Walras' Law and the IS-LM Model.
A Tale of Progress and Regress.

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Abstract:

This paper deals with the integration of Walras' law into Keynesian macroeconomics and the attempts at a consistent specification of period models (beginning- vs. end-of-period-equilibrium). Three examples are examined where neglect of a consistent specification led to erroneous results: (1) the identification of the IS-condition with equilibrium of the "flow capital market", (2) superficial treatments of the liquidity trap, and (3) the assumptions on the stochastic structure of monetary and real shocks when determining the optimal monetary instrument.

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1. Introduction

The integration of macroeconomics into the framework of general equilibrium analysis marked a vital step from the "Economics of Keynes" to the neoclassical-synthesis version of "Keynesian economics". Two crucial elements in this process were the translation of the "General Theory" into the terms of the IS-LM-model and the application of Walras' law. It is not merely by accidence that Hicks – whose approach towards the "General Theory" was shaped by "Value and Capital" – was the most influential protagonist in the early development of both these ideas. Thereby, the neoclassical synthesis provided the unifying framework for analysing and discussing Keynesian topics, as for example the controversy between the liquidity preference and the loanable funds approach to the determination of the rate of interest. However, in the course of this development a host of subtle analytical problems emerged – e.g. with respect to the time structure of period analysis and the proper handling of stocks and flows. Starting with the early attempts by Hicks and Patinkin and culminating in the final accomplishment of a consistent stock-flow version of IS-LM by Foley and Buiter, most of these problems could be resolved within the general equilibrium framework. In this sense one may consider the steady refinement of the analytical tools used in IS-LM analysis as an example of "theoretical progress".[1] Yet, curiously enough, side by side with this increasing analytical sophistication simplified and superficial treatments have persisted up to the present, not only in many textbooks but also in more "high-brow" contributions.[2] Thus it seems that many adherents (and foes alike) to the neoclassical synthesis have neglected these lessons from the debates on the integration of Walras' law into macroeconomic theory.

It is this theme of regress after progress on which the following contribution will concentrate. It consists of five sections. The next (second) section gives a brief historical overview on the progress towards a consistent version of IS-LM. The third section sketches as a point of reference the two alternative period equilibrium versions of IS-LM. Then the fourth section provides three examples of regress in examining (i) the meaning of the flow equilibrium

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[1] For the notion of "theoretical progress" (in distinction to "empirical progress") cf. Blaug (2000, 3ff.).

[2] Although traditional Keynesian macroeconomics experienced a dramatic loss of reputation in the last decades, the works referred to above remain relevant as the reduction of macroeconomics to the interaction of the goods and the money market (as in IS-LM) still permeates many contributions.

condition of the market for bonds, (ii) the derivation of the liquidity trap, and (iii) the validity of Poole's rule (introducing a kind of Walras' law for shocks).

2. Progress in modelling IS-LM and Walras' law

In the following we will give a short overview on the progress in developing a consistent general equilibrium framework for stock-flow-analysis with references to the IS-LM-model.

As is well known, from the beginning the attempt to interpret Keynes's theory in terms of general equilibrium introduced Walras' law into the debate. The crucial conflict in this regard referred to the determination of the rate of interest by the money vs. the bond market, or what turned out as the controversy between liquidity preference and loanable funds theory. Already in his first review of the "General Theory" Hicks (1936, 92) pointed out: "If ... the demand for every commodity and factor equals the supply, and if the demand for money equals the supply of money, it follows by mere arithmetic that the demand for loans must equal the supply of loans ...", where "mere arithmetic" is a reference to Walras' law as the aggregation of budget restrictions. Therefore he claims: "The choice between them [the method of eliminating the loan market or the money market] is purely a question of convenience." (ibid., 93)[3]

Leaving the liquidity-preference-loanable-funds controversy aside[4] and concentrating on the development of IS-LM, the next vital step towards the integration of Walras' law was taken by Patinkin. In the first and second edition of "Money, Interest and Prices" (Patinkin 1956, 1965) he supplements the usual equilibrium loci of the goods and of the money market by another explicit one for the bond market. First, this is applied to the full employment case (Patinkin 1965, 253ff.) and then to the case of involuntary unemployment (ibid., 328ff.), where the goods market equilibrium is carefully reinterpreted as the condition of equality between income and expenditure. Thereby Patinkin assumes, for the sake of simplicity, that changes in income have an identical effect on bond demand and bond supply, so that the bond-market-equilibrium (BB-) curve is horizontal (cf. ibid., 281f.). Obviously, general equilibrium is where the three curves intersect and from that follows that the equilibrium interest rate cannot be said to be determined

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[3] For a similar formulation cf. also Hicks (1939, 158). – It should be noted that in the 1937-version of Hicks's IS-LM-model income is in nominal and not in real terms. On the consistency of this formulation (as contrary to the usual one) cf. Barens (2000).

[4] Cf. for a more thorough investigation Klausinger (1992).

by a single market. Furthermore, Patinkin is well aware that the ways in which e.g. additional investment demand is financed will determine which of these curves are shifted and that consequently any shock that shifts one curve must at least shift one of the other two curves, too. As a consequence from Walras' law follows a restriction on the derivatives of the relevant excess demand functions, a point later on elaborated by Tobin/Brainard (1968) with regard to asset markets.[5] Patinkin is also quite explicit in stating "that the savings = investment condition is *not* an alternative statement of the equilibrium condition in the bond market." (Patinkin 1965, 272). In this regard, Patinkin's construction can be said to have anticipated vital elements of the consistent integration of Walras' law; however, what is missing is a clear period structure of the model when it refers to stocks and flows.

A related debate that came to a provisional solution by Patinkin's device was that about the dynamics of the rate of interest. Following Samuelson's statement of the law of demand and supply by the formula $\dot{p} = H[D(p) - S(p)]$, $H(0) = 0, H' > 0$ (Samuelson 1946, 263), the obvious question was which market (i.e. the excess demand of the money or the bond market) would then govern the movements of the rate of interest. In this context liquidity preference theory (as favoured by Klein 1950a, b) would thus make the movements of the interest rate depend on the (stock) excess demand in the money market and loanable funds theory (as favoured by Fellner/Somers 1949, 1950a, b) on the (flow) excess demand of bonds. As Brunner (1950) pointed out by invoking Walras' law both formulations imply non-intuitive reactions with respect to the market eliminated from the analysis. For example the liquidity preference formulation implies that the rate of interest might change although there is no excess demand in the bond market (ibid., 250f.). Eventually Patinkin (1958) supplied a unifying framework in which to discuss the different dynamic specifications by interpreting the IS-LM-BB-curves as the isokines of the corresponding phase diagram. Thereby he confirmed the different and non-intuitive results of the two competing specifications and he agreed with most participants of the debate that ultimately the question could only be settled on empirical grounds.[6]

Another issue that figured prominently in the debate was that of stock vs. flow analysis. In a partial equilibrium framework the liquidity preference approach was identified with stock

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[5] In a careful if somewhat eclectic analysis of the IS-LM-BB-model McCaleb/Sellon (1980, 403ff.) examined the restrictions on the slopes of these curves implied by Walras' law.

equilibrium whereas the loanable funds approach apparently dealt with flow equilibrium (cf. again the contributions by Klein and Fellner/Somers). Brunner (1950, 249) was able to reconcile these divergent views by demonstrating that stock and flow equilibrium led to the same results for a given period provided that there had been stock equilibrium in the preceding period, so that stock equilibrium turned out as the more natural specification. Patinkin (1965) in his general equilibrium formulation resorted to stock equilibrium for money and bonds combined with flow equilibrium for goods; obviously (but without an explicit justification) equilibrium in his account refers to the end of the period considered. Another step at disentangling stock-and-flow dynamics was made by taking account of the government-budget-constraint (cf. for example Christ 1968, Blinder/Solow 1973). This constraint linked flows (e.g. budget deficits) with changes in asset supply (e.g. money or bonds); obviously a similar constraint applied to the private sector as well.

However, a kind of breakthrough was only achieved by Foley's distinction between two consistent formulations of stock-and-flow equilibrium in discrete time, namely beginning- and end-of-period equilibrium (Foley 1975). These two specifications as applied to the IS-LM-BB-model will be replicated in section 3 below. According to which specification is used we will arrive at different versions of Walras' law and, as Foley (ibid, 320f.) demonstrated, the determination of the rate of interest conforms to the liquidity preference or the loanable funds approach, respectively, when the beginning or the end-of-period formulation is used. A series of contributions followed, dealing for example with the relationship between the specification in discrete and in continuous time and with the conditions for equivalence between different specifications, culminating in Buiter (1979, 1980). Furthermore, the consistency of this approach was authoritatively proved (for Keynesian balance-of-payments models) in the interchange between Kuska (1978) and Buiter/Eaton (1981).

Finally in the discussion of the period version of Patinkin's model May (1970) and Woglom (1980) noticed the crucial connection between the parameters of the model and the length of the period considered. When there are parameters that denote a stock-flow relationship, as for example the so-called "Cambridge-k", then their (absolute) value will decrease with the length of the period. Therefore by making the period sufficiently long one can force the BB-curve to become downward sloping. Similarly if the period length goes to zero, so that time becomes a

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[6] For an attempt at such an empirical investigation cf. for example Ferguson/Hart (1980).

continuous variable, k will increase without bound and the LM- and the BB-curve will coincide (as in the beginning-of-period-equilibrium version of the period model).[7]

3. A consistent period version of IS-LM

The IS-LM model consists of a condition for flow equilibrium, income equals expenditure or, equivalently, $I = S$ (investment = saving), and two conditions for stock equilibrium, an explicit one for the money market, $M = L$ (money supply = money demand), and one for the bond market, $B = B^d$ (bond supply = bond demand), usually left implicit. Because of the co-existence of flows and stocks the time dimension of the variables must be carefully specified: The model refers to a period of time $t_0 \equiv [0, 1]$, that starts at $t = 0$ and ends with $t = 1$. This time period is the one relevant for the specification of the flow variables, whereas the relevant point of time for the specification of the stock variables is either 0 or 1 and is indicated by a subscript. During the time period wealth can be accumulated or decumulated, as per definition: $\Delta W \equiv W_1 - W_0 = S$ (the change in wealth = saving). Correspondingly changes in stocks are denoted by $\Delta B^d \equiv B_1^d - B_0^d$ and $\Delta B \equiv B_1 - B_0$ etc.

The following models are simplified versions in the spirit of Foley (1975) and Buiter (1980).[8] Accordingly, there are two ways for consistently specifying IS-LM, namely end-of-period and beginning-of-period equilibrium. These will be examined in turn.

3.1 End-of-period equilibrium

In the end-of-period specification the equilibrium of investment and savings refers to the flows within the period (t_0), the equilibrium of the money and the bond market to the stocks at the end of the period ($t = 1$). Therefore the existing stocks at the end of the period, after taking account of the changes (accumulation or decumulation) within the period, must equal stock demand. We formulate the model with the three equilibrium conditions, (1) to (3), and with the usual parameter restrictions:

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[7] However, as Hellwig (1975) pointed out, the compatibility of models in continuous time, which imply continuous transactions, with the foundations of the transactions demand for money is dubious.

[8] The basic simplification consists in neglecting the distinction between current and expected variables and between the different points in time when expectations are formed.

(1) $\qquad I(i,u) = S(Y,W_1); \quad I_i < 0, \ I_u = 1, \ S_Y > 0, \ S_W < 0.$

(2) $\qquad L_1(Y,i,W_1,v) = M_1 \equiv M_0 + \Delta M; \quad L_Y > 0, \ L_i < 0, \ 0 \le L_W \le 1, \ L_v = 1.$

(3) $\qquad B_1^d = B_1 \equiv B_0 + \Delta B.$

Here and in the following Y denotes income, i the rate of interest and u and v are shift variables in investment and money demand. Notice that the stocks at the beginning of the period, M_0 and B_0, and therefore also wealth $W_0 \equiv M_0 + B_0$, are exogenously given, and that $W_1 = W_0 + S$.[9] The shape of the functions of bond demand and supply is left open, as it will be shown that they cannot be determined independently of (1) and (2).[10]

We start by deriving the effects of Y, i and W_0 on the respective excess demands, that is the slopes and shifts of the IS-, LM- and BB-curve. As these excess demands depend on end-of-period wealth, we must distinguish between the direct effects (as determined by the above parameter restrictions) and the indirect effects operating on saving and thereby on wealth. For example, differentiating S leads to:

$$dS = S_Y dY + S_W(dW_0 + dS) = \frac{1}{1-S_W}(S_Y dY + S_W dW_0),$$

where S_Y is the direct effect of income and $S_Y/(1-S_W)$ is the total effect.

By similar calculations we obtain from differentiating (1), (2) and (3):

(4) $\qquad d(I-S) = -\frac{S_Y}{1-S_W} dY + I_i di - \frac{S_W}{1-S_W} dW_0 + du = 0,$

(5) $\qquad d(L_1 - M_1) = \left(L_Y + L_W \frac{S_Y}{1-S_W} \right) dY + L_i di + \left(\frac{L_W}{1-S_W} - M_{0W} \right) dW_0 + dv - d\Delta M = 0,$

(6) $\qquad d(B_1^d - B_1) = \left(B_Y^d + B_W^d \frac{S_Y}{1-S_W} \right) dY + B_i^d di + \left(\frac{B_W^d}{1-S_W} - B_{0W} \right) dW_0 - d\Delta B = 0,$

$$\text{where} \quad M_{0W} \equiv \frac{dM_0}{dW_0}, \ B_{0W} \equiv \frac{dB_0}{dW_0}, \ M_{0W} + B_{0W} = 1.$$

In the above equations it must be carefully distinguished between dM_0 and $d\Delta M$, the former

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[9] For simplicity here and in the following capital gains and losses from holding bonds are disregarded, that is bond supply is specified as B_0 instead of B_0/i.

[10] Subscripts indicate partial derivatives.

referring to a change in the stock at the beginning of the period, the latter to a change during the period. As usual, with the given parameter restrictions the IS-curve is downward sloping and the LM-curve is upward sloping in *Y-i*-space.

Now we introduce Walras' law, which states the following restriction:

(7) $\qquad B_1^d - B_0 - \Delta B + L_1 - M_0 - \Delta M + I - S = 0$.

Therefore excess demand in the bond market is determined by the excess demands of the two other markets, so that the equilibrium conditions (1) and (2) imply (3):

(8) $\qquad B_1^d - B_0 - \Delta B = M_0 + \Delta M - L_1 + S - I$.

Taking account of the definition of wealth, (8) can be expressed equivalently as:

(9) $\qquad L_1 + B_1^d - \Delta B - \Delta M + I = M_0 + B_0 + S \equiv W_1$.

By differentiating (9) in turn with respect to W_0, Y, i, v and u, we arrive at the following set of parameter restrictions:

(10) $\qquad \dfrac{L_W}{1 - S_W} + \dfrac{B_W^d}{1 - S_W} = 1 + \dfrac{S_W}{1 - S_W} \Rightarrow L_W + B_W^d = 1$;

(11) $\qquad L_Y + L_W \dfrac{S_Y}{1 - S_W} + B_Y^d + B_W^d \dfrac{S_Y}{1 - S_W} = \dfrac{S_Y}{1 - S_W} \Rightarrow L_Y + B_Y^d = 0$;

(12) $\qquad L_i + B_i^d - \Delta B_i - \Delta M_i + I_i = 0$;

(13) $\qquad 1 + B_v^d - \Delta B_v - \Delta M_v = 0$;

(14) $\qquad 1 + B_u^d - \Delta B_u - \Delta M_u = 0$.

For the sake of simplicity we assume that there are no endogenous changes to the money supply during the period, that investment demand is financed by supplying bonds and that autonomous changes to money demand mean a switch to or from bond demand. That is:

(A1) $\qquad \Delta M = 0$; $\quad \Delta M_i = \Delta M_u = \Delta M_v = 0$;

(A2) $\qquad \Delta B = I + u$; $\quad \Delta B_i = I_i, \Delta B_u = 1, \Delta B_v = 0$;

(A3) $\qquad B_v^d = -1$.

As can be seen from (12) to (14) these assumptions are mutually consistent. From these

assumptions (12) simplifies to:

(12') $L_i + B_i^d = 0.$

Inserting these restrictions into (6) gives:

(15)
$$d(B_1^d - B_1) = \left[-L_Y + (1 - L_W)\frac{S_Y}{1 - S_W} \right]dY - (L_i + I_i)di +$$
$$+ \left(\frac{1 - L_W}{1 - S_W} - \frac{dB_0}{dW_0} \right)dW_0 - du - dv = 0,$$

so that:

(16) $$\left. \frac{dY}{di} \right|_{BB} = \frac{L_i + I_i}{(1 - L_W)\dfrac{S_Y}{1 - S_W} - L_Y} = ?$$

Because of the indeterminate sign of (16) both upward and downward sloping BB-curves are consistent with restriction (9).[11] However, it is easily proved, that the slope of the BB-curve cannot be steeper than that of LM nor flatter than that of IS. Furthermore, shifts in IS and LM, respectively, cause compensating shifts in BB.

3.2 Beginning-of-period equilibrium

In this specification the equilibrium of investment and savings refers again to the flows within the period (t_0), however the equilibrium of the money and the bond market to the stocks at the beginning of the period ($t = 0$). The three equilibrium conditions, (1*) to (3*), are then:

(1*) $I(i, u) = S(Y, W_0),$

(2*) $L_0(Y, i, W_0, v) = M_0,$

(3*) $B_0^d = B_0.$

The total differentials of the excess demands are:

(4*) $d(I - S) = -S_Y dY + I_i di - S_W dW_0 + du = 0,$

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[11] Note that the parameter L_Y refers to a stock-flow-relation whereas S_Y to a relation between two flows. Therefore the value of L_Y will depend on the length of the period (decrease with increasing period length), but S_Y will not. Consequently, the slope of the BB-curve will depend on the length of the period, too. For example, for very short periods BB must be upwards sloping.

$$(5^*) \qquad d(L_0 - M_0) = L_Y dY + L_i di + (L_W - M_{0W})dW_0 + dv = 0,$$

$$(6^*) \qquad d(B_0^d - B_0) = B_Y^d dY + B_i^d di + (B_W^d - B_{0W})dW_0 + B_v^d dv = 0.$$

Qualitatively similar to the end-of-period case we obtain again a downward sloping IS-curve and an upward sloping LM-curve.

As equilibrium in the two markets for stocks is realised before the flow market for the period is activated, Walras' law is split up into a wealth restriction (7*) on the one hand and a budget restriction for the flow variables (14*) on the other hand. The wealth restriction is given by:

$$(7^*) \qquad L_0 + B_0^d = M_0 + B_0 \equiv W_0.$$

Therefore the excess demand in the bond market is identical to the excess supply in the money market, so that the equilibrium conditions are identical, too:

$$(8^*) \qquad B_0^d - B_0 = M_0 - L_0.$$

By differentiating (7*) we again get a set of parameter restrictions, that is:

$$(9^*) \qquad L_i + B_i^d = 0,$$

$$(10^*) \qquad L_Y + B_Y^d = 0,$$

$$(11^*) \qquad L_W + B_W^d = 1,$$

$$(12^*) \qquad 1 + B_v^d = 0.$$

Inserting these results into (6*) gives:

$$(13^*) \qquad d(B_0^d - B_0) = -L_Y dY - L_i di - \left(L_W - \frac{dM_0}{dW_0}\right)dW_0 - dv,$$

so that, of course, the LM-curve and the BB-curve coincide in the case of beginning-of-period equilibrium.

The remaining flows within the period, that is I and S as well as ΔB and ΔB^d, ΔM and ΔL, must fulfil a budget restriction. Thereby an excess of expenditure over income is mirrored by a respective excess of the sum of the flows of money and bond supply over the sum of the flows of money and bond demand:

$$(14^*) \qquad I - S = \Delta B + \Delta M - \Delta B^d - \Delta L.$$

We may stipulate the assumptions equivalent to the end-of-period version:

(A1*) $\Delta M = 0$;

(A2*) $\Delta B = I$.

Furthermore, we see that what had to be assumed in the end-of-period version as (A3) now follows from the restriction (12*).

Inserting (A1*) and (A2*) into (14*) we arrive at

(15*) $S = \Delta B^d + \Delta L$,

which just means that the whole of planned saving is split up between increasing the demand for bonds or that for money. Note however, that in this model there is no equilibrium condition involving these asset flows.

4. Some examples of regress

In the following we will provide three examples where analysis within the IS-LM framework did lack the consistency achieved in the contributions surveyed above.

4.1 Equilibrium of the "flow capital market"

There is a long and time-honoured tradition in identifying the flow demand and supply of the bond market with investment and saving. This identification (or one might say, confusion) of demand and supply of the flow capital market with investment and savings, respectively, has been ascribed by Keynes (1936, ch. 14) to the "classical approach" (which he rejected). It is, however, not part of the "loanable funds approach" as formulated by Robertson (1940, 2ff.) and Ohlin (1937), that takes into account such monetary factors as hoarding and money creation as components of the flow demand or supply of the capital market. Yet, recently the classical approach has been revived, now in the guise of an interpretation of the IS-curve of Keynesian macroeconomics, by Felderer/Homburg (1986; 1994, 128ff.).[12] A similar argument can be found in the textbook of Mankiw (1997, 62ff.) where the demand and supply of loanable funds are equated, in his model of long-run equilibrium, with investment and savings.

Specifically, Felderer/Homburg (1986, 458) proposed to distinguish within the IS-LM-model between two types of the bond market: a market for the existing stocks (the bond market in the

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[12] For early critiques cf. Maussner (1988) and Barens/Caspari (1995).

usual sense) on the one hand and a market for flows (sometimes termed "capital market"), with ΔB^d and ΔB as demand and supply, on the other hand. Furthermore, the condition $I = S$ was identified with the equilibrium condition for the "capital market". This proposition will now be examined within the period equilibrium framework as established above.

For the specification of end-of-period equilibrium the distinction between a stock and a flow market for bonds makes no sense: There are not two different markets or equilibrium conditions, but only the single equilibrium condition for the single bond market at the end of the period, which refers to the sum of the bonds existing at the beginning of the period and of the flow of newly supplied bonds (and similarly so for bond demand). Consequently, if the bond market was in equilibrium at the end of the preceding period (i.e. the beginning of the current period), then obviously end-of-period equilibrium of stocks implies also equilibrium of the flows during the current period. Moreover, it was shown above that the conditions for investment-savings and bond market equilibrium (the IS- and the BB-curve) differ in this model.

Therefore, as noticed by Felderer/Homburg (1986, 465f.), the equilibrium condition of the "capital market" is only meaningful for the specification of beginning-of-period equilibrium. The relation between investment-savings equilibrium and the flow demand and supply of bonds, ΔB^d and ΔB, and of money, ΔL and ΔM, is then determined by the restriction (14*).

From (14*) and from assumption (A1*) follows $\Delta B^d - \Delta B = S - I - \Delta L = 0$ as the equilibrium condition for the flow capital market. Taking total differentials in the neighbourhood of equilibrium, where $\Delta \tilde{L} = 0$, we obtain

$$(1.1) \quad \begin{aligned} d(\Delta B^d - \Delta B) &= dS - dI - dL = \\ &= (S_Y - L_Y)dY - (I_i + L_i)di + (S_W - L_W)dW_0 - dv - du = 0, \end{aligned}$$

which is definitely not identical with the condition for $I = S$. For example, the slope in Y-i-space of the curve (let's call it FF) for equilibrium in the flow capital market is

$$(1.2) \quad \left. \frac{\partial Y}{\partial i} \right|_{FF} = \frac{I_i + L_i}{S_Y - L_Y} = ?,$$

which is not necessarily downward sloping as for IS.

Furthermore, we can look at the model in beginning-of-period equilibrium for period t_0, so that the values for Y and i are determined for this period. Disregarding shocks, from (14*) and (A1*)

and because of $I = S$ in equilibrium the condition for flow equilibrium in the capital market $\Delta B^d - \Delta B$ implies $\Delta L = 0$. However, as $\Delta L = L_Y \Delta Y + L_i \Delta i + L_W \Delta W = L_W \Delta W$ and $\Delta W = S > 0$, $\Delta L = 0$ implies $L_W = 0$ or consequently $B_W = 1$. Only under this condition will beginning-of-period equilibrium of the model also imply flow equilibrium in the capital market (with unchanging Y and i). Or expressed differently, the FF-curve is not only different from the IS curve, the IS-, LM- and FF-curve will only under this special condition have a common point of intersection.

Therefore, it must be concluded that in the case of end-of-period equilibrium the stock equilibrium condition for the bond market implies the flow equilibrium condition, yet it is different from the condition for investment-savings equilibrium. And in the case of beginning-of-period equilibrium the equilibrium condition for the "capital market" is neither part of the model, nor in general identical with the condition for investment-savings equilibrium, nor is it necessarily implied by beginning-of-period equilibrium conditions.

4.2 The liquidity trap

Within the Keynesian macro-model of the neoclassical synthesis the liquidity trap constitutes a special case that challenges the existence (and stability) of full employment equilibrium. The trap is usually identified with a horizontal LM-curve at a positive rate of interest. However, more often than not the characterisation of the trap – in textbooks and even in more scientific contributions – is rather superficial: It is said that in the liquidity trap money demand will not only become infinitely interest elastic but money demand itself infinite; or the trap is described verbally as a situation of "absolute liquidity preference" where all asset-owners prefer to hold money instead of bonds. Examples of such statements abound in the literature.

With regard to the early literature it should be noted that Hicks was more cautious than later popularisers of the liquidity trap in stating that the LM-curve "will probably tend to be *nearly* horizontal on the left ... we can think of the curve as approaching these limits asymptotically" (Hicks 1937 [1982], 109) and that "if the supply of money is increased, the curve LL moves to the right ..., but the horizontal parts of the curve are *almost* the same." (ibid., 111; my emphasis in both quotations). It was again Patinkin (1965, 223 and 349ff.), who criticised careless representations of the trap by invoking Walras' law: For example he noticed that an infinite demand for money in the trap would imply an infinite supply of bonds.

Nevertheless careless treatments of the liquidity trap are still common. Examples of the kind of reasoning criticised above are provided by Branson (1989, 154ff. and 181), Gordon (1993, 176f.), Dornbusch/Fischer (1998, 243 and 249), as well as by Felderer/Homburg (1994, 121ff., 144ff.), Klatt (1995, 61ff.) and Jarchow (1998, 76, 196).

Outside the textbooks, too, despite its longevity, controversy on the liquidity trap has not subsided since Patinkin, as is demonstrated by a host of new contributions. One strand of the literature has reopened the debate on the consistent integration of the bond market into the IS-LM-model and thereby questioned the existence or stability of the liquidity trap, see for example Größl-Gschwendtner (1991, 171; 1993, 174f.), Barens/Caspari (1992, 343f.; 1995), Ernst (1992, 337), Ernst/Walpuski (1993a; 1983b, 583ff.; 1996, 249f.) and Klausinger (1995, 232ff.). Another strand dealt with the different versions of the trap to be found in the "General Theory". However, in their careful study Beranek/Timberlake (1987) committed a vital error by confusing the horizontal part of the money demand function (as implied when money and bonds become perfect substitutes) with a discontinuity of the money demand function. Obviously, the latter interpretation would endanger the existence of asset equilibrium in the trap at all. The erroneous nature of this interpretation was pointed out by Cushman (1990) (cf. also Beranek/Timberlake (1990)); it will also be neatly demonstrated by the derivation below.

In the following we formulate a consistent derivation of the liquidity trap by means of the Keynes-Tobin version of the speculative demand for money. As we shall also examine the relation between the liquidity trap and the Pigou effect, it is most convenient to use the end-of-period equilibrium framework. (However all results can be easily translated into the version of beginning-of-period equilibrium.)

Taking the equilibrium conditions (1) to (3) as the point of departure, we give now a more concrete specification of the money demand function as:

$$(2.1) \qquad L_1(Y, i, W_1) = kY + L_s(Y, i, W_1),$$

where L_s refers to speculative demand and end-of-period wealth is given by (9).

The speculative component of money demand is derived from the individual decisions of the asset-owners. The asset-owner j decides how to allocate disposable wealth, $W_{s,j}$, between money and bonds, according to whether the actual rate of interest i exceeds the expected rate of interest $i*$ or not, so that:

$$(2.2) \qquad L_{s,j} = \begin{Bmatrix} 0 \\ [0, W_{s,j}] \\ W_{s,j} \end{Bmatrix}, \text{ if } i \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} i_j{}^*, \quad W_{s,j} \equiv W_j - kY_j.$$

Obviously, for $i = i^*$ the asset-owner is indifferent between money and bond holdings.[13]

The individual interest rate expectations (weighted by the respective disposable wealth) are assumed to be distributed according to a probability density function $f(i)$ such that:

$$(2.3) \qquad f(i) \begin{Bmatrix} = \\ > \\ = \end{Bmatrix} 0, \text{ if } \begin{Bmatrix} i > i_+ \\ i_+ \geq i \geq i_- \\ i_- > i \end{Bmatrix},$$

where $f(i)$ is the share of wealth of asset-owners with $i = i^*$. It is assumed that there is an upper and a lower bound to the expected interest rate, $i_+ \geq i_- > 0$.

The corresponding probability distribution function is:

$$(2.4) \qquad F(i) = \int_{-\infty}^{i} f(i)\, di \text{ with } \int_{-\infty}^{+\infty} f(i)\, di = \int_{i-}^{i+} f(i)\, di = 1.$$

Consequently, the share $l(i)$ of aggregate disposable wealth that is held in cash is $1 - F(i)$ and the effect of i on l, $l_i = -f$.

The aggregate speculative demand for money and the corresponding demand for bonds are therefore given by

$$(2.5) \qquad L_s = l(i)W_s, \quad l(i) \equiv \begin{Bmatrix} 0 \\ 0 \leq 1 - F(i) \leq 1 \\ 1 \end{Bmatrix}, \text{ if } \begin{Bmatrix} i > i_+ \\ i_+ \geq i \geq i_- \\ i_- > i \end{Bmatrix}.$$

$$(2.6) \qquad B^d = [1 - l(i)]W_s.$$

The distinct values of the rate of interest in (2.5) may be identified as belonging to the "classical", "normal" and "Keynesian" region.

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[13] This formulation follows Keynes (1936, 166ff.) and Tobin (1958). For sake of simplicity we concentrate on expected capital gains and losses and disregard the interest income from bonds. Cf. Chick (1983, 200ff. and 219ff.), who similarly uses Keynes's normal rate instead of Tobin's critical rate of interest in deriving the speculative demand for money.

Then $i = i_-$ represents a situation of "absolute liquidity preference". However, none of the usual statements on the liquidity trap is true for this case. Neither is money demand infinite at $i = i_-$ nor is the interest elasticity:

$$(2.7) \qquad L(i_-) = kY + W_s = W,$$

$$(2.8) \qquad \boldsymbol{e}(i_-) \equiv L_i(i_-) \frac{i_-}{L(i_-)} = -i_- \frac{f(W - kY)}{W} = 0 \text{ because of } f = 0.$$

As can be seen money demand equals (but cannot exceed) total wealth, which is finite, and the interest rate elasticity (evaluated at the left-hand-side derivative) is therefore zero.

Furthermore, and more importantly, the state of absolute liquidity preference at $i = i_-$ does not correspond to a point on the LM-curve. This can be easily verified by deriving the LM curve. The total differential of the money market equilibrium condition, $M_0 = L_1$, renders:

$$(2.9) \qquad dM_0 = 0 = (1 - l)kdY - fW_s.$$

Therefore the slope of the LM curve is given by:

$$(2.10) \qquad \left. \frac{\partial Y}{\partial i} \right|_{LM} = \frac{fW_s}{(1 - l)k} \begin{Bmatrix} = \\ > \end{Bmatrix} 0 \text{ if } \begin{Bmatrix} i > i_+ \\ i_+ \geq i \geq i_{\min} \end{Bmatrix},$$

which is finite throughout (the curve is not horizontal). Here $i_{\min}$ is the value of i (on the LM curve), calculated at $Y = 0$. Again substituting into the equilibrium condition for $Y = 0$ we see:

$$(2.11) \qquad M_0 = L_1 = l(i_{\min})W_1 = l(i_{\min})(M_0 + B_0 + S), \text{ or}$$

$$l(i_{\min}) = \frac{M_0}{M_0 + B_0 + S} < 1 \Rightarrow i_{\min} > i_-.$$

Therefore, the only possibility for establishing a "true" liquidity trap within this model is to assume identical interest rate expectations, so that from the asset-owners' point of view money and bonds become perfect substitutes.[14] Then aggregate money demand is:

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[14] For simplicity we neglect the possibility that f is monotonically, but not strict monotonically decreasing so that $L(i)$ could become a kind of step-function. This would allow the possibility that LM would be horizontal in the neighbourhood of equilibrium but not throughout. This (not very interesting) possibility is analysed by Beranek/Timberlake (1987, 391ff.) as the so-called "strong version".

$$(2.12) \qquad L = kY + l(i)W_s, \quad l(i) = \begin{Bmatrix} 0 \\ [0,1] \\ 1 \end{Bmatrix} \text{ if } i \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} i*.$$

This implies a horizontal LM curve at $i = i*$ for $Y \leq M_1/k$.[15]

As the next step we calculate the partial derivatives of the (excess) demand functions in the trap region. The relevant results are:

$(2.13) \qquad L_i \to -\infty, \quad B_i^d \to +\infty$;

$(2.14) \qquad L_Y = (1-l)k, \quad B_Y^d = -(1-l)k$;

$(2.15) \qquad L_W = l, \quad B_W^d = 1-l$.

Upon substitution it turns out that the consistency restrictions (10) to (12') are fulfilled as required.

This brings us to the question of the Pigou effect, $S_W < 0$. Rabin/Keilany (1986) have argued that in the liquidity trap any increase in wealth (or of real balances) would be fully absorbed by money demand so that there cannot be a Pigou effect. The way out of the liquidity trap would thus be blocked as an increase in real balances could not spill over to consumption (and decrease saving).[16]

Obviously, this argument is invalid for the beginning-of-period specification: As can be seen from (8*), an excess demand for money is just compensated by an excess supply of bonds of the same magnitude, with no effect on expenditure. Yet, also for the end-of-period specification the consistency restrictions (10) to (12), as satisfied in the liquidity trap by (2.13) to (2.15), are compatible with any value of S_W, in particular with $S_W < 0$. This means, of course, that Walras' law does not rule out the existence of the Pigou effect in the liquidity trap (if consistently specified).

The formal results are straightforward. We model the Pigou effect by an equiproportionate increase in the real value of money and bonds, assuming both to be fixed in nominal terms, so

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[15] Obviously, the LM curve is continuous, in particular it does not contain "holes" in its horizontal part, as suggested erroneously by Beranek/Timberlake (1987, 394).

[16] For the ensuing debate cf. Rabin/Keilany (1987, 1988), Coulombe (1987) and Mayer (1988).

that:

$$(2.16) \quad \frac{dW_0}{W_0} = \frac{dM_0}{M_0} = \frac{dB_0}{B_0} \equiv \boldsymbol{p} \Rightarrow M_{0W} = \frac{M_0}{W_0}, B_{0W} = \frac{B_0}{W_0}.$$

As the interest rate is fixed in the trap at $i = i^*$, the effect on income can be calculated from the IS-curve:

$$(2.17) \quad \frac{di}{d\boldsymbol{p}} = 0; \frac{dY}{d\boldsymbol{p}} = -\frac{S_W}{S_Y} W_0 > 0.$$

Finally, money demand passively adapts to the increase in the real money supply:

$$(2.18) \quad \frac{dL_1}{d\boldsymbol{p}} = M_0.$$

We may thus conclude that the assertion of the logical impossibility of a Pigou effect in the liquidity trap is plainly wrong.

4.3 The optimal monetary instrument: Poole's rule

The third example concerns the analysis of the optimal monetary instrument. In a seminal paper Poole (1970) examined this question within an IS-LM framework and derived "Poole's rule", according to which the relative efficiency of the money supply as a monetary instrument, as compared with the rate of interest, depends positively on the ratio of the variance of real relative to monetary shocks. The underlying assumptions are that the central bank knows the deterministic components of the IS- and LM-equations and that it can control the expected value of output by the suitable choice of its instruments. However, the actual value of output may deviate from its expected value because of (unanticipated) random shocks to commodity and money demand. As these deviations differ according to the monetary instrument chosen, the optimal monetary instrument can be determined by minimising the variance of output (around its expected value). Despite the numerous "revolutions" in macroeconomic theory Poole's is still considered a "useful model", even by high-brow monetary theorists (cf. Blanchard/Fischer 1989, 575ff.).

We start by recapitulating Poole's model. The equations (3.1) and (3.2) are to be interpreted as linear approximations in the neighbourhood of equilibrium to the IS- and LM-curve (1) and (2) with $S_W, L_W = 0$. To economise on notation we introduce the following abbreviations: $S_Y \equiv s$, $I_i \equiv -g$, $L_Y \equiv k$, $L_i \equiv -h$, so that all parameters are positive. (3.3a) and (3.3b) define monetary policy

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with a money-supply and an interest-rate-target, respectively.

(3.1) $sY = a - gi + u$,

(3.2) $M = kY - hi + v$,

(3.3a) $M = \overline{M} \equiv \dfrac{(kg + sh)\overline{Y} - ah}{g}$,

(3.3b) $i = \bar{i} \equiv \dfrac{a - s\overline{Y}}{g}$.

The symbols are the same as used above, $\overline{Y}$ denotes the target value of output, $\overline{M}$ and $\bar{i}$ the corresponding values of the monetary instruments. u and v are random components in commodity and money demand, respectively, the so-called "real" and "monetary" shocks, with $u, v \equiv N(0, \boldsymbol{s}_j^2)$, $j = u, v$.

The model can be solved for the variance of output in each of the two regimes. For the money-supply regime (denoted by M) we arrive at:

(3.4) $\boldsymbol{s}_Y^2(M) = \dfrac{1}{m^2}(h^2\boldsymbol{s}_u^2 - 2hg\boldsymbol{s}_{uv} + g^2\boldsymbol{s}_v^2), \quad m \equiv sh + kg > 0$,

whereas for the interest-rate regime (denoted by i):

(3.5) $\boldsymbol{s}_Y^2(i) = \dfrac{1}{s^2}\boldsymbol{s}_u^2$.

Although Poole (1970, 206f.) noted the possibility of a non-zero covariance between real and monetary shocks, usually the analysis proceeds from the assumption that $\boldsymbol{s}_{uv} = 0$. This results in the following condition for the superiority of the money-supply regime:

(3.6) $\dfrac{\boldsymbol{s}_v^2}{\boldsymbol{s}_u^2} < \dfrac{m^2 - s^2h^2}{s^2g^2} = \dfrac{kg(kg + 2sh)}{s^2g^2} \equiv \boldsymbol{l} > 0$.

The relative performance of the money-supply regime is the better the smaller the ratio of the variance of monetary to real shocks and it becomes definitely superior to the interest-rate regime if this ratio falls below $\boldsymbol{l}$, which is determined by the parameters of the model – Poole's rule.

At this stage of the argument we again introduce Walras' law and apply it to the end-of-period equilibrium version of the model. At first, we rewrite Walras' law (7) as:

(3.7) $(I - S) + (L - M) + (B^d - B) = 0$,

where all the variables include the relevant stochastic shocks.[17] We define the respective deterministic components of demand as $\hat{I}, \hat{L}$ and $\hat{B}^d$, so that $I - \hat{I} = u$, $L - \hat{L} = v$ and $B^d - \hat{B}^d = w$, where w is another random variable, a "financial" shock. Then, we examine the deterministic part of the model where all random shocks are set to zero. Obviously, Walras' law applies also to this version and it renders:

$$(3.8) \qquad (\hat{I} - S) + (\hat{L} - M) + (\hat{B}^d - B) = 0.$$

At last, subtracting (3.8) from (3.7) we see that Walras' law implies a restriction on the stochastic shocks, namely:

$$(3.9) \qquad u + v + w = 0.$$

From equation (3.9) we can derive the following three symmetrical restrictions on the variances and covariances of the shock variables[18]

$$(3.10a) \qquad s_u^2 + 2s_{uv} + s_v^2 = s_w^2,$$

$$(3.10b) \qquad s_u^2 + 2s_{uw} + s_w^2 = s_v^2,$$

$$(3.10c) \qquad s_v^2 + 2s_{vw} + s_w^2 = s_u^2.$$

The stochastic structure of the three shocks, u, v and w, is represented by their variances and covariances, that is in sum by six parameters. Taking account of the restrictions (3.10a-c), all feasible structures can be fully described by the three parameters s_u^2, s_v^2 and s_{uv}. (For example, $s_u^2, s_v^2 > 0; s_{uv} = 0$ implies $s_w^2 = s_u^2 + s_v^2$, $s_{uw} = -s_u^2$, $s_{vw} = -s_v^2$.) Thus, equation (3.4) specifies the variance of output under the money-supply regime for any such structure. However, the usual assumption $s_{uv} = 0$ is only one among infinitely many, that are all feasible.

The crucial question then is, if Poole's rule remains valid for other feasible stochastic structures. We shall demonstrate in the following by way of a counter-example that this need not be the case.

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[17] Note that all excess demands are measured in nominal units.

[18] It should be noted that equations (3.9) and consequently (3.10a, b, c) only apply to the money-supply regime, as with an interest-rate target there is an endogenous money supply disturbance to be taken into account. However, for the specific comparison undertaken below we can make use of equations (3.10) as these are only needed for computing the results of the money-supply regime.

A characteristic feature of the usual assumption, $s_{uv} = 0$, is that the origins of shocks are located exclusively in the commodity and money market. The financial shock in the bond market just mirrors the real and monetary shocks, but there are no autonomous shocks in the bond market. This implies that e.g. an increase in commodity demand due to positive real shock, $u > 0$, is financed by a simultaneous decrease in the demand for bonds, $w < 0$, i.e. by "selling bonds". However, this assumption that it is the demand for bonds that acts as a kind of buffer stock is not necessarily the most plausible one. To the contrary, with money balances as buffer stocks – a notion propagated for example by Laidler (1984) and Bain/McGregor (1985) – real shocks would be absorbed by deviations of money demand from its target level, $u > 0$ implying $v < 0$. In this case real and financial shocks would be autonomous and the monetary shock just their mirror image. It is this counter-example that we examine in the following.

Now suppose s_u^2 and $s_w^2 > 0$ are given and $s_{uw} = 0$. From the restrictions (3.10a-c) follows:

(3.11) $s_v^2 = s_u^2 + s_w^2$ and

(3.12) $s_{uv} = -s_u^2$.

Inserting (3.11) and (3.12) into (3.4) renders the variance of output under a money-supply regime:

(3.13) $s_Y^2(M) = \dfrac{1}{m^2}\left[(h+g)^2 s_u^2 + g^2 s_w^2\right]$,

whereas the results of the interest-rate regime are still given by (3.5). By comparing (3.13) and (3.5) we obtain the condition for the superiority of the money-supply regime:

(3.14) $\dfrac{s_w^2}{s_u^2} < \dfrac{m^2 - s^2(h+g)^2}{g^2} \equiv \boldsymbol{m}$.

Obviously, the left hand side of (3.14) will be positive, so that the condition can only be fulfilled if $\boldsymbol{m} > 0$. This is, however, not guaranteed by the parameter restrictions:

(3.15) $\boldsymbol{m} = [m - s(g+h)]\,[m + s(g+h)] = (k-s)g[2sh + g(k+s)]$, so that

(3.16) $\boldsymbol{m} > 0 \Leftrightarrow k - s > 0$.

Looking at equation (14), it turns out the sign of $k - s$ cannot be determined a priori, so that $k - s < 0$, and therefore $\boldsymbol{m} < 0$, is at least logically possible. (In fact, it corresponds to the case of a

downward sloping BB-curve.) Yet, in this case the interest-rate regime is always superior to a money supply regime, irrespective of the variances of the shocks.

This can easily be seen by transforming (3.16) so that Poole's rule can be applied directly:

$$(3.17) \qquad \frac{s_v^2}{s_u^2} = \frac{s_w^2}{s_u^2} + 1 < m + 1.$$

Decreasing the variance of monetary relative to real shocks still improves the performance of the money-supply regime. However, as the ratio of these variances cannot fall below one, the money-supply regime will never dominate as long as $m < 0$.

Therefore, taking account of the restrictions imposed by Walras' law focuses attention on the diversity of feasible stochastic structures and on the crucial dependence of Poole's rule on the specific structure assumed. As the assumption of the statistical independence of monetary and real shocks has persisted more or less unquestioned in the models of the new classical macroeconomics (cf. e.g. Sargent/Wallace 1975 and Parkin 1978) as well as in the more recent literature on monetary targeting (cf. Svensson 1999), there seem still to be some lessons to be learnt from the not so recent literature on Walras' law.

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