraints in a general equilibrium framework where exogenous volatility changes induce endogenous leverage changes that affect asset prices. Fostel and Geanakoplos (2014, 2015) put forward the theory of leverage cycles in collateral equilibrium models and showed, among others, how leverage can be endogenously defined in equilibrium in absence of information asymmetry.

Finally, the paper is related to a broad strand of research focusing on asset liquidity, liquidity hoarding and market freezes. Bolton et al. (2011) and Malherbe (2014) model a situation of asymmetric information in the sense that the buyer cannot observe the reason why assets are sold, which leads to an inefficient premature sale and excessively high cash reserves in the former and to a self-fulfilling loop between future market illiquidity and cash storing in the latter. Recent work of Heider et al. (2015) pointed out that the interbank market can break down because of lower opportunity costs in liquidity holding arising from adverse selection. Asset illiquidity arises because of asymmetric informations between buyer and seller instead of the assumption of the limited set of natural buyers (Shleifer and Vishny (1992)). More importantly, though, endogenous funding constraints triggered by collateral runs as well as explicit leverage impacts are not considered.

Acharya and Skeie (2011) showed that the inability of rolling over short-term debt forces highly leveraged banks to reduce term lending and hoard liquidity, which is induced by agency problems associated with great maturity mismatches. High leverage ratios drive the results but are exogenous to the model and are not derived from balance sheet principles. More interestingly, though, precautionary motives are the reason for liquidity hoarding which differs from the speculative motives in this paper. Gale and Yorulmazer (2013) focus on the efficiency impacts of liquidity hoarding, where the motives for hoarding are either speculative or precautionary. The authors state that in case of sufficiently high demand for cash, illiquid institutions default whereas liquid institutions hoard liquidity instead of supplying it to the market – a source of significant inefficiency caused by market incompleteness. The funding constraint arises exogenously on the demand side, that is, illiquid bankers can either sell assets or use their assets as collateral in order to raise cash. Contrary to that, the model in this paper shows that a financial institution can be endogenously affected by funding shocks even if it is not directly exposed to them, which is, among others, probably the most interesting divergence.

5 Concluding Remarks

This paper develops an extended version of the model by Diamond and Rajan (2011). It examines the amplification mechanisms between market and funding liquidity given that ex ante liquid financial institutions are endogenously exposed to collateral runs via increased haircuts. Asset liquidity is determined by the fear of potential future fire sales which endogenously affects the maximum amount that can be borrowed using the assets as collateral. As the return on buying such illiquid assets increases, collateralized funding constrained institutions will reduce their credit supply more than those not being constrained. The explanation offered by the model developed in this article is based on the speculative motive for liquidity hoarding, that is, in order to have enough cash to gain higher returns on illiquid assets, financially constrained institutions will limit their current credit supply even more. Similarly, financial institutions having such illiquid assets on their balance sheets will risk-shift by refusing to sell more assets at fires sale prices than necessary to meet premature liquidity needs – the speculative motive for illiquidity seeking.

Combining these seemingly unrelated impacts leads to self-reinforcing liquidity and funding spirals resulting in a greater drop in fire sale prices, a more pronounced deleveraging and a more severe decline in credit supply if collateral constraints are endogenously affected by liquidity shocks. Simultaneous market and credit freezes are more likely and funding liquidity shrinks steadily. The extended model in this paper potentially provides a novel integrated explanation for some of the most important striking features in the recent financial crisis 2007/08 and is well-suited for policy and efficiency analyses. This interesting and important task, however, is left for further research.

Appendix A Derivations

Derivation of seller banks' date-2 equity and solvency condition: Consider the case the seller bank survives conditional on the date-1 liquidity shock if it sells some assets at date 0 but it fails if it has not sold assets previously. The seller banks' expected payoff from selling assets at date 0 is

$$q\left[\beta\left(1-\eta_0\right)Z+(1-\beta)P_1Z-(1-f)D\right]+(1-q)\left[\beta\left(1-\eta_0\right)Z+\beta\eta_0P_0Z+(1-\beta)Z-D\right]. \quad \text{(A.1)}$$

Substituting η_0 by $\frac{fD-(1-\beta)\frac{Z}{2}\left(1-P_1^2\right)}{P_0\beta Z}$ yields

$$q\left[\beta Z+(1-\beta)P_1Z-\left(\frac{fD-(1-\beta)\frac{Z}{2}\left(1-P_1^2\right)}{P_0}\right)-(1-f)D\right] \quad \text{(A.2)}$$

conditional on the liquidity shock and

$$(1-q)\left[Z-D+\left(fD-(1-\beta)\frac{Z}{2}(1-P_1^2)\right)\left(1-\frac{1}{P_0}\right)\right] \quad \text{(A.3)}$$

in the absence of the shock. By rearranging (2.5) such that

$$fD-(1-\beta)\frac{Z}{2}(1-P_1^2)=\left[\theta+(1-h)C-I\left(\frac{1}{P_0}\right)\right],$$

the date-2 equity values $E_S^{2,0}$ and $E_{NS}^{2,0}$ can be easily obtained from (A.2) and (A.3) respectively.

In contrast, the banks' expected payoff from selling assets at date 1 is given by

$$q\left[\beta\left(1-\eta_1\right)Z+(1-\beta)P_1Z-(1-f)D\right]+(1-q)\left[\beta Z+(1-\beta)Z-D\right]. \quad \text{(A.4)}$$

Substituting η_1 by $\frac{fD-(1-\beta)\frac{Z}{2}(1-P_1^2)}{P_1\beta Z}$ yields

$$q\left[\beta Z+(1-\beta)P_1Z-\left(\frac{fD-(1-\beta)\frac{Z}{2}(1-P_1^2)}{P_1}\right)-(1-f)D\right] \quad \text{(A.5)}$$

conditional on the shock and

$$(1-q)\left[Z-D\right] \quad \text{(A.6)}$$

in the absence of the shock. Based on previous analyses, deriving date-2 equity values $E_S^{2,1}$ and $E_{NS}^{2,1}$ from (A.5) and (A.6) is straightforward.

To specify the solvency condition explicitly please note that the seller bank is willing to sell assets at date 0 for P_0^{seller} if $qE_S^{2,0} + (1-q)E_{NS}^{2,0} \geq (1-q)[Z-D]$, that is

$$q\left[\beta Z + (1-\beta)P_1 Z - \left(\frac{fD - (1-\beta)\frac{Z}{2}1 - P_1^2}{P_0^{\text{seller}}}\right) - (1-f)D\right] +$$
$$(1-q)\left[Z - D + \left(fD - (1-\beta)\frac{Z}{2}(1-P_1^2)\right)\left(1 - \frac{1}{P_0^{\text{seller}}}\right)\right] \geq (1-q)[Z-D], \tag{A.7}$$

where the right hand side is the bank's expected payoff if it fails conditional on the shock and not having sold assets previously at date 0. Solving (A.7) for P_0^{seller} yields

$$P_0^{\text{seller}} \geq \frac{fD - (1-\beta)\frac{Z}{2}(1-P_1^2)}{fD - (1-\beta)\frac{Z}{2}(1-P_1^2) + q\left[(1-\beta)\frac{Z}{2}(1-P_1^2) - D + \beta Z + P_1 Z - \beta P_1 Z\right]}. \tag{A.8}$$

As long as the buyer bank's date-0 price from (2.3) is no less than (A.8), the seller bank is willing to sell assets at P_0^{seller}, that is

$$\frac{1}{q\frac{1}{P_1} + (1-q)} \geq \frac{fD - (1-\beta)\frac{Z}{2}(1-P_1^2)}{fD - (1-\beta)\frac{Z}{2}(1-P_1^2) + q\left[(1-\beta)\frac{Z}{2}(1-P_1^2) - D + \beta Z + P_1 Z - \beta P_1 Z\right]}. \tag{A.9}$$

Rearranging (A.9) gives the solvency condition

$$P_1\beta Z - \left[\theta + (1-h)C - I\left(\frac{1}{P_0}\right)\right] + (1-\beta)P_1^2 Z - (1-f)DP_1 \geq 0, \tag{A.10}$$

where $\left[\theta + (1-h)C - I\left(\frac{1}{P_0}\right)\right] = fD - (1-\beta)\frac{Z}{2}(1-P_1^2)$ from (2.5). Condition (2.6) is violated for every $P_0^{\text{seller}} > P_0^{\text{buyer}}$ and date-0 asset trading will cease, i.e. a market freeze. ■

Appendix B Proofs

Proof of Proposition 1: Let us start with (i) and the seller bank's slightly modified liquidity constraint

$$(1-\beta)\frac{Z}{2}\left(1-P_1^2\right) + \left[\theta + (1-h)C - I\left(\frac{1}{P_0(P_1)}\right)\right] = fD,$$

where $P_0(P_1)$ denotes P_0 as a function depending on P_1 satisfying (2.3). By total differentiation and rearranging (2.5) we find

$$\begin{aligned}\left[-(1-\beta)ZP_1 - I'\left(\frac{1}{P_0(P_1)}\right)\frac{\partial\frac{1}{P_0(P_1)}}{\partial P_0(P_1)}\frac{\partial P_0(P_1)}{\partial P_1}\right]dP_1 - Ddf &= 0\\ \frac{dP_1}{df} = \frac{D}{-(1-\beta)ZP_1 - \underbrace{I'\left(\frac{1}{P_0(P_1)}\right)}_{<0}\underbrace{\frac{\partial\frac{1}{P_0(P_1)}}{\partial P_0(P_1)}}_{<0}\underbrace{\frac{\partial P_0(P_1)}{\partial P_1}}_{>0}} &< 0,\end{aligned} \tag{B.1}$$

which is negative since the denominator is negative. Thus, an increase in f leads to a reduction in P_1. Consequently follows

$$\frac{dP_1}{dD} = \frac{f}{-(1-\beta)ZP_1 - I'\left(\frac{1}{P_0(P_1)}\right)\frac{\partial \frac{1}{P_0(P_1)}}{\partial P_0(P_1)}\frac{\partial P_0(P_1)}{\partial P_1}} < 0 \tag{B.2}$$

conditional on a change in the face value D and

$$\frac{dP_1}{dh} = \frac{C}{-(1-\beta)ZP_1 - I'\left(\frac{1}{P_0(P_1)}\right)\frac{\partial \frac{1}{P_0(P_1)}}{\partial P_0(P_1)}\frac{\partial P_0(P_1)}{\partial P_1}} < 0 \tag{B.3}$$

the effect on P_1 based on the funding shock h. Simultaneously, the effect on P_1 due to a change in C is

$$\frac{dP_1}{dC} = \frac{-1+h}{-(1-\beta)ZP_1 - I'\left(\frac{1}{P_0(P_1)}\right)\frac{\partial \frac{1}{P_0(P_1)}}{\partial P_0(P_1)}\frac{\partial P_0(P_1)}{\partial P_1}} \geq 0, \tag{B.4}$$

as $h \in [0,1]$. Finally, a change in θ affects date-1 fire sale prices P_1 such that

$$\frac{dP_1}{d\theta} = \frac{-1}{-(1-\beta)ZP_1 - I'\left(\frac{1}{P_0(P_1)}\right)\frac{\partial \frac{1}{P_0(P_1)}}{\partial P_0(P_1)}\frac{\partial P_0(P_1)}{\partial P_1}} > 0, \tag{B.5}$$

that is, a drop in the amount of cash endowment θ reduces date-1 fire sale prices P_1.

We now turn to the second part of (i). In order to derive the overall effects on seller bank's leverage ratio consider (2.7) first. Based on

$$L_s^{1,1} = \frac{P_1\beta Z - \left[\theta + (1-h)C - I\left(\frac{1}{P_0(P_1)}\right)\right] + (1-\beta)P_1^2 Z}{P_1\beta Z - \left[\theta + (1-h)C - I\left(\frac{1}{P_0(P_1)}\right)\right] + (1-\beta)P_1^2 Z - (1-f)DP_1}$$

directly follows

$$\frac{\partial L_s^{1,1}}{\partial f} = \frac{-DP_1\left\{P_1\beta Z - \left[\theta + (1-h)C - I\left(\frac{1}{P_0(P_1)}\right)\right] + (1-\beta)P_1^2 Z\right\}}{\left\{P_1\beta Z - \left[\theta + (1-h)C - I\left(\frac{1}{P_0(P_1)}\right)\right] + (1-\beta)P_1^2 Z - (1-f)DP_1\right\}^2} < 0 \tag{B.6}$$

that an increase in f forces the bank to reduce their leverage and

$$\frac{\partial L_s^{1,1}}{\partial D} = \frac{P_1(1-f)\left\{P_1\beta Z - \left[\theta + (1-h)C - I\left(\frac{1}{P_0(P_1)}\right)\right] + (1-\beta)P_1^2 Z\right\}}{\left\{P_1\beta Z - \left[\theta + (1-h)C - I\left(\frac{1}{P_0(P_1)}\right)\right] + (1-\beta)P_1^2 Z - (1-f)DP_1\right\}^2} \geq 0 \tag{B.7}$$

that a reduction in seller bank's equity induces asset sales and loan recalls to return on target leverage conditional on the solvency condition (2.6). Using the same approach we find

$$\frac{\partial L_s^{1,1}}{\partial h} = \frac{-CDP_1(1-f)}{\left\{P_1\beta Z - \left[\theta + (1-h)C - I\left(\frac{1}{P_0(P_1)}\right)\right] + (1-\beta)P_1^2 Z - (1-f)DP_1\right\}^2} \leq 0, \tag{B.8}$$

$$\frac{\partial L_s^{1,1}}{\partial C} = \frac{DP_1(1-f)(1-h)}{\left\{P_1\beta Z - \left[\theta + (1-h)C - I\left(\frac{1}{P_0(P_1)}\right)\right] + (1-\beta)P_1^2 Z - (1-f)DP_1\right\}^2} \geq 0 \tag{B.9}$$

and

$$\frac{\partial L_s^{1,1}}{\partial \theta} = \frac{(1-f)DP_1}{\left\{P_1\beta Z - \left[\theta + (1-h)C - I\left(\frac{1}{P_0(P_1)}\right)\right] + (1-\beta)P_1^2 Z - (1-f)DP_1\right\}^2} \geq 0 \tag{B.10}$$

as $f \in [0,1]$ and $h \in [0,1]$. ■

Let us now turn to part (ii). Based on (B.1), (B.2), (B.3) and (B.5) it is straightforward that

$$\left|\frac{dP_1}{df} + \frac{dP_1}{dh}\right| > \left|\frac{dP_1}{df}\right|, \tag{B.11}$$

$$\left|\frac{dP_1}{dD} + \frac{dP_1}{dh}\right| > \left|\frac{dP_1}{dD}\right| \tag{B.12}$$

the effect on P_1 triggered by f or D is amplified by a simultaneous increase in the haircut and

$$\left|\frac{dP_1}{d\theta}_{(-1)} + \frac{dP_1}{dh}_{(1)}\right| > \left|\frac{dP_1}{d\theta}_{(-1)}\right| \tag{B.13}$$

the effect of a reduction in the amount of buyer banks's cash endowment on date-1 fire sale prices P_1 is aggravated when the haircut increases simultaneously. Note that the different signs in (B.13) indicate that buyer bank's cash endowment θ is decreased by one unit and the haircut h is increased by one unit. Thus, (B.13) can be equally written as

$$\left|\frac{\partial P_1}{\partial \theta}d\theta + \frac{\partial P_1}{\partial h}dh\right| > \left|\frac{\partial P_1}{\partial \theta}d\theta\right| \tag{B.14}$$

given $d\theta = -1$ and $dh = 1$.

Conditional on (B.6) and (B.8) consequently follows

$$\left|\frac{\partial L_s^{1,1}}{\partial f} + \frac{\partial L_s^{1,1}}{\partial h}\right| > \left|\frac{\partial L_s^{1,1}}{\partial f}\right| \tag{B.15}$$

that the seller bank is forced to reduce their leverage ratio more compared to the case of constant haircuts.

Finally, we focus on the amplifying impacts on seller bank's leverage ratio triggered by an increase in the amount of outstanding debt D and a simultaneous funding shock. Let L be the target leverage in the absence and $L^h < L$ in the presence of the funding shock conditional on (B.8). Let $L^D > L$ be the leverage after an increase of the seller bank's amount of debt D conditional on (B.7). In order to return to the target (or maximum permitted) leverage, the seller bank is forced to reduce their leverage ratio even more in case of a simultaneous funding shock, that is,

$$L^D - L < L^D - L^h. \tag{B.16}$$

■

Part (iii) is straightforward. Partial differentiation of (2.2) gives $\frac{\partial L_b}{\partial h} < 0$ and $\frac{\partial L_b}{\partial C} \geq 0$. ■

Proof of Proposition 2: Given that

$$\frac{\partial I\left(\frac{1}{P_0(P_1)}\right)}{\partial P1} = \underbrace{I'\left(\frac{1}{P_0(P_1)}\right)}_{<0} \underbrace{\frac{\partial \frac{1}{P_0(P_1)}}{\partial P_0(P_1)}}_{<0} \underbrace{\frac{\partial P_0(P_1)}{\partial P_1}}_{>0} > 0 \tag{B.17}$$

and (B.3), the first part of (i) is straightforward. Note that $P_0(P_1)$ denotes P_0 as a function depending on P_1 satisfying (2.3).

In order to show the existence of a market freeze (second part of (i)) it is important to recall that the condition for a complete halt in date-0 asset trading is determined by seller bank's insolvency conditional on the shock. To derive the price dependence of the solvency constraint explicitly, (2.6) will be rewritten as

$$\begin{aligned} P_1\beta Z - \left[\theta + (1-h)C - I\left(\frac{1}{P_0(P_1)}\right)\right] + (1-\beta)P_1^2 Z &\geq (1-f)DP_1 \\ P_1\beta Z - \left[fD - (1-\beta)\frac{Z}{2}(1-P_1^2)\right] + (1-\beta)P_1^2 Z &\geq (1-f)DP_1 \end{aligned}$$

given the liquidity constraint (2.5) and hence

$$P_1\left[\beta Z + (1-\beta)\frac{Z}{2}P_1 - D\right] \geq fD(1-P_1) - (1-\beta)\frac{Z}{2}, \tag{B.18}$$

where the LHS is increasing and the RHS is decreasing in P_1. This implies that a drop in date-1 fire sale prices makes it harder to meet (B.18) and date-0 asset trading becomes even more unlikely.

Assume that for all $P_1 \geq \bar{P}_1$, (B.18) is satisfied. Suppose now that there is a combination (f^X, h^X) where $f^X > f$ and $h = h^X$ such that

$$P_1^X\left[\beta Z + (1-\beta)\frac{Z}{2}P_1^X - D\right] \geq f^X D(1-P_1^X) - (1-\beta)\frac{Z}{2}, \tag{B.19}$$

given that $P_1 > P_1^X \geq \bar{P}_1$ conditional on (B.1). This implies that the reduction in date-1 fire sale prices based on the liquidity shock is not sufficient for a market freeze. Consider a simultaneous funding shock with $h^Y > h^X$ and $P_1 > P_1^X \geq \bar{P}_1 > P_1^Y$ given (B.11), that is,

$$P_1^Y \left[\beta Z + (1-\beta)\frac{Z}{2}P_1^Y - D\right] < f^X D(1-P_1^Y) - (1-\beta)\frac{Z}{2}, \tag{B.20}$$

the seller bank becomes insolvent, no assets are traded at date-0 and hence a market freeze occurs. This result suggests that if simultaneous funding shocks are considered additionally, date-0 asset trading becomes more unlikely and a sudden stop of date-0 asset trading is more probable. ■

Let us now turn to (ii). By partial differentiation and several algebraic rearrangements, $\partial L_s^{1,1}/\partial P_1$ equals

$$\frac{-(1-f)D\left\{(1-\beta)P_1^2 Z + \left[\theta + (1-h)C - I\left(\frac{1}{P_0(P_1)}\right)\right] + I'\left(\frac{1}{P_0(P_1)}\right)\frac{\partial \frac{1}{P_0(P_1)}}{\partial P_0(P_1)}\frac{\partial P_0(P_1)}{\partial P_1}P_1\right\}}{\left\{P_1\beta Z - \left[\theta + (1-h)C - I\left(\frac{1}{P_0(P_1)}\right)\right] + (1-\beta)P_1^2 Z - (1-f)DP_1\right\}^2} < 0, \tag{B.21}$$

conditional on the solvency condition (2.6) and (B.17). Based on (B.3), the amplification of (B.21) is straightforward. ■

Proof of Lemma 3: Based on the seller bank's slightly modified liquidity constraint

$$(1-\beta)\frac{Z}{2}\left(1-P_1^2\right) + \left[\theta + (1-H(P_0(P_1)))C - I\left(\frac{1}{P_0(P_1)}\right)\right] = fD \tag{B.22}$$

we find $\theta + C - \bar{I} = fD$ for $P_0 = P_1 = 1$ given (3.1). Note that $P_0(P_1)$ denotes P_0 as a function depending on P_1 satisfying (2.3). Consider now a liquidity shock s.t. $\theta + C - \bar{I} < fD$ and assume that

$$\left|H'(P_0(P_1))\frac{\partial P_0(P_1)}{\partial P_1}C\right| > \left|I'\left(\frac{1}{P_0(P_1)}\right)\frac{\partial \frac{1}{P_0(P_1)}}{\partial P_0(P_1)}\frac{\partial P_0(P_1)}{\partial P_1}\right| + |(1-\beta)P_1 Z| \tag{B.23}$$

the reduction of buyer bank's disposable cash triggered by an endogenous increase of the haircut is greater than the joint effect of a reduction in date-0 lending to industrials and an increase in the seller bank's liquidation value of recalled loans. Conditional on Assumption 1 this would imply that the liquidity constraint is violated, that is,

$$(1-\beta)\frac{Z}{2}\left(1-P_1^2\right) + \left[\theta + (1-H(P_0(P_1)))C - I\left(\frac{1}{P_0(P_1)}\right)\right] < fD \tag{B.24}$$

and hence a contradiction. ■

Proof of Proposition 4: Let us start with (i) and the seller bank's slightly modified liquidity constraint

$$(1-\beta)\frac{Z}{2}\left(1-P_1^2\right) + \left[\theta + (1-H(P_0(P_1)))C - I\left(\frac{1}{P_0(P_1)}\right)\right] = fD, \tag{B.25}$$

where $P_0(P_1)$ denotes P_0 as a function depending on P_1 satisfying (2.3). (Note that it would be obviously possible to explicitly substitute for $H(P_0(P_1))$ conditional on (3.1). I deliberately choose the general representation for clarity reasons.)

By total differentiation and rearranging (B.25) we find

$$\frac{dP_1}{df} = \frac{D}{-(1-\beta)ZP_1 - \underbrace{H'(P_0(P_1))\frac{\partial P_0(P_1)}{\partial P_1}C}_{<0} - \underbrace{I'\left(\frac{1}{P_0(P_1)}\right)\frac{\partial \frac{1}{P_0(P_1)}}{\partial P_0(P_1)}\frac{\partial P_0(P_1)}{\partial P_1}}_{>0}} < 0, \tag{B.26}$$

which is negative since the denominator is negative according to Lemma 3. Thus, consequently follows

$$\frac{dP_1}{dD} = \frac{f}{-(1-\beta)ZP_1 - H'(P_0(P_1))\frac{\partial P_0(P_1)}{\partial P_1}C - I'\left(\frac{1}{P_0(P_1)}\right)\frac{\partial \frac{1}{P_0(P_1)}}{\partial P_0(P_1)}\frac{\partial P_0(P_1)}{\partial P_1}} < 0. \tag{B.27}$$

To show the amplification, suppose that the haircut in case (A) is more sensitive conditional on a change in date-1 fire sale prices than in case (B), that is,

$$\left| H'(P_0(P_1))\frac{\partial P_0(P_1)}{\partial P_1}C \right|_{(A)} > \left| H'(P_0(P_1))\frac{\partial P_0(P_1)}{\partial P_1}C \right|_{(B)}. \tag{B.28}$$

Conditional on (B.28) obviously follows that

$$\begin{aligned} |dP_1/df|_{(A)} &> |dP_1/df|_{(B)} \\ |dP_1/dD|_{(A)} &> |dP_1/dD|_{(B)}. \end{aligned} \tag{B.29}$$

Focusing on the second part of (i), buyer bank's leverage L_b is increasing in P_1 conditional on (3.5) and (B.17), that is,

$$\frac{\partial L_b}{\partial P_1} = \frac{-H'(P_0(P_1))\frac{\partial P_0(P_1)}{\partial P_1}C\,[\theta + C]}{[\theta + C]^2} > 0. \tag{B.30}$$

Using the same method and several algebraic rearrangements the impact on seller bank's leverage $\partial L_s^{1,1}/\partial P_1$ will be

$$\frac{-(1-f)D\left\{\left[\theta + (1-H(P_0(P_1)))C - I\left(\frac{1}{P_0(P_1)}\right)\right] + P_1\left[(1-\beta)P_1 Z + \frac{\partial I\left(\frac{1}{P_0(P_1)}\right)}{\partial P_1} + H'(P_0(P_1))\frac{\partial P_0(P_1)}{\partial P_1}C\right]\right\}}{\left\{P_1\beta Z - \left[\theta + (1-H(P_0(P_1)))C - I\left(\frac{1}{P_0(P_1)}\right)\right] + (1-\beta)P_1^2 Z - (1-f)DP_1\right\}^2} \tag{B.31}$$

which is negative conditional on (3.4), (B.17) and Lemma 3.

To show the amplification consider (B.28) for case (A) and (B). Buyer bank's leverage ratio is lower in case (A) than in (B) since the maximum amount of permitted leverage decreases, that is,

$[\partial L_b/\partial P_1]_{(A)} > [\partial L_b/\partial P_1]_{(B)}$ given (B.30). The seller bank has to delever more in case (A) than in (B) to target leverage, i.e., $\left|\partial L_s^{1,1}/\partial P_1\right|_{(A)} > \left|\partial L_s^{1,1}/\partial P_1\right|_{(B)}$ conditional on (B.31). ■

Turning to (ii), the first part is straightforward conditional on (B.17) and (B.29). Based on (B.18) and (B.29), the solvency condition is harder to meet and date-0 asset trading becomes even more unlikely. In order to show a potential market freeze according to (B.20) the condition $P_1 > P_1^X \geq \bar{P}_1 > P_1^Y$ is now endogenously induced conditional on (B.29). Hence part (ii) consequently follows. ■

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