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DOI:
10.57938/93c047bd-e98a-4ffc-970c-280ee33716bc

Published: *14/01/2014*

Document Version:
Publisher's PDF, also known as Version of record

Department of Economics
Working Paper No. 164

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January 2014

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November 21, 2013

Abstract

This paper presents a 3-Region footloose-entrepreneur new economic geography model. Two symmetric regions are part of an economically integrated area (the Union), while the third region represents an outside trade partner. We explore how the spatial allocation of industrial production and employment within the Union is affected by changes in two aspects of trade liberalisation: regional integration and globalisation. Our main contribution pertains to the analysis of the local and global dynamics of the specified factor mobility process. We show that significant parameter ranges exist for which asymmetric distribution of economic activities is one of the possible long-run outcomes. This is a remarkable result within the NEG literature. We then analyse the impact of international trade liberalisation on the dynamics of agglomeration conditional on the endowments of skilled and unskilled labour of the ouside region.

Keywords: Industrial agglomeration, New Economic Geography, footloose entrepreneurs, local and global dynamics, bifurcation scenarios

JEL classification: C62, F12, F2, R12.

1 Introduction

New Economic Geography (NEG) models do not typically account for the presence of regions other than the ones involved in the economic integration process. Nevertheless, a vast body of empirical evidence reveals the ongoing long-term parallel trends of increasing regional integration and globalisation. The EU is a part of this phenomenon: on the one hand, within-EU integration has become more important over the last decades and, on the other, the EU as a whole has gained greater exposure to the world economy (Foster et al. [2013]).

The analytic structure of NEG models is intrinsically complex, therefore many NEG models are actually confined to the analysis of two regions, aiming to predict the impact of stronger integration on industrial agglomeration in a given economically integrated area (e.g., EU regions). However, the understanding of agglomeration and dispersion forces stemming from stronger exposure of the integrated area to the rest of the world (e.g., EU integration into the world economy) requires a more general set up including (at least) an "outside" region.

Scholars dealing with 3-region NEG models – see, among others, Paluzie [2001]; Krugman and Elizondo [1996]; Brülhart et al. [2004] – typically explore how the spatial distribution of economic activities in a given home country is affected by international trade liberalisation. On the other hand, as pointed out by Behrens [2011], a large part of this literature underplays the role of regional integration. That is, one relevant aspect of economic integration – globalisation – is studied, while the second one – regional integration – is left out of the picture.

Inspired by the case of the EU, the main aim of this paper is to explore how the spatial allocation of industrial production and employment within an economically integrated area (the Union) is affected by changes in both aspects of trade liberalisation: regional integration and globalisation. Our main objective is to study the effects of higher integration within the Union (reduced internal transport costs), and those due to higher economic integration of the Union as a whole with the rest of the World (reduced external transport costs). This is the first contribution of the paper. Furthermore, motivated by the changing picture of the main trade partners of the EU, we study the impact of both aspects of integration under alternative assumptions on the industrialisation level of the Union's trade partners. In particular, we will show that integration with less industrialised regions will make agglomeration of industrial activity within the Union less likely. In addition, we also analyse the effects of international integration under alternative assumptions about the size of the outside region. For many parameter values, trade liberalisation ultimately leads to agglomeration of economic activity with the Union. However, the pattern of the transition to agglomeration depends upon the size of the outside region. When integrating with a small outsider region, catastrophic agglomeration will be observed; instead when integrating with a large outsider country, the transition path to full agglomeration will be smooth.

We depart from the existing multi-region NEG models in three ways. In contrast with most previous contributions, we assume that unskilled workers are immobile both domestically and internationally. This assumption makes our model closer to the reality of the EU where labour mobility plays a relatively unimportant role as compared to other economically integrated areas such as the US (Gáková and Dijkstra [2008]). On the other hand, we will maintain that the interregional mobile factor is human / knowledge capital embodied in skilled workers and entrepreneurs (Forslid and Ottaviano [2003]). A second important departure is the specification of our model in discrete time. This represents an easy way to account for delays in the dynamic process (that are obviously involved in firm relocations). Finally, we try to fill a relevant gap in the NEG literature: the lack of explicit dynamic analysis. This is a particularly relevant issue as many core results of the NEG depend on the properties of dynamic processes, such as multiple equilibria, change in stability properties, the nature of the basins of attraction. We carefully analyse the emerging bifurcation scenarios – detecting a typical sequence – and show that coexistence of equilibria is much more pervasive than in standard NEG models. We show that in some cases – due to the complex structure of the basins of attraction – it is even impossible to predict the long-run spatial distribution of economic activity.

The remainder of the paper is structured as follows. Section 2 provides some stylised facts on the case of EU which inspired our work and reviews the main findings of the literature most related to our contribution. Section 3 presents the general framework of the model including the definition of short-run and long-run equilibria. Section 4 describes the equilibrium properties of the model. Section 5 presents results on local and global dynamics of the model. Section 6 concludes.

2 Stylised facts and Related Literature

The departure point for our analysis are three stylised facts:

- Trade barriers among European regions have been lowered by the long term process of EU integration;
- globalisation has produced greater exposure of the EU to the world economy, leading to higher dependency of the Union (and of each Member State) on final demand outside the EU;
- the deeper integration into the world economy of the EU is currently characterised by an increasing weight of big, less industrialised trade partners.

The strenghtening of the EU internal integration is a well documented fact. Figure 1 outlines the evolution of economic integration within the EU from 1957 to 2001 based on the composite index developed by Dorrucci et al. [2002]. This is a numerical composite index based on scores attributed to each single event of European integration grouped according to Balassa [1961] five main stages of regional integration: a) Free Trade Area (FTA) where internal tariffs and quotas are abolished for imports from area members; b) Customs Union (CU): a FTA setting up tariffs and quotas for trade with non-members; c) Common Market (CM): a CU where restrictions on factor movements as well as non-tariff barriers to trade are abolished; d) Economic Union (EUN): a CM with a significant degree of co-ordination of national policies and harmonisation of relevant domestic laws; and (e) Total Economic Integration (TEI), an EUN with all relevant economic policies conducted at a supra-national level.

Looking at Figure 1 one can identify three sub-periods. The first period goes from March 1957 (Treaty of Rome) to July 1968 (completion of the CU) and is characterised by faster integration as, by the end of this period, more than half of the overall institutional integration process had been already completed. In the late 1960s, the EU was indeed much more than a CU, having already some genuine characteristics of subsequent Balassa stages. The second period (between the early 1970s and the mid-1980s) is characterised by sluggish integration, with the noteworthy exception of the European Monetary System start in March 1979. In the third period, the creation of the CM and the Monetary Union has led to considerable acceleration in regional integration. As a result, the EU/euro area in early 2000s could already be classified somewhere between an EUN and a TEI. The 2004 and 2007 EU enlargements to Eastern countries has then pushed this process forward.

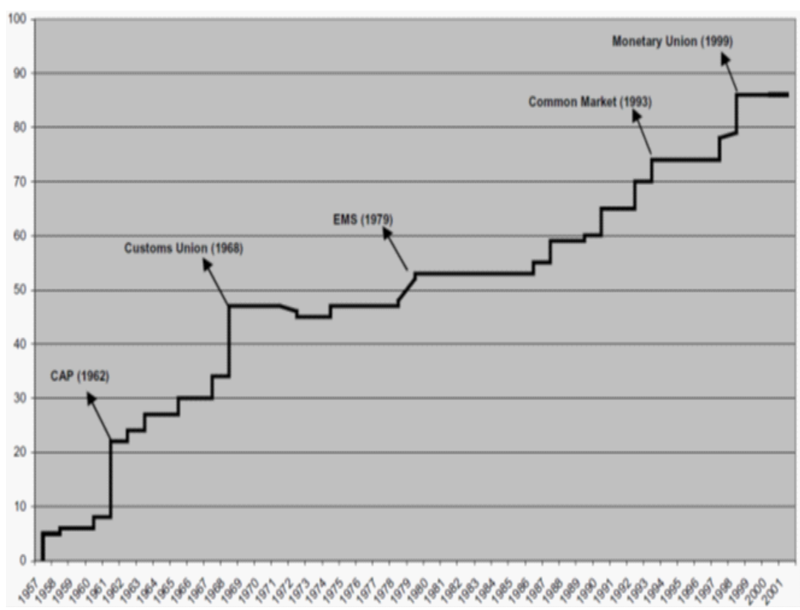

Figure 1: The Index of EU Economic Integration (1957-2001); Source: Dorrucci et al. [2002]; Notes: The index is defined for the EU-6 founding members; Highest score possible for regional integration: 100; 1957 = 0.

Turning to the second stylised fact, note that the EU launched in 2006 its new "Global Europe" strategy aiming at further integrating the EU into the world economy (see for a discussion of the institutional progress, Kleimann

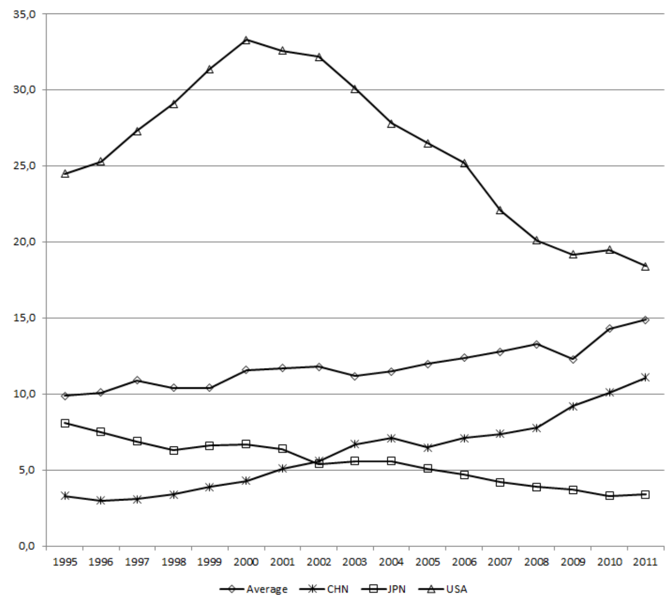

Figure 2: EU-27 value added due to foreign demand by partner (% GDP). Source: Foster et al. [2013] elaboration on WIOD database

[2013]). One visible consequence of increased globalisation is that production taking place in an integration area relies more heavily on foreign final demand. Such a process has been at the core of European countries' recent economic performances since the exponential expansion of emerging economies such has China, India and Brazil has provided an increasing demand for others' countries products. In a detailed study, Foster et al. [2013] calculate the EU value added due to foreign final demand in % of GDP for the 1995-2011 period[1] and point out that since the mid-1990s the dependency of the EU economy on foreign demand has significantly increased: in 1995 9.9% of GDP of the EU-27 was produced to satisfy – directly and indirectly – foreign demand abroad, while this share has increased to almost 15% in 2011.[2] Interestingly, such an increasing trend has continued during the years of the great recession.

The trend of increasing dependency of EU income level upon foreign demand is far from being at rest. According to EC [2013], "over the next two years, 90% of world demand will be generated outside the EU. That is why it is a key priority for the EU to open up more market opportunities for European

[1] The value added created in an economy due to demand for final products in other economies – the so-called "value added exports (VAX)" is described in Johnson and Noguera [2012]. Foster et al. [2013] calculate the VAX for the set of countries included in the WIOD database.

[2] The same holds for employment. EU employment due to foreign demand in % of total employment has risen from 9.3% in 1995 to 11.6% in 2011.

business by negotiating new Free Trade Agreements with key countries. If we were to complete all our current free trade talks tomorrow, we would add 2.2% to the EU's GDP or €275 billion. This is equivalent of adding a country as big as Austria or Denmark to the EU economy. In terms of employment, these agreements could generate 2.2 million new jobs or additional 1% of the EU total workforce."[3]

Turning to our third stylised fact, note that the figures for the valued added due to foreign demand disaggregated by trading partners, reveal striking regional disparities (see Figure 2): China's share increased from 3.3% in 1995 to more 11.1% in 2011 at the expense of Japan (8.1% in 1995 and 3.4% in 2011) and the US (24.5% in 1995 compared to 18.4% in 2011).

A recent OECD study (Woo [2012]) shows that these changes in the trading partners involve also a change in the technology level. Based on growth accounting, (Woo [2012], p15) uncovers a considerable technology gap between China and the US and Japan respectively. Labour productivity in China as measured by output per worker is 16% of that of US workers in 2007 and the total factor productivity level as a measure of technology (or overall efficiency) in China are 25% of the US counterpart. Japan's labour productivity (total factor productivity) is 69% (54%) of the respective US value.

These facts imply relevant research questions to be answered in multi-region NEG models. That is, how are both agglomeration and dispersion forces that drive agglomeration of economic activities affected once deeper integration within the union and within the world economy is considered? And how relevant are the size and the industrialisation level of the trade partners?

So far a small strand of literature has developed 3-region models within the NEG literature. Inspired by the debate on the role of protectionist policies in the development of a pattern of striking regional inequalities during the Spanish industrialisation process, Paluzie [2001] proposes a standard core periphery model accommodating for the presence of a third region. She considers a world economy consisting of two domestic regions and one external economy, with labour being mobile only domestically. In line with Krugman [1991], the centripetal forces that produce agglomerations are represented by the interaction of economies of scales, market size and transport costs, while centrifugal forces that tend to weaken agglomerations are the pull of a dispersed rural market. The main result in the model is that a reduction in the external trade cost strengthens the agglomerative forces in the home country with two regions. In other words, external trade liberalisation is expected to increase regional inequalities in the country that opens up to trade. Similar results are put forward by Alonso-Villar [2001], and Monfort and Nicolini [2000].

Krugman and Elizondo [1996], obtain the opposite result, in the same context of a model with three regions, two domestic and one external, where the domestic dispersion force is due to land rent and commuting costs and it is thus exogenous and independent of trade costs. The authors study the impact of

[3]For an overview of the most important forthcoming and on-going free trade negotiations see EC [2013].

trade liberalisation on the distribution of economic activities within the home country and conclude that opening up external trade favours dispersion of economic activity between the two internal regions. It is claimed in the paper that such a result explains the rise of large metropolis in developing countries (Mexico City is the case discussed by the authors) and its progressive loss of importance after the implementation of trade liberalisation policies.

Brülhart et al. [2004] and Crozet and Soubeyran [2004] introduce more geographical structure into the analysis, as they assume that one of the home regions is a border region, i.e. that it has lower transport cost with respect to the outside region than the other home region. Also in these frameworks, a reduction of the international transport cost favours agglomeration in the 2-region home country.

Brülhart et al. [2004] present a footloose entrepreneur model – a 3-region version of Pflüger [2004] – where two of the three regions are relatively integrated. The aim of the authors is to track how the economies of these two regions are affected by an opening towards the third region. The real world case that motivate this work is the 2004 EU enlargement, which integrated ten Central and Eastern European countries (the third region) fully into the EU's internal market. The research question is then linked to the implications for the distribution of economic activities in the incumbent EU countries of the improved access to and from the third region. In the model, the production of the manufactured good requires one unit of human capital and a variable amount of labour. Human capital is mobile between the two regions in one country but immobile with respect to the third region. The sectorial location is determined endogenously through the interplay of agglomeration and dispersion forces. External market opening has a bearing on several spatial forces. Forces related to better access to foreign export markets and cheaper imports enhance the locational attraction of the border region. Conversely, forces related to import competition from foreign firms enhance the locational attraction of the interior region. The interplay of these forces in the non-linear setup of the model can lead to a variety of equilibria. The main results are such that the range of parameter values, for which domestic manufacturing agglomerates in only one region, increases as external trade costs fall. The same result obtains if – given constant external trade costs – the foreign country gets bigger, i.e., the larger the outside economy, ceteris paribus, the greater the probability that domestic manufacturing agglomerates in one region. Hence, the size of the third region matters for the results.

Wang and Zheng [2013] notice that in most developing economies, like China, a country includes a gate region and the hinterland, where the gate region has better access to overseas markets and the hinterland has a greater share of unskilled workers. Hence, they extend the standard framework of Krugman [1991] by assuming that domestic regions are asymmetrical in terms of their sizes and accessibilities. They obtain two key results. First, when international trade liberalisation continues but domestic regions remain poorly integrated, the gate region experiences a change from partial to full agglomeration. Second, when a country is closed to global markets, regional integration makes the hinterland attractive to a greater share of manufacturing firms. However, when a country is extremely open to global markets, full agglomeration of manufacturing firms occurs in the gate region.

With the exception of Brülhart et al. [2004] and Crozet and Soubeyran [2004], in the above mentioned contributions, labour mobility is the dynamic process bringing agglomeration about. This is the case of Krugman and Elizondo [1996] for Mexico, Paluzie [2001] for Spain and Wang and Zheng [2013] for China. In all these cases labour mobility is plausible whereas this is not the case for the EU.

As shown by Gáková and Dijkstra [2008], labour mobility between the regions of the EU at NUTS 2 level is relatively low. This seems to be a common feature in the EU as it applies to both the old and the new Member States, irrespective of their economic development or the openness of their labour market. In particular, the analysis provided by Gáková and Dijkstra [2008] shows that the share of working age residents moving in another EU region represents, on average, less than 1% of the EU's working age population (vs 2% in the US).[4] As migration in Europe is rather weak, as far as the EU is concerned, the mobility of unskilled workers does not really appear to play the role of an adjustment process to wage differential among countries (Siebert [1997]; Obstfeld and Peri [1998]; Puga [2002]). On the other hand, firm mobility has been achieving an increasing role since the EU enlargement to the Eastern European countries from the mid-1990s.

The contributions whose main results have been summarised earlier in this section share the common feature of addressing only one part of the issues at hand as they only analyse the effects of a closer integration of the home economy with the rest of the world, independently of the transportation costs within the home economy itself. Nevertheless, confining our attention to the EU, two parallel trends have been taking place in the last decades – gaining momentum with the EU enlargement to the Eastern European countries – and provide the first two stylised facts for our theoretical framework. Moreover, the higher importance gained by China at the expenses of the US provides a motivation for studying the effects of globalisation conditional upon the size and the output composition of the external commercial partners.

3 General framework

In this section we present a variant of the 3-region NEG model developed in Forslid and Ottaviano [2003] where the mobile factor is human / knowledge capital, embodied in skilled workers or, equivalently in their framework, 'entrepreneurs'. This is particularly convenient in a multiregional framework for two reasons. First, the assumption of mobile skilled workers / entrepreneurs implies

[4] The analysis presented in this paper is based on the average share of the working age residents in 2005-2006 who had changed their region of residence during the previous year.

analytical tractability and it matches the empirical evidence (esp. for Europe) according to which skilled workers / entrepreneurial undertakings are typically more mobile than less specialised labour (Forslid and Ottaviano [2003], p. 230). Differently from Forslid and Ottaviano [2003], we envisage a situation in which the entrepreneurs mobility could be limited by various types of barriers (for example: national, cultural, language and legal barriers), even stronger than those constraining the unskilled workforce migration. Specifically, in our framework, entrepreneurial migration occurs only within an economically integrated area (a Union composed of two regions) and no factor movements take place between the Union and the rest of the World.

3.1 Basic assumptions

We consider an economy composed of 3 regions ($r = 1, 2, 3$). Regions 1 and 2 are part of an economically integrated area (the Union); region 3 instead represents an outside trade partner. There are two sectors, agriculture (A) and manufacturing (M). There is a unique homogeneous agricultural good produced under perfect competition, while manufacturing involves n differentiated varieties produced by monopolistically competitive firms. Unskilled workers (L) and entrepreneurs (E) are endowed with (unskilled) labour and human capital, respectively. Workers are immobile (but can be reallocated across sectors), whereas entrepreneurs can migrate only between region 1 and 2, i.e. within the Union. We assume that there is not factor mobility between the Union and region 3.

L is the amount of unskilled labour in the overall economy and θ_r is the share of labour located in region $r(= 1, 2, 3)$; it follows that $\theta_r L$ is the endowment of unskilled workers of region r. With immobile unskilled workers and a constant and equal to one wage rate (see below), $\theta_r L$ can also be interpreted as the size of local demand (θ_r representing its share) which is not affected by entrepreneurial migration. When regions 1 and 2 are symmetric, we have that $\theta_1 = \theta_2 = \theta$ and $\theta_3 = 1 - 2\theta$. Moreover, E represents the overall number of entrepreneurs in the economy and $\widetilde{E}$ the number of entrepreneurs that are free to move between regions 1 and 2. We denote by $\widetilde{n} = \frac{\widetilde{E}}{E}$ the corresponding share. Consequently, $\overline{E} = E - \widetilde{E}$ represents the number of immobile entrepreneurs located in region 3 and $1 - \widetilde{n} = \frac{\overline{E}}{E} = 1 - \frac{\widetilde{E}}{E}$ is the corresponding share.

3.2 Consumers' preferences

The three regions are homogeneous in terms of tastes. Individual (entrepreneur or unskilled worker) preferences are expressed by a two-tier utility function. The upper-tier concerns the choice between agricultural and manufactured goods according to the following Cobb-Douglas utility function:

$$U = C_M^{\mu} C_A^{1-\mu}$$

where C_A is the consumption of the agricultural good. The lower-tier concerns the consumption of the composite of manufactured varieties, C_M, given the following CES function:

$$C_M = \left(\sum_{i=1}^{n} c_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

where c_i represents the quantity consumed of the variety i, with $i = 1, ..., n$; σ the constant elasticity of substitution / taste for variety: the closer σ to 1, the greater is consumer's taste for variety, with $\sigma > 1$; and μ and $1-\mu$ represent the income shares devoted to the manufactured varieties and to the homogeneous agricultural good, respectively, with $0 < \mu < 1$.

The budget constraint of an individual resident in region r is

$$\sum_{i=1}^{N} \widetilde{p}_i c_i + p_A C_A = y \tag{1}$$

where p_A is the price of the homogeneous agricultural good; $\widetilde{p}_i$ is the price of variety i inclusive of transport costs and y is the income of the individual agent (unskilled worker or entrepreneur).

3.3 Production

The A sector is characterised by perfect competition and constant returns to scale. Production of 1 unit of output requires only α unit of L; without loss of generality, we set $\alpha = 1$. Moreover, we assume that none of the regions has enough labour to engage exclusively in the production of the agricultural good, that is, the so-called "non-full-specialisation condition" holds.

The M sector is (Dixit-Stiglitz) monopolistically competitive. It is modelled according to a few basic characteristics: identical firms produce differentiated goods / varieties with the same production technology involving a fixed component (one entrepreneur), and a variable component (unskilled workers), with β units of L required for each unit of the differentiated good.

The total cost of producing the quantity q_i of a variety i corresponds to:

$$CT(q_i) = \pi_i + w\beta q_i$$

where π_i represents the fixed cost component and the remuneration of the entrepreneur, with $i = 1, ..., n$.

Given consumers' preference for variety and increasing returns, each firm will always produce a variety different from those produced by the other firms. Moreover, since one entrepreneur is required for each manufacturing firm, the total number of firms / varieties, n, always equates the total number of entrepreneurs, $E = n$. Denoting by x_t the share of entrepreneurs located in region 1 during the time unit t, and recalling the notation introduced above, the number of regional varieties produced in region $r(=1,2,3)$ during that period can be expressed as

$$\begin{aligned} n_{1,t} &= x_t\widetilde{E} = x_t\widetilde{n}E \\ n_{2,t} &= (1-x_t)\widetilde{E} = (1-x_t)\widetilde{n}E \\ n_{3,t} &= \overline{E} = (1-\widetilde{n})E \end{aligned}$$

where $0 \leq x_t \leq 1$.It also follows that $n_{r,t}$, $\widetilde{n}$ and $1-\widetilde{n}$ correspond to the size of the manufacturing sector in region r, in the Union and in the outside region, respectively.

3.3.1 Trade costs

Distance plays a crucial role in NEG models. Trade between regions can be inhibited by various types of costs that can involve transportation and / or (tariffs or non tariff) barriers and / or other types of impediments / frictions. We adopt a broad definition of trade costs. Following Anderson and van Wincoop [2004], pp. 691-692): "[t]rade costs broadly defined, include all costs incurred in getting a good to a final user other than the marginal cost of producing the good itself: transportation costs (both freight costs and time costs), policy barriers (tariffs and nontariff barriers), information costs, contract enforcement costs, costs associated with the use of different currencies, legal and regulatory costs, and local distribution costs (wholesale and retail)." Typically, NEG models assume that transportation of the agricultural good is costless. On the other hand, trade costs for manufacturers take an iceberg form: if one unit is shipped from region s to region r, only $\frac{1}{T_{rs}}$ arrives at destination, where $T_{rs} \geq 1$ and $r, s = 1, 2, 3$.

Region 1 and 2 (the Union) are involved in a trade agreement whereas the economic integration with region 3 (the outside region) is less deep. We model this spatial arrangement as follows: the three regions are located on the vertices of a isosceles triangle. The "internal distance" (trade barriers) between regions 1 and 2 is S (short); the "external distance" between 1 and 3 and 2 and 3 is the same and it is equal to L (long). Moreover, trade costs do not depend on the direction of thet trade flow ($T_{rs} = T_{sr}$). Trade costs between region 1 and 2 are

$$T_{12} = T_S$$

and between regions 1 and 3 and regions 2 and 3 are

$$T_{13} = T_{23} = T_L$$

where $T_L > T_S \geq 1$. Finally, in order to simplify the notation, we introduce the standard transformation of trade costs into the following "trade freeness" parameters: $\phi_{12} = \phi_S$ and $\phi_{13} = \phi_{23} = \phi_L$, where $\phi_S \equiv T_S^{1-\sigma}$ and $\phi_L \equiv T_L^{1-\sigma}$ and where $\phi_L < \phi_S \leq 1$.

3.4 Short-run general equilibrium

The short-run general equilibrium (SRGE) in period t is defined by a given spatial allocation of entrepreneurs across regions, $x_t\widetilde{n}$ and $(1- x_t)\widetilde{n}$ in regions 1 and 2 and the invariant share $1-\widetilde{n}$ in region 3. In a SRGE, which is established istantaneously in each period, supply equals demand for the agricultural commodity and each manufacturer meets the demand for its variety. Moreover, as a result of Walras's law, simultaneous equilibrium in the product markets implies equilibrium in the regional labour markets.

With zero transport costs, the agricultural price p_A is the same across regions. Since competition results in zero agricultural profits, the short-run equilibrium nominal wage w is equal to the agricultural product price and it is also equalised across regions. Setting this wage / agricultural price equal to 1, it becomes the numeraire in terms of which the other prices are defined, $w = p_A = 1$.[5]

Facing a wage of 1, each manufacturer has a marginal cost of β. Each maximizes profit on the basis of a perceived price elasticity of $-\sigma$ and sets a local (mill) price p for its variety, given by

$$p = \frac{\sigma}{\sigma - 1}\beta \tag{2}$$

The demand facing a producer located in region r (where it is also taken into account the part that is "melting along the way" because of iceberg trade costs) corresponds to:

$$\begin{aligned} d_{r,t} &= \left(\sum_{s=1}^{3} \mu Y_{s,t} P_{s,t}^{\sigma-1} T_{rs}^{1-\sigma}\right) p^{-\sigma} = \left(\sum_{s=1}^{3} s_{s,t} P_{s,t}^{\sigma-1}\phi_{rs}\right) p^{-\sigma}\mu Y \\ &= \left(\sum_{s=1}^{3} \frac{s_{s,t}}{\Delta_{s,t}}\phi_{rs}\right) p^{-1}\frac{\mu Y}{E} \end{aligned} \tag{3}$$

where

$$P_{r,t} = \left(\sum_{s=1}^{R} n_{s,t}^{1-\sigma} p^{1-\sigma} T_{rs}^{1-\sigma}\right)^{\frac{1}{1-\sigma}} = \Delta_{r,t}^{\frac{1}{1-\sigma}} E^{\frac{1}{1-\sigma}} p, \tag{4}$$

is the price index facing consumers in region r; $Y_{s,t}$ represents income and expenditure in region s; $s_{s,t} = \frac{Y_{s,t}}{Y}$ denotes region s's share in expenditure and $s = 1, ..., 3$. Moreover, we have defined

$$\Delta_{r,t} = x_t\widetilde{n}\phi_{r1} + (1 - x_t)\widetilde{n}\phi_{r2} + (1 - \widetilde{n})\phi_{r3}$$

[5] Denoting by Y the income of the overall economy, that (as confirmed below) is invariant over time, total expenditure on the agricultural product is $(1-\mu)Y$. Assuming $(1-\mu)Y > \max(2\theta L, (1-\theta)L)$ all regions produce the agricultural commodity, whereas $(1-\mu)Y > \max(\theta L, (1-2\theta)L)$ implies that no single region is able to satisfy all the demand for the agricultural good.

SRGE in region r requires that each firm meets the demand for its variety. For a variety produced in region r,

$$q_{r,t} = d_{r,t} \tag{5}$$

where $q_{r,t}$ is the output of each firm located in region r. From equation (2), the short-run equilibrium operating profit / entrepreneur remuneration per variety in region r is

$$\pi_{r,t} = pq_{r,t} - \beta q_{r,t} = \frac{pq_{r,t}}{\sigma}. \tag{6}$$

Since profit equals the value of sales time $1/\sigma$ and since total expenditure on manufacturers is μY, the total profit received by entrepreneurs is $\mu Y/\sigma$. Total income is $Y = L + \mu Y/\sigma$, so that

$$Y = \frac{\sigma L}{\sigma - \mu}. \tag{7}$$

Total profit is therefore $\mu L/(\sigma - \mu)$.[6]

Using (2) to (7), the short-run equilibrium profit in region r is determined by the spatial distribution and by the regional expenditure shares

$$\pi_{r,t} = \left(\sum_{s=1}^{3} \mu Y_{s,t} P_{s,t}^{\sigma-1} T_{rs}^{1-\sigma}\right) \frac{p^{1-\sigma}}{\sigma} = \left(\sum_{s=1}^{R} \frac{s_{s,t}}{\Delta_{s,t}} \phi_{rs}\right) \frac{\mu Y}{\sigma E} \tag{8}$$

Under our assumptions on trade costs across regions, we can write

$$\begin{aligned}
\pi_{1,t} &= \left(\frac{s_{1,t}}{\Delta_{1,t}} + \frac{s_{2,t}}{\Delta_{2,t}}\phi_S + \frac{s_{3,t}}{\Delta_{3,t}}\phi_L\right)\frac{\mu Y}{\sigma N} \\
\pi_{2,t} &= \left(\frac{s_{1,t}}{\Delta_{1,t}}\phi_S + \frac{s_{2,t}}{\Delta_{2,t}} + \frac{s_{3,t}}{\Delta_{3,t}}\phi_L\right)\frac{\mu Y}{\sigma N} \\
\pi_{3,t} &= \left(\frac{s_{1,t}}{\Delta_{1,t}}\phi_S + \frac{s_{2,t}}{\Delta_{2,t}}\phi_S + \frac{s_{3,t}}{\Delta_{3,t}}\right)\frac{\mu Y}{\sigma N}
\end{aligned} \tag{9}$$

where
$\Delta_{1,t} = x_1\widetilde{n} + (1 - x_t)\widetilde{n}\phi_S + (1 - \widetilde{n})\phi_L$,
$\Delta_{2,t} = x_t\widetilde{n}\phi_S + (1 - x_t)\widetilde{n} + (1 - \widetilde{n})\phi_L$,
$\Delta_{3,t} = x_t\widetilde{n}\phi_L + (1 - x_t)\widetilde{n}\phi_L + 1 - \widetilde{n} = 1 - \widetilde{n}(1 - \phi_L)$.

Regional incomes / expenditures are:

[6] Equation (7) confirms that total income is invariant over time. From (7), $(1-\mu)Y > \max(2\theta L, (1-\theta)L)$ is equivalent to $\min(2\theta\mu + (1-2\theta)\sigma - \mu\sigma, 2[(1-\theta)\mu + \theta\sigma - \mu\sigma]) > 0$; and $(1-\mu)Y > \max(\theta L, (1-2\theta)L)$ is equivalent to $\min(2[\theta\mu + (1-\theta)\sigma - \mu\sigma], (1-2\theta)\mu + 2\theta\sigma - \mu\sigma) > 0$. The former is a sufficient non-full-specialisation condition and the latter is a necessary one, where both are expressed in terms of the utility parameters.

$$Y_{r,t} = L_{r,t} + n_{r,t}\pi_{r,t}$$

Under our assumptions on trade costs and unskilled labour endowments, we can write:

$$\begin{aligned}
Y_{1,t} &= \theta L + x_t \widetilde{n} E \\
Y_{2,t} &= \theta L + (1-x_t)\widetilde{n} E \\
Y_{3,t} &= (1-2\theta)L + (1-\widetilde{n})E
\end{aligned} \qquad (10)$$

Using (9) and (10) the regional income shares can be expressed in terms of x_t:

$$s_{1,t} = \frac{(\sigma-\mu)\theta + \mu\widetilde{n}x_t\left[\frac{\phi_L}{\Delta_{3,t}} - \frac{\left(\frac{\phi_L}{\Delta_{3,t}}-\frac{\phi_S}{\Delta_{2,t}}\right)\left((\sigma-\mu)\theta+\frac{\mu\widetilde{n}(1-x_t)\phi_L}{\Delta_{3,t}}\right)}{\sigma-\mu\widetilde{n}(1-x_t)\left(\frac{1}{\Delta_{2,t}}-\frac{\phi_L}{\Delta_{3,t}}\right)}\right]}{\left[\sigma-\mu\widetilde{n}x_t\left(\frac{1}{\Delta_{1,t}}-\frac{\phi_L}{\Delta_{3,t}}\right)\right]\left[1-\frac{\left(\frac{\phi_L}{\Delta_{3,t}}-\frac{\phi_S}{\Delta_{2,t}}\right)\left(\frac{\phi_L}{\Delta_{3,t}}-\frac{\phi_S}{\Delta_{1,t}}\right)\mu^2\widetilde{n}^2x_t(1-x_t)}{\left[\sigma-\mu\widetilde{n}(1-x_t)\left(\frac{1}{\Delta_{2,t}}-\frac{\phi_L}{\Delta_{3,t}}\right)\right]\left[\sigma-\mu\widetilde{n}x_t\left(\frac{1}{\Delta_{1,t}}-\frac{\phi_L}{\Delta_{3,t}}\right)\right]}\right]}$$

$$s_{2,t} = \frac{(\sigma-\mu)\theta + \mu\widetilde{n}(1-x_t)\left[\frac{\phi_L}{\Delta_{3,t}} - \frac{\left(\frac{\phi_L}{\Delta_{3,t}}-\frac{\phi_S}{\Delta_{1,t}}\right)\left((\sigma-\mu)\theta+\frac{\mu\widetilde{n}x_t\phi_L}{\Delta_{3,t}}\right)}{\sigma-\mu\widetilde{n}x_t\left(\frac{1}{\Delta_{1,t}}-\frac{\phi_L}{\Delta_{3,t}}\right)}\right]}{\left[\sigma-\mu\widetilde{n}(1-x_t)\left(\frac{1}{\Delta_{2,t}}-\frac{\phi_L}{\Delta_{3,t}}\right)\right]\left[1-\frac{\left(\frac{\phi_L}{\Delta_{3,t}}-\frac{\phi_S}{\Delta_{2,t}}\right)\left(\frac{\phi_L}{\Delta_{3,t}}-\frac{\phi_S}{\Delta_{1,t}}\right)\mu^2\widetilde{n}^2x_t(1-x_t)}{\left[\sigma-\mu\widetilde{n}(1-x_t)\left(\frac{1}{\Delta_{2,t}}-\frac{\phi_L}{\Delta_{3,t}}\right)\right]\left[\sigma-\mu\widetilde{n}x_t\left(\frac{1}{\Delta_{1,t}}-\frac{\phi_L}{\Delta_{3,t}}\right)\right]}\right]}$$

$$s_{3,t} = 1 - s_{1,t} - s_{2,t}$$

Given that the agricultural price is 1, the real income / indirect utility of an entrepreneur in region r is:

$$V_{r,t} = \frac{\pi_{r,t}}{P_{r,t}^{\mu}}$$

Notice that the real income of an entrepreneur located in region 3 is affected by the distribution of the manufacturing activity between region 1 and 2, even though no migration takes place from that region towards the other two.

Letting $\widetilde{n} = 1$ an interesting result can be shown:

Proposition 1 *In the absence of a manufacturing sector in region* 3, $\widetilde{n} = 1$, *real incomes in region* 1 *and* 2 *are not affected by the distance from region* 3.

This can be easily checked in (4), in (9) and in the expressions for the income shares considering that, when $\widetilde{n} = 1$, $\Delta_{1,t} = x_1 + (1-x_t)\phi_S$, $\Delta_{2,t} = x_t\phi_S + 1 - x_t$ and $\Delta_{3,t} = \phi_L$.

According to Proposition 1, external trade liberalisation has no impact on the locational choices of entrepreneurs within the Union in the absence of a manufacturing sector in the outside region. This result follows from three features of the model set up: (i) the demand for the manufactured goods is unitary elastic: the change in trade costs, via ϕ_L,determines a proportional change in the price index in region 3 and a similar but inversely proportional change in the quantity demanded, so the overall change of expenditures on manufacturing in this region is zero; (ii) since region 3 does not produce manufactured varieties, a change in ϕ_L does not impact on price indices in regions 1 and 2; (iii) the distance between regions 1 and 3 and that between 2 and 3 are the same, so that prices of imported manufactured goods in region 3 do not depend on entrepreneurs' locational choice between region 1 and region 2.

3.5 The entrepreneurial migration hypothesis and the complete dynamical model

Since the share of entrepreneurs located in region 3 is given, migration only involves regions 1 and 2. The central dynamic equation is analogous to the replicator dynamics, widely used in evolutionary game theory:

$$Z_{t+1} = x_t \left[1 + \gamma \left(\frac{V_{1,t} - [x_t V_{1,t} + (1-x_t)V_{2,t}]}{x_t V_{1,t} + (1-x_t)V_{2,t}} \right)\right] \tag{11}$$

where γ represents the migration speed. According to (11), the share of entrepreneurs in region 1, Z_{t+1}, depends on a comparison between the real income / indirect utility gained in that region and the weighted average of the incomes / indirect utilities in region 1 and 2. Expression (11) can be reformulated in terms of the relevant state variable x_t:

$$Z_{t+1} = Z(x_t) = \gamma \left(1 + (1-x_t)\frac{T(x_t)}{1 + x_t T(x_t)}\right)$$

where $T(x_t) = \frac{V_{1,t}}{V_{2,t}} - 1 = \frac{\frac{s_{1,t}}{\Delta_{1,t}} + \frac{s_{2,t}}{\Delta_{2,t}}\phi_S + \frac{1-s_{1,t}-s_{2,t}}{\Delta_3}\phi_L}{\frac{s_{1,t}}{\Delta_{1,t}}\phi_S + \frac{s_{2,t}}{\Delta_{2,t}} + \frac{1-s_{1,t}-s_{2,t}}{\Delta_3}\phi_L} \left(\frac{\Delta_{2,t}}{\Delta_{1,t}}\right)^{\frac{\mu}{1-\sigma}} - 1$

Taking into account the constraints, $0 \leq x_t \leq 1$, the full dynamical model corresponds to the map:

$$x_{t+1} = \Lambda(x_t) = \begin{cases} 0 & if \quad Z(x_t) < 0 \\ Z(x_t) & if \quad 0 \leq Z(x_t) \leq 1 \\ 1 & if \quad Z(x_t) > 1 \end{cases} \tag{12}$$

In what follows, for convenience, we drop the subscript t.

A long-run stationary equilibrium involves $\Lambda(x^*) = x^*$, where x^* represents a fixed point of the map (12). There are three types of fixed points, concerning the location of the manufacturing secor within the Union (given the share of entrepreneurs located in region 3):

1. the Core-Periphery equilibria are characterised by full agglomeration in region 1 or in region 2. These are: $x^{CP(0)}$, corresponding to complete agglomeration in region 2, which gives $Z(0) = 0$; and $x^{CP(1)}$, corresponding to complete agglomeration in region 1, which gives $Z(1) = 1$.

2. the symmetric equilibrium is characterised by an equal split of the manufacturing sector between region 1 and 2: $x^* = \frac{1}{2}$, that gives $Z\left(\frac{1}{2}\right) = \frac{1}{2}$. It also implies $T\left(\frac{1}{2}\right) = 0$.

3. the asymmetric interior equilibria are characterised by incomplete agglomeration in one of the two regions of the Union, with some industry still present in the other region. The following cases are possible depending on parameters configuration:

Case 1: no asymmetric fixed point exists;

Case 2: two asymmetric fixed points exist which are symmetric around $\frac{1}{2}$: x_a, $1 - x_a$;

Case 3: four asymmetric fixed points exist two by two around $\frac{1}{2}$: x_a, $1 - x_a$ and x_b, $1 - x_b$.

These equilibria are obtained by solving the conditions $Z(x_i) = x_i$ and $Z(1 - x_i) = 1 - x_i$, which imply $T(x_i) = 0$ and $Z(1 - x_i) = 0$, where $i = a, b$.

4 Properties of the equilibrium

In the following we take a closer look at the properties of the fixed point, in particular at the sectoral employment structure measured by the share of unskilled workers that are employed in the manufacturing sector (skilled workers are entirely employed in manufacturing). Note that our model allows for differences between the regions in relative skill endowment (i.e. the ratio between skilled and unskilled workers residing in the region under consideration) and for differences in skill requirements between the two industries (skilled workers are used only in the manufacturing sector). This allows to analyse the determinants of the sectoral employment structure from an Heckscher-Ohlin perspective in the sense of establishing the link between sectoral employment structure and relative skill endowment comparing all three regions. Has the region with a higher skill endowment also the bigger manufacturing sector (measured by the share of unskilled employment)?

We start with the symmetric equilibrium. Recall that the endowment of unskilled workers in region 1 is equal to θL (which is also the endowment of region 2); the endowment of region 3 equals $(1-2\theta)L$. Note that in a symmetric equilibrium $\Delta_1 = \Delta_2 = 0.5\tilde{n} + \phi_L + 0.5\tilde{n}\phi_S - \tilde{n}\phi_L$ and $\Delta_3 = 1 - \tilde{n}(1-\phi_L)$ holds; as well as $s_1 = s_2$ and $s_3 = 1 - 2s_1$. Therefore, the ratio of unskilled employment in manufacturing and total endowment (employment) of unskilled labour in region r, USR_r, is equal to

$$USR_1 = USR_2 = \frac{\left(\frac{s_1(1+\phi_S)}{0.5\tilde{n}+\phi_L+0.5\tilde{n}\phi_S-\tilde{n}\phi_L} + \frac{(1-2s_1)\phi_L}{1-\tilde{n}(1-\phi_L)}\right)\frac{\sigma-1}{\beta\sigma}\frac{\mu Y}{E}\beta\frac{\tilde{n}}{2}E}{\theta L}$$

$$USR_3 = \frac{\left(\frac{2s_1\phi_L}{0.5\tilde{n}+\phi_L+0.5\tilde{n}\phi_S-\tilde{n}\phi_L} + \frac{1-2s_1}{1-\tilde{n}(1-\phi_L)}\right)\frac{\sigma-1}{\beta\sigma}\frac{\mu Y}{E}\beta(1-\tilde{n})E}{(1-2\theta)L}$$

The relative employment ratio is

$$\frac{USR_1}{USR_3} = \frac{Num_1}{Num_3}\frac{\frac{0.5\tilde{n}E}{\theta L}}{\frac{(1-\tilde{n})E}{(1-2\theta)L}}$$

with

$$Num_1 = 2\left(\phi_S - 2(\phi_L)^2 + 1\right)(1-\tilde{n})(\sigma-\mu)\theta + \sigma\phi_L(\tilde{n} + 2\phi_L - \tilde{n}\phi_L) + \sigma\phi_L\tilde{n}(\phi_S - \phi_L)$$

and

$$Num_3 = \left(\phi_S - 2(\phi_L)^2 + 1\right)\tilde{n}(\sigma-\mu)(1-2\theta) + 2\sigma\phi_L + 2\tilde{n}\sigma\phi_L(\phi_L - 1)$$

Note that $\frac{\frac{0.5\tilde{n}E}{\theta L}}{\frac{(1-\tilde{n})E}{(1-2\theta)L}}$ is the relative endowment ratio (skilled labour to unskilled labour in region 1 and region 3 resp.).

Therefore, *relative employment ratio* $\leqslant$ *relative endowment ratio* if $Num_1 \leqslant Num_3$ or if $Num_1 - Num_3 \leqslant 0$.

It can be shown that the relative employment ratio is equal to the relative endowment ratio, i.e. that

$$Num_1 - Num_3 = \left(\phi_S - 2(\phi_L)^2 + 1\right)(\sigma-\mu)(2\theta-\tilde{n}) + \sigma\phi_L(1-\phi_L)(3\tilde{n}-2) + \tilde{n}\sigma\phi_L(\phi_S-\phi_L) = 0$$

if the following 3 conditions hold:

1. if $\tilde{n} = \frac{2}{3}$; i.e. if the skilled workers/units of human capital/entrepreneurs/firms are equally distributed over all 3 regions.

2. if $\theta = \frac{\tilde{n}}{2}$ (implying that $1 - 2\theta = 1 - \tilde{n}$ holds as well); i.e. if each firm/ skilled workers/units of human capital/entrepreneurs located in any region has the same number of unskilled workers residing in this region; taking the number of unskilled workers in each region as a measure of the size of the home market this condition holds, if no location offers advantages wrt local market size.

3. if $\phi_L = \phi_S$; i.e. if no location offers an advantage wrt transport costs.

If any of these 3 conditions is not satisfied, then the relative employment ratio is not equal to the relative endowment ratio. Note that condition 2 and condition 3 have a direct NEG interpretation.

It can be shown that $\frac{USR_1}{USR_3}$ decreases with θ, and that it increases with $\tilde{n}$ - both results are in line with an Heckscher Ohlin intuition: increasing θ, i.e. the relative endowment of unskilled workers, reduces the relative share of employment in the manufacturing sector (the two regions integration area specialize in agriculture); increasing $\tilde{n}$, i.e. increasing the endowment with skilled labour, leads to a relative specialization in the manufacturing sector.

Wrt the transport cost, it can be shown that $\frac{USR_1}{USR_3}$ increases with ϕ_S: a reduction of the internal trade costs fosters industry in the 2-regions integration area and this area specializes in manufacturing (and the third region in agriculture).

Wrt ϕ_L, only local results at $\theta = \frac{\tilde{n}}{2}$ and $\tilde{n} = \frac{2}{3}$ can be obtained:

$\frac{USR_1}{USR_3}$ has a parabolic shape in ϕ_L with $\frac{USR_1}{USR_3} = 1$ for $\phi_L = \phi_S$ and $\phi_L = 0$, and with $\frac{USR_1}{USR_3} > 1$ for $0 < \phi_L < \phi_S$.

Starting with prohibitive trade costs wrt the third country, a slight reduction in the trade barriers (a slight increase in the trade freeness) fosters industry in the 2-region country (that specialises in industry); if we start from identical trade costs between all three regions, reducing the trade freeness wrt the third country again fosters manufacturing in the two regions country (which specialises in manufacturing).

Let us now turn to the Core-Periphery equilibrium in which the mobile firms are agglomerated in region 1. This case turns out to be analytically more complex and only local results can be obtained. With $\phi_L = \phi_S$, $\tilde{n} = 0.5$ and $\theta = \frac{1}{3}$ region 1 and region 3 are identical; and the relative employment ratio equals the endowment ratio. For that point, we have (local) results for the derivatives: Wrt endowment variation, the Heckscher Ohlin intuition still applies - the relative employment ratio decreases with θ, and it increases with $\tilde{n}$. Wrt the transport cost, it can be shown that the relative employment ratio increases with ϕ_S and it decreases with ϕ_L. The last result is a bit more general, it holds for all $0 \leq \phi_L \leq \phi_S$.

The analysis in this section is centered on how the employment structure inside the 2-region Union is affected by changes in trade costs and in relative endowments with entrepreneurs or labour, always in comparison to the mirror developments in the outside region. It turned out that the Heckscher-Ohlin intuition carries over to an astonishingly large extent. However, at the heart of a New Economic Geography perspective are not so much comparative static properties as the dynamic processes: Under what conditions the symmetric equilibrium inside the Union is destabilized and a self-reinforcing agglomeration process sets in leading to a Core-periphery pattern? The next section explicitly addresses the properties of the dynamic process. Note that the dynamic process is based on the mobility of firms; it thus involves only the two regions inside the Union on which the subsequent analysis focuses.

5 Local and global dynamics

In the present section we discuss some analytical and numerical results related to local and global dynamics of the map Λ defined in (12). Indeed, in spite of the fact that this map is one-dimensional, its complicated form allows to obtain only a few analytical results, therefore numerical investigation is quite important. It helps, in particular, to get an idea about the overall bifurcation structure of the parameter space of the map and about bifurcation sequences which can be observed under variation of its parameters.

5.1 Preliminaries

We begin the dynamic analysis by exploring the local stability of the fixed points of the map Λ listed in the previous section. Recall that the map Λ always has two Core-Periphery fixed points, $x^{CP(0)} = 0$ and $x^{CP(1)} = 1$, as well as one symmetric fixed point $x^* = \frac{1}{2}$. An important property of the map Λ is related to its *symmetry with respect to* x^*: it can be checked that $\Lambda(1-x) = 1 - \Lambda(x)$. Thus, any invariant set A of the map Λ (such as fixed points, cycles, chaotic attractors, basins of attraction, etc.) is either symmetric itself with respect to x^*, or there exists one more invariant set A' which is symmetric to A. In particular, if the map Λ has an asymmetric fixed point $x = x_a$, then the fixed point symmetric to it $x = 1 - x_a \equiv x'_a$ also exists. As already mentioned, Λ can have one or two couples of asymmetric fixed points. In Fig.3 we show examples of the map Λ and its fixed points $x^{CP(0)}$, $x^{CP(1)}$, and x^* for different parameter values.

Let us first summarize stability properties of the CP fixed point $x^{CP(1)}$. The same conclusions hold for $x^{CP(0)}$ due to the symmetry of the map Λ. Recall that $x^{CP(1)}$ is a *border point* at which the map Λ is not differentiable, thus, one can only discuss a one-side stability of $x^{CP(1)}$. In fact, this fixed point is always one-side superstable with the related one-side eigenvalue $\Lambda'_+(1) = 0$. Due to the upper constraint of the map Λ we have obviously $\Lambda'_-(1) \geq 0$ (in fact, if $Z'(1) < 0$

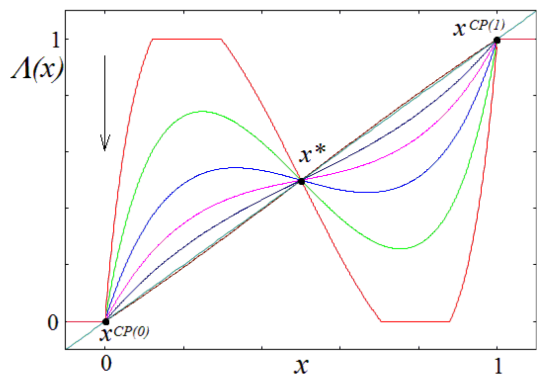

Figure 3: The map T for $\sigma = 6$, $\mu = 0.45$, $\gamma = 10$, $\theta = 0.25$, $\phi_L = 0.1$, $n = 0.8$ and $\phi_S = 0.11$, 0.2, 0.3, 0.4, 0.5 and 0.7 (the direction indicated by an arrow corresponds to increasing ϕ_S).

then $\Lambda'_-(1) = 0$), and if $\Lambda'_-(1) < 1$ (or > 1) then $x^{CP(1)}$ is one-side attracting (one-side repelling, respectively). To shorten, using the notion of stability with respect to the CP fixed points from now on we mean one-side stability. Thus, stability loss of $x^{CP(1)}$ can occur if its eigenvalue passes through 1, that is, $\Lambda'_-(1) = 1$. We have

$$\Lambda'_-(1) = Z'(1) = 1 - \gamma \frac{T(1)}{1 + T(1)},$$

so that stability condition of $x^{CP(1)}$ is

$$0 \le 1 - \gamma \frac{T(1)}{1 + T(1)} < 1, \tag{13}$$

from where we get the following inequalities:

$$0 < T(1) \le \frac{1}{\gamma - 1}.$$

Here the condition

$$T(1) = \frac{1}{\gamma - 1} \tag{14}$$

corresponds to $\Lambda'_-(1) = 0$, so that $x^{CP(1)}$ is both sides superstable, and for

$$T(1) > \frac{1}{\gamma - 1}$$

the flat branch $\Lambda(x) = 1$ 'enters' the interval $I = [0, 1]$, while the condition

$$T(1) = 0 \tag{15}$$

is related to $\Lambda'_{-}(1) = 1$, that is, to the stability loss of $x^{CP(1)}$. Given that this fixed point always exists, this bifurcation cannot be related to a (one-side) fold bifurcation. As we shall see the fixed point $x^{CP(1)}$ losses stability due to a 'one-side' transcritical bifurcation which we call *border-transcritical bifurcation* (see, for example, Fig.4 where the border-transcritical bifurcation is indicated by green circles). It is associated with an asymmetric fixed point with which $x^{CP(1)}$ merges at the bifurcation value and changes stability. The asymmetric fixed point can be born either due to a *fold bifurcation*, in which case it appears in a couple with one more asymmetric fixed point (as, for example, in Fig.4c where the fold bifurcation is indicated by black circles), or due to *pitchfork bifurcation* of the symmetric fixed point x^* (indicated in Fig.4 by red circles). In fact, the stability condition of x^* defined by $-1 < \Lambda'\left(\frac{1}{2}\right) < 1$ is satisfied for

$$-1 < 1 + \frac{\gamma}{4} T'\left(\frac{1}{2}\right) < 1 \tag{16}$$

(to get this condition we've used the equality $T\left(\frac{1}{2}\right) = 0$ which is easy to check), or

$$-\frac{8}{\gamma} < T'\left(\frac{1}{2}\right) < 0. \tag{17}$$

If

$$T'\left(\frac{1}{2}\right) = -\frac{8}{\gamma} \tag{18}$$

then $\Lambda'\left(\frac{1}{2}\right) = -1$, so that x^* undergoes a *flip bifurcation*, while the equality

$$T'\left(\frac{1}{2}\right) = 0 \tag{19}$$

is related to $\Lambda'\left(\frac{1}{2}\right) = 1$, so that x^* undergoes a *pitchfork bifurcation* (due to the symmetry of the map fold or transcritical bifurcations cannot occur). In case of *supercritical* pitchfork bifurcation of x^* the asymmetric fixed points $x = x_a$ and $x = 1 - x_a \equiv x'_a$ born due to this bifurcation are attracting (see, e.g., Fig.4a). Then the attracting fixed point x'_a collides with repelling fixed point $x^{CP(1)}$ due to the border-transcritical bifurcation. In contrast, in the *subcritical* case two repelling fixed points $x = x_b$ and $x = 1 - x_b \equiv x'_b$ are born, and then the repelling fixed point x'_b collides with the attracting fixed point $x^{CP(1)}$ due to the border-transcritical bifurcation (see, e.g., Fig.4b).

In Sec.5.3 we present numerical results related to the mentioned above local bifurcations, as well as results on global dynamics of the map Λ for the general case $0 < \widetilde{n} \leq 1$, while in the next subsection we assume $\widetilde{n} = 1$ in which case it is possible to get more analytical results on local stability of the fixed points (see also Commendatore et al. [2012]).

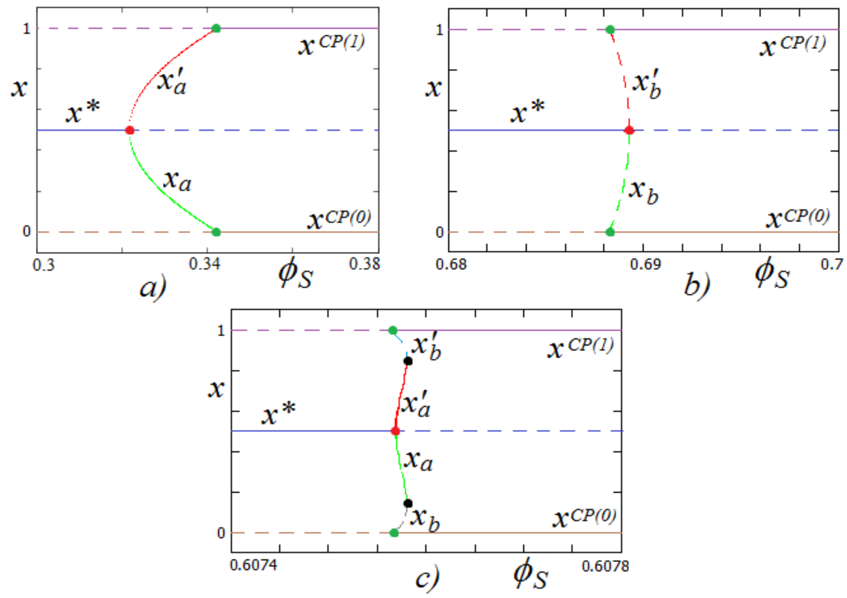

Figure 4: 1D bifurcation diagrams for $\phi_L = 0.1$, $n = 0.8$, $\sigma = 6$, $\mu = 0.45$, $\gamma = 10$ and $\theta = 0.05$, $\phi_S \in [0.3, 0.38]$ in *a)*, $\theta = 0.35$, $\phi_S \in [0.68, 0.7]$ in *b)*, $\theta = 0.18$, $\phi_S \in [0.6074, 0.6078]$ in *c)*.

5.2 Stability of the fixed points: the case $\widetilde{n} = 1$

For $\widetilde{n} = 1$ the condition (13) related to the stability of the CP fixed point $x^{CP(1)}$ can be written as

$$\frac{\gamma - 1}{\gamma}\varphi < \delta < \varphi \tag{20}$$

where

$$\varphi = \frac{\sigma}{(\sigma - \mu)[\theta + \phi_S(1 - 2\theta)] + [\mu(1 - \theta) + \theta\sigma]\phi_S^2} \quad \textit{and} \quad \delta = \phi_S^{\frac{\mu - \sigma + 1}{\sigma - 1}}$$

For $1 < \sigma < 1 + \mu$ and $0 \leq \phi_S < 1$, the right hand side inequality in (20) is always satisfied; for $\sigma > 1 + \mu$ it can be shown that the right hand side inequality in (20) is satisfied for sufficiently high values of ϕ_S and violated for low values (hint: we are dealing with two monotonically decreasing function of ϕ_S, the first, $\delta(\phi_S)$, tends to infinity for $\phi_S \to 0$ and it is equal to 1 at $\phi_S = 1$; the second, $\varphi(\phi_S)$, is positive (and larger than 1) but finite at $\phi_S = 0$ and it is equal to 1 at $\phi_S = 1$. Since at $\phi_S = 1$ the first derivative of the first function is smaller in absolute value than the derivative of the second function, the two necessarily cross at some $\phi_S = \phi_S^{tr}$, where $0 < \phi_S^{tr} < 1$). It is not possible to specify the corresponding bifurcation value for the (internal) trade freeness parameter ϕ_S explicitly, however, it is possible to find an explicit expression for θ, see (28).

For $\sigma > 1+\mu$, as ϕ_S crosses ϕ_S^{tr} from left to right, the map Λ undergoes a border-transcritical bifurcation: the CP fixed point $x^{CP(1)}$ merges with the asymmetric fixed point gaining stability. Symmetrically, $x^{CP(0)}$ meets the other asymmetric fixed point gaining stability. From this we infer that the asymmetric fixed points must have always the same local stability properties in the neighborhood of the CP fixed points (as can be seen, for example, in Fig.7).

The left hand side inequality in (20) holds for a sufficiently small value of γ:

$$\gamma < \frac{1}{1-\delta\varphi^{-1}}.$$

When this latter condition does not hold, $x^{CP(1)}$ becomes both-side superstable.

Moving on to the symmetric fixed point x^*, recall that its local stability requires that the condition (17) holds. Concerning the inequality on the right hand side of (17), we denote by ϕ_S^{pf} that value of ϕ_S for which the condition $T'\left(\frac{1}{2}\right)=0$ is satisfied. Moreover, $\phi_S < \phi_S^{pf}$ implies $T'\left(\frac{1}{2}\right)<0$. For $0<\widetilde{n}<1$, ϕ_S^{pf} corresponds to the positive root of a quadratic equation whose expression is quite complicated. From simulations we find that for some meaningful parameter combinations, but not for all, $0<\phi_S^{pf}<1$. For $\widetilde{n}=1$, we are able to obtain the following, relatively simple, expression:

$$\phi_S^{pf} = \frac{(\sigma-\mu)[2\theta(\sigma-1)-\mu]}{2\theta(\sigma-\mu)(\sigma-1)+\mu(3\sigma-2+\mu)} < 1.$$

(In (27) we give the same condition solved with respect to the parameter θ). One can show that

- if $1<\sigma<1+\frac{\mu}{2\theta}$, it follows that $\phi_S^{pf}<0$. Therefore this inequality is never satisfied;

- if $\sigma > 1+\frac{\mu}{2\theta}$, as ϕ_S crosses ϕ_S^{pf} from left to right, the map Λ undergoes a pitchfork bifurcation.

A first interesting result is that, when $\widetilde{n}=1$, the local stability of the symmetric fixed point does not depend on the trade distance with respect to the outside region, as measured by the parameter ϕ_L. This result is a direct consequence of Proposition 1. The size of local demand (outside demand) independent of entrepreneurial migration (i.e. originating from immobile unskilled workers), instead plays a relevant role, affecting positively ϕ_S^{pf}:

$$\frac{\partial\phi_S^{pf}}{\partial\theta} = \frac{4\mu(\sigma-\mu)(2\sigma-1)(\sigma-1)}{\left[2\theta(\sigma-\mu)(\sigma-1)+\mu(3\sigma-2+\mu)\right]^2} > 0.$$

This implies that increasing the size of local demand vis-à-vis that of outside demand has a stabilizing effect on the symmetric fixed point and tends to favour dispersion (the opposite holds true by increasing outside demand with respect to local demand which has a destabilizing effect on x^* and tends to favour agglomeration).

The local stability properties of the symmetric equilibrium and, therefore, the characteristics of the bifurcation value determine the specific location pattern (see Pflüger and Südekum [2008]) as trade integration between region 1 and 2 intensifies. The typical bifurcation scenario of a standard 2-region FE model is catastrophic agglomeration, with an immediate jump to a Core-Periphery equilibrium, corresponding to a subcritical pitchfork bifurcation. However, in our framework, a smoother agglomeration process can also emerge in correspondence of a supercritical pitchfork bifurcation with the emergence of two (locally) stable interior asymmetric equilibria.

In order to study in detail the properties of the pitchfork bifurcation, even though only for the case $\widetilde{n} = 1$, we first redefine our central map to highlight the control parameter we are interested in, the trade freeness parameter ϕ_S, and we verify how these may change when another crucial parameter, the size of local demand (at least, that part that it is not affected by entrepreneurial movements and that originates from immobile unskilledworkers) θ, varies. The redefined map is:

$$Z(\phi_S, x) = \gamma \left(1 + (1-x)\frac{T(\phi_S, x)}{1 + xT(\phi_S, x)}\right)$$

From the theory of dynamical systems (see e.g., Wiggins [2013]), in correspondence of a pitchfork bifurcation, that is, when $\phi_S = \phi_S^{pf}$ and $x = \frac{1}{2}$, the following conditions must hold:

1. $\frac{\partial Z}{\partial \phi_S}\left(\phi_S^{pf}, \frac{1}{2}\right) = 0 \quad \Leftrightarrow \quad \frac{\partial T}{\partial \phi_S}\left(\phi_S^{pf}, \frac{1}{2}\right) = 0;$
2. $\frac{\partial^2 Z}{\partial x^2}\left(\phi_S^{pf}, \frac{1}{2}\right) = 0 \quad \Leftrightarrow \quad \frac{\partial^2 T}{\partial x^2}\left(\phi_S^{pf}, \frac{1}{2}\right) = 0;$
3. $\frac{\partial^2 Z}{\partial x \partial \phi_S}\left(\phi_S^{pf}, \frac{1}{2}\right) \neq 0 \quad \Leftrightarrow \quad \frac{\partial^2 T}{\partial x \partial \phi_S}\left(\phi_S^{pf}, \frac{1}{2}\right) \neq 0;$
4. $\frac{\partial^3 Z}{\partial x^3}\left(\phi_S^{pf}, \frac{1}{2}\right) \neq 0 \quad \Leftrightarrow \quad \frac{\partial^3 T}{\partial x^3}\left(\phi_S^{pf}, \frac{1}{2}\right) \neq 0.$

Moreover, the sign of the following expression can be used to determine on which side of ϕ_S^{pf} the asymmetric fixed points, at least initially, lie:

5. $-\frac{\frac{\partial^3 Z}{\partial x^3}\left(\phi_S^{pf}, \frac{1}{2}\right)}{\frac{\partial^2 Z}{\partial x \partial \phi_S}\left(\phi_S^{pf}, \frac{1}{2}\right)} > (<) 0 \Leftrightarrow -\frac{\frac{\partial^3 T}{\partial x^3}\left(\phi_S^{pf}, \frac{1}{2}\right)}{\frac{\partial^2 T}{\partial x \partial \phi_S}\left(\phi_S^{pf}, \frac{1}{2}\right)} > (<) 0.$

When this expression is larger (less) than zero, the pitchfork bifurcation is supercritical (subcritical); for the particular case

$$\frac{\partial^3 Z}{\partial x^3}\left(\phi_S^{pf}, \frac{1}{2}\right) = 0 \tag{21}$$

the pitchfork bifurcation is critical[7] (for example, in Fig.6 the intersection point of the pitchfork and fold bifurcation curves $\theta = \theta_{pf}$ and $\theta = \theta_f$ is a codimention-2 bifurcation point at which the pitchfork bifurcation is critical: it occurs simultaneously with two fold bifurcations). We have that:

- Condition 1 is verified due to the fact that at the symmetric equilibrium $V_1 = V_2$ and $\frac{\partial V_1}{\partial \phi_S} = \frac{\partial V_2}{\partial \phi_S}$ for any ϕ_S;

- Condition 2 is verified due to the symmetric properties of the map $Z(x)$. Indeed, at the bifurcation point, this condition can be reduced to:

$$\frac{\partial^2 V_1}{\partial x^2} - \frac{\partial^2 V_2}{\partial x^2} = \frac{\partial^2 \left(\frac{\pi_1}{P_1^\mu}\right)}{\partial x^2} - \frac{\partial^2 \left(\frac{\pi_2}{P_2^\mu}\right)}{\partial x^2} = \mu(1+\mu)\left(\frac{\left(\frac{\partial \pi_1}{\partial x}\right)^2}{P_1^{\mu+2}}\pi_1 - \frac{\left(\frac{\partial \pi_2}{\partial x}\right)^2}{P_2^{\mu+2}}\pi_2\right) -$$
$$2\mu\left(\frac{\frac{\partial \pi_1}{\partial x}\frac{\partial P_1}{\partial x}}{P_1^{\mu+1}} - \frac{\frac{\partial \pi_2}{\partial x}\frac{\partial P_2}{\partial x}}{P_2^{\mu+1}}\right) + \left(\frac{\frac{\partial^2 \pi_1}{\partial x^2}}{P_1^\mu} - \frac{\frac{\partial^2 \pi_2}{\partial x^2}}{P_2^\mu}\right) - \mu\left(\frac{\frac{\partial^2 P_1}{\partial x^2}}{P_1^{\mu+1}}\pi_1 - \frac{\frac{\partial^2 P_2}{\partial x^2}}{P_2^{\mu+1}}\pi_2\right) = 0,$$

which holds, given that at $x = \frac{1}{2}$:

$\pi_1 = \pi_2$, $P_1 = P_2$, $\frac{\partial \pi_1}{\partial x} = -\frac{\partial \pi_2}{\partial x}$, $\frac{\partial P_1}{\partial x} = -\frac{\partial P_2}{\partial x}$, $\frac{\partial^2 \pi_1}{\partial x^2} = \frac{\partial^2 \pi_2}{\partial x^2}$ and $\frac{\partial^2 P_1}{\partial x^2} = \frac{\partial^2 P_2}{\partial x^2}$.
Note that both conditions 1 and 2 hold also for the case $0 < \widetilde{n} < 1$;

- Concerning condition 3 we obtain the following result:

$$\frac{\partial^2 T}{\partial x \partial \phi_S}\left(\phi_S^{pf}, \frac{1}{2}\right) =$$
$$= \frac{2\mu(2\sigma-1)[(\sigma-1)(\sigma-\mu)2\theta + \mu(\mu+3\sigma-2)]^2}{(\sigma-\mu)(\sigma-1)^2(\mu+2\sigma\theta)[(\sigma-1)(\sigma-\mu)2\theta + \mu(\sigma-1+\mu)]} > 0$$

which is always satisfied;

- Concerning condition 4 we obtain the following result:

$$\frac{\partial^3 T}{\partial x^3}\left(\phi_S^{pf}, \frac{1}{2}\right) =$$
$$= \frac{32\mu^4(2\sigma-1)^3\left\{12(\sigma-1)^2(\sigma-\mu)\theta^2 + [2(2\sigma-3)\mu^2 + 4(3\mu-\sigma)(\sigma-1)^2]\theta - \mu(\sigma-1+\mu)[2(\sigma-1)+\mu]\right\}}{(\sigma-1)^3(\mu+2\sigma\theta)[(\sigma-1)(\sigma-\mu)2\theta+\mu(\sigma-1+\mu)]^3}$$

According to this expression condition 4 may or may not hold depending on parameter combinations;

- Finally, considering condition 5, the sign of $\frac{\partial^3 T}{\partial x^3}\left(\phi_S^{pf}, \frac{1}{2}\right)$ coincides with that of the following expression:

$$\begin{aligned} K = {} & 12(\sigma-1)^2(\sigma-\mu)\theta^2 + [2(2\sigma-3)\mu^2 + \\ & +4(3\mu-\sigma)(\sigma-1)^2]\theta - \mu(\sigma-1+\mu)[2(\sigma-1)+\mu]. \end{aligned} \quad (22)$$

This allows us to state the following

Proposition 2 *Let's assume that $\sigma > 1 + \mu$. Then there exists a value of the parameter $\theta = \overline{\theta}$ with $0 < \overline{\theta} < \frac{1}{2}$ such that $K < 0$ for $\theta < \overline{\theta}$, $K > 0$ for $\theta > \overline{\theta}$, and $K = 0$ for $\theta = \overline{\theta}$, where K is given in (22).*

[7] If the condition (21) holds not only for $x = \frac{1}{2}$, but for also in some neighborhood of $x = \frac{1}{2}$, this means so-called degenerate pitchfork bifurcation (see Sushko and Gardini [2010]) as sketched, for example, in Fig.7, that cannot occur in the considered map.

Proof. First we consider the following second degree equation based on (22):

$$a\theta^2 + b\theta + c = 0, \tag{23}$$

where

$$\begin{aligned} a &= 12(\sigma-1)^2(\sigma-\mu) > 0; \\ b &= 2(2\sigma-3)\mu^2 + 4(3\mu-\sigma)(\sigma-1)^2 \geq (<)0; \\ c &= -\mu(\sigma-1+\mu)[2(\sigma-1)+\mu] < 0. \end{aligned}$$

Therefore (23) admits one positive and one negative solution. In order to have real roots, it must be:

$$\Delta = b^2 - 4ac = (12\sigma - 8\sigma^2 - 3)\mu^4 + 4(10\sigma^2 - 12\sigma + 3)(\sigma-1)^2\mu^2 + 4\sigma^2(\sigma-1)^4 > 0.$$

$\Delta = 0$ is a quartic equation that admits four solutions of which only two at most are real (or none). Define $y = \mu^2$. Then we can write:

$$\Delta = b^2 - 4ac = (12\sigma - 8\sigma^2 - 3)y^2 + 4(10\sigma^2 - 12\sigma + 3)(\sigma-1)^2 y + 4\sigma^2(\sigma-1)^4 = 0. \tag{24}$$

This is now a second degree equation whose solutions, y_1 and y_2, are real since

$$\widetilde{\Delta} = 144(3\sigma-1)(2\sigma-1)^2(2\sigma-1)^5 > 0.$$

Moreover, these solutions are both negative for $1 < \sigma < \frac{3+\sqrt{3}}{4}$ and one positive and one negative for $\sigma > \frac{3+\sqrt{3}}{4}$, i.e. $y_2 < 0 < y_1$. Therefore, for $1 < \sigma < \frac{3+\sqrt{3}}{4}$, $\Delta > 0$ always; and for $\sigma > \frac{3+\sqrt{3}}{4}$, $\Delta > 0$ as long as $0 < x < 1 < x_1$. Therefore also in this case $\Delta > 0$ for all relevant values of μ.[8]

Therefore, (23) admits two real solutions, one positive and one negative. Let's call the positive solution $\overline{\theta}$, where $\overline{\theta} = -\frac{b-\sqrt{\Delta}}{2a}$. Given that $a > 0$, we have $K < 0$ for $0 < \theta < \overline{\theta}$ and $K > 0$ for $\theta > \overline{\theta}$; and $K = 0$ when $\theta = \overline{\theta}$, where K is given in (22).

Finally, notice that the condition $\overline{\theta} < \frac{1}{2}$ corresponds to

$$4(\sigma+\mu)(\mu+\sigma-1)[\sigma-(1+\mu)] > 0.$$

That can be further reduced to $\sigma - (1+\mu) > 0$.

Q.E.D.

In summary, the size of local (immobile) demand matters with respect to outside (immobile) demand determining the location pattern at the bifurcation

[8] Note that $x_2 > 1$ as long as the sum of the coefficients in the equation (24) is positive, that is,

$$4\sigma^6 - 16\sigma^5 + 64\sigma^4 - 144\sigma^3 + 144\sigma^2 - 60\sigma + 9 > 0$$

Looking at the solutions of the six degrees equation, none of them is real. It follows that this inequality is always satisfied or never satisfied. It is immediate to check, by substituting any number for σ, that the condition holds.

point, which is catastrophic when the size of local demand is sufficiently large in relative terms and it is smooth when it is the size of outside demand that becomes sufficiently large in relative terms (see Fig.7c and b, where we assume $\theta = 0.4$ and $\theta = 0.32$, respectively). We envisage that, by continuity, a similar result must hold also when we allow for a local manufacturing sector in region 3. Indeed Fig. 4 confirms for the case $0 < \widetilde{n} < 1$ the possible occurrence of these bifurcation scenarios.

Finally, concerning the inequality on the left hand side of (17), it holds for a sufficiently small value of γ or for a sufficiently high value of θ. When this condition does not hold, a flip bifurcation scenario emerges, with the possible occurrence of complex behavior, involving the global properties of dynamics. We turn now to such type of analysis.

5.3 Numerical results

In this section we study local and global dynamics of the map Λ in generic case. Recall that Λ depends on 7 parameters satisfying the following conditions:

$$\sigma > 1,\ \gamma > 0,\ 0 < \mu, \phi_S, \phi_L, \widetilde{n} < 1,\ 0 < \theta < 1/2,\ \phi_L \leq \phi_S.$$

Our main interest is related to the influence on dynamics of the parameters ϕ_S, ϕ_L, n and θ, so, we can fix the other parameters as follows:

$$\sigma = 6, \quad \mu = 0.45, \quad \gamma = 10, \tag{25}$$

and investigate bifurcation structure of the remaining parameter space by means of 1D and 2D bifurcation diagrams. This analysis extends the results of the case $\widetilde{n} = 1$ to that of $\widetilde{n} < 1$, when the manufacturing sector is also located in the outside region.

First, in Fig.5 we show a 2D bifurcation diagram in the (ϕ_S, θ)-parameter plane for $\phi_L = 0.1$ and $\widetilde{n} = 0.8$. Here different colors are related to different attracting cycles, namely, the red region S to the symmetric fixed point x^*; the pink region AS to coexisting asymmetric fixed points x_a and x'_a; the blue region CP_1 to the Core-Periphery fixed points $x^{CP(0)}$ and $x^{CP(1)}$; the blue region CP_2 to the fixed points $x^{CP(0)}$ and $x^{CP(1)}$ which are locally repelling but represent attractors in Milnor sense; the green region to 2-cycles; the other colors correspond to cycles of periods $k \leq 30$ and white region is related either to higher periodicity or to chaotic attractors. Clearly, the upper boundary of the region S in Fig.5 is related to the flip bifurcation of x^*, while its lower boundary is the pitchfork bifurcation curve. In the following subsections we discuss both scenarios in detail.

Before doing so, let us briefly summarize the economic interpretations that can be drawn from Fig. 5: for high free tradess inside the Union (i.e. for a high value of ϕ_S) agglomeration of economic activities in one of the two regions of the Union is a likely outcome; lowering this parameter leads to an equal distribution of economic activities (if ϕ_S enters in the red area S) or to an uneven distribution but with manufacturing production in both regions (if ϕ_S enters in the pink area AS); these configurations lose stability for further lower values of internal trade freeness giving rise to cyclical attractors. For very low values of ϕ_S agglomeration of the industrial activity of the Union might again be the long-run outcome. It is interesting to note that the bifurcation lines are positively sloped, i.e. the bifurcation values for ϕ_S depend positively upon θ. Lower values of θ shrink the range for which the symmetric equilibrium is stable (i.e. the red area S), shifting it to the left ; i.e. reducing trade distance with a big country (which reduces the share of unskilled workers in the Union and increases the demand from outside) favours agglomeration processes and Core-Periphery patterns inside the Union even at lower level of trade integration.

A similar pattern is actually found wrt the external trade freeness ϕ_L (in numerical simulations not incuded in the paper).

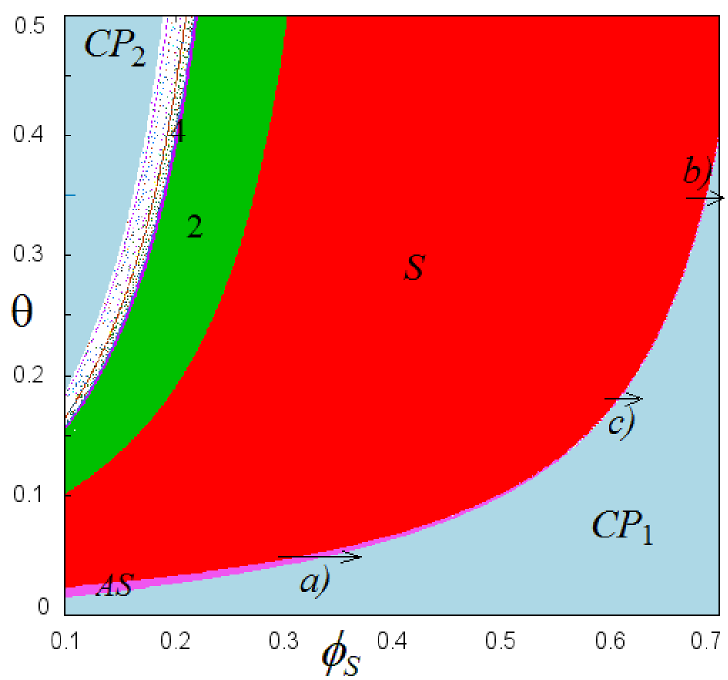

Figure 5: 2D bifurcation diagram in the (ϕ_S, θ)-parameter plane for $\phi_L = 0.1$, $\widetilde{n} = 0.8$, $\sigma = 6$, $\mu = 0.45$, $\gamma = 10$. The 1D bifurcation diagrams related to the straight lines with arrows are shown in Fig.4.

5.3.1 Bifurcation scenario via a pitchfork bifurcation

Let us clarify which kind of pitchfork bifurcation occurs if the (ϕ_S, θ)-parameter point crosses the lower boundary of the region S. In Fig.4 we present 1D bifurcation diagrams corresponding to cross-section of this boundary for fixed θ varying ϕ_S as indicated by the straight lines with arrows. In particular, in Fig.4*a,* where $\theta = 0.05$, one can see that for increasing ϕ_S the fixed point x^* undergoes supercritical pitchfork bifurcation (marked by red circle) leading to two attracting asymmetric fixed points, x_a and x'_a (their basins are separated by the repelling fixed point x^*), which then merge with the repelling fixed points $x^{CP(0)}$ and $x^{CP(1)}$, respectively, due to the border-transcritical bifurcation (marked by green circles). In Fig.4*b,* where $\theta = 0.35$, the pitchfork bifurcation is subcritical leading for decreasing ϕ_S to two repelling asymmetric fixed points x_b and x'_b, which soon after merge with $x^{CP(0)}$ and $x^{CP(1)}$ due to border-transcritical bifurcation. In the small parameter interval bounded by the values related to border-transcritical and pitchfork bifurcations, the attracting fixed point x^* coexists with the attracting fixed points $x^{CP(0)}$ and $x^{CP(1)}$, and their basins are separated by the repelling fixed points x_b and x'_b.

The question arises about dynamics of the map at the moment when the pitchfork bifurcation of x^* and transcritical bifurcation of $x^{CP(0)}$ and $x^{CP(1)}$ occur simultaneously. As can be seen in Fig.4*c* where $\theta = 0.18$, for decreasing ϕ_S at first two couples of asymmetric fixed points are born due to fold bifurcations (marked by black circles), namely, attracting fixed points x_a, x'_a and repelling fixed points x_b, x'_b. Then the points x_b and x'_b merge with $x^{CP(0)}$ and $x^{CP(1)}$, respectively, due to the border-transcritical bifurcation, while the points x_a and x'_a merge with x^* due to supercritical pitchfork bifurcation.

In order to clarify the location of the fold bifurcation curve let us come back to the case $\widetilde{n} = 1$ (see Commendatore et al. [2012]). As we already mentioned, in such a case expressions of the bifurcation curves of the fixed points x^*, $x^{CP(0)}$ and $x^{CP(1)}$ can be obtained analytically. In particular, the flip bifurcation boundary is defined by the condition (18) which can be written as

$$\theta = \theta_{fl} \equiv \frac{(\sigma(1+\phi_S) - \mu(1-\phi_S))}{2(\sigma-\mu)(1-\phi_S)} \left(\frac{\mu}{(\sigma-1)} + \frac{2(1+\phi_S)}{(1-\phi_S)\gamma} \right) + \frac{\mu\phi_S}{(\sigma-\mu)(1-\phi_S)}, \tag{26}$$

the pitchfork bifurcation boundary is defined by the condition (19) which can be written as

$$\theta = \theta_{pf} \equiv \frac{\mu}{2(\sigma-\mu)(1-\phi_S)} \left(\frac{\sigma(1+\phi_S) - \mu(1-\phi_S)}{(\sigma-1)} + 2\phi_S \right), \tag{27}$$

and the border-transcritical bifurcation boundary satisfies (15) that can be written as

$$\theta = \theta_{tr} \equiv \frac{\phi_S(\sigma(\phi_S^{\mu/(1-\sigma)} - 1) + \mu(1-\phi_S))}{(\sigma-\mu)(1-\phi_S)^2}. \tag{28}$$

The curves $\theta = \theta_{pf}$ and $\theta = \theta_{tr}$ can intersect each other as it occurs, for example, in the (ϕ_S, θ)-parameter plane for $\sigma = 2$, $\mu = 0.45$, $\gamma = 20$ (see Fig.6). Here the region marked as $S + CP_1$ is related to coexisting attracting symmetric and Core-Periphery fixed points, x^*, $x^{CP(0)}$ and $x^{CP(1)}$, the region AS corresponds to two asymmetric attracting fixed points x_a and x'_a, and the

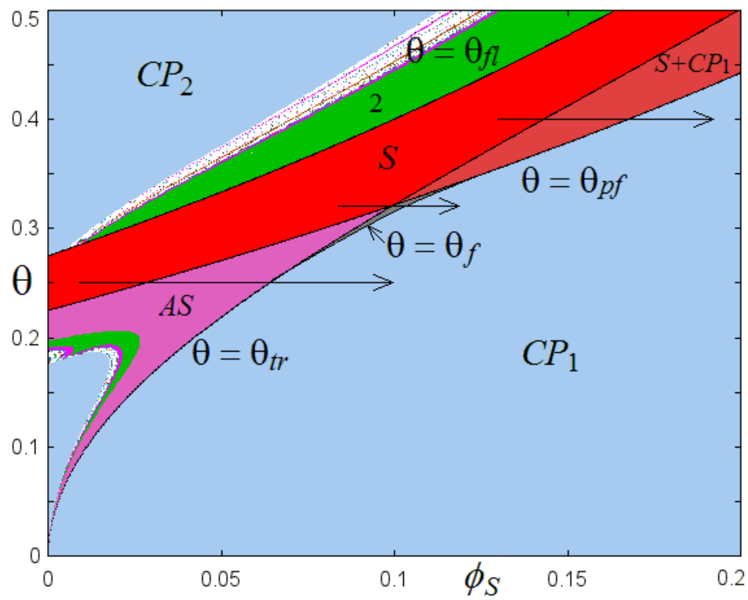

Figure 6: 2D bifurcation diagram in the (ϕ_S, θ)-parameter plane for $\sigma = 2$, $\mu = 0.45$, $\gamma = 20$, $\widetilde{n} = 1$. 1D bifurcation diagrams related to the straight lines with arrows are shown in Fig.7.

gray region bounded by the fold bifurcation curve $\theta = \theta_f$ is related to coexisting attracting fixed points $x^{CP(0)}$, $x^{CP(1)}$ and x_a, x'_a.

The 1D bifurcation diagrams, related to super- and subcritical pitchfork bifurcations of x^* are shown in Fig.7*a* and *b*, respectively. If the (ϕ_S, θ)-parameter point crosses the bifurcation curves at the intersection point of θ_{pf} and θ_{tr}, one could expect a 1D bifurcation diagram like the one sketched in Fig.7*d*. However, such a diagram is impossible because it would mean that a degenerate pitchfork bifurcation occurs (see Sushko and Gardini [2010]), with $\Lambda(x) \equiv x$ in the complete interval $[0, 1]$. In fact, the true transition is as shown in Fig.7*c* where the fixed points x_a, x'_a and x_b, x'_b are born due to the fold bifurcations (the related curve $\theta = \theta_f$ obtained numerically is indicated in Fig.6). As one can see, such a transition occurs not only at the intersection point of the curve θ_{pf} and θ_{tr} but also in some its neighborhood. It is clear that the bifurcation structure similar to the one shown in Fig.6 exists also in case related to Fig.5, however, this structure is very tight and difficult to be recognized. The effect of ϕ_L, the external trade distance, is similar to that of ϕ_S: increasing trade ingeration with the outside region destabilizes the symmetric fixed point and favours agglomeration in one of the two regions of the Union.

In this subsection we have shown that the symmetric equilibrium may lose stability via a pitchfork bifurcation. We paid special attention to the various

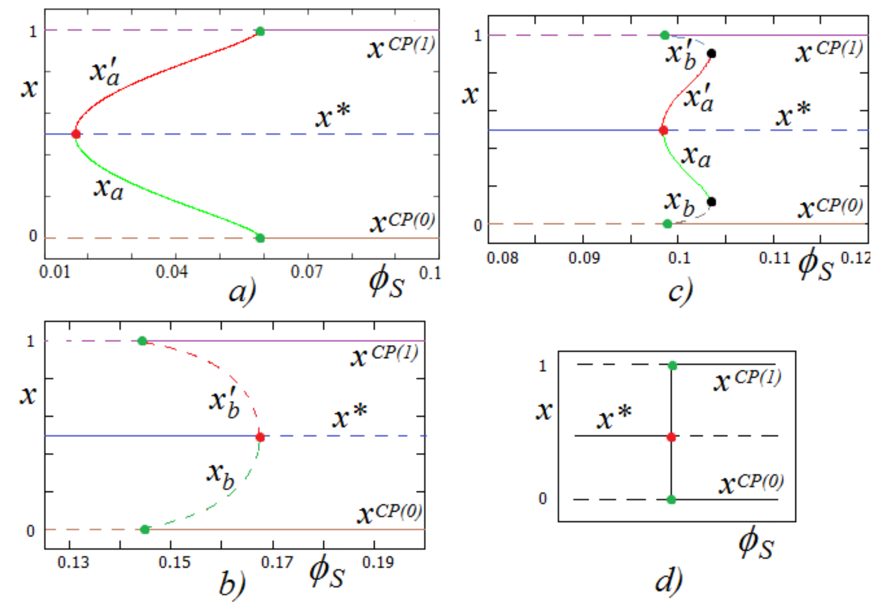

Figure 7: 1D bifurcation diagram for $\sigma = 2$, $\mu = 0.45$, $\gamma = 20$, $\widetilde{n} = 1$ and $\theta = 0.25$, $\phi_S \in [0.01, 0.1]$ in *a)*, $\theta = 0.4$, $\phi_S \in [0.08, 0.12]$ in *b)*, $\theta = 0.32$, $\phi_S \in [0.13, 0.19]$ in *c)*. Degenerate pitchfork bifurcation (which cannot occur in the map Λ) is shown schematically in *d)*.

bifurcation scenarios that my occur afterwards and distinguished three cases. The subcritical bifurcation (depicted in Figs 7b and 4b) is in some sense the standard case in the New Economic Geography: Between the break and sustain point values for the trade freeness, both the symmetric and the Core periphery fixed points are stable and their basins of attractions are delimited by unstable asymmetric fixed points. Fixed points coexist and the long-run pattern of regional industry location is highly sensitive to initial conditions and to parameters. However, we were able to show that also a supercritical pitchfork bifurcation is possible (as depicted in Figs 7a and 4a): Interestingly, in this case asymmetric stable fixed points are born after the bifurcation - the model, which is entirely based on standard NEG assumptions, is thus able to generate endogenously interior asymmetric outcomes (in which economic activity is neither symmetrically distributed between the two regions nor full agglomerated in one of the regions). This feature greatly improves the applicability of the models to real world developments (and is seldom found in other standard NEG models). The limiting case (depicted in Figs 7c and 4c) shows that also for these asymmetric stable equilibrium coexistence with the Core periphery equilibria is possible (and the related basins of attraction are delimited by additional asymmetric equilibria that are unstable). Also in this case, the longrun regional distribution of industrial activity is highly sensitive upon initial conditions and on parameters.

Note that we discussed these phenomena wrt a change in the interior trade freeness (between the two regions of our integration area). However, qualitatively similar results can be obtain for a variation of the exterior trade freeness and for the (mobile) share of entrepreneurs and of unqualified workers. Changing any of these parameters may lead to asymmetric stable equilibria that may coexist with stable CP equilibria.

5.3.2 Bifurcation scenario related to a flip bifurcation

In this section we discuss the second type of bifurcation that may lead to a stability loss of the symetric fixed point - the flip bifrucation. We show that the map can have attracting cycles and chaotic attractors, and that coexistence of attractors may actually involve either cycles or chaotic attractors with complex basins of attractions.

In order to comment bifurcation scenario which is observed in the map Λ if the parameter point crosses the flip bifurcation boundary of the parameter region S, that is, when the symmetric fixed point x^* undergoes the flip bifurcation, we show in Fig.8 the bifurcation structure for fixed σ, μ, γ (see (25)) in the $(\phi_S, \widetilde{n})$-parameter plane for $\theta = 0.25$, $\phi_L = 0.01$ in *a)*, and in the $(\phi_L, \widetilde{n})$-parameter plane for $\theta = 0.25$, $\phi_S = 0.15$ in *b)*. Note from these Figures a similar effect of varying the internal and external trade freeness as in Figure 5 thus corroborating the conclusions drawn above: For high trade freeness (in particular inside the Union) agglomeration is the most likely outcome for industrial location; reducing trade freeness leads first to a stable symmetric equilibrium; then it gives rise to cyclical solutions; and finally to a full agglomeration outcome. It is interesting to see that the effect of increasing $\widetilde{n}$ (i.e. the relative size of the industrial sector inside the Union) has a non-monotonic effect: starting from a low (high) value, increasing $\widetilde{n}$ shifts the stable range for the symmetric equilibrium towards higher (lower) values of trade freeness. For a highly industrialized Union ($\widetilde{n}$ close to 1) the bifurcation curves are negatively sloped, implying that the bifurcation values for both dimensions of trade freeness (internal and external) depend negatively upon its industrial share in the overall economy. Thus, if there is a shift in the Union's trading partners towards lower industrialized regions ($\widetilde{n}$ getting even closer to 1), a Core-Periphery long-run position becomes less likely.

Let us consider the 1D bifurcation diagram related to the cross-section indicated in Fig.8 by the straight line with an arrow. It is shown in Fig.9 togheter with an enlargement.

One can see in Fig.9a that for decreasing ϕ_S the fixed point x^* undergoes a supercritical flip bifurcation (at the point marked by black circle) leading to an attracting 2-cycle $g_2 = \{x_0, x_1\}$, whose points are symmetric with respect to x^*. Then g_2 undergoes a supercritical pitchfork bifurcation (at the point marked by red circle), due to which two new attracting 2-cycles q_2 and q'_2 are born, points of which are symmetric to each other with respect to x^*. If we

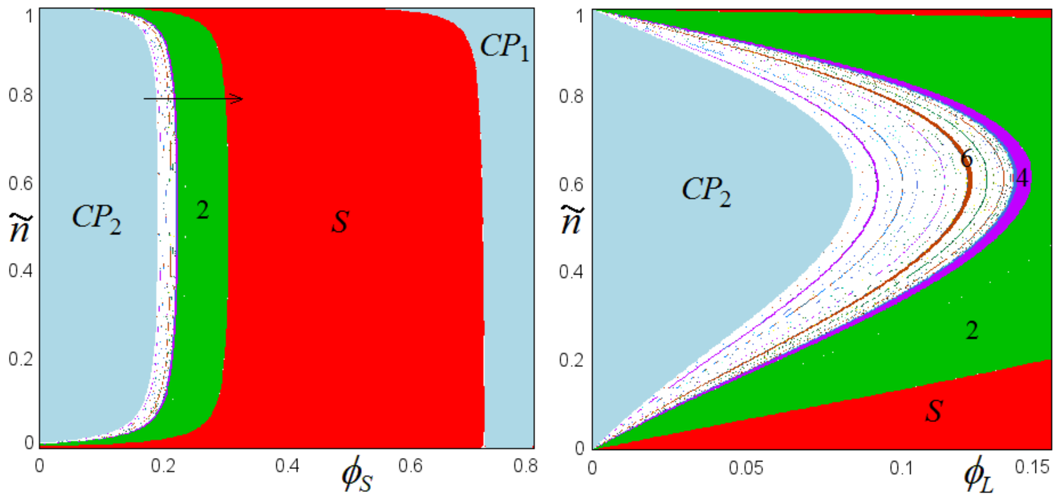

Figure 8: 2D bifurcation diagrams in $(\phi_S, \widetilde{n})$- and $(\phi_L, \widetilde{n})$-parameter plane.for $\sigma = 6$, $\mu = 0.45$, $\gamma = 10$, $\theta = 0.25$ and $\phi_L = 0.01$ in *a)*, $\phi_S = 0.15$ in *b)*.

continue to decrease ϕ_S each of the 2-cycles q_2 and q_2' undergoes a sequence of bifurcations following well-known logistic bifurcation scenario starting with a cascade of flip bifurcations up to a homoclinic bifurcation (marked by blue points) of 2-cycle g_2 (see Fig.9*b*).

Thus, for the parameter interval bounded by the points related to homoclinic and pitchfork bifurcations the map Λ has two coexisting attractors (cycles or cyclic chaotic intervals shown in red and green). As an example, we show in Fig.10 two coexisting attracting 2-cycles together with their immediate basins bounded by the points x_0 and x_1 of the 2-cycle g_2, their preimages by the middle branch of Λ, denoted $x_{0,-m}$ and $x_{1,-m}$ and their preimages by the left and right branches, respectively, denoted $x_{0,-ml}$ and $x_{1,-mr}$. All the preimages of the immediate basins constitute the total basins of attraction of coexisting 2-cycles, and these preimages for increasing rank are accumulating to the fixed points $x^{CP(0)}$ and $x^{CP(1)}$ as well as to the fixed point x^* and its preimage by the left and right branches of Λ, denoted x^*_{-l} and x^*_{-r}.

In Fig.11 the map Λ is at the moment of homoclinic bifurcation of g_2 (see the blue point in Fig.9*b*), at which two 2-cyclic chaotic attractors merge into one 2-cyclic chaotic attractor. In fact, the boundary of one chaotic attractor is formed by the first critical point[9] of the map Λ, denoted c, and its images $c_i = \Lambda^i(c)$, $i = 1, 2, 3$, while the boundary of the second chaotic attractor is formed by the second critical point, denoted c', and its images $c_i' = \Lambda^i(c')$. The first homoclinic bifurcation of g_2 is defined by the condition $\Lambda^2(c) = x_0$, or $\Lambda^2(c') = x_1$, in which

[9]Recall that for a 1D continuous noninvertible map $f : I \to I$, $I \subseteq \mathbb{R}$, its local extrema are called critical points (see Mira et al. [1996]).

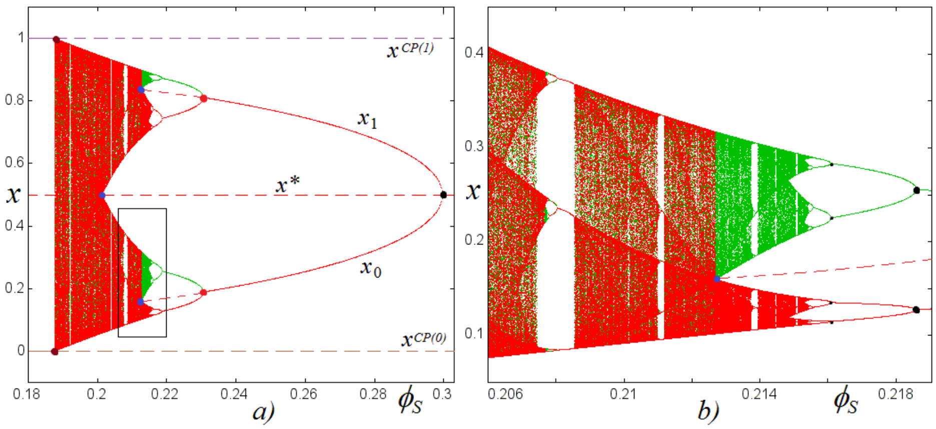

Figure 9: In *a)*: 1D bifurcation diagram of the map Λ for $\sigma = 6$, $\mu = 0.45$, $\gamma = 10$, $\theta = 0.25$, $\phi_L = 0.01$ and $\widetilde{n} = 0.8$ related to crossection indicated in Fig.8 by the straight line with an arrow. In *b)*: An enlargement of the window indicated in *a)*.

case we have $c_2 = c_3' = x_0$ and $c_2' = c_3 = x_1$, that is, two attractors merge at the points of g_2. After this bifurcation, if we continue to decrease ϕ_S, one can observe other coexisting attractors (as already mentioned, if an attractor is not symmetric with respect to x^* then there exists one more attractor, symmetric to it). For example, one can see in Fig.9b a periodic window related to 6-cycle g_6 whose points are symmetric with respect to x^*, and its pitchfork bifurcation leads to two new symmetric attracting 6-cycles q_6 and q_6'.

Let us comment now a bifurcation marked in Fig.9a by brown circles. It is a contact bifurcation of a one-piece chaotic attractor, bounded by the critical points c and c', with its basin confined by the fixed point $x^{CP(0)}$ and $x^{CP(1)}$. Such a contact occurs if a parameter point crosses the boundary of the region CP_2 (see Fig.8). In Fig.12a the map Λ is shown at the moment of such a contact defined by the condition $c = 0$ or $c' = 1$. After this bifurcation the locally repelling fixed points $x^{CP(0)}$ and $x^{CP(1)}$ become attractors in Milnor sense because almost all the initial points of the interval $I = (0, 1)$ are mapped into these fixed points in finite number of iterations.

Recall that according to the most spread definition, an *attractor* $\mathcal{A}$ of a map $f : I \to I$, $I \subseteq \mathbb{R}$, is an attracting set with a dense orbit. An attracting set A is defined as a closed invariant set for which a neighborhood U exists such that $f(U) \subset U$ and $f^n(x) \to A$ as $n \to \infty$ for *any* $x \in U$. A *Milnor attractor* is defined as a closed invariant set $A \subset I$ such that its stable set $\rho(A)$ (consisting of all points $x \in \rho(A)$ for which ω-limit set[10] is a subset of A) has a strictly positive

[10] *ω-limit set* $\omega(x)$ is the set of all accumulation points under forward iterations of the orbit with initial point x.

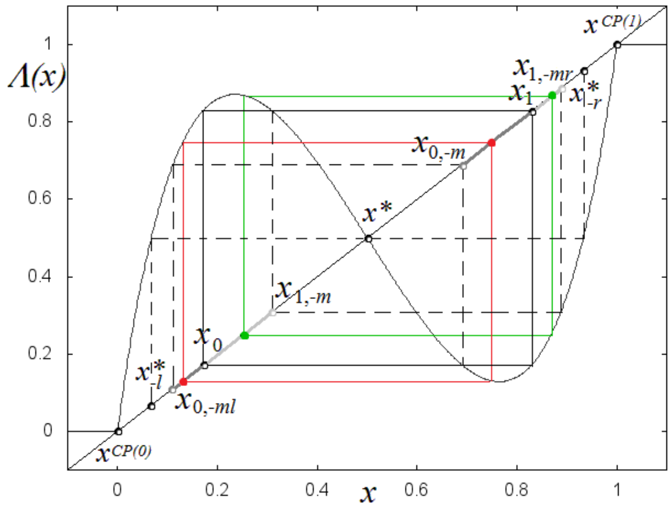

Figure 10: The map Λ and its two coexisting attracting 2-cycles (red and green circles) whose immediated basins indicated by dark and light gray, respectively, are bounded by the repelling 2-cycle $g_2 = \{x_0, x_1\}$ and its preimages. Here $\sigma = 6$, $\mu = 0.45$, $\gamma = 10$, $\theta = 0.25$, $\phi_L = 0.01$, $\widetilde{n} = 0.8$, $\phi_S = 0.22$.

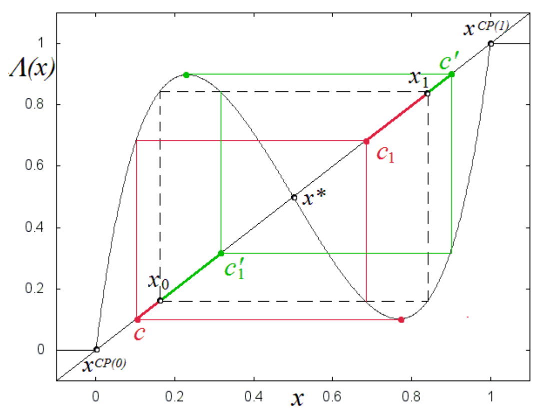

Figure 11: The map Λ at the moment of homoclinic bifurcation of the 2-cycle $g_2 = \{x_0, x_1\}$ leading to merging to two 2-cyclic chaotic intervals shown in red and green. Here $\sigma = 6$, $\mu = 0.45$, $\gamma = 10$, $\theta = 0.25$, $\phi_L = 0.01$, $n = 0.8$, $\phi_S = 0.2127$ (the related point is indicated in Fig.9b by blue circle).

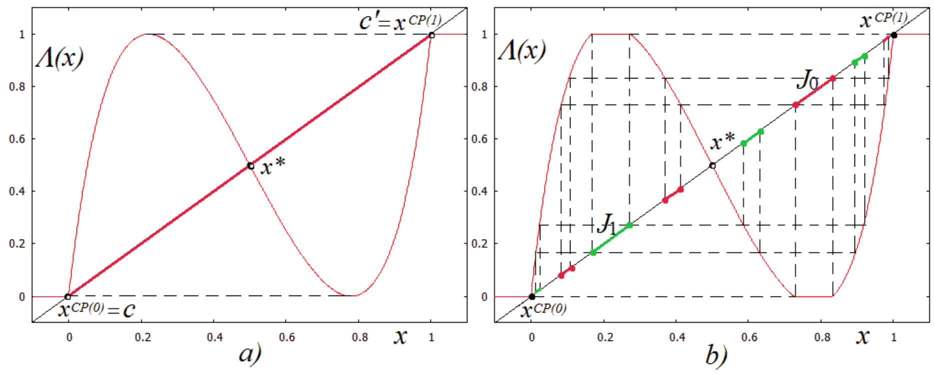

Figure 12: The map Λ in *a)* at the moment and in *b)* after the contact bifurcation of the chaotic attractor $A = [c, c']$ with its basin confined by the fixed points $x^{CP(0)}$ and $x^{CP(1)}$. Here $\sigma = 6$, $\mu = 0.45$, $\gamma = 10$, $\theta = 0.25$, $\phi_L = 0.01$, $\widetilde{n} = 0.8$ and $\phi_S = 0.187626$ in *a)* (the related point is indicated in Fig.9a by brown circle), $\phi_S = 0.18$ in *b)*.

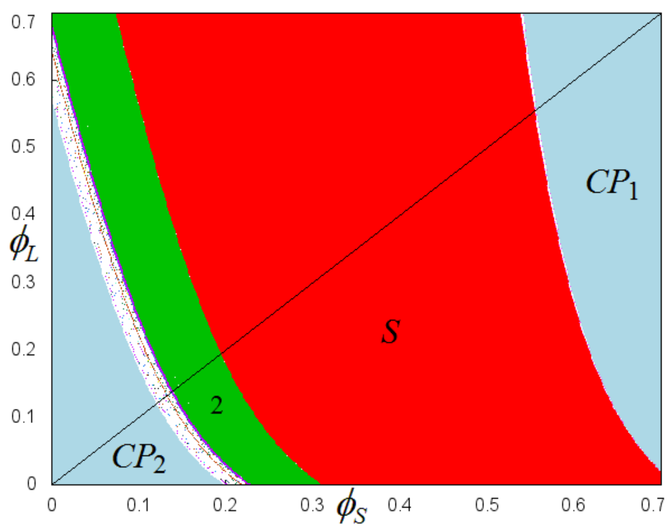

Figure 13: 2D bifurcation diagram in the (ϕ_S, ϕ_L)-parameter plane for $\sigma = 6$, $\mu = 0.45$, $\gamma = 10$, $\theta = 0.25$ and $\widetilde{n} = 0.8$.

measure, and there is no strictly smaller closed subset A' of A such that $\rho(A')$ coincides with $\rho(A)$ up to a set of measure zero (see Milnor [1985]). As we see, the basic difference of these two definitions of an attractor is related to the fact that in the first case *any* point from a neighborhood of $\mathcal{A}$ is attracted to $\mathcal{A}$, while in case of a Milnor attractor not necessarily all the points from a neighborhood of $\mathcal{A}$ are attracted to it (in fact, the class of Milnor attractors is wider and it includes the sets which are attractors according to the first definition).

For example, one can see in Fig.12b that the points of the green intervals are mapped into $x^{CP(1)}$ and the points of the red intervals are mapped into $x^{CP(0)}$. In fact, the interval J_1 related to the left flat branch of Λ and all its preimages (a few of which are shown in Fig.12b) constitute the stable set of the fixed point $x^{CP(1)}$. There is a sequence of preimages of J_1 accumulating to $x^{CP(0)}$, and there is also a sequence of preimages of J_1 accumulating to $x^{CP(1)}$. The same can be said about sequences of preimages of J_2 related to the second flat branch of Λ. Thus, in any (one-side) neighborhood of $x^{CP(0)}$ or $x^{CP(1)}$ there is a set of positive measure of points which first escape from this neighborhood and then eventually are mapped to $x^{CP(0)}$, as well a symmetric set of positive measure of points mapped to $x^{CP(1)}$. Clearly, not all the point of I are mapped to $x^{CP(0)}$ or $x^{CP(1)}$: a chaotic repellor, separating the basins of the CP fixed points, remains in I, which is a Cantor set formed by all the repelling cycles and their preimages, as well as uncountably many aperiodic orbits.

In Fig.13 we show bifurcation structure of the (ϕ_S, ϕ_L)-parameter plane for fixed σ, μ, γ (see (25)) and $\theta = 0.25$, $\widetilde{n} = 0.8$. Recall that the inequality $\phi_S \geq \phi_L$ has to be satisfied, thus, the region above the straight line $\phi_S = \phi_L$ is not valid. As one can see, qualitative structures of all the presented 2D bifurcation diagrams are quite similar, so that we can suggest that the bifurcation scenario observed crossing the pitchfork bifurcation boundary of the region S is qualitatively similar to the one shown in Fig.4, while if the flip bifurcation boundary of the region S is crossed then the scenario is qualitatively similar to the one illustrated in Fig.9a.

This Figure allows us to comment on the interaction between the two dimensions of trade freeness (external and internal): the negative slope of the bifurcation curves implies that the bifurcation values for ϕ_S is decreasing in ϕ_L and the stable range of the symmetric equilibrium (the area S) shifts to the left for higher values of ϕ_L. This gives a clear hint that agglomeration is the most likely outcome of a symultaneous process of regional integration within the Union and external trade liberalisation.

6 Conclusions

The EU has experienced a strong trend of deepening internal economic integration, including trade and factor mobility. At the same time, the ongoing strategy "Global Europe" aims at reducing trade barriers with respect to the rest of the World. Furthermore, deeper EU integration into the world economy is currently characterised by the increasing importance of trade partners endowed with relatively lower levels of technology and higher shares of unskilled workers.

Given these stylised facts, our research question was to analyse the effects of deeper regional integration and international liberalisation within the same theoretical framework, especially shedding light on the importance of the endowments of the external trade partners.

To this end, this paper has presented a 3-region FE-NEG model aiming to study the interaction of commodity trade and factor mobility under the assumption – that is in line with stylized facts for EU – of mobile entrepreneurs/high qualified workers and immobile less qualified workers. The model has been set in discrete time and we studied the dynamic processes involved.

In the model, the world economy includes an economically integrated area composed of two regions – the Union – and an outside region. The two regions of the Union are closely integrated by means of lower mutual trade costs and mobile entrepreneurs. The outside region is separated from the Union due to high trade costs and immobile entrepreneurs. We paid particular attention to regional endowments of entrepreneurs and low qualified workers, considering the outside region not having a manufacturing sector as limiting case. Despite the analytical complexity typical for NEG models, we derived some results analytically while, for other results, we had to use simulations as standard in the NEG literature.

A first set of results is related to the existence and properties of the multiple equilibria which are a typical outcome of NEG models:

Most remarkably, our model – although being entirely based on standard NEG assumption – is able to endogenously generate, for significant parameter ranges, asymmetric stable equilibria, i.e. long-term positions in which the economic activity is still present in both regions of the Union even though it is not distributed symmetrically. This result is analytically derived for the special case in which the outside region is a pure rural economy without a manufacturing sector. Simulations show that it carries over to the general case.

For the standard NEG long-run equilibria, i.e. for the symmetric and the core periphery equilibria, we take a closer look at the employment structure: We can show that they respond in a usual Heckscher-Ohlin way to changes in the relative endowments: increasing the share of the mobile high qualified entrepreneurs or decreasing the share of low qualified labor favors the employment share of the manufacturing sector in the resp region. This result holds although the model is specified in the NEG perspective. Also, it can be shown that trade costs matter – e.g. a reduction in internal trade costs fosters manufacturing in the two regions' Union (driving the economic activities away from the outside region).

Another set of results pertains to the analysis of the local and global dynamics of the specified factor mobility process within the Union.

Surprisingly, we found that the basic bifurcation scenarios are qualitatively similar in all the parameter planes which we considered. It is initiated by a pitchfork bifurcation of the symmetric fixed point on one side and its flip bifurcation on the other side. The existence of the constraints in the model give rise to a particular bifurcation of the CP fixed points which we call "border-transcritical" bifurcation.

More specific, varying separately the two parameters of trade integration, internal trade costs / trade freeness or external trade costs / trade freeness leads to the follwoing bifurcation scenario: For low trade costs, manufacturing is expected to agglomerate in one region; for intermediate trade cost, a stable symmetric allocation is the long-run spatial pattern; further increasing the trade costs leads to cyclical and chaotic attractors which give rise to agglomeration again for very high trade costs. Given our stylized facts it is interesting to ask how this pattern changes when the outside region has a lower share of high qualified entrepreneurs or a higher share of less qualified workers (note that the latter corresponds also to a higher local demand in the outside region):

We found that the bifurcation values for ϕ_S (and ϕ_L) depend positively upon θ; a lower value of θ shrinks the range of trade costs (trade freness) for which the symmetric equilibrium is stable and shifts it to higher values of trade costs (lower values of the trade freeness). In this sense, a higher share of less qualified workers in the outside region, i.e. a lower value of θ, favours agglomeration.

In addition, the size of outside demand as measured in θ plays also a role in determining the transition pattern to agglomeration pattern as trade freeness is increased (we show results for ϕ_S but the same applies for ϕ_L): the pitchfork bifurcation is subcritical for θ above a certain threshold – in that we observe a catastrophic agglomeration pattern; instead, the pitchfork bifurcation is supercritical for θ below a certain value and we observe a smooth transition to agglomeration within the Union.

Instead, the effect of increasing the relative size of the industrial sector within the union has a non-monotonic effect: starting from a low (high) value, the bifurcation lines are positively (negatively) sloped, implying that the bifurcation values for both dimensions of trade freeness depend positively (negatively) upon the share of industry within the union; increasing the relative size of the industrial sector within the union has a destabilizing (stabilizing) effect on the symmetric allocation, in the sense of shifting the stable range for the symmetric equilibrium to higher (lower) values of the trade freeness. Thus, a highly industrialized union experiencing a shift in their trading partners towards lower industrialized regions will experience a shift of the stability ranges for the symmetric equilibrium towards lower values of the trade freeness, which makes agglomeration of economic activities within the Union less likely.

Finally, we comment on the interaction between the two aspects of the trade freeness: The respective bifurcation lines are negatively sloped implying that the bifurcation values for ϕ_S is decreasing in ϕ_L (and vice versa) and the stable range of the symmetric equilibrium shifts again to the left for higher values of ϕ_L. Simultaneously opening up internally and externally, both contribute to strengthening the agglomerative forces within the Union.

In addition to the bifurcation scenarios discussed so far, bistability, i.e. coexistence of attractors, is also a characteristic feature of the model. This goes well beyond the standard result in NEG models concerning the possible coexistence of the symmetric equilibrium and the Core-periphery equilibria; in our model it applies also to cyclical attractors. In particular, two different attractors, namely, two cycles or two cyclic chaotic attractors may coexist, with quite complicated structures of the respective basins of attraction. In some cases it is even impossible to predict to which attractor the system will converge: For low values of ϕ_S or ϕ_L the CP fixed points, being locally repelling, attract almost all the initial points (which is possible due to the flat branches of the map). Moreover, their basins are separated by such a complicated set as a chaotic repellor. Taking an initial point as close as we wish to one CP point, it cannot be stated a priori to which of the two CP fixed points the system will converge. History matters and a small perturbation may alter significantly the long-run allocation of industrial capital.

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