

## ECONOMIC POLICY IN A MODEL OF ENDOGENOUS GROWTH\*

CHRISTIAN RAGACS  
MARTIN ZAGLER

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*We develop a model of endogenous growth based on the division of labour in order to discuss policy issues. The engine of growth is the worker's incentive to achieve higher income, thereby inducing an increase in the degree of specialisation. The genuine contribution of this paper is that both supply side and demand side policies may stimulate long-run economic growth, and do not only induce level shifts. On the supply side, an increase in productivity of innovative workers, alongside with investment in infrastructure, human capital, and improvements in the market setting may stimulate growth. On the demand side, we find that transfers to innovative workers, a reduction in consumption taxes, an increase of labour income taxation of the specialised workforce, and a redistribution towards specialised workers will foster economic growth.*

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**Address of the authors:**

Vienna University of Economics & B.A.  
Augasse 2-6, 1090 Vienna, Austria  
Ragacs@wu.edu and Zagler@wu.edu  
Phone: +43-1-31336-4530, Fax: +43-1-31336-727  
<http://www.wu-wien.ac.at/inst/vw1/papers/>

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## **Motivation**

A vast literature on endogenous growth theory has emerged in recent years<sup>1</sup>. Whereas most papers in the field are theory oriented, we focus on policy issues. The theory of endogenous growth has succeeded the old exogenous growth theory, which had a strand of literature on economic policy. Most recommendations lacked theoretical foundations<sup>2</sup>, focusing on investment and human capital accumulation, in clear contradiction to the suggestions of the model, which assumed that growth is exogenous in the long-run. Policy advice of the new growth theory would focus on discussing supply side policies.

The genuine contribution of this paper is that both supply side and demand side policies may stimulate long-run economic growth, and do not only induce level shifts. On the supply side, an increase in productivity of innovative workers, alongside with investment in infrastructure, human capital, and improvements in the market setting may stimulate growth. On the demand side, we find that transfers to innovative workers, a reduction in consumption taxes, an increase of labour income taxation of the specialised workforce, and a redistribution towards the specialised workers will foster economic growth.

## **The Basic Model Setup**

We develop a model of endogenous growth based on the division of labour. The central economic agent of the model is the innovative worker. In contrast to Adam Smith's concept, here it is these worker's incentive to achieve higher income that increases the degree of specialisation. These innovative workers lucrates economic rents as they invent or innovate

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<sup>1</sup> See for instance Romer (1990), Grossman, Helpman (1991), Grossman, Helpman (1994), and Barro/Sala-i-Martin (1995)

<sup>2</sup> See, e.g., Streißler/Neudeck (1997), p. 166 - 213.

new specialisations or production processes, similar to a Schumpeterian entrepreneur<sup>3</sup>. As workers are not aware that new specialisations typically increase productivity and reduce costs of production, we are facing Marshallian external economies of scale<sup>4</sup>.

The basic setting of the model is that of a closed economy. The model economy is populated by professions, innovative workers, firms and a government. There are four types of markets. The first is for the sole consumption good. The consumption good is produced by firms and demanded by both kinds of workers - professions and innovative workers - and the public sector. Firms produce this good with a homothetic, isoelastic technology, using only specialised labour as an input. These specialised labour inputs are provided by professions under the conditions of monopolistic competition on a continuum of labour markets, which can all be described by the second type of markets. Alternatively, specialised workers may earn a reservation wage.

The third is a market for new specialisations, which are demanded by workers in existing professions, who would like to form a new profession. New specialisations are competitively provided by the innovative workers at some cost, which may be interpreted as costs for teaching and training. Labour markets are completely segmented, hence innovative workers do not compete with workers in the specialised labour markets and vice-versa.

Finally there is an implicit asset market. Government expenditures are financed by taxes on consumption and all kinds of income. The government budget is assumed to be balanced. All variables are denoted in real terms.

The following chapter describes the cost minimisation problem of firms and derives the supply function of consumer goods and the demand function for specialised labour inputs

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<sup>3</sup> Schumpeter (1912)

<sup>4</sup> Marshall (1890)

offered by professions. The next chapter analysis the behaviour of professions. Taking into account the monopolistic behaviour of specialised workers, we derive the demand function for the consumption good and the demand function for new specialisations. Thereafter, we discuss the behaviour of innovative workers. We receive their supply function for new specialisations and their consumption demand function. Closing the model, these chapters are followed by a brief discussion of the public sector, the derivation of aggregate demand and of steady-state growth rates. Afterwards various policy options are discussed. The last chapter concludes the paper.

## Firms

Firms producing consumer goods are assumed to minimise costs of production  $k_t$ , with respect to every specialised labour input  $L_{i,t}$

$$\text{Min}_{L_{i,t}} k_t = \int_0^{n_t} w_{i,t} L_{i,t} di, \quad (1)$$

where  $w_{i,t}$  is the real price of specialised labour input  $L_{i,t}$ .  $t$  denotes time and  $i$  is an index of

specialised labour inputs, ranging from zero to  $n_t$ .  $n_t$  is a measure of the division of labour<sup>5</sup>.

A homothetic, isoelastic production technology is adopted,

$$y_t = \left[ \int_0^{n_t} L_{i,t}^{\frac{e-1}{e}} di \right]^{\frac{e}{e-1}}, \quad (2)$$

where  $e > 1$  is the elasticity of substitution. The production technology shows decreasing marginal productivity in every specialised labour input, constant returns to scale at a specific point-in-time (holding  $n_t$  constant), but exhibits increasing returns to scale through time (increasing the number of specialisations  $n_t$ ). As can be seen, we ignore raw capital and

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<sup>5</sup> One may interpret  $n_t$  as knowledge, which, does not depreciate over time.

unspecialised labour. However, these variables could be implemented without loss of contents. The first-order-conditions all read,

$$w_{j,t} - \mathbf{m}_t \left[ \int_0^{n_t} L_{i,t}^e di \right]^{\frac{1}{e-1}} L_{j,t}^e = 0, \quad (3)$$

where  $\mathbf{m}_t$  is the Lagrange multiplier. Multiplying both sides with  $L_{j,t}$  and integrating over all inputs results in,

$$\int_0^{n_t} w_{j,t} L_{j,t} dj - \mathbf{m}_t \left[ \int_0^{n_t} L_{i,t}^e di \right]^{\frac{1}{e-1}} \int_0^{n_t} L_{j,t}^e dj = 0. \quad (4)$$

Substitution of production technology helps to describe costs as a function of output,

$$\mathbf{k}_t = \mathbf{m}_t y_t. \quad (5)$$

Differentiating for output yields marginal costs for a single firm,

$$\frac{\mathcal{M}\mathbf{k}_t}{\mathcal{M}y_t} = \mathbf{m}_t, \quad (6)$$

where  $\mathbf{m}_t$  equals the marginal (average) cost of providing an additional unit of output, or the shadow price of additional output. So we are facing a horizontal supply schedule, equation (6). Integration over all wages, equation (3), helps to calculate  $\mathbf{m}_t$ ,<sup>6</sup>

$$\mathbf{m}_t = \left[ \int_0^{n_t} w_{i,t}^{1-e} di \right]^{\frac{1}{1-e}}, \quad (7)$$

where  $\mathbf{m}_t$  can be interpreted as an employment cost index. Note that in the case of perfect competition firms could ask a price equal to unit costs. Normalising prices in the goods market to unity, the no-profit-condition implies  $\mathbf{m}_t = 1$ .

We can then continue to derive conditional factor demand  $L_{i,t}^d$  of a single, representative firm from the first-order-condition (3) as,

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<sup>6</sup> See appendix.

$$L_{i,t}^d = w_{i,t}^{-\varepsilon} y_t. \quad (8)$$

We can therefore interpret  $\varepsilon$  as the wage elasticity of labour demand,

$$-\frac{\partial L_{i,t}^d}{\partial w_{i,t}} \frac{w_{i,t}}{L_{i,t}^d} = \varepsilon. \quad (9)$$

As the number of firms is indeterminate, we normalise the number of firms to unity, without loss of generality. Equation (6) then also represents aggregate supply,  $Y_t^s = y_t$ .

## Professions

The model is driven by the division of labour. The central agents of the model are the professional groups, which are the unique suppliers of a specific specialisation or skill  $i$ . By definition, a profession consists of a range of identical households, which maximise life-time utility  $U^i$ , with separable point-in-time felicity,  $u_s$ , which is log-linear<sup>7</sup>,

$$U^i = \int_t^\infty u_s(c_s, L_{i,s}) e^{-q(s-t)} ds = \int_t^\infty (\ln c_s - g \ln L_{i,s}) e^{-q(s-t)} ds, \quad (10)$$

where  $c_s$  denotes consumption at time  $s$ ,  $L_{i,s}$  stands for labour supply (hence this term enters with a negative sign into the felicity function),  $0 < L_{i,s} < 1$ , the intertemporal rate of time preference is  $q$ , and  $g$  describes the elasticity of substitution between labour and consumption<sup>8</sup>, subject to the intertemporal budget constraint,

$$\dot{a}_t = (1 - t^w) w_{i,t} (L_{i,t}) L_{i,t} + (1 - t^r) r_t a_t - (1 + t^c) c_t - q_t, \quad (11)$$

where  $a_t$  is nonhuman wealth („assets“),  $w_t$  is the real wage,  $r_t$  is the interest rate, and  $t^w$ ,  $t^r$ , and  $t^c$  are constant proportional tax rates (negative, if transfers) on wage income, interest earnings, and consumption. Within a profession, households interact collusively when

<sup>7</sup> This specification is a special case of the CES felicity function with unit elasticity of substitution, as used for instance by Eriksson (1996).

<sup>8</sup> In order to obtain sensible results,  $0 < g < (e-1)/e < 1$ , is required.

supplying labour. Hence the specific professional group is a wage setter on their specific factor market. We assume that they behave as monopolists for the sake of simplicity. Membership within a profession depends upon the knowledge of a specific skill, whose acquisition is costly. We assume that  $q_t$  is the price of the purchase for the entire professional group, shared amongst its members. Whilst paying for the specialisation only once, agents may draw upon the skill for the entire time being.

Optimisation with respect to consumption, labour supply, asset demand, and the Hamiltonian multiplier  $I_t$  yields the following first-order-conditions,

$$\frac{\partial U^i}{\partial c_t} = \frac{1}{c_t} - I_t(1 + t^c), \quad (12)$$

$$\frac{\partial U^i}{\partial L_{i,t}} = -\frac{g}{L_{i,t}} + (1 - t^w) [w'_{i,t}(w_{i,t})L_{i,t} + w_{i,t}(L_{i,t})] I_t, \quad (13)$$

$$\frac{\partial U^i}{\partial a_t} = q_t - \dot{I}_t = r_t(1 - t^r) I_t, \quad (14)$$

and the budget constraint (11) with equality. The first equation states that the consumption equivalent of an additional unit of the asset must equal the marginal utility from consumption. The second equation states that the net wage in terms of utility must equal the marginal utility from leisure. The third is the dynamic equation of motion.

From equation (12) and (14) follows in optimum,

$$\frac{\dot{c}_t}{c_t} = -\frac{\dot{I}_t}{I_t} = (1 - t^r)r_t - q, \quad (15)$$

the Keynes-Ramsey-Rule. Substituting (12) into (13) yields,

$$\frac{e-1}{e}(1 - t^w)w_{i,t}L_{i,t} = g(1 + t^c)c_t, \quad (16)$$

where the left hand side describes the marginal revenue from supplying an additional unit of specialised labour, and  $(e-1)/e$  describes the wage decline due to an increase in supply. The



right hand side represents the corresponding increase in utility from consumption necessary to equate the loss of leisure.

From the budget constraint (11) and the Keynes-Ramsey-Rule (15) follows, after some algebraic transformations<sup>9</sup>, the consumption demand function of the individual professional group,

$$c_t = \frac{a_t}{1 + t^c} \frac{q(e-1)}{e - e g^{-1}}. \quad (17)$$

Consumption is an increasing function of wealth and a decreasing function of the consumption tax, which seems intuitive.

As mentioned above, professions earn economic rents. They can be calculated as the difference to the alternative labour income, determined by the exogenous reservation wage<sup>10</sup>  $\hat{w}_t$  and the labour supply  $\hat{L}_t$ , which follows from the subsequent first order condition,

$$\frac{\partial U^i}{\partial \hat{L}_t} = -\frac{g}{\hat{L}_t} + \hat{w}_t I_t \quad \forall i. \quad (18)$$

Eliminating the Lagrange multiplier by combining equation (13) and (18), and substituting equation (9), we obtain,

$$\frac{(1 - t^w) w_{i,t} L_{i,t}}{\hat{w}_t \hat{L}_t} = \frac{w_{i,t}(L_{i,t})}{w'_{i,t}(L_{i,t}) L_{i,t} + w_{i,t}(L_{i,t})} = \frac{e}{e-1}, \quad (19)$$

which is larger than unity. Equation (19) describes the additional income of supplying one unit of specialised labour in relation to the reservation income, also representing individual demand for an additional specialisation. This concludes the discussion of the individual professional groups.

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<sup>9</sup> See appendix.

<sup>10</sup> This reservation wage may be interpreted as the intrinsic utility value of household production in units of income. Therefore taxes do not interfere in the decision problem.

Next we focus on the deepening of the division of labour itself. A more recent specialisation is adopted by individuals leaving existing specialisations in order to obtain higher life-time income. This leads to a reduction of income in the more recent specialisations, whilst at the same time increasing wages of existing specialisations, as the labour supply declines.

In every instant of time, a specific quantity of new specialisations denoted by  $\dot{n}_t$  is introduced to the economy, which are supplied at the identical price  $q_t$ . Market revenue at a point in time equal the discounted stream of excess earnings of new specialised workers, indexed from  $n_t$  to  $n_t + \dot{n}_t$ ,

$$\dot{n}_t q_t = \int_{n_t}^{n_t + \dot{n}_t} \int_t^{\infty} \left[ w_{i,s}(L_{i,s})L_{i,s} - \hat{w}_s \hat{L}_s \right] e^{-\int_t^s r_v(1-t^v)dv} ds di \quad (20)$$

Integrating over time and specialisations, using the cost function (5) and the Keynes-Ramsey-Rule (15), the demand function for new specialisation equals<sup>11</sup>,

$$\dot{n}_t = \frac{Y_t^s}{q_t} \frac{1-q}{qe}, \quad (21)$$

which is positive since  $0 < q < 1$ . The demand for new specialisations increases with aggregate supply, and declines with an increase in its price.

### Innovative Workers

New specialisations are typically obtained during vocational training, apprenticeships or formal training. In all of these cases, those who engage in training and research face opportunity costs in the form of foregone wages. This trade-off is subject to economic rationality on behalf of the agents. Instead of adding this decision to the optimisation

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<sup>11</sup> See appendix.

problem of the professions, we assume that new specialisations are introduced by specific agents, the innovative workers, and offered to professions under competitive market conditions.

Assume that the innovative worker can add incrementally to the existing number of specialisations  $n_t$  by devoting  $\tilde{L}_t$  units of labour,

$$\dot{n}_t = f_t n_t \tilde{L}_t. \quad (22)$$

The preceding technology assumes that the current division of labour has a positive effect on future innovations.  $n_t$  is assumed to enter linearly in the production function<sup>12</sup> for convenience<sup>13</sup>.  $f_t$  is a productivity term<sup>14</sup>. In analogy to a large number of contributions in the literature, we assume that productivity can be influenced by either fostering human capital  $H_t$ , or by improving the working conditions of the innovative workers via infrastructure  $I_t$  or the institutional setting  $S_t$  of the markets she is operating in<sup>15</sup>. For simplicity, and without loss of economic intuition, we assume the special functional form,

$$f_t = H_t^a I_t^b S_t^{1-a-b} \quad (23)$$

The innovative workers maximise life-time utility,

$$\tilde{U} = \int_t^{\infty} u_s(\tilde{c}_s, \tilde{L}_s) e^{-q(s-t)} ds = \int_t^{\infty} (\ln \tilde{c}_s - g \ln \tilde{L}_s) e^{-q(s-t)} ds, \quad (24)$$

where tilde denotes variables for the innovative workers, subject to technology (22) and an intertemporal budget constraint,

$$\dot{\tilde{a}}_t = (1 - t^n) q_t \tilde{h}_t + (1 - t^r) r_t \tilde{a}_t - (1 + t^c) \tilde{c}_t, \quad (25)$$

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<sup>12</sup> Grossman / Helpman (1991) show that more general specifications do not change general results.

<sup>13</sup> Not taking account of his own contribution to the number of specialisations, the model exhibits an additional externality, which implies that the division of labour is suboptimally low.

<sup>14</sup> An alternative interpretation for  $f_t$  is a success probability of innovation.

<sup>15</sup> All of these parameters can be exogenously influenced by policymakers via investment into education, infrastructure or changes in legislation.

where  $q_t$  is the price of a new specialisation,  $\hat{n}_t$  is the quantity of new innovations supplied, and  $t^n$  is the tax or transfer (if negative) on the returns from additional specialisations. Innovative workers receive income from wealth and the revenues of new innovations, and spend it on consumption and wealth accumulation. The first-order conditions with respect to consumption and wealth are equivalent to equations (12) and (14). Maximising with respect to innovative labour supply yields,

$$\frac{\partial \tilde{U}}{\partial \tilde{L}_t} = -\frac{g}{\tilde{L}_t} + (1-t^n)q_t f_t n_t l_t. \quad (26)$$

Combining equation (26) with the equivalent of (12) and substituting into the budget constraint, using the Keynes-Ramsey-Rule, adopted for innovative workers consumption (15), yields after some rearrangements the consumption demand function,

$$\tilde{c}_t = \frac{\tilde{a}_t}{1+t^c} \frac{q}{1-g}, \quad (27)$$

Substituting back into equation (25) gives,

$$q_t = \frac{q \tilde{a}_t}{(1-g)\tilde{L}_t} \frac{1}{(1-t^n) f_t n_t} = \frac{b_t}{(1-t^n) f_t n_t}, \quad (28)$$

where  $b_t$  is the alternative non-working income, with  $q \tilde{a}_t / (1 - F_t)$  representing the asset yield in utility terms per unit of leisure  $F_t = (1 - \tilde{L}_t)$ , and where  $1 - g$  evaluates this expression in consumption units. We can interpret (28) as the supply function for new specialisations.

### Closing the Model

Consumption has been in terms of individual professional groups (17) or individual workers (27). From now on, the number of innovative workers is normalised to unity, hence individual demand and demand of all innovative workers are formally identical. Aggregating demand of professions yields  $n_t c_t$ . Note that at every point in time there exists the incentive

to create new specialisations, as the demand for new professions is greater than zero for any positive price. In optimum every innovative worker will receive a wage above the reservation wage, the goods demand function of innovative workers is therefore given by equation (27). A similar argument holds for professional groups. Marginal revenue for labour supply at every point in time is greater than the opportunity costs, so the goods demand function of professional groups is described by equation (17). For the following, we denote aggregate demand by  $C_t = n_t c_t + \bar{c}_t$ . We also obtain aggregate wealth as the sum of wealth holdings by professions and innovative workers,  $A_t = n_t a_t + \bar{a}_t$ .

The government budget is assumed to be balanced at every instant. This implies that government spending,  $G_t$ , is financed by taxes on consumption, innovation, asset and labour income,

$$G_t = I_t + H_t + G_t^c = t^c C_t + t^n q_t \dot{n}_t + t^r r_t A_t + t^w \int_j^n w_{i,t} L_{i,t} dj. \quad (29)$$

Government spending is divided into public infrastructure investment, public investment in human capital formation and nonproductive government consumption expenditures. The latter can be interpreted as a negative flat tax. We use changes in government consumption as the unbiased benchmark for policy conclusions. The balanced budget condition endogenises government spending.

After some manipulations, we are able to formulate aggregate demand  $Y_t^d$  on the goods market as follows,

$$Y_t^d = C_t + G_t = \frac{qA_t}{(1-t^c)(1-g)} + \frac{g}{(1-g)} \frac{q(A_t - \bar{a}_t)}{(1-t^c)(e-eg-1)} + G_t, \quad (30)$$

where we assume that any kind of government activity uses the sole consumption good. The goods market supply function (6) and equation (30) determine goods market equilibrium, which is our first type of market. Equilibrium in the second type of markets are well described by a an interval of  $n_t$  demand functions for specialised labour (8) and the

according monopoly supply of professions. The market for new specialisations is described by its demand (21) and supply function (28). Finally the asset market, which is our fourth type of market, is in equilibrium due to Walras' law.<sup>16</sup>

The engine of growth in this paper is the division of labour as seen from equation (22). The growth rate of specialisations is given by  $f_t \tilde{L}_t$ . The increase in the division of labour leads to increasing returns to scale in the goods market production function (2). Assuming identical quantities of specialised labour inputs, conventionally defined total factor productivity ( $Y_t/n_t L_t$ ) is given by  $n_t^{1/(e-1)}$ , which increases in specialisations. This implies that the growth rate of output is given by  $1/(e-1)$  times the growth rate in the division of labour, despite a constant quantity of factor inputs. The growth rate of government spending is driven by the growth rate of consumption, as can be seen from the government budget constraint (30).<sup>17</sup>

### Economic Policy

Growth is induced in the market for specialisations, with quantities  $\hat{n}_t$  and prices  $q_t$ , so this is the crucial market for policy issues. Equilibrium in the market for specialisations is given by the demand function for specialisations (21), derived from marginal revenue, and its respective supply function (28), derived from marginal costs. Hence the relationship of marginal cost and revenue determines growth,

$$MC = \frac{b_t}{(1-t^n)f_t n_t} \begin{matrix} > \\ < \end{matrix} \frac{Y_t}{\hat{n}_t} \frac{1-q}{qe} = MR. \quad (31)$$

Growth can be stimulated either by a reduction in marginal costs, a downward shift of the

<sup>16</sup> It is beyond the scope of this paper to explicitly specify the asset market supply and demand functions.

<sup>17</sup> We find that labour income is proportional to consumption, equation (17). The same holds for innovation revenues by looking at equations (18), (22) and (28), and asset yield, from equation (17).

MC schedule, or by an increase in marginal revenue, corresponding to a shift of the MR schedule to the left. When marginal cost exceed marginal revenue, the incentive to innovate declines, thereby reducing the equilibrium rate of growth, or vice-versa. Rearrangement of equation (31) yields,

$$h_t \equiv \frac{\dot{n}_t}{n_t} = (1-t^n) \frac{1-q}{qe} \frac{f_t Y_t}{b_t}. \quad (32)$$

An increase in patience evidently increases the equilibrium growth rate, whilst an increase in the elasticity of substitution decreases it, as rents for the particular specialisation decline.

From equation (32), we observe that policy can influence the growth performance of the economy by changing either the tax on revenue from innovation, or productivity of the innovative workers, or aggregate demand. As aggregate demand itself depends upon government spending and the various tax rates, any shift in policy would also alter aggregate demand. The linearity of the model suggests that in most cases we should only investigate the direction and magnitude of marginal changes in policy.

The effect of an increase in the tax on innovation revenue is twofold. First, it directly reduces the innovation rate, as seen from equation (32). Second, it increases government revenue, which is assumed to lead to an increase in government spending. The aggregate effect is as follows,

$$\frac{dh_t}{dt^n} = \frac{\partial h_t}{\partial t^n} + \frac{\partial h_t}{\partial Y_t} \frac{\partial Y_t}{\partial G_t} \frac{\partial G_t}{\partial t^n} = \frac{h_t}{Y_t} \dot{n}_t q_t - \frac{h_t}{1-t^n}.$$

Substituting equation (21) in the above expression, we see that a reduction in the tax on innovation revenue has a positive effect on the rate of growth if and only if,

$$t^n > \frac{1-q(e+1)}{1-q},$$

which is strictly below unity. This implicitly defines the growth maximising tax rate on innovation revenue. Another obvious economic policy<sup>18</sup> is to increase the success probability of the innovative workers, since

$$\frac{\mathcal{H}_t}{\mathcal{F}_t} = \frac{h_t}{f_t} > 0.$$

Despite different possible policy strategies to increase the probability of success, the concept behind all of them is to try „picking the winning ideas“. As it is difficult if not impossible for public institutions to chose particular "ideas" to subsidise, one may attempt to increase innovation by improving research productivity instead.

We have previously argued<sup>19</sup> that the institutional setting, human capital, or public infrastructure exhibit positive effects on productivity. Increasing any of these parameters will augment productivity, hence reduce marginal costs in innovation, and thereby, as indicated above, foster economic growth. Examples for an improved institutional setting for the innovative workers are policies that facilitate the transmission from innovation to implementation of a new specialisation, and the easier access for new professions in the labour markets due to a dismantling of craft legislation. From equations (23) and (32), it can be shown that the growth elasticity of the institutional setting is given by,

$$\frac{\mathcal{H}_t}{\mathcal{S}_t} \frac{S_t}{h_t} = 1 - a - b.$$

In order to achieve positive growth effects, these growth benefits have to exceed the cost of changing the institutional setting.

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<sup>18</sup> In the following discussion of supply side policies, we increase productive government spending,  $H_t$  and  $I_t$ , by reducing consumptive government spending,  $G_t^C$ , holding the budget balanced.

<sup>19</sup> See equation (23).



To foster human capital accumulation, many different policies as mentioned in the tradition of endogenous growth theory seem to be possible and suggestive. The growth elasticity of human capital is given by,

$$\frac{\mathcal{H}_t H_t}{\mathcal{H}_t h_t} = a ,$$

which is unambiguously positive.

In our model the third channel to increase economic growth due to an increase of productivity is to improve the production conditions of the innovative workers via infrastructure investment, financed by a reduction of other government expenditures, with the growth elasticity of infrastructure investment equalling,

$$\frac{\mathcal{H}_t I_t}{\mathcal{H}_t h_t} = b ,$$

which is positive. Note that according to our assumptions about technology in the innovative sector, equation (23), all elements influencing economic growth on the supply side, such as human capital, infrastructure expenditure and the institutional setting of the markets exhibit diminishing marginal productivity. Hence all efforts of unbalanced supply side growth policy will be less and less effective.

Whilst the supply side effects above are well known, the model does exhibit potential for demand side policies. Note that the monopolistic competition setting in the specialised labour markets implies that output is below its optimal level. Policies that increase output will improve welfare and foster economic growth, as seen from equation (32). Whilst traditional Keynesian models would only argue for level shifts of demand stimulating policies, in our model growth effects are induced by the positive effect of aggregate demand on rents in innovation.

In contrast to the previous analysis, here we focus on the change of nonproductive government consumption only. An increase in the consumption tax in order to finance an increase in government spending has an unambiguously negative effect on economic growth,

$$\frac{\partial h_t}{\partial t^c} = \frac{\partial h_t}{\partial Y_t} \frac{\partial Y_t}{\partial t^c} = \frac{\partial h_t}{\partial Y_t} \left[ \frac{\partial Y_t}{\partial G_t} \frac{\partial G_t}{\partial t^c} + \frac{\partial Y_t}{\partial C_t} \frac{\partial C_t}{\partial t^c} \right] = \frac{h_t}{Y_t} \left[ C_t - \frac{C_t}{1-t^c} \right] = -\frac{h_t}{Y_t} C_t \frac{t^c}{1-t^c} < 0.$$

which is unambiguously negative.<sup>20</sup>

Substituting production costs for the last term in the government budget constraint (29), the effect of an increase in government spending financed by an increase in the tax on specialised labour income equals,

$$\frac{\partial h_t}{\partial t^w} = \frac{\partial h_t}{\partial Y_t} \frac{\partial Y_t}{\partial t^w} = \frac{\partial h_t}{\partial Y_t} \left[ \frac{\partial Y_t}{\partial t^w} + \frac{\partial Y_t}{\partial G_t} \frac{\partial G_t}{\partial t^w} \right] = \frac{h_t}{Y_t} \left[ \frac{Y_t}{1-t^w} + Y_t \right] = h_t \frac{2-t^w}{1-t^w} > 0,$$

which is unambiguously positive. An increase in the tax reduces rents of specialised labour and induces higher labour supply, as can be seen by combining equations (16) and (17),

$$\frac{\partial g}{\partial e - e g - 1} a_t = (1 - t^w) w_{i,t} L_{i,t},$$

and noting that an increase in net wages reduces labour supply. This implies that the income effect dominates the substitution effect. The tax reduces the economic rents of specialised workers, which may be interpreted as an increase in horizontal distributional justice. The tax also increases labour supply, which is the key to the growth effect as described above. Having previously discussed the effects of an increase in the tax on innovation, this completes discussion on demand management, as the lack of an asset market does not allow us to treat the tax on asset income.

An increase in the alternative income of innovative workers reduces the economic rate of growth, as,

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<sup>20</sup> However, there may be a positive effect on welfare, if government spending enters the social welfare function.

$$\frac{\partial h_t}{\partial b_t} = -\frac{h_t}{b_t} < 0.$$

This implies that an increase in income from wealth decreases economic growth, as seen from equation (28).

Finally, *incomes policy* in our setting focuses on the endowments of economic agents. Redistributing wealth from innovative workers to the specialised workers, holding total wealth constant yields,

$$\left. \frac{\partial h_t}{\partial (A_t - \tilde{a}_t)} \right|_{A_t} = \frac{g}{(1-g)} \frac{q}{(1-t^c)(e-eg-1)},$$

which is unambiguously positive.

## Conclusions and Perspectives

We have developed a model of endogenous growth based on the division of labour. The novel contribution of this paper is that both supply side and demand side policies may stimulate long-run economic growth. On the supply side, an increase in productivity of innovative workers, alongside with investment in infrastructure, human capital, and improvements in the market setting may stimulate growth. On the demand side, we find that a reduction of taxes on innovative revenues, a reduction in consumption taxes, an increase of labour income taxation of the specialised workforce, and a redistribution of wealth towards specialised labour will foster economic growth. Summarising, we find that whilst the old-fashioned Keynesian policy prescriptions of increasing unproductive government spending - digging holes - exhibits negative growth effects, demand management may yield higher growth rates, if government spending is allocated towards productive uses.



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## Mathematical Appendix

Derivation of equation (7):

$$\int_0^{n_t} w_{j,t}^{1-e} dj = m_t^{1-e} y_t^e \int_0^{\frac{1-e}{e} n_t} L_{j,t}^e dj$$

$$\Rightarrow \int_0^{n_t} w_{j,t}^{1-e} dj = m_t^{1-e}$$

$$\Rightarrow m_t = \left[ \int_0^{n_t} w_{i,t}^{1-e} di \right]^{\frac{1}{1-e}}$$

Derivation of equation (17):

$$\int_t^{\infty} \frac{q}{q-t} a_s e^{-\int_t^s r_v(1-t^r) dv} ds = \int_0^{\infty} \left[ \dot{a}_s - (1-t^r) r_s a_s \right] e^{-\int_t^s r_v(1-t^r) dv} ds \Rightarrow$$

$$\Rightarrow a_s e^{-\int_t^s r_v(1-t^r) dv} \Big|_t^{\infty} = \int_t^{\infty} \left[ (1-t^w) w_{i,s} (L_{i,s}) L_{i,s} - (1-t^c) c_s \right] e^{-\int_t^s r_v(1-t^r) dv} ds$$

$$\Rightarrow -a_t = \int_t^{\infty} \frac{1-e+eg}{e-1} (1-t^c) c_s e^{-\int_t^s r_v(1-t^r) dv} ds$$

$$\Rightarrow a_t = -\int_t^{\infty} \frac{1-e+eg}{e-1} (1-t^c) c_t e^{-q(s-t)} ds$$

$$\Rightarrow a_t = \frac{e-eg-1}{q(e-1)} (1-t^c) c_t$$

$$\Rightarrow c_t = \frac{a_t}{1-t^c} \frac{q(e-1)}{e-1-eg}$$

Derivation of equation (22):

$$\dot{n}_t q_t = \int_{n_t}^{n_t + \dot{n}_t} \int_t^\infty \left[ w_{i,s}(L_{i,s}) L_{i,s} - \hat{w}_s \hat{L}_s \right] e^{-\int_t^s r_v(1-t^r) dv} ds di = \int_{n_t}^{n_t + \dot{n}_t} \int_t^\infty \frac{1}{e^{-\int_t^s r_v(1-t^r) dv}} w_{i,s}(L_{i,s}) L_{i,s} e^{-\int_t^s r_v(1-t^r) dv} ds di$$

$$\Rightarrow \dot{n}_t q_t = \int_t^\infty \int_0^{n_t + \dot{n}_t} \frac{1}{e^{-\int_t^s r_v(1-t^r) dv}} w_{i,s}(L_{i,s}) L_{i,s} di e^{-\int_t^s r_v(1-t^r) dv} ds - \int_t^\infty \int_0^{n_t} \frac{1}{e^{-\int_t^s r_v(1-t^r) dv}} w_{i,s}(L_{i,s}) L_{i,s} di e^{-\int_t^s r_v(1-t^r) dv} ds$$

$$\Rightarrow \dot{n}_t q_t = \frac{1}{e^{-\int_t^\infty r_v(1-t^r) dv}} \int_t^\infty (\mathbf{k}_s + \mathbf{k}_s) e^{-\int_t^s r_v(1-t^r) dv} ds - \frac{1}{e^{-\int_t^\infty r_v(1-t^r) dv}} \int_t^\infty \mathbf{k}_s e^{-\int_t^s r_v(1-t^r) dv} ds$$

$$\Rightarrow \dot{n}_t q_t = \frac{1}{e^{-\int_t^\infty r_v(1-t^r) dv}} \int_t^\infty \dot{y}_s e^{-\int_t^s r_v(1-t^r) dv} ds = \frac{1}{e^{-\int_t^\infty r_v(1-t^r) dv}} \int_t^\infty y_s e^{-\int_t^s r_v(1-t^r) dv} ds \Bigg|_t^\infty + \frac{1}{e^{-\int_t^\infty r_v(1-t^r) dv}} \int_t^\infty y_s e^{-\int_t^s r_v(1-t^r) dv} ds$$

$$\Rightarrow \dot{n}_t q_t = -\frac{1}{e} y_t + \frac{1}{e} y_t \int_t^\infty e^{-q(s-t)} ds = y_t \frac{1-q}{qe}$$

$$\Rightarrow \dot{n}_t = \frac{y_t}{q_t} \frac{1-q}{qe}$$