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Trend Fundamentals and Exchange Rate Dynamics*

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Abstract

We estimate a multivariate unobserved components stochastic volatility model to explain the dynamics of a panel of six exchange rates against the US Dollar. The empirical model is based on the assumption that both countries' monetary policy strategies may be well described by Taylor rules with a time-varying inflation target, a time-varying natural rate of unemployment, and interest rate smoothing. The estimates closely track major movements along with important time series properties of real and nominal exchange rates across all currencies considered. The model generally outperforms a benchmark model that does not account for changes in trend inflation and trend unemployment.

JEL classification: F31, E52, F41, C5, E31.

Keywords: Exchange rate models, trend inflation, natural rate of unemployment, Taylor rule, unobserved components stochastic volatility model.

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1 Introduction

To what extent do economic fundamentals explain exchange rate movements? Following the seminal work by Meese and Rogoff (1983), a wealth of studies has aimed to answer this question by comparing the out-of-sample predictive ability of economic exchange rate models to random walk forecasts, with mixed success (see Rossi 2013, for an overview). However, as Engel and West (2005) show, the random walk property of the exchange rate does not imply that economic fundamentals are irrelevant for exchange rate movements. In fact, they stress that the current exchange rate depends on future expected fundamentals. If some fundamentals are non-stationary, and the discount factor associated with these expectations is large, unpredictable shocks to the non-stationary fundamentals will dominate and lead to a persistent process for the exchange rate, which is almost indistinguishable from a random walk in finite samples.

In this paper, we build on this insight and show that changes in non-stationary trend inflation plays a relevant role for explaining bilateral exchange rate dynamics. To do so, we derive a partial equilibrium expression for the bilateral real exchange rate assuming that each countries' central bank targets short-term interest rates according to a Taylor rule with a time-varying inflation target and a time-varying natural rate of unemployment. Combining these Taylor rules with a no-arbitrage condition reveals that the current real exchange rate is determined by future expected trend inflation, inflation gaps, unemployment gaps, and short-term interest rates. To estimate these trends and gaps and to derive the corresponding expectations, we use an unobserved components stochastic volatility (UC-SV) model, similar to Stock and Watson (2007), in which trend inflation and trend unemployment follow non-stationary processes with stochastic volatility. This choice is common in the recent literature estimating trend inflation over different monetary policy regimes (see Ascari and Sbordone 2014, and references therein).

Our findings can be summarized as follows. First, the UC-SV model captures the major up- and downturns of bilateral real exchange rates against the US Dollar for a panel comprising of six economies during the post-Bretton Woods era. In fact, the correlations between the model-based predictions and the actual real exchange rates are as high as 0.56. A benchmark model, which is estimated on the same information set but does not discriminate between trend and gap components, yields significantly lower correlations comparable to existing studies (see Engel and West 2006, Mark 2009). Second, the UC-SV model is capable of reproducing all major long-run trends of the nominal exchange rates over the last 40 years. Third, the model successfully mimics the actual exchange rates with respect to several key time series properties. More specifically, we accurately reproduce the persistence of the real exchange rates and the correlations with other macroeconomic variables.

In what follows, Section 2 motivates the UC-SV model by deriving a partial equilibrium expression for the real exchange rate in terms of future expected fundamentals. Then, Section 3 outlines the empirical strategy adopted along with the corresponding prior specification. Finally, Section 4 presents the empirical results and the last section concludes.

2 Theoretical framework

Following Engel and West (2006), we derive an expression for the real exchange rate in terms of future expected fundamentals if monetary policy in two countries is characterized by Taylor rules. All equations are shown in log-linearized terms. Let the short-term policy interest rate i_t in the home economy be determined as

$$i_t = i_{t-1} + \gamma_\pi E_t \hat{\pi}_{t+1} + \gamma_u E_t \hat{u}_{t+1} + \gamma_q q_t + \varepsilon_t. \tag{1}$$

The central bank in the home economy targets the short-term interest rate as a function of deviations of expected inflation from the target ($E_t \hat{\pi}_{t+1}$), deviations of the expected unemployment rate from its natural level ($E_t \hat{u}_{t+1}$) and of the lagged interest rate, whereas ε_t is a monetary policy innovation.[1] The inflation and unemployment gaps are defined as $\hat{\pi}_t = \pi_t - \bar{\pi}_t$ and $\hat{u}_t = u_t - \bar{u}_t$, respectively. Therefore, the inflation target ($\bar{\pi}_t$) as well as the natural rate of unemployment ($\bar{u}_t$) change over time. As is standard in the literature $\gamma_\pi > 0,\ \gamma_u < 0$ such that the central bank increases its policy interest rate in response to a higher inflation gap or a lower unemployment gap.

[1] This specification reflects studies that find movements in trend inflation over time (Ascari and Sbordone 2014), changes in the non-accelerating inflation rate of unemployment over time (Gordon 1998) and relevant interest rate smoothing behavior of central banks (Coibion and Gorodnichenko 2012).

We follow Engel and West (2006) and assume that the home central bank responds to the real exchange rate defined as $q_t = e_t - p_t + p_t^*$. The nominal exchange rate (e_t) is expressed as the price of one unit of the foreign currency in terms of domestic currency such that a rise in the exchange rate implies a depreciation of the home currency. Furthermore, p_t and p_t^* denote the domestic and foreign price levels, respectively. We assume that $\gamma_q > 0$, implying that the central bank lowers the interest rate when the exchange rate appreciates in real terms.

The central bank in the foreign economy targets the short-term interest rate using an analogous rule, except that it does not respond to the real exchange rate, where foreign variables are labeled by an asterisk:[2]

$$i_t^* = i_{t-1}^* + \gamma_\pi E_t \hat{\pi}_{t+1}^* + \gamma_u E_t \hat{u}_{t+1}^* + \varepsilon_t^*. \tag{2}$$

Furthermore, we assume that an uncovered interest parity relationship holds period-by-period:[3]

$$i_t - i_t^* = E_t[\Delta q_{t+1} + \pi_{t+1} - \pi_{t+1}^*]. \tag{3}$$

Replacing the interest rate differential by the two policy rules and rearranging terms we obtain:

$$\begin{aligned} q_t \quad = \quad & \rho E_t q_{t+1} + (1-\gamma_\pi)\rho E_t(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^*) + \rho E_t(\bar{\pi}_{t+1} - \bar{\pi}_{t+1}^*) \\ & -\gamma_u \rho E_t(\hat{u}_{t+1} - \hat{u}_{t+1}^*) - \rho(i_{t-1} - i_{t-1}^*) - \rho(\varepsilon_t - \varepsilon_t^*). \end{aligned} \tag{4}$$

with $\rho = \frac{1}{1+\gamma_q}$. Solving the equation forward allows to express the real exchange rate in terms of future expected fundamentals. This expression is the present-value solution of Engel and West (2005).

[2]The Taylor rule parameters in the home and foreign economy are homogeneous for ease of exposition. In the empirical application we relax this restriction.

[3]A risk premium term would be straightforward to incorporate, see Engel and West (2006).

$$
\begin{aligned}
q_t &= \rho^{J+1} E_t q_{t+J+1} + (1-\gamma_\pi) E_t \sum_{j=0}^{J} \rho^{j+1} (\hat{\pi}_{t+j+1} - \hat{\pi}^*_{t+j+1}) \\
&\quad + E_t \sum_{j=0}^{J} \rho^{j+1} (\bar{\pi}_{t+j+1} - \bar{\pi}^*_{t+j+1}) - E_t \sum_{j=0}^{J-1} \rho^{j+1} (i_{t+j} - i^*_{t+j}) \\
&\quad - \gamma_u E_t \sum_{j=0}^{J} \rho^{j+1} (\hat{u}_{t+j+1} - \hat{u}^*_{t+j+1}) - \rho(i_{t-1} - i^*_{t-1}) - \rho(\varepsilon_t - \varepsilon^*_t).
\end{aligned} \tag{5}
$$

Despite the partial equilibrium nature of the analysis some interesting insights emerge. Reflecting the findings by Engel and West (2005), this equation suggests that changes in trend inflation, if they occur, will dominate fluctuations of the real exchange rate. To see this, assume that the home inflation gap follows a stationary AR(1) process with autoregressive parameter φ and home trend inflation follows a random walk. For simplicity, we also assume that $\gamma_\pi = 0$. For $J \to \infty$, it is straightforward to show that the partial equilibrium effect of a change in the inflation gap and trend inflation amount to

$$
\begin{aligned}
\frac{\partial q_t}{\partial \hat{\pi}_t} &= \frac{1}{1-\rho\varphi}, \\
\frac{\partial q_t}{\partial \bar{\pi}_t} &= \frac{1}{1-\rho}.
\end{aligned} \tag{6}
$$

As long as φ is smaller than unity and positive, a reasonable assumption for the inflation gap, the model implies that changes in trend inflation will have a larger effect on the exchange rate in absolute value than changes in the inflation gap. Moreover, the discrepancy between the response to a one unit change in the gap and trend, respectively, increases as the discount factor increases.

A second insight is that whether rising inflation leads to an appreciation or depreciation may depend on whether the gap or the trend changes. The sign of the effect depends on the specific value of γ_π. Engel et al. (2008) emphasize that, if the Taylor principle holds in a Taylor rule without interest rate smoothing, an increase in the expected inflation gap at home relative to the foreign economy implies a real appreciation. However, in our model we see that an increase in the trend inflation rate is unambiguously associated with a real depreciation, everything else being equal.

Intuitively, if the inflation trend rises while future expected interest rates are assumed to remain unchanged, future real interest rates will also decline, leading to a real deprecation. Because the random walk property of the trend inflation rate leads to a lower real exchange rate for a longer period of time, such changes have a larger effect on the current real exchange rate than a change in the inflation gap.

To map the theoretical equation to empirical data, we need to form expectations about nominal short-term rates, the inflation and unemployment gaps, as well as future trend inflation. In what follows we outline the empirical strategy to model the decomposition and the future expected evolution of these measures.

3 Empirical strategy

We propose a simple multivariate unobserved components stochastic volatility (UC-SV) model to describe the dynamics of the fundamentals. The model may be viewed as an open economy variant of earlier UC-SV specifications that aim to model inflation and unemployment dynamics by decomposing the respective variables in non-stationary trend and stationary gap components (see e.g. Gordon 1998, Stock and Watson 2007, Stella and Stock 2012).

3.1 The unobserved components stochastic volatility model

Let us store the observed inflation and unemployment series measured at time $t = 1, \ldots, T$ in a 4×1 vector $\boldsymbol{x}_t = (\pi_t, \pi_t^*, u_t, u_t^*)'$. We assume $\boldsymbol{x}_t$ may be decomposed as follows

$$\boldsymbol{x}_t = \overline{\boldsymbol{f}}_t + \hat{\boldsymbol{f}}_t + \boldsymbol{\varepsilon}_t, \tag{7}$$

$$\overline{\boldsymbol{f}}_t = \overline{\boldsymbol{f}}_{t-1} + \overline{\boldsymbol{\eta}}_t, \tag{8}$$

$$\hat{\boldsymbol{f}}_t = \boldsymbol{\Phi} \hat{\boldsymbol{f}}_{t-1} + \hat{\boldsymbol{\eta}}_t, \tag{9}$$

with $\overline{\boldsymbol{f}}_t = [\overline{\pi}_t, \overline{\pi}_t^*, \overline{u}_t, \overline{u}_t^*]'$ being a 4×1 vector of latent trend components of inflation and unemployment at home and abroad. Similarly, $\hat{\boldsymbol{f}}_t = [\hat{\pi}_t, \hat{\pi}_t^*, \hat{u}_t, \hat{u}_t^*]'$ denotes a 4×1 vector of (stationary) latent gap components of inflation and unemployment. We assume that

$\boldsymbol{\Phi} = \text{diag}(\phi_\pi, \phi_\pi^*, \phi_u, \phi_u^*)$ is a 4×4 dimensional matrix of autoregressive coefficients with absolute value below unity. This ensures that $\hat{\boldsymbol{f}}_t$ is mean reverting and thus permits us to interpret $\hat{\boldsymbol{f}}_t$ as a vector containing the inflation and unemployment gap, respectively.

Finally, $\boldsymbol{\varepsilon}_t$ and $\boldsymbol{\zeta}_t = [\overline{\boldsymbol{\eta}}_t', \hat{\boldsymbol{\eta}}_t']'$ are normally distributed vector white noise errors with time-varying variance covariance matrices $\boldsymbol{\Sigma}_t$ and $\boldsymbol{V}_t$. We assume that $\boldsymbol{\Sigma}_t$ is a diagonal matrix with typical element σ_{jt}^2 $(j = 1, \ldots, 4)$ and $\boldsymbol{V}_t$ is a full matrix that can be decomposed as

$$\boldsymbol{V}_t = \boldsymbol{A}\boldsymbol{S}_t\boldsymbol{A}', \tag{10}$$

where $\boldsymbol{A}$ is a 8×8 dimensional lower triangular matrix with unit diagonal and typical non-zero off-diagonal element a_j and $\boldsymbol{S}_t = \text{diag}(s_{1t}, \ldots, s_{8t})$ contains the stochastic volatilities of the latent factors on its main diagonal. This specification assumes that the errors of the state equations in Eq. (8) and Eq. (9) are correlated.

We complete the description of our empirical model by stacking the logarithm of the volatilities in $\boldsymbol{\Sigma}_t$ and $\boldsymbol{S}_t$ in a generic vector $\boldsymbol{h}_t$, with typical element denoted by h_{it}. Following Kastner and Frühwirth-Schnatter (2014), we assume that each h_{it} evolves according to

$$h_{it} = \mu_i + \rho_i(h_{it-1} - \mu_i) + \sqrt{\vartheta_i}v_{it}, \tag{11}$$

where μ_i is the level of the log-volatility, $\rho_i \in (-1, 1)$ denotes the autoregressive parameter and ϑ_i denotes the variance of the log-volatility. This choice ensures that the volatility is bounded in the limit and rules out odd behavior related to random walk state equations for log-volatilities.

The UC-SV model explicitly discriminates between components that are non-stationary, capturing trends in the respective macroeconomic variable, and stationary processes that capture the high frequency behavior. To improve the fit of the model we moreover assume that all components are allowed to follow distinct stochastic volatility processes. The specification described by Eqs. (7) to (9) is closely related to the model put forward by Stella and Stock (2012). However, while they assume that the inflation gap is proportional to the unemployment gap, we allow for more flexibility by assuming that the inflation gap evolves independently from the unemployment gap.

3.2 Relation to the real exchange rate

We can derive an approximation that maps the empirical model described in subsection 3.1 to the theoretical exchange rate model. If we assume that the discount factor ρ is close to unity and assume that the expectations hypothesis holds, $E_t \sum_{j=0}^{J-1}(i_{t+j} - i^*_{t+j})$ is approximately J times the interest rate differential for J-period bonds which we denote as $J(b_{J,t} - b^*_{J,t})$. Furthermore, for a discount factor close but below unity we have for large J that $E_t \rho^{J+1} q_{t+J+1} \approx 0$. Finally, under the structure of the UC-SV model we have that expectations of the gap components are formed as $E_t \hat{\pi}_{t+j} = \phi_\pi^j \hat{\pi}_t, E_t \hat{\pi}^*_{t+j} = \phi_\pi^{*j} \hat{\pi}^*_t$, $E_t \hat{u}_{t+j} = \phi_u^j \hat{u}_t$ and $E_t \hat{u}^*_{t+j} = \phi_u^{*j} \hat{u}^*_t$. Since the trend components follow a random walk process the expectations are given by $E_t \bar{\pi}_{t+j} = \bar{\pi}_t, E_t \bar{\pi}^*_{t+j} = \bar{\pi}^*_t, E_t \bar{u}_{t+j} = \bar{u}_t$ and $E_t \bar{u}^*_{t+j} = \bar{u}^*_t$. For large J and a discount factor close to unity we can approximate the exchange rate relationship in Eq. (5) as:

$$\begin{aligned} q_t \approx \; & \frac{1-\gamma_\pi}{1-\phi_\pi}(\hat{\pi}_t - \hat{\pi}^*_t) - \frac{\gamma_u}{1-\phi_u}(\hat{u}_t - \hat{u}^*_t) \\ & +(J+1)(\bar{\pi}_t - \bar{\pi}^*_t) - J(b_{J,t} - b^*_{J,t}) - (i_{t-1} - i^*_{t-1}) - (\varepsilon_t - \varepsilon^*_t). \end{aligned} \tag{12}$$

The terms involving the gap components are exact for $J \to \infty$ and $\rho \to 1$. However, it is worth noting that these approximating assumptions are accurate even for finite J and relatively persistent processes.[4] In the empirical specification, we relax the assumption of parameter homogeneity across both countries' Taylor rules. The empirical model that relates the system described in the previous subsection to Eq. (12) is therefore given by

$$q_t = \boldsymbol{X}_t \boldsymbol{\beta} + \nu_t, \tag{13}$$

[4]For an AR(1) process with autoregressive parameter $\rho = 0.97$ and forecast horizon $J = 120$, implying that $b_{J,t} - b^*_{J,t}$ is the difference in a ten-year government bond yield, the approximation error for the gap components amounts to 2.6% in terms of the correct finite-horizon expectation.

with

$$\boldsymbol{X}_t = [1, i_{t-1}, i^*_{t-1}, \frac{\hat{\pi}_t}{1-\phi_\pi}, \frac{\hat{\pi}^*_t}{1-\phi^*_\pi}, \frac{\hat{u}_t}{1-\phi_u}, \frac{\hat{u}^*_t}{1-\phi^*_u}, \\ (J+1)\bar{\pi}_t, (J+1)\bar{\pi}^*_t, Jb_{Jt}, Jb^*_{Jt}], \tag{14}$$

and $\nu_t \sim \mathcal{N}(0, \sigma^2_\nu)$ being a homoscedastic white noise error term. While it would be straightforward to allow for stochastic volatility in Eq. (13) we leave this possibility aside because we are mainly interested in capturing the dynamics of the exchange rate related to the first moment of the corresponding predictive density.

3.3 Prior setup and posterior simulation

The approach to estimation and inference is Bayesian. We thus have to specify suitable prior distributions for all coefficients of the UC-SV model.

Point of departure is a normally distributed prior for the initial value of $\boldsymbol{f}_t = (\overline{\boldsymbol{f}}'_t, \hat{\boldsymbol{f}}'_t)'$,

$$\boldsymbol{f}_1 \sim \mathcal{N}(\boldsymbol{0}, \underline{\boldsymbol{V}}_f). \tag{15}$$

Here $\underline{\boldsymbol{V}}_f$ is a diagonal prior variance-covariance matrix where we set the diagonal elements equal to ten, implying that we are relatively uninformative about the specific value of the initial state of the system.

For the diagonal elements of $\boldsymbol{\Phi}$ we also impose a normally distributed prior. More specifically, we set

$$\phi_{ii} \sim \mathcal{N}(\underline{\phi}_{ii}, \underline{v}_{\phi ii}) \text{ for } i = 1, \ldots, 4, \tag{16}$$

with $\underline{\phi}_{ii}$ and $\underline{v}_{\phi_{ii}}$ denoting prior mean and variance, respectively. We center the prior means associated with the inflation gap to 0.75 and the corresponding prior variance to $(0.1)^3$.[5] In addition, we set the prior mean related to the unemployment gap to 0.99, with prior variance set equal to $(0.1)^3$. This tight prior implies that the inflation gap is less persistent than the unemployment gap. A prior setup that is relatively uninformative on the autoregressive coefficients of the gap components yields results that are qualitatively

[5] This is broadly consistent with findings on the persistence of the inflation gap for the US before the Great Moderation (see Cogley and Sbordone 2008, Cogley et al. 2010).

similar. However, inspection of the posterior draws reveals that the likelihood is relatively uninformative on the persistence, and we thus experimented with different values of the parameters for the US to match the results presented in Stella and Stock (2012).

We use a Gaussian prior for the free elements of $\boldsymbol{A}_t$,

$$a_j \sim \mathcal{N}(\underline{a}_j, \underline{v}_{a_j}) \tag{17}$$

where we set $\underline{a}_j$ equal to zero and $\underline{v}_{a_j}$ equal to $(0.1)^3$. Again, this prior specification places considerable mass on the prior view that the shocks to the state equations are uncorrelated. Being effectively uninformative about a_j yields similar results but at the cost that the MCMC algorithm mixes somewhat slower.

For the priors on the level of the log-volatility μ_i we impose a normal prior with mean $\underline{\mu}_i$ and variance $\underline{v}_{\mu i}$,

$$\mu_i \sim \mathcal{N}(\underline{\mu}_i, \underline{v}_{\mu_i}). \tag{18}$$

We set $\underline{\mu}_i = 0$ and $\underline{v}_{\mu_i} = 10^2$ for $i = 1, \dots, 9$ to render this prior effectively uninformative. In addition, we impose a Beta prior on the persistence parameter ρ_i

$$\frac{\rho_i + 1}{2} \sim \mathcal{B}(b_0, b_1), \tag{19}$$

where we set $b_0 = 25$ and $b_1 = 5$ for all i leading to a prior mean of 0.83 with prior standard deviation of 0.07, thus placing considerable prior mass on high persistence regions of ρ_i. Note that this choice proves to be quite influential in practice since the likelihood typically carries little information about the persistence of the log-volatility.

Following Kastner and Frühwirth-Schnatter (2014) we use a non-conjugate Gamma prior on the variance of the log-volatility,

$$\vartheta_i \sim \mathcal{G}(1/2, \frac{1}{2B_\vartheta}). \tag{20}$$

The hyperparameter B_ϑ controls the tightness of the prior. It is straightforward to show that this prior implies

$$\pm\sqrt{\vartheta_i} \sim \mathcal{N}(0, B_\vartheta). \tag{21}$$

In the empirical application we set B_ϑ equal to unity. After experimenting with different values of B_ϑ, the specific choice of this hyperparameter proves to be rather unimportant in the present application. This prior setup has been motivated in Frühwirth-Schnatter and Wagner (2010) and provides several convenient properties. For instance, the Gamma prior does not bound ϑ_i away from zero and thus induces more shrinkage as the typical conjugate inverted Gamma prior.

For the elements of $\boldsymbol{\beta}$, denoted as β_i, we use a normal prior with mean $\underline{\beta}_i$ and variance $\underline{v}_{\beta_i}$,

$$\beta_i \sim \mathcal{N}(\underline{\beta}_i, \underline{v}_{\beta_i}). \tag{22}$$

We center the prior on the values analysed by Giannoni (2014) for a quasi-optimal Taylor rule with interest rate smoothing ($\gamma_\pi = \gamma_\pi^* = 0.64,\ \gamma_u = \gamma_u^* = -0.33$). For the remaining coefficients, we center the prior on the values implied by Eq. (12). However, because this is only a partial equilibrium expression for the real exchange rate we set the prior variance equal to ten to be rather uninformative.

Finally, we use an inverted Gamma prior for σ_ν^2,

$$\sigma_\nu^2 \sim \mathcal{IG}(c_0, c_1), \tag{23}$$

where c_0 and c_1 are set equal to $(0.1)^3$, rendering this prior effectively non-influential.

The Markov chain Monte Carlo algorithm iterates between the following steps:

- Simulate the full history of $\boldsymbol{f}_t$, denoted as $\boldsymbol{f}^T = (\boldsymbol{f}_1, \dots, \boldsymbol{f}_T)'$ conditional on all other parameters and the data using the well-known algorithm developed by Carter and Kohn (1994) and Frühwirth-Schnatter (1994).
- The parameters of the log-volatility in Eq. (11) and the full history of log-volatilities $h_i^T = (h_{i1}, \dots, h_{iT})'$ are simulated by means of the algorithm provided in Kastner and Frühwirth-Schnatter (2014), which proves to be an efficient alternative to other popular algorithms.[6]
- The autoregressive parameters of the state equations in Eq. (8) and Eq. (9) are sampled through Gibbs steps, sampling from Gaussian distributions. To ensure stationarity we impose the constraint that all draws have to be smaller than unity

[6]This step is implemented using the `R` package stochvol (Kastner 2015a,b).

in absolute values.

- Similarly, given the conjugacy of the prior setup employed, $\boldsymbol{\beta}$ is simulated from a normal distribution with well-known posterior mean and variance.
- For the covariance parameters a_j we follow Cogley and Sargent (2005) and rewrite the reduced-form errors as a set of simple regression models with innovations that are standard normally distributed. The normal prior on each a_j then yields a well-known Gaussian posterior density with known moments that can be used to simulate a_j.
- Finally, σ_ν^2 is sampled with a Gibbs step by noting that the conditional posterior is of a well-known form, namely an inverted Gamma distribution.

In the empirical application we repeat this algorithm 30,000 times and discard the first 15,000 iterations as burn-ins. Moreover we impose the restriction that the variance of the unemployment gap at home and abroad equals to 0.3. Since allowing for stochastic volatility in the measurement error and the errors of the gap components separately typically leads to empirical problems, we fix the variance of $\hat{u}_t$ and $\hat{u}_t^*$. Again, setting the variance equal to 0.3 is predicated by calibrating the model to match the trend unemployment rate and unemployment gap estimated by previous studies for the US.

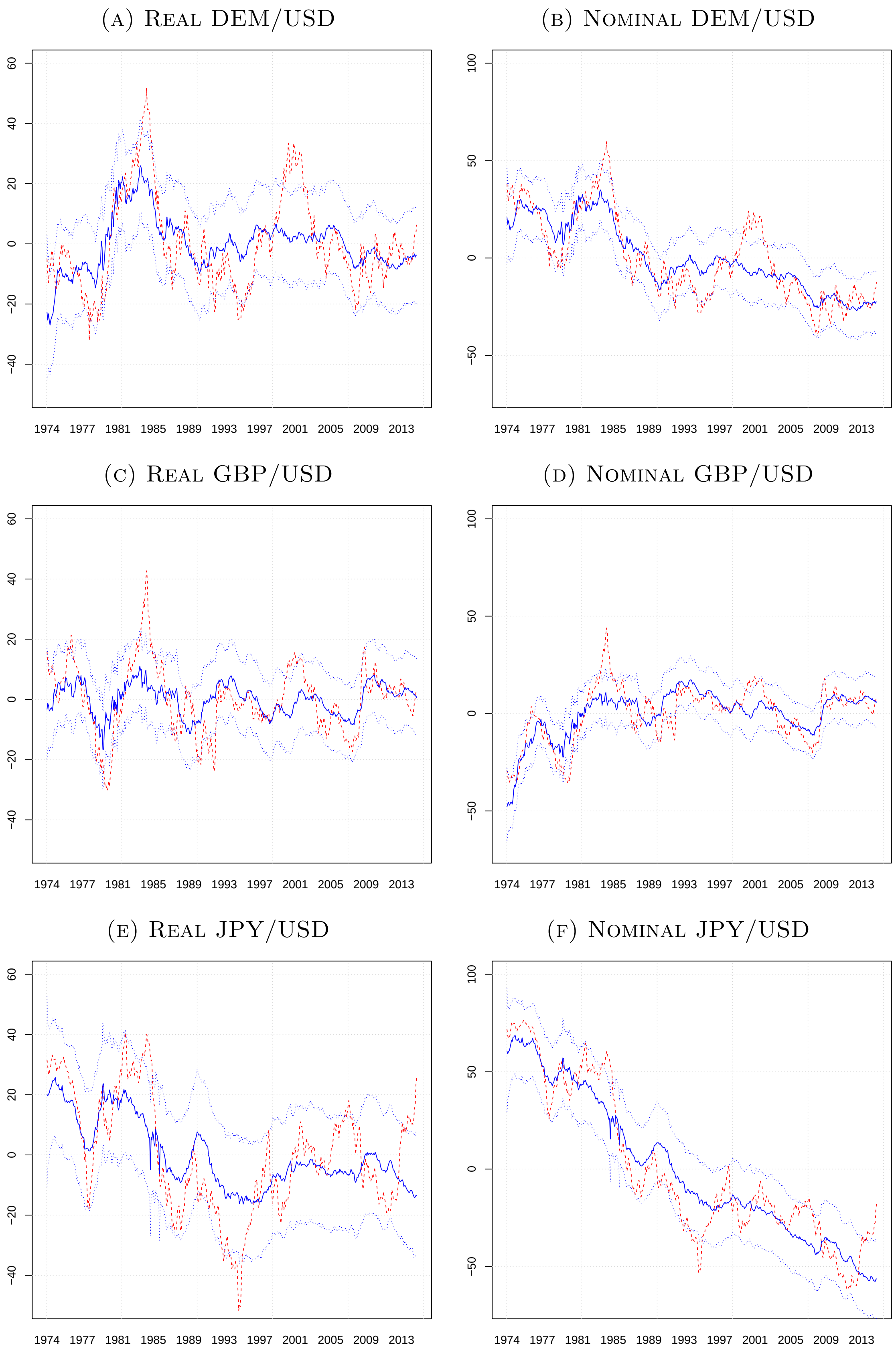
4 Results

We estimate the model for the US Dollar against the currencies of a panel of six economies: Germany, UK, Japan, Canada, Sweden and Switzerland (see Appendix A for a detailed description of the data). For the DEM/USD exchange rate, the series is linked with the EUR/USD exchange rate after the introduction of the Euro.[7] The real exchange rate is calculated using the same consumer price indices that are used in the estimation for the trend inflation rate. We use 10-year government bond yields to approximate the sum of future expected short-term interest rates and thus set $J = 120$ months. As short-term interest rates we use 3-month interbank or T-Bill rates. Finally, we use civilian unemployment rates to estimate the unemployment gaps.

Figures 1 and 2 show the actual real and nominal exchange rates along with the mean and the 5th and 95th percentiles of the posterior distribution from the UC-SV model. The

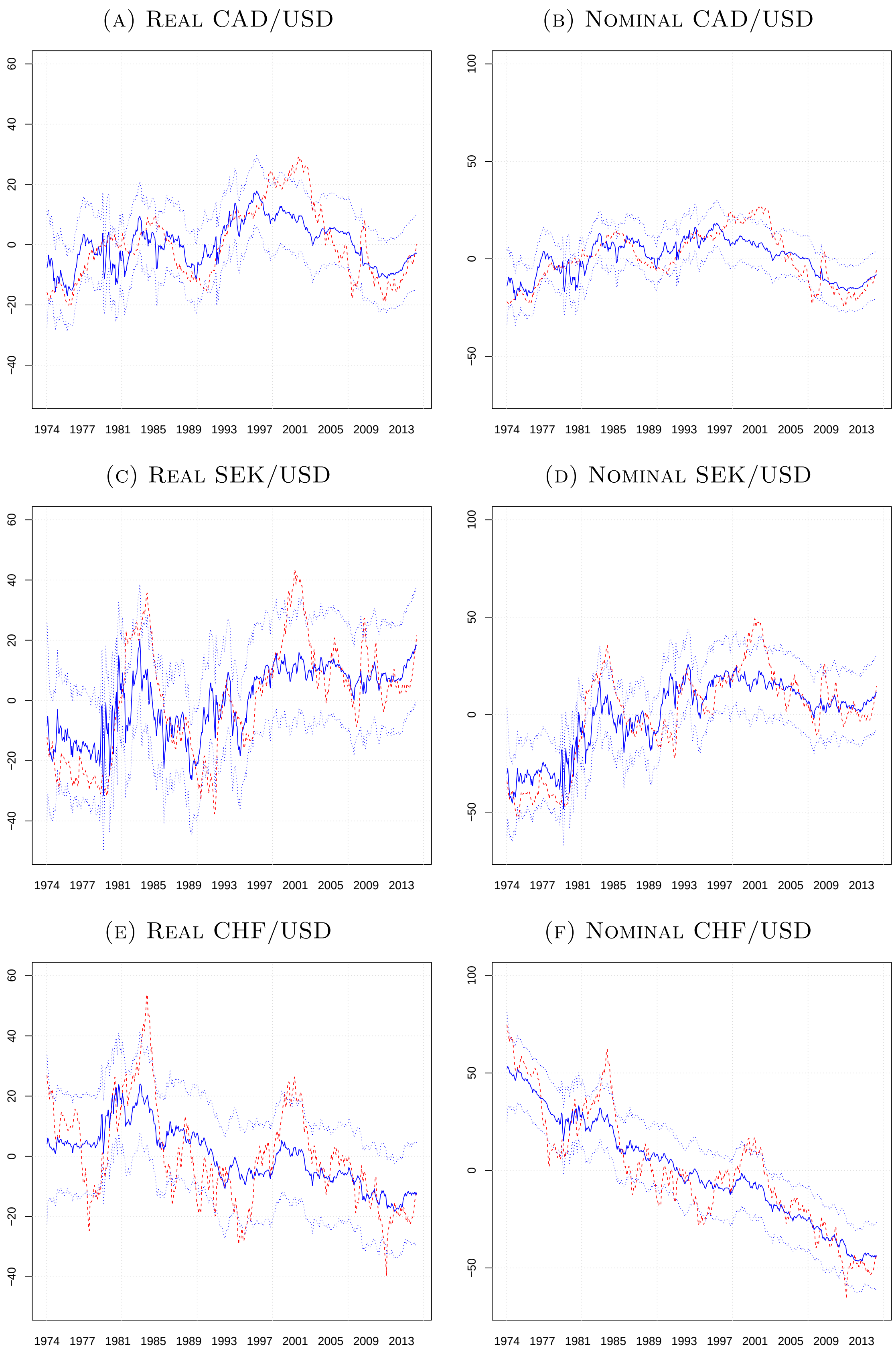
[7] We experimented with linking all data with euro area aggregates after the euro changeover and the results prove to be robust to this alternative.

Figure 1 — Model predictions for large economies



Notes: Actual real and nominal US Dollar exchange rates are given by dashed red lines (in logarithms times 100, centered around 0). The posterior median is given by the solid blue lines and the dashed blue lines correspond to 5th and 95th percentiles. The results are based on 15,000 posterior draws.

Figure 2 — Model predictions for small economies



Notes: Actual real and nominal US Dollar exchange rates in dashed red lines (in logarithms times 100, centered around 0). The posterior median is given by the solid blue lines and the dashed blue lines correspond to 5th and 95th percentiles. The results are based on 15,000 posterior draws.

posterior distribution reflects uncertainty associated with the estimation of the UC-SV model as well as due to the estimation of the linear relationship of the exchange rate equation. For all countries, the posterior mean tracks major exchange rate movements well. The majority of turning points of the real exchange rate are captured by our model. Moreover, we match the appreciation trends of the nominal exchange rate, in particular, for Japan and Switzerland well.

In what follows, we discuss the episodes when the actual exchange rate moves outside of the 5th and 95th percentiles. In the mid-1980s, the real exchange rate leaves the credible bands for all countries except Canada. Similar problems of matching the strong US Dollar during this period are reported by Engel and West (2006), where they note that this period has been frequently labelled a US Dollar "bubble". This is in line with the idea that the fundamentals included in the extended model do not explain the strong Dollar.

Starting in 1998, the US Dollar appreciated and rose outside of the 95th percentile for most currencies under consideration. We conjecture that this is closely related with several major economic crises that forced investors to reduce their non-USD exposure ("flight to safety"). More specifically, the Asian financial crisis, that hit the region between 1997 and 1998, was closely followed by the sovereign default of Russia and the unwind of Long-Term Capital Management. Beside these developments in Asia, increased uncertainty surrounding the Argentinian crisis between 1998 and 2002 presumably contributed to the upward pressure on the US Dollar. Such save-haven considerations are probably not well captured in the factors affecting short-term interest rates via the Taylor rule.

Generally speaking, significant deviations from the model predictions occur when the Taylor rule is a poor approximation to monetary policy, for example, at the effective lower bound on short-term interest rates and during unconventional monetary policy actions. In 1978 and 2011, the real exchange rate leaves the credible bands for Switzerland when the short-term interest rate was constrained by the effective lower bound. Bäurle and Kaufmann (2014) argue that a currency is likely to appreciate strongly at the effective lower bound in response to modestly deflationary risk premium shocks because of increasing instead of declining real interest rates. In the late 1970s as well as in 2011, the SNB counteracted the appreciation by introducing a minimum exchange rate against the German Mark and the Euro, respectively. Also for Japan, we observe a substantial deviation from the prediction in 1995 when short-term interest rates fell to very low levels (to 0.4% in September 1995).

Similarly, the UC-SV approach may not fully include unconventional monetary policy actions and sharp and sudden changes in inflation expectations. For Japan, the real and nominal exchange rates leave the percentiles in 2014. But the posterior mean moves into the opposite direction of the actual exchange rate already since 2012. This episode was governed by exceptional policy actions due to Abenomics which may not be appropriately reflected in the empirical UC-SV model: a higher inflation target, quantitative easing and an expansionary fiscal policy stance.

Using the posterior distribution of the exchange rate prediction, we may investigate the model fit more formally by calculating the posterior distribution of the correlation with the actual exchange rate. The model predictions match the dynamics of the level of the exchange rate well, however, they do not explain exchange rate changes. Table 1 shows the posterior mean and percentiles for the correlation with the actual real and nominal exchange rates for each country. The first line is a benchmark model where we do not control for the fact that trend inflation and trend unemployment may change over time. This specification includes the same information set as the UC-SV model, however, without decomposing inflation and unemployment into trends and cycles. The second line gives the UC-SV model specification with the decomposition. Using the benchmark model we obtain correlations between 0.22 for the UK and 0.48 for Canada. The correlation for Germany at 0.35 is close to existing estimates by Engel and West (2006) and Mark (2009).

If we include trend inflation rates, the inflation gaps and the unemployment gaps separately, the correlation rises to 0.33 for the UK and even to 0.56 for Canada. For Germany, the posterior mean correlation amounts to 0.47. The model thus improves existing predictions for the real exchange rate and this can be traced back to accounting for changes in trend inflation and the trend unemployment rate. For the nominal exchange rate, the correlation is generally higher reflecting that we match the trends for Japan and Switzerland particularly well. But also, the correlation is substantial for Canada where the nominal exchange rate does not exhibit a strong secular trend.

For changes in exchange rates, the model does not outperform the benchmark.

Table 1 — Correlation with actual exchange rate

		(A) Real		(B) Nominal	
		Log-level	Log-change	Log-level	Log-change
DEM/USD	Benchmark	0.35	0.01	0.66	0.02
		$[0.26, 0.43]$	$[-0.09, 0.12]$	$[0.61, 0.80]$	$[-0.09, 0.12]$
	UC-SV	0.47	0.02	0.75	0.02
		$[0.38, 0.59]$	$[-0.05, 0.09]$	$[0.71, 0.80]$	$[-0.05, 0.10]$
GBP/USD	Benchmark	0.22	0.02	0.55	0.03
		$[0.12, 0.31]$	$[-0.08, 0.10]$	$[0.49, 0.69]$	$[-0.06, 0.11]$
	UC-SV	0.33	0.02	0.64	0.03
		$[0.24, 0.45]$	$[-0.04, 0.08]$	$[0.59, 0.69]$	$[-0.04, 0.09]$
JPY/USD	Benchmark	0.36	0.00	0.85	0.02
		$[0.27, 0.44]$	$[-0.08, 0.09]$	$[0.83, 0.92]$	$[-0.07, 0.10]$
	UC-SV	0.50	0.01	0.90	0.01
		$[0.38, 0.64]$	$[-0.05, 0.07]$	$[0.87, 0.92]$	$[-0.05, 0.07]$
CAD/USD	Benchmark	0.48	0.03	0.59	0.04
		$[0.40, 0.55]$	$[-0.07, 0.12]$	$[0.52, 0.75]$	$[-0.05, 0.13]$
	UC-SV	0.56	0.03	0.65	0.04
		$[0.40, 0.68]$	$[-0.03, 0.10]$	$[0.52, 0.75]$	$[-0.03, 0.10]$
SEK/USD	Benchmark	0.44	0.03	0.65	0.04
		$[0.36, 0.51]$	$[-0.05, 0.11]$	$[0.60, 0.78]$	$[-0.04, 0.12]$
	UC-SV	0.52	0.02	0.72	0.02
		$[0.43, 0.63]$	$[-0.04, 0.07]$	$[0.66, 0.78]$	$[-0.04, 0.08]$
CHF/USD	Benchmark	0.39	0.02	0.82	0.03
		$[0.31, 0.47]$	$[-0.06, 0.11]$	$[0.79, 0.89]$	$[-0.06, 0.12]$
	UC-SV	0.48	0.02	0.86	0.03
		$[0.40, 0.60]$	$[-0.04, 0.09]$	$[0.84, 0.89]$	$[-0.04, 0.09]$

Notes: Posterior mean correlation with actual US Dollar exchange rate. 5th and 95th percentiles in brackets. The benchmark model does not take into account changes in the inflation and unemployment trends.

While the posterior mean correlation is usually higher for the UC-SV model when compared with the benchmark. In fact, the percentiles always include zero for both specifications. This suggests that we mainly capture the major exchange rate movements while month-to-month movements are not very well captured.

An important aspect for an exchange rate model to match is the high persistence or near random walk properties of the real exchange rate. Table 2 shows the sample autocorrelation up to the third order for the actual real exchange rate along with the autocorrelation of the posterior means of the predictions. The benchmark model already

Table 2 — Autocorrelation real exchange rate

		Log-level			Log-change		
		1st	2nd	3rd	1st	2nd	3rd
DEM/USD	Actual	0.98	0.96	0.93	0.01	0.04	0.04
	Benchmark	0.94	0.90	0.88	−0.14	−0.24	−0.02
	UC-SV	0.98	0.96	0.94	0.13	−0.17	0.05
GBP/USD	Actual	0.97	0.94	0.90	0.33	0.02	0.05
	Benchmark	0.90	0.82	0.78	−0.12	−0.17	−0.07
	UC-SV	0.96	0.93	0.89	0.02	−0.06	−0.09
JPY/USD	Actual	0.99	0.96	0.94	0.33	0.08	0.05
	Benchmark	0.90	0.89	0.87	−0.41	−0.02	−0.02
	UC-SV	0.99	0.98	0.97	−0.17	0.09	−0.04
CAD/USD	Actual	0.99	0.98	0.97	0.21	0.05	0.03
	Benchmark	0.95	0.91	0.89	−0.17	−0.11	−0.09
	UC-SV	0.98	0.95	0.92	0.10	−0.03	−0.13
SEK/USD	Actual	0.99	0.97	0.95	0.36	0.04	0.05
	Benchmark	0.86	0.76	0.73	−0.12	−0.26	−0.10
	UC-SV	0.95	0.89	0.86	0.04	−0.24	−0.19
CHF/USD	Actual	0.98	0.96	0.93	0.27	0.03	0.02
	Benchmark	0.94	0.91	0.90	−0.17	−0.26	−0.03
	UC-SV	0.99	0.97	0.96	0.11	−0.21	−0.08

Notes: Sample autocorrelation function for the actual real US Dollar exchange rate and sample autocorrelation function for the posterior mean of the model predictions up to 3rd order. The benchmark model does not take into account changes in the inflation and unemployment trends.

implies a highly persistent real exchange rate. Nevertheless, the persistence of the exchange rate based on the benchmark model is lower than that of the actual real exchange rate for all countries and all lags. The UC-SV model is capable of explaining the higher persistence of the real exchange rate and matches the actual persistence closely. The only countries where the model does not quite match the persistence are Canada and Sweden. Nevertheless, the model still outperforms the benchmark for both countries.

Similarly, we also make progress of matching the persistence of exchange rate changes. In the actual data the first order autocorrelation is larger than zero for all countries. By contrast, the benchmark model implies a negative first order autocorrelation for exchange rate changes. Although the UC-SV model does not exactly reproduce the pattern in the data, first order autocorrelations are mostly positive (except for Japan and UK) and second order correlations are closer to the actual values.

Table 3 — Correlation of real exchange rate with fundamentals

		$\pi_t - \pi_t^*$	$u_t - u_t^*$	$i_t - i_t^*$	$b_t - b_t^*$
DEM/USD	Actual	0.15	0.19	−0.20	−0.44
	UC-SV	0.08	0.21	−0.24	−0.53
		[0.01, 0.15]	[0.12, 0.31]	[−0.30, −0.17]	[−0.58, −0.47]
GBP/USD	Actual	0.11	0.04	−0.06	−0.02
	UC-SV	0.04	0.02	−0.07	−0.02
		[−0.04, 0.12]	[−0.10, 0.13]	[−0.15, 0.00]	[−0.10, 0.06]
JPY/USD	Actual	0.17	−0.39	−0.07	0.20
	UC-SV	0.07	−0.47	−0.09	0.24
		[0.01, 0.14]	[−0.58, −0.36]	[−0.15, −0.02]	[0.18, 0.30]
CAD/USD	Actual	−0.03	0.54	−0.38	−0.18
	UC-SV	−0.11	0.54	−0.44	−0.21
		[−0.17, −0.05]	[0.36, 0.66]	[−0.50, −0.39]	[−0.28, −0.15]
SEK/USD	Actual	−0.06	0.38	−0.07	−0.43
	UC-SV	−0.14	0.42	−0.08	−0.50
		[−0.20, −0.08]	[0.31, 0.51]	[−0.14, −0.01]	[−0.56, −0.44]
CHF/USD	Actual	0.08	−0.32	−0.30	−0.47
	UC-SV	−0.03	−0.37	−0.36	−0.56
		[−0.10, 0.05]	[−0.49, −0.26]	[−0.42, −0.29]	[−0.61, −0.50]

Notes: Correlations of the actual and predicted real US Dollar exchange rate with differences in the fundamentals. The 5th and 95th percentiles are given in brackets. The benchmark model does not take into account changes in the inflation and unemployment trends.

As a further check whether the exchange rate model predicts a real exchange rate with reasonable properties, we compare correlations of the real exchange rate with the fundamentals. Table 3 shows that the model closely matches the correlation between the actual exchange rate and the fundamentals. The posterior mean is close to the actual correlation and the 5th and 95th percentiles mostly include the actual value. The actual value of the correlation lies outside of the percentiles for inflation in Japan, Canada and Switzerland and long-term bonds in Germany and Switzerland. However, even for these correlations the sign of the posterior mean is consistent with the sign of the actual correlation.

Finally, we performed robustness checks with respect to the model specification. In particular, we experimented with simpler detrending methods to construct a measure of trend inflation and the unemployment gap. Applying a Hodrick and Prescott (1997) filter and performing the corresponding linear regression yields similar correlations between the actual and predicted real exchange rate as in the UC-SV model. Moreover, versions of the regression that include the differences in domestic and US variables directly reveals that the coefficient on trend inflation differential is always significantly positive, except for Canada. Although we refrain from reading too much into the actual coefficients due to the reduced-form nature of the regression equation, we take this feature as evidence that expectations about trend inflation rate are essential to explaining the current real exchange rate.

5 Closing remarks

Recent research has documented that trend inflation changes over time. We add an international dimension to this line of research and highlight that changes in trend inflation explain important aspects of exchange rate dynamics. We develop a multivariate UC-SV model that is theoretically motivated by assuming that both countries' central banks follow Taylor rules, but the inflation target as well as the natural rate of unemployment may change over time.

The UC-SV model succeeds in capturing major up- and downturns of the real US Dollar exchange rate against the currencies of six economies. In fact, the correlations of the model predictions with the actual real exchange rates are higher than in existing studies. While a benchmark model performs comparatively well, the improvements obtained by explicitly discriminating between non-stationary trend and stationary gap components are significant for all currencies under consideration. Looking at nominal exchange rates reveals that we are able to accurately reproduce major exchange rate trends observed over the last 40 years. Finally, the model successfully captures several key time series characteristics commonly found for real exchange rates. More specifically, we accurately reproduce the persistence of the real exchange rate and its correlation with other macroeconomic variables.

Our discussion shows that, although the model explains a larger share of exchange rate fluctuations than previous studies, it fails during episodes when the Taylor rule is unlikely to be an accurate description of the central banks' conduct of monetary policy. Improving the model predictions by accounting for unconventional monetary policy actions and constraints on the operational targets of central banks might be a promising avenue for future research.

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Appendix A   Data

Table 4 — Data, sources, transformations

	Country	FRED identifier	Source	Comments
Exchange rates	CA	EXCAUS	FRB	
	JP	EXJPUS	FRB	
	SE	EXSDUS	FRB	
	CH	EXSZUS	FRB	
	UK	EXUSUK	FRB	Inverted
	DE	CCUSSP01DEM650N	MEI	Inverted, EUR/USD after euro changeover
CPI	CA	CANCPIALLMINMEI	MEI	Census X13 seas. adj.
	JP	JPNCPIALLMINMEI	MEI	Census X13 seas. adj.
	SE	SWECPIALLMINMEI	MEI	Census X13 seas. adj.
	CH	CHECPIALLMINMEI	MEI	Census X13 seas. adj.
	UK	GBRCPIALLMINMEI	MEI	Census X13 seas. adj.
	US	CPIAUCSL	BLS	
	DE	DEUCPIALLMINMEI	MEI	Census X13 seas. adj.
Unemployment rates	CA	LRUNTTTTCAM156S	MEI	
	JP	LRUN24TTJPM156N	MEI	Census X13 seas. adj.
	SE	LRHUTTTTSEM156S, SWEURHARMMDSMEI	MEI	Sources linked in 1983
	CH	LMUNRRTTCHM156N	MEI	Census X13 seas. adj.
	UK	LMUNRRTTGBM156S	MEI	
	US	UNRATE	BLS	
	DE		BA	Downloaded from Datastream
Short rates	CA	IR3TIB01CAM156N	MEI	Interbank rate
	JP	INTGSTJPM193N	IFS	T-Bill rate
	SE	IR3TIB01SEM156N	MEI	Linked with Riksbank data (see notes)
	CH	IR3TIB01CHM156N	MEI	Interbank rate
	UK	IR3TTS01GBM156N	MEI	T-Bill rate
	US	IR3TIB01USM156N	MEI	Interbank rate
	DE	IR3TIB01DEM156N	MEI	Interbank rate
Long rates	CA	IRLTLT01CAM156N	MEI	
	JP	INTGSBJPM193N	IFS	
	SE	IRLTLT01SEM156N	MEI	Linked with Riksbank data (see notes)
	CH	IRLTLT01CHM156N	MEI	
	UK	IRLTLT01GBM156N	MEI	
	US	IRLTLT01USM156N	MEI	
	DE	IRLTLT01DEM156N	MEI	

Notes: All data, unless otherwise indicated, was retrieved from FRED, Federal Reserve Bank of St. Louis `https://research.stlouisfed.org/fred2/`. Data for short-term and long-term interest rates for Sweden was downloaded from `http://www.riksbank.se/en/The-Riksbank/Research/Historical-Monetary-Statistics-/Interest-and-stock-returns/`.