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Debt Maturity and the Dynamics of Leverage

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This paper shows that short debt maturities commit equityholders to leverage reductions when refinancing expiring debt in low-profitability states. However, shorter maturities lead to higher transaction costs since larger amounts of expiring debt need to be refinanced. We show that this trade-off between higher expected transaction costs against the commitment to reduce leverage in low-profitability states motivates an optimal maturity structure of corporate debt. Since firms with high costs of financial distress and risky cash flows benefit most from committing to leverage reductions, they have a stronger motive to issue short-term debt. Evidence supports the model's predictions. (*JEL* G3, G32)

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Significant progress has been made toward understanding firms' dynamic financing decisions. Major contributions to this literature model a firm's assets or cash flows as a stochastic process and assume that debt generates some benefit, such as a tax advantage, but also generates dead weight costs associated with excessively high leverage, such as bankruptcy costs.[1] While these models have successfully explained firms' optimal target leverage ratios and their decisions to dynamically increase debt levels in response to increases in their asset values or cash flows, they have much less successfully explained leverage reductions. In fact, these models generally imply that equityholders never find it optimal to use retained earnings or to issue equity to reduce

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[1] See, for example, Fischer, Heinkel, and Zechner (1989), Leland (1994a), Leland and Toft (1996), Goldstein, Ju, and Leland (2001), Dangl and Zechner (2004), and Strebulaev (2007).

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leverage. This is due to a version of Myers (1977) underinvestment problem: equityholders “underinvest”in leverage reductions and do not inject capital or reduce dividends to repurchase debt, since this would transfer wealth to the remaining bondholders. As shown by Admati et al. (2018), equityholders not only lack any incentive to actively repurchase outstanding debt but also frequently have incentives to increase debt even if increasing their debt reduces their firm’s total value. Thus, in these models, debt reductions only occur following bankruptcy.[2]

This implication is in contrast to evidence that debt reductions frequently occur even in the absence of bankruptcy or negotiated debt forgiveness.[3] In this paper, we develop a dynamic capital structure model that features such voluntary leverage reductions induced by firms’ finite debt maturities. The key insight is that equityholders of a firm with perpetual debt do not find it optimal to repurchase debt to reduce leverage, due to a version of the Myers (1977) underinvestment problem. However, a firm that must repay some of its debt due to its finite maturity may not want to issue new debt with the same face value, when its profitability is low. Essentially, a firm with a higher fraction of maturing debt (shorter maturity), has a greater flexibility to manage its leverage in relatively bad states.[4] Thus, we identify and analyze a largely unexplored aspect of debt maturity, namely, its effect on future capital structure dynamics. We specifically address the following questions. How is debt maturity related to equityholders’ dynamic leverage adjustments? How do firms optimally refinance expiring debt? What is the optimal debt maturity structure given its implications for dynamic capital structure adjustments and which firms are most likely to issue short-term debt? We analyze these questions in a framework in which equityholders are allowed to optimize the mix of debt and equity used to refinance maturing debt, but they cannot commit ex ante to a particular refinancing mix. Firms can also increase the face value of debt at any point in time, but, to do so, they first need to repurchase the existing debt. This can be interpreted as eliminating debt covenants, which prevent firms from diluting existing debtholders by issuing more debt.

2 Leland and Hackbarth (2019) extend the analysis in Admati et al. (2018) and analyze multiple rounds of debt issues when existing debt is senior to new debt. They provide results on how maturity affects incentives for subsequent debt issues. Other models consider debt renegotiations and derive partial debt forgiveness outside of bankruptcy (see, e.g., Anderson and Sundaresan 1996; Mella-Barral and Perraudin 1997; Mella-Barral 1999; Christensen et al. 2014). Mao and Tserlukevich (2015) present a model in which noncoordinated debt holders may accept repurchase offers below the market price when firms pay with existing safe assets or cash. Lehar (2018) considers multilateral bargaining and explicitly considers renegotiation breakdowns and subsequent inefficient liquidation. In contrast to these papers, we focus on situations in which coordination problems among bondholders prevent renegotiation solutions.

3 Surveying 392 CFOs, Graham and Harvey (2001) report that 81% of firms in their sample use target leverage ratios. If highly levered, firms tend to issue equity to maintain their target ratios. Hovakimian, Opler, and Titman (2001) find strong evidence that firms use (time-varying) target leverage ratios. They identify deviation from this target as the dominant economic factor in determining whether a firm retires debt. DeAngelo, Gonçalves, and Stulz (2018) report that firms deleverage regularly, but that this deleveraging is typically small when compared to retained earnings or new equity issues.

4 We are grateful to an anonymous referee for providing this intuition.

We find that firms' equityholders may not wish to rollover maturing debt by issuing a new bond with the same face value. Instead, it may be optimal for them to issue a new bond with lower face value and at least partly refinance expiring debt with equity. This happens after a deterioration in the firm's profitability and debt maturity is sufficiently short. In this case new debt can only be issued at a high credit spread, since the price of the new bonds reflects the increased default probability and the resultant increase in expected costs of financial distress. Equityholders and may therefore wish to only partially rollover maturing debt.

If, by contrast, debt maturity is sufficiently long, then replacing maturing debt with equity always leads to a significant wealth transfer to the remaining bonds outstanding, since debt with a longer maturity is subject to more credit risk. This makes the rollover decision subject to a more severe debt overhang problem and makes the use of equity to refinance maturing debt suboptimal for equityholders. We find that, for sufficiently long debt maturities, firms always prefer to rollover debt at the maximum rate, that is, to issue a new bond with a face value that corresponds to the face value of the maturing bonds. This result is in accordance with evidence reported by Hovakimian, Opler, and Titman (2001) and Jungherr and Schott (2020b), who find that long debt maturities majorly impede debt reductions.

We show that shorter debt maturities lead to more pronounced debt reductions since they require the firm to refund a larger fraction of its debt during any given period of time. Thus, ceteris paribus, the shorter the maturity, the faster the debt face value declines in response to deteriorating firm cash flows.

In accordance with Admati et al. (2018) and DeMarzo and He (forthcoming), firms never wish to actively repurchase nonexpiring debt. Instead, debt reduction is determined by the ex post decision how to repay maturing debt. Short-term debt, therefore, can be interpreted as an ex ante commitment to engage in debt reductions when the firm's profitability deteriorates.

Equityholders' incentives to refund maturing debt with equity are nonmonotonic in the firm's profitability and thus in firm value. For values around the initial cash flow level it is optimal to fully rollover maturing debt by issuing new bonds with the same face value. If the firm's profitability drops sufficiently, then equityholders reduce the rollover rate, as explained above. However, if the firm's cash flows continue to deteriorate and the firm is pushed toward the default boundary, equityholders eventually find it optimal again to choose the maximum rollover rate. Since the firm is close to bankruptcy a reduction in leverage largely benefits the remaining bondholders, even if the maturity of the remaining debt is short. Thus, the resultant debt overhang problem implies that equityholders are no longer willing to contribute capital to reduce debt.

Our analysis is closely related to that of DeMarzo and He (forthcoming), who analyze capital structure dynamics in the absence of any commitment or debt covenants. They show that in such a setting equityholders' incentives to dilute existing bondholders lead to rollover and debt issuing decisions that make the net present value of all future debt issues exactly zero. As a result, relatively simple valuation expressions and a rich set of results are obtained. The focus of our paper is very different. Whereas in DeMarzo and He (forthcoming) debt maturity is irrelevant and the value of the optimally levered firm equals that of a firm that never issues any debt, we focus precisely on the optimal maturity choice and the trade-offs that motivate it. In doing so, we allow for two realistic features, namely, transaction costs associated with debt rollover and a covenant that protects existing bondholders from being diluted via leverage increasing new pari passu issues. This generates an interior optimal debt maturity and also results in a positive net benefit of leverage.

Hovakimian, Opler, and Titman (2001) present strong empirical support for this nonmonotonicity in voluntary debt reductions. Our own empirical results, presented in this paper, confirm these findings and also provide novel support for our theoretical results. Interestingly, existing literature, such as Welch (2004), has interpreted the fact that highly levered firms issue debt as evidence *against* the trade-off theory of capital structure choice, since it moves the leverage ratio away from the optimal target ratio. Our analysis demonstrates that this behavior can be in full accordance with a dynamic trade-off paradigm once new debt issues and rollover decisions are considered.

In our setting, debt maturity significantly influences the expected probability of bankruptcy since short maturities lead to more rapid debt reductions when the firm's profitability starts to decrease. Investors take this into account when they price the debt initially. This implies that firms' debt capacity generally *increases* as they choose shorter debt maturities. This result is in contrast to the earlier literature (e.g., Leland 1994b; 1998; Leland and Toft 1996), which predicts that short-term debt leads to early and inefficient default and therefore reduces debt capacity, as measured by the firm's initial target leverage.[5]

Our analysis therefore generates a novel theory of optimal debt maturity where, for reasonable parameter values, total firm value is maximized at an interior debt maturity. Firms hereby trade off an increased flexibility in future leverage reductions induced by short-term debt against the additional transaction costs incurred when refinancing expiring debt.

We show that firm value has two local maxima when plotted against its debt maturity. One local maximum is obtained when debt maturity goes to infinity. This is the case since firms with debt maturities beyond a critical threshold

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5 This result can be understood as follows. In bad states of the world, issuing a bond with the same face value to refinance expiring debt leads to a funding gap, since the new bond must be sold below face value. This funding gap increases with the fraction of debt that must be rolled over. As a result, firms with short maturities default sooner, that is, at more profitable states, since equityholders would have to cover this wider funding gap. In contrast to these papers, we do not force firms to always keep constant the face value of debt, and, thus, we capture the effect of short debt maturity on future leverage reductions. This reverses the relation between debt capacity and debt maturity.

never engage in leverage reductions when rolling over debt. Increasing maturity beyond this critical threshold therefore no longer has any effect on leverage dynamics, but leads to a reduction in transaction costs since a smaller fraction of debt must be refinanced at any given period of time. Therefore, total firm value is locally maximized for infinite-maturity debt. However, when shortening debt maturity below the critical threshold, firms start to engage in debt reductions when their profitability decreases, thereby reducing the probability of financial distress. Over this maturity range, shortening maturity increases debt capacity and total firm value starts to rise, until the increase in transaction costs associated with refinancing maturing debt outweigh the increased benefits due to faster debt reductions along unfavorable cash flow paths. Thus, total firm value exhibits another local maximum at an interior value of debt maturity. The exact location of this maximum depends on the parameters of the firm's cash flow process, such as its growth rate and its volatility, as well as on the transaction costs associated with rolling over debt and the magnitude of bankruptcy costs. For empirically reasonable model parameterizations we find that firm value is indeed maximized for interior debt maturities. Infinite-maturity debt maximizes firm value globally only if the costs of financial distress and/or the tax advantage of debt are very low and/or transaction costs associated with rolling over debt are very high. In this case the benefit from increasing debt capacity and reducing the bankruptcy probability by committing to future leverage reductions via short-term debt is too low compared to the additional transactions.

Existing evidence as well as our own empirical results accord well with our theoretical predictions. When we double-sort firms by market leverage and the fraction of short-term debt into quintiles, we find that more highly levered firms subsequently delever more when a larger fraction of their debt is short term. We also find evidence that the incentive to reduce leverage is nonmonotonic, as predicted by the model. Furthermore, our regression results imply that firms with high cash flow volatilities and low bankruptcy costs prefer debt with shorter maturities, in line with our model results.

Leland (1994b), Leland and Toft (1996), and Leland (1998) were the first to analyze debt maturity in a dynamic trade-off setting. Titman and Tsyplakov (2007) extended this literature by endogenizing investment decisions. Their contributions have led to the development of important modeling approaches allowing for the analysis of debt maturity in a tractable continuous-time framework. These papers also provide insights into the interplay between leverage and debt maturity. However, they cannot explain interior optimal debt maturities since in all these models it would be optimal to issue perpetual debt.

We extend the above literature along one crucial aspect: we allow firms to choose how they refinance expiring debt, whereas firms in the papers discussed above *must* always issue new bonds with a face value that equals the face value of the expiring debt. Thus, the total face value of debt remains constant. This paper differs from this literature, since it focuses on the role debt maturity to mitigate conflicts of interest between debtholders and equityholders that arise in the context of subsequent capital structure adjustments and find that this may generate an interior optimal debt maturity.[6]

The growing interest in the interaction between debt maturity and leverage dynamics by corporate theorists has recently inspired several papers related to our work. Benzoni et al. (2020) focus on the main result of DeMarzo and He (forthcoming), namely, that in the absence of any commitment or bond covenants, agency problems imply that the present value of future tax benefits of debt are perfectly offset by the present value of expected future bankruptcy costs. They show that this result may not be robust once one allows for fixed costs of debt issuance. Such costs lead to a more conservative debt strategy, which in fact reestablishes a net benefit of debt, even without commitment. Benzoni et al. (2020) conclude that this result holds even in the limit, as fixed issuance costs go to zero. Our model allows for debt covenants, proportional costs of debt rollover, and fixed costs of discrete debt restructurings. This setting motivates a net benefit of leverage and generates insights into the trade-offs between debt maturity, rollover costs, and the resultant leverage dynamics and into the ways firm characteristics affect all three.

Geelen (2016) studies a firm that issues noncallable debt with a single finite maturity. After having repaid the maturing debt, firms implement a new, total firm value-maximizing capital structure. This setting also captures some of the benefits of short-term debt that we identify, since firms issue a lower amount of debt after having repaid outstanding debt in a bad state of the world. As in our model, the trade-off between the flexibility of finite maturity debt, which reduces bankruptcy costs, on the one hand, and issuance costs on the other hand motivates an optimal debt maturity. However, in Geelen (2016), leverage adjustments cannot take place prior to the lumpy debt maturity, and the model cannot account for the agency conflicts that lead to, what we refer to as underinvestment in leverage reduction, which arises in the presence of multiple debt maturity issues. As we will discuss in the conclusion, extending our setting, where maturity structure is perfectly granular, to allow for some lumpiness of maturities, while still allowing for heterogeneous debt maturities, is an interesting avenue for future research.

In a somewhat different vein, Chen, Xu, and Yang (2020) study debt maturity over the business cycle and model optimal maturity choice as a trade-off between higher liquidity discounts associated with long-term debt and higher

[6] Childs, Mauer, and Ott (2005) and Ju et al. (2005) also explore debt maturity. However, in these models, firms only have a single bond outstanding, and they can only change their debt levels after the entire existing debt has matured. In our model, firms are allowed to change the debt level at any point in time. As a result, we are able to isolate the commitment effect of debt maturity on equityholders' willingness to adjust debt levels downward after a decrease in profitability. Furthermore, firms in our model have many bonds with different maturities outstanding, as is frequently the case in practice. At any point in time, firms retire only a fraction of all outstanding bonds. Therefore, when some bonds mature and are refinanced with new debt or via equity, this influences the value of the remaining bonds outstanding.

rollover costs associated with short-term debt. As in Geelen (2016), debt maturity is lumpy, that is, perfectly correlated across all units of outstanding debt. The authors are able to link maturity choice to firms' systematic risk and to the business cycle. Our model does not generate predictions for the interaction between systematic risk and maturity choice but reveals that the disadvantage of long-term debt arises endogenously from agency conflicts.

Our paper is also related to debt maturity theories, which are driven by informational asymmetries. As demonstrated by the seminal work of Diamond (1991, 1993) and Flannery (1986, 1994), short-term debt maturities signal positive inside information. Other authors, such as Calomiris and Kahn (1991) and Diamond and Rajan (2001), have emphasized the disciplinary role of short-term debt. Debt maturity also has been linked to the debt overhang or underinvestment problem. While the original work by Myers (1977) concludes that short-term debt mitigates these problems, Diamond and He (2014) show that maturing short-term debt can lead to more severe debt overhang than nonmaturing long-term debt. Different from these papers, we analyze a debt overhang problem that is not related to the asset side of the corporate balance sheet, but instead to the liability side. Long-term debt in our context leads equityholders to underinvest in leverage reductions.[7] Gomes, Jermann, and Schmid (2016) emphasize the role of long-term nominal corporate debt in the presence of unexpectedly low inflation and the resultant debt overhang problem in a general equilibrium model. They identify corporate long-term debt as an important channel that transmits inflationary shocks into the real economy.

An interesting, related literature tackles the interaction between debt maturity, rollover risk, and capital structure. Examples include He and Xiong (2012a,b), He and Milbradt (2014), Cheng and Milbradt (2012), and Chen et al. (2017). In a similar vein, He and Xiong (2012a) and Acharya, Gale, and Yorulmazer (2011) analyze debt maturity when short-term debt can lead to early and inefficient asset liquidation. Some papers, such as Brunnermeier and Oehmke (2013) and He and Milbradt (2016), analyze debt maturity adjustments over time, but in these papers the initial maturity is exogenous.

Finally, our paper is also related to an emerging literature that analyzes rollover risk and the volatility of credit spreads and the optimal dispersion of debt maturities (see Choi, Hackbarth, and Zechner 2018; Chaderina 2018). While credit spreads at future rollover dates are also stochastic in our model and therefore affect optimal maturity choices, we do not explicitly model the dispersion of debt maturities.[8] None of the contributions discussed above shares the focus of our paper, namely, the effect of debt maturity on equityholders' future incentives to delever.

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[7] Jungherr and Schott (2020a) analyze the effects of Myers' underinvestment problem and the incentive to dilute long-term debt via additional pari passu issues on maturity in a dynamic framework.

[8] In fact, the maturity dispersion in our model is characterized by a constant proportion of bonds expiring in each instant of time.

1. The Model

We consider a firm with unlevered instantaneous free cash flow after corporate tax, c_t, following a geometric Brownian motion

$$\begin{aligned} \frac{dc_t}{c_t} &= \mu dt + \sigma dW_t, \\ c_0 &= c(0), \end{aligned} \tag{1}$$

where dW_t is the increment to a standard Wiener process. Table 1 summarizes the notation used throughout this paper.

Let B_t denote the firm's face value of debt outstanding at time t, with fixed coupon rate i. Coupon payments are deductible from the corporate tax base. In the spirit of Leland (1994b), Leland (1998), and Ericsson (2000), a constant fraction m of the outstanding debt matures at any instant of time. Ignoring default and debt repurchase, the average maturity of a debt contract is then $1/m$ years.

Retired debt may be replaced with a new debt issue, but the face value of the new bond may not exceed the face value of the retired debt, mB_t. Thus, the bond indenture ensures that the rate, δ_t, at which the firm issues new debt satisfies $\delta_t \leq m$. The new debt issue generates proportional transaction costs, k_i, has the same priority as existing debt and is amortized at the same constant rate m.

If the firm wishes to increase its face value of debt, it must first remove the existing debt covenants by repurchasing all outstanding debt. The subsequent new bond issue is also associated with proportional transaction costs, denoted by k_r. The coupon rate of the new issue is set so that the bond can be sold at par.

Table 1
Notation

A firm's instantaneous free cash flow after corporate tax	c_t
Expected rate of change of c_t	μ
Risk-adjusted drift of the cash flow process	$\hat{\mu}$
Riskless rate of interest	r
Instantaneous variance of the cash flow process	$c_t^2\sigma^2$
Face value of debt	B_t
Debt retirement rate	m
Average debt maturity	$T=1/m$
Debt rollover rate	δ
Value of equity	E
Value of debt	D
Total value of the firm	V
Instantaneous coupon rate	i
Firm's inverse leverage ratio	y_t
Personal tax rate on ordinary income	τ_p
Corporate tax rate	τ_c
Proportional bankruptcy costs	g
Proportional transaction costs for rolling over debt	k_i
Proportional transaction costs for issuing debt after Recapitalization	k_r
Proportional call premium	λ

The important difference to existing models with finite average maturity is that the firm is not forced to rollover the entire amount of maturing debt. While firms in Leland (1994b), Leland (1998), and Ericsson (2000) must always set the rollover rate $\delta_t = m$, we allow the firm to choose δ optimally. In some states of the world, the firm may find it optimal to replace only part of the retired debt with new debt or it might entirely abstain from issuing new debt contracts. If the firm does not fully replace retired debt, then the face value of debt outstanding is reduced at a rate $m - \delta_t$, which, in turn, may help the firm avoid future financial distress.

If the firm's equityholders stop coupon and principal repayments and thereby trigger bankruptcy, all control rights over the firm's productive assets are transferred to debtholders who will then optimally re-lever the firm. As in Fischer, Heinkel, and Zechner (1989), this transfer of control rights is associated with bankruptcy costs, assumed to be a fraction g of the outstanding face value of the firm's debt.[9]

Our model captures two central features of many countries' tax systems. First, we assume that interest payments are deductible from the corporate tax base while dividends are not.[10] Second, we allow for interest income to be taxed more heavily at the personal level than equity income. To capture this feature in a parsimonious way, we assume that equity income is not taxed at the personal level, whereas debt income is taxed at rate τ_p. Therefore the appropriate discount rate for expected after-corporate-tax income for equityholders under the risk-neutral probability measure is given by $r(1-\tau_p)$, see Section 2. For a discussion of the calibration of the tax parameters and how they relate to the current U.S. tax code, we refer to Section 3.

Since the values of equity and debt will be shown to be linear homogeneous in the face value of debt, we can redefine the state variable as the inverse leverage ratio with respect to the unlevered firm value, y_t:

$$y_t = \frac{1}{B_t} \frac{c_t}{r(1-\tau_p) - \hat{\mu}}, \tag{2}$$

where τ_p is the personal income tax rate on debt and $\hat{\mu}$ is the risk-neutral drift rate of the cash flow c_t.

Since firms can, at any point in time, either increase debt by a discrete amount from B_t to B_t^* or rollover maturing debt at a rate of $\delta_t \leq m$, the dynamics of the

9 Alternatively, we could assume that bankruptcy costs are a proportion of the unlevered assets in bankruptcy. This alternative assumption does not qualitatively change our results.

10 We assume instant tax refunds for coupon payments and do not explicitly model any loss carryback and loss carryforward due to limited corporate taxable income.

face value of debt are given by

$$\frac{dB_t}{B_t} = \begin{cases} \dfrac{B_t^*}{B_t} - 1 & : \text{debt is increased from } B_t \text{ to } B_t^* \\ & \quad \text{at time } t, \\ -(m-\delta_t)dt & : \text{firm replaces maturing debt at} \\ & \quad \text{a rate } \delta_t \in [0,m] \text{ at time } t, \end{cases} \tag{3}$$
$$B_0 = B(0).$$

As a result, the risk-neutral dynamics of y_t are

$$\frac{dy_t}{y_t} = \begin{cases} \dfrac{B_t}{B_t^*} - 1 & : \text{debt is increased from} \\ & \quad B_t \text{ to } B_t^* \text{ at time } t, \\ (\hat{\mu}+(m-\delta_t))dt + \sigma\, dW_t & : \text{maturing debt is replaced} \\ & \quad \text{at a rate } \delta_t \text{ at time } t \end{cases}$$

$$y_0 = y(c_0, B_0) = \frac{1}{B_0} \frac{c_0}{r(1-\tau_p)-\hat{\mu}}. \tag{4}$$

(For a derivation of the dynamics, see Appendix A.1.)

Thus, a discrete adjustment of the debt level following a debt repurchase leads to an immediate jump in the inverse leverage ratio. Alternatively, if the face value of debt is maintained at a constant level (i.e., $\delta_t = m$), then the inverse leverage ratio follows a geometric Brownian motion with the same drift rate and volatility as the original cash flow process for c_t. When only part of the maturing debt is rolled over ($\delta_t < m$), then the drift rate of the inverse leverage ratio is $\hat{\mu}+(m-\delta_t) > \hat{\mu}$, that is, because of the shrinking debt level, the firm's leverage ratio tends to fall, and, thus, the inverse leverage ratio, y, tends to rise.

2. Claim Valuation and Optimal Capital Structure Strategies

In this section we derive the valuation equations for the firm's debt and equity, given the state variables B and y, as well as propositions for the optimal refinancing mix for maturing debt. Consider first the value of equity, E_t. Equityholders have a claim on the firm's after corporate tax cash flow, c_t, reduced by the required payments to debtholders, that is, after-tax coupon payments, $(1-\tau_c)iB_t$ plus principal repayment, mB_t. The cash flow to equityholders also includes the proceeds from (partially) rolling over maturing debt, $(1-k_i)\delta_t D_t$, where k_i represents the proportional transaction costs associated with the new debt issue, δ_t is the rate at which debt is rolled over and D_t denotes its market value. When the resultant dividends to equityholders become negative, they can be interpreted as seasoned equity issues.[11] Equityholders default when it is optimal for them to do so by stopping

[11] We do not consider equity issuance costs since this would considerably complicate the analysis but would not qualitatively alter the main drivers of our results. Short-term debt would still reduce agency conflicts in bad states, but the net benefits are likely to be smaller. This is consistent with Chen, Xu, and Yang (2020), who model bondholders' liquidity preferences over the business cycle and find that in their model higher equity issuance costs lead to less short-term debt and higher default probabilities.

coupon and principal payments. At the resultant random default time, T_D, the equity claim becomes worthless.

When the firm decides to increase the face value of debt at time T_K, it must repurchase its outstanding debt at a price $(1+\lambda)B_{T_K}$. Following such a call, equityholders hold a claim to the value of the firm, V_K, endogenously determined later, which anticipates optimal re-levering. For a given contingent strategy on rollover, recapitalization, and bankruptcy, the value of the firm's equity is

$$\begin{aligned} E_t &= \mathbb{E}_t^Q \int_t^{T_D \wedge T_K} \Big(\underbrace{c_s(B_s, y_s)}_{\substack{\text{after tax}\\ \text{cash flow}}} - \underbrace{B_s(i(1-\tau_c)+m)}_{\text{coupons and amortization}} \\ &\qquad + \underbrace{(1-k_i)\delta_s D_t(B_s, y_s)}_{\substack{\text{proceeds from}\\ \text{reissuing debt}}} \Big) e^{-r(1-\tau_p)(s-t)} \mathrm{d}s \\ &\quad + \mathbb{E}_t^Q \Big(\mathbf{1}_{\{T_K \le T_D\}} \Big[\underbrace{V_K - (1+\lambda)B_{T_K}}_{\substack{\text{optimally restructured firm}\\ \text{after debt repurchase}}} \Big] e^{-r(1-\tau_p)(T_K-t)} \Big), \end{aligned} \tag{5}$$

where $\mathbf{1}_{\{.\}}$ is the indicator function, which equals one if the expression in brackets is true and zero otherwise.

Next, we derive the valuation equation for the debt claim. Debtholders receive an after-tax coupon flow of $i(1-\tau_p)B_t$ and a flow of principal repayment mB_t. Depending on the firm's choice of the rollover rate δ_t, debtholders buy the new issues at market value D_t, resulting in a cash flow of $-\delta_t D_t$ as well as a change of the firm's debt level at a rate $-(m-\delta_t)$. Debtholders receive this cash flow until either the firm defaults at random time T_D or the entire debt is called at a random time, T_K. At default, debtholders receive the value D_D, equal to the value of the firm's re-levered productive assets net of bankruptcy costs, which will be determined later. When debt is called, debtholders receive $(1+\lambda)B_{T_K}$.

Anticipating the firm's future decisions about rollovers, recapitalization, and bankruptcy, debtholders price the firm's debt as the expected present value of the total after-tax cash flow under the risk-neutral measure

$$\begin{aligned} D_t &= \mathbb{E}_t^Q \int_t^{T_D \wedge T_K} \Big(\underbrace{B_s(i(1-\tau_p)+m)}_{\substack{\text{coupons and}\\ \text{amortization}}} - \underbrace{\delta_s D_t(B_s, y_s)}_{\substack{\text{purchase of}\\ \text{reissued debt}}} \Big) e^{-r(1-\tau_p)(s-t)} \mathrm{d}s \\ &\quad + \mathbb{E}_t^Q \Big(\mathbf{1}_{\{T_D < T_K\}} \underbrace{D_D}_{\text{vaule in default}} e^{-r(1-\tau_p)(T_D-t)} \\ &\qquad + \mathbf{1}_{\{T_K \le T_D\}} \underbrace{(1+\lambda)B_{T_K}}_{\substack{\text{debt repurchase prior to}\\ \text{discrete leverage increase}}} e^{-r(1-\tau_p)(T_K-t)} \Big). \end{aligned} \tag{6}$$

2.1 Equityholders' Optimal Capital Structure Strategy

In this subsection we derive a Markov perfect Nash equilibrium capital structure strategy where investors price debt based on rational beliefs about the firm's debt rollover, default and recapitalization decisions and, given these prices, firms have no incentive to deviate from the conjectured capital structure strategy. There is no precommitment, except that firms must repurchase the existing debt (i.e., eliminate a debt covenant) before they can issue new debt with a higher face value.

Anticipating that debt and equity values are homogenous of degree one in the face value of debt, B, we derive time-invariant dynamic equilibrium capital structure strategies that only depend on the firm's inverse leverage ratio, y. Specifically, equityholders' capital structure strategy, S, consists of the initial leverage, $\acute{y}$ and debt maturity, m, a time-invariant rollover schedule $\delta(y)$, which determines the refinancing of expiring debt as a function of leverage as well as two time-invariant leverage thresholds, $\underline{y}$ and $\overline{y}$. As mentioned above, the specific choices of $\delta(y)$, $\underline{y}$ and $\overline{y}$ must be time consistent (no precommitment). S therefore can be written as

$$\begin{aligned} S &= \{\delta(y), \underline{y}, \overline{y} \,|\, \acute{y}, m\}, \qquad (7)\\ \text{with} \quad & 0 \leq \underline{y} \leq \acute{y} \leq \overline{y},\\ & 0 \leq \delta \leq m. \end{aligned}$$

Default is triggered at random time T_D at which y first hits the lower threshold $\underline{y}$. In this case debtholders become the owners of the firm's productive assets, which they can re-lever optimally. If y first hits the upper threshold, $\overline{y}$, at random time T_K, then all debt is repurchased and the firm is subsequently re-levered optimally.[12] Thus, both at the default boundary and at the recapitalization boundary the firm becomes unlevered and its owners will therefore find it optimal to choose the initial leverage $\acute{y}$ and maturity m.

Next, we present the values of equity and debt for a given strategy of type (7). We hereby use the fact that the value of equity and debt are functions of the state variables B and y, and that valuations are homogeneous in B. Therefore, we write $E(B,y) = B\tilde{E}(y)$ and $D(B,y) = B\tilde{D}(y)$.

Proposition 1. For a given capital structure strategy (7), the value of equity and debt per unit of face value of debt, $\tilde{E}(y)$ and $\tilde{D}(y)$, must satisfy the following

[12] Equityholders' recapitalization and default strategies can be interpreted as indicator functions $I_{y \geq \overline{y}}$ and $I_{y \leq \underline{y}}$, which map the state variable y onto zero or one.

valuation equations (Hamilton-Jacobi-Bellman)

$$\big(r(1-\tau_p)+[m-\delta(y)]\big)\tilde{E}(y) = \frac{1}{2}\sigma^2 y^2 \frac{\partial^2 \tilde{E}(y)}{\partial y^2} + (\hat{\mu}+[m-\delta(y)])y\frac{\partial \tilde{E}(y)}{\partial y} + (1-k_i)\delta(y)\tilde{D}(y) - (i(1-\tau_c)+m) + (r(1-\tau_p)-\hat{\mu})y \quad \text{(equity valuation)} \tag{8}$$

$$\text{subject to} \quad \tilde{E}(\underline{y}) = 0 \quad \text{(equity at default)} \tag{9}$$

$$\tilde{E}(\overline{y}) = \Big[\tilde{E}(\acute{y})+\tilde{D}(\acute{y})-k_r\Big]\frac{\overline{y}}{\acute{y}} - (1+\lambda) \quad \text{(equity at recap.)} \tag{10}$$

$$(r(1-\tau_p)+m)\tilde{D}(y) = \frac{1}{2}\sigma^2 y^2 \frac{\partial^2 \tilde{D}}{\partial y^2} + (\hat{\mu}+(m-\delta_t))y\frac{\partial \tilde{D}}{\partial y} + (i(1-\tau_p)+m) \quad \text{(debt valuation)} \tag{11}$$

$$\text{subject to} \quad \tilde{D}(\underline{y}) = \max\Big\{\big[\tilde{E}(\acute{y})+\tilde{D}(\acute{y})-k_r\big]\underline{y}\frac{1}{\acute{y}} - g, 0\Big\} \quad \text{(debt at default)} \tag{12}$$

$$\tilde{D}(\overline{y}) = 1+\lambda \quad \text{(debt at recapitalization)} \tag{13}$$

Proof: See Appendix A.2.

Before determining the optimal equilibrium rollover rate δ, we discuss the system of equations stated in Proposition 1. Valuation equations for equity and debt, (8) and (11), are standard HJB equations, with the exception that the rollover rate $\delta(y)$ determines the dynamics of the state y cash flows to investors. They have analytically tractable solutions for all possible ranges of rollover rates, that is, $0 \leq \delta(y) \leq m$.

Equation (9) reflects the absolute priority rule that makes equity worthless in default, occurring at $\underline{y}$. At the upper restructuring threshold $\overline{y}$, all debt is repurchased at a premium over face value equal to $(1+\lambda)$ and the firm recapitalizes to the initial inverse leverage ratio of $\acute{y}$. The value of equity at $\overline{y}$ is therefore endogenously determined by the value of the total firm minus costs associated with the recapitalization, stated in Equation (10). Boundary condition (12) defines the value of debt in default as the value of the newly re-levered firm (at inverse leverage ratio $\acute{y}$) minus bankruptcy costs, g. If this does not result in a positive value, the firm is liquidated, and debtholders get zero. Finally, condition (13) states that at the upper restructuring threshold, debtholders get paid the face value plus a call premium λ.

In the setup explained above, the valuation of corporate debt depends on the capital structure strategy, conjectured by bond investors. At the same time, the firm's (i.e., equityholders') capital structure decisions depend on the valuation of the debt claims. This is so since equity value and debt value are interrelated, as can be seen from Equations (5) and (6). In particular, the equity value in Equation (5) depends on debt valuation via the proceeds from debt issuance, $(1-k_i)\delta_s D(B_s, y_s)$.[13] Similarly, debt value in Equation (6) depends on equity valuation via the equityholders' rollover decision, δ, and via the value given default, D_D, which constitutes the total value of the optimally restructured firm.

Consequently, the optimal capital structure strategy and securities valuation must satisfy equilibrium conditions. That is, investors price securities based on a conjectured capital structure strategy, and firms must have no incentive to deviate from the conjectured strategy if securities are priced in this way. Employing the theory of stochastic control, we therefore model a dynamic game and derive a Markov-perfect Nash equilibrium in which debtholders price debt based on rational beliefs about equityholders' capital structure strategy and equityholders correctly anticipate the pricing of debt claims when making capital structure decisions.[14]

Appendixes A.3 and A.4 formally derive the optimal equilibrium strategy. The key feature of the equilibrium strategy is that the optimal rollover depends on the value of debt relative to the value of equity (and its first derivative), which can be expressed as the critical threshold, $\tilde{D}^I(y)$, given in the following proposition.

Proposition 2. In any state y, the critical threshold $\tilde{D}(y)$

$$\tilde{D}^I(y)=\frac{1}{1-k_i}\left(y\frac{\partial \tilde{E}}{\partial y}(y)-\tilde{E}(y)\right), \tag{14}$$

determines equityholders' optimal rollover decision such that

$$\delta^*(y)=\begin{cases} m, & \text{if} \quad \tilde{D}(y)>\tilde{D}^I(y), \\ 0, & \text{if} \quad \tilde{D}(y)<\tilde{D}^I(y). \end{cases} \tag{15}$$

Equityholders are indifferent between all feasible rollover rates, $\delta(y)$, if $\tilde{D}(y)=\tilde{D}^I(y)$. Proof: See Appendix A.3.

According to Proposition 2, equityholders wish to fully rollover expiring debt and therefore set $\delta^*(y)=m$ if debt value is above the critical threshold $\tilde{D}^I(y)$. For debt values below this threshold, equityholders do not rollover debt at all by

[13] In addition, the value V_K, which represents the value of the optimally re-levered firm after restructuring debt, depends on the market value of debt at the optimal initial leverage ratio.

[14] See, for example, the chapter on optimal stochastic control in Björk (2004) or the chapter on stochastic differential games in Dockner et al. (2000).

setting $\delta^*(y)=0$. Finally, when debt value matches the threshold value $\tilde{D}^I(y)$, equityholders are indifferent between alternative values of $\delta(y)$.

The economic intuition behind this optimal decision rule can be seen by considering equityholders' costs and benefits from issuing an additional unit of debt. Issuing an additional unit of debt generates net proceeds of $(1-k_i)\tilde{D}(y)$. On the other hand, the additional debt issue increases the firm's leverage (decreases y) and, thus, reduces the valuation of equity. This valuation effect is given by $\frac{dE}{dB}=\tilde{E}(y)-y\frac{\partial \tilde{E}(y)}{\partial (y)}$. Equating these costs and benefits and solving for the critical debt value yields $\tilde{D}^I(y)$ in Equation (14).

One might suspect that the optimal debt rollover is of a bang-bang type, that is, that δ^* is either at its minimum, $\delta^*=0$, or at its maximum, $\delta^*=0$ and that the states y at which debt value exactly equals $\tilde{D}^I(y)$ serve as singular switching points where the optimal debt strategy jumps from $\delta^*=0$ to $\delta^*=m$ or vice versa. However, we will demonstrate below that, for sufficiently short debt maturities, there exist extended regions of firm leverage, where an interior choice $0<\delta^*<m$ is the equilibrium. In this region, the optimal rollover rate $\delta^*(y)$ is set such that the resultant debt value exactly matches the critical threshold (14), which in turn makes equityholders indifferent between alternative rollover rates.

Interior optima arise in state y, where neither $\delta=0$ nor $\delta=m$ are feasible equilibrium solutions. This is the case if the choice $\delta=0$ results in a debt valuation $\tilde{D}(y)>\tilde{D}^I(y)$, which according to the decision rule (15) implies an optimal choice of $\delta=m$ or, alternatively, if full rollover, that is, $\delta=m$, implies a debt value $\tilde{D}(y)$ below the critical threshold $\tilde{D}^I(y)$, and therefore a rollover choice $\delta=0$.

The following proposition summarizes valuation equations of equity and debt in the interior equilibrium as well as the equilibrium rollover rate together with conditions that ensure the stability of the equilibrium, that is, conditions that ensure that a small perturbation of the conjectured rollover rate does not destroy the equilibrium.

Proposition 3. In an interior equilibrium $0<\delta^*<m$ the value of equity and debt as well as the optimal rollover rate must satisfy the systems of equations

$$
\begin{aligned}
[r(1-\tau_p)+m]\tilde{E}(y) &= \frac{1}{2}\sigma^2 y^2 \frac{\partial^2 \tilde{E}(y)}{\partial y^2}+(\hat{\mu}+m)y\frac{\partial \tilde{E}(y)}{\partial y} \\
&\quad -(i(1-\tau_c)+m)+(r(1-\tau_p)-\hat{\mu})y, \qquad (16)
\end{aligned}
$$

$$
\begin{aligned}
\tilde{D}(y) &= \tilde{D}^I(y), \\
0<\delta^* &= \frac{1}{y\frac{\partial \tilde{D}}{\partial y}}\left[\frac{1}{2}\sigma^2 y^2\frac{\partial^2 \tilde{D}}{\partial y^2}+(\hat{\mu}+m)y\frac{\partial \tilde{D}}{\partial y}\right. \qquad (17) \\
&\quad \left. +(i(1-\tau_p)+m)-(r(1-\tau_p)+m)\tilde{D}\right]<m.
\end{aligned}
$$

In particular, equity valuation (16) is independent of the rollover rate, while δ^* is strictly positive in this region. Furthermore, the existence of an interior equilibrium implies that equity is convex in y and that the value of debt per unit of face value increases with decreasing leverage (increasing y),

$$\frac{\partial^2 \tilde{E}}{\partial y^2}(y) > 0,$$

$$\frac{\partial \tilde{D}}{\partial y}(y) > 0.$$

Proof: See Appendix A.4.

We can now derive equity and debt valuation expressions for the three strategy regions. In the first one, $\delta(y)=m$, that is, equityholders fully rollover expiring debt, since proceeds from issuing new debt at market value are sufficiently high to make the choice of the maximum rollover rate that the covenant allows optimal. In the second one $\delta(y)=0$, proceeds from reissuing expiring debt at market value excessively dilutes the firm's equity and, thus, equityholders do not wish to issue any new debt. Finally, in the region with interior rollover we know from Proposition 2 that equityholders are indifferent between all feasible choices of $\delta(y)$. Substituting $\tilde{D}(y)=\tilde{D}(y)^I$ into the valuation equation (8) lets δ vanish from the valuation equation, that is, the value of equity is independent of the choice of δ. The following proposition derives the resultant valuation expressions.

Proposition 4. In a region at which the firm fully rolls over its debt, that is, $\delta=m$, the value of equity and debt are given by

$$\begin{aligned}\tilde{E}(y) &= E_1 y^{\beta_{m1}} + E_2 y^{\beta_{m2}} - \frac{i(1-\tau_c)+m}{r(1-\tau_p)} \\ &\quad + m(1-k_i)\left[\frac{1}{r(1-\tau_p)}\frac{i(1-\tau_p)+m}{(r(1-\tau_p)+m)}\right. \\ &\qquad + \frac{D_1 y^{\gamma_1}}{r(1-\tau_p)-\hat{\mu}\gamma_1-\frac{1}{2}\sigma^2\gamma_1(\gamma_1-1)} \\ &\qquad \left. + \frac{D_2 y^{\gamma_2}}{r(1-\tau_p)-\hat{\mu}\gamma_2-\frac{1}{2}\sigma^2\gamma_2(\gamma_2-1)}\right] + y, \\ \tilde{D}(y) &= D_1 y^{\gamma_1} + D_2 y^{\gamma_2} + \frac{i(1-\tau_p)+m}{r(1-\tau_p)+m}\end{aligned}$$

In a region at which the firm rolls over its debt at an interior optimum δ^*, the value of equity and debt are given by

$$\tilde{E}(y) = E_1 y^{\beta_{01}} + E_2 y^{\beta_{02}} - \frac{i(1-\tau_c)+m}{r(1-\tau_p)+m} + y,$$

$$\tilde{D}(y) = \tilde{D}^I(y).$$

In the region in which the firm funds repayment of retiring debt entirely with equity, that is, where $\delta = 0$, the values of equity and debt are given by

$$\tilde{E}(y) = E_1 y^{\beta_{01}} + E_2 y^{\beta_{02}} - \frac{i(1-\tau_c)+m}{r(1-\tau_p)+m} + y,$$

$$\tilde{D}(y) = D_1 y^{\beta_{01}} + D_2 y^{\beta_{02}} + \frac{i(1-\tau_p)+m}{r(1-\tau_p)+m}.$$

The exponents β and γ are the characteristic roots of the homogeneous differential equations given by

$$\beta_{m1,m2} = \frac{1}{2} - \frac{\hat{\mu}}{\sigma^2} \pm \sqrt{(\frac{1}{2} - \frac{\hat{\mu}}{\sigma^2})^2 + \frac{2(r(1-\tau_p))}{\sigma^2}},$$

$$\beta_{01,02} = \frac{1}{2} - \frac{\hat{\mu}+m}{\sigma^2} \pm \sqrt{(\frac{1}{2} - \frac{\hat{\mu}+m}{\sigma^2})^2 + \frac{2(r(1-\tau_p)+m)}{\sigma^2}},$$

$$\gamma_{1,2} = \frac{1}{2} - \frac{\hat{\mu}}{\sigma^2} \pm \sqrt{(\frac{1}{2} - \frac{\hat{\mu}}{\sigma^2})^2 + \frac{2(r(1-\tau_p)+m)}{\sigma^2}}.$$

The constants $E_{1,2}$ and $D_{1,2}$ can be determined for each region by the proper boundary conditions that ensure that value functions are continuous across changes in the rollover regime.

(See Appendix A.5 for the proof of Proposition 4.)

2.2 Endogenous Bankruptcy and Optimal Discrete Recapitalization

While the choice of the rollover threshold $\delta(y)$ constitutes instantaneous stochastic control, the selection of the recapitalization threshold $\overline{y}$ as well as the choice of the bankruptcy threshold $\underline{y}$ are optimal stopping problems.[15] Proper boundary conditions (9), (10), (12), and (13) determine equity and debt values at these critical thresholds.

First-order conditions of optimality at the upper and the lower reorganization thresholds follow from inspecting boundary conditions (10) and (12) with respect to an equity value-maximizing choice of the reorganization thresholds

[15] Optimal instantaneous rollover, $\delta(y)$, and discrete reorganization at $\underline{y}$, $\overline{y}$ are optimized simultaneously.

$\underline{y}$ and $\overline{y}$. This leads to the following "smooth pasting" or "super contact" conditions (for a discussion of these optimality conditions, see Dixit 1993; Dumas 1991),

$$\frac{\partial \tilde{E}}{\partial y}(y)|_{y=\underline{y}} = 0, \tag{18}$$

$$\frac{\partial \tilde{E}}{\partial y}(y)|_{y=\overline{y}} = \frac{1}{\acute{y}}\left[\tilde{E}(\acute{y})+(1-k_r)\right]. \tag{19}$$

These conditions together with optimal rollover determine equityholders' optimal capital structure strategy contingent on all-equity firm owners' initial choice of leverage and debt maturity, $S=\{\delta(y),\underline{y},\overline{y}\,|\,\acute{y},m\}$. As discussed above, discrete refinancing at $\underline{y}$ or $\overline{y}$ creates an unlevered firm, which all-equity owners will re-lever immediately.

A discussion of optimal debt rollover, δ, near bankruptcy closes this section. In Proposition 5 below, we will prove that in any state y, which is sufficiently close to the bankruptcy state $\underline{y}$, firms find it optimal to fully rollover expiring debt, that is, to set δ equal to m. In other words, when cash flows are sufficiently low, setting $\delta<m$ and thus reducing leverage would transfer wealth from equityholders to remaining debtholders. To see this, note that setting $\delta<m$ reduces the overall face value of debt outstanding and therefore increases the value of the remaining bonds in bankruptcy. For sufficiently bad states, this value transfer to remaining bondholders dominates any potential benefits to equityholders, since bankruptcy occurs in any case with very high probability.[16] Thus, in bad states the firm faces a classical debt overhang problem where equityholders do not engage in debt reducing activities. Instead, they wish to minimize their investment in debt reduction near the bankruptcy boundary, by choosing the highest feasible debt rollover rate. This is formally stated in the following proposition:

Proposition 5. If loss-given-default is strictly less than 100%, it is optimal to rollover debt at the maximum rate $\delta=m$ for y in a neighborhood above the bankruptcy threshold $\underline{y}$.

(See Appendix A.6 for the proof of Proposition 5.)

2.3 Optimal Initial Leverage

All-equity firms choose target leverage $\acute{y}$ and average debt maturity m to maximize total firm value, fully anticipating the effects on equityholders'

[16] This can be seen most intuitively if one considers the effects of a debt reduction for given default decisions. Remaining bondholders benefit from the debt reduction since they receive a higher value per unit of face value in default. Equityholders do not enjoy a similar benefit, since they receive zero in bankruptcy, independent of the shortfall of firm value in bankruptcy relative to the total face value of debt.

optimal capital structure strategy S_E. We assume that i is set such that debt is initially issued at par,[17]

$$\text{choose } i \text{ such that} \qquad D(\acute{y}, B) = B. \tag{20}$$

The maximization problem of all-equity owners is therefore given by

$$\max_{\acute{y},m}(V(y,B;\acute{y},m) - k_r B|_{y=\acute{y}}), \tag{21}$$

with the first-order conditions

$$\frac{\partial V}{\partial m}(\acute{y},B;\acute{y},m) = 0, \tag{22}$$

$$\frac{\partial V}{\partial y}(y,B;\acute{y},m)|_{y=\acute{y}} + \frac{\partial V}{\partial \acute{y}}(\acute{y},B;\acute{y},m) - \frac{1}{\acute{y}}(V(\acute{y},B;\acute{y},m) - k_r B) = 0. \tag{23}$$

The optimization problem of all-equity owners after recapitalization is identical to the problem of the initial firm owners who decide over initial leverage and maturity since capital structure strategies S preserve the model's linear homogeneity in the debt level B. Hence, the initial choice of $\acute{y}$, m, and i is identical to the choice made by all-equity owners after reorganization at $\underline{y}$ or $\overline{y}$.

3. Calibration

For our numerical analysis our parameters are calibrated to capture important features of the U.S. tax code, which are also shared by many other countries.[18] First, debt coupons are deductible from the corporate tax base. Second, income from interest bearing investments is more heavily taxed than income from equity investments. In the United States, interest income is treated as ordinary income and we calibrate this to the maximum tax rate on wage income. Prior to the 2018 tax reform, this amounted to 39.6% plus a 3.8% Medicare surtax on investment income. By contrast, high-income earners only pay 20% tax on dividend income plus 3.8% for Medicare, that is, 23.8%. To keep the analysis tractable, our model only explicitly considers a personal tax on debt income, but not on equity income. To capture the tax disadvantage of interest income over income from equity in our reduced-form model, we set the personal tax rate on debt equal to the difference between the 43.4% tax on debt income and the 23.8% tax on equity income, that is, τ_p=19.6%.

[17] Without such an assumption, the joint choice of face value and coupon rate is ambiguous. (20) resolves this ambiguity and preserves homogeneity in B.

[18] Table 2 lists base case parameters used in our numerical analysis.

At the corporate level, we assume that income is taxed at a constant statutory rate, τ_c, which we calibrate to empirically effective marginal tax rates. For this purpose, we use two sources of information. First, we use marginal tax rates from COMPUSTAT MTR database, which employs the nonparametric estimation method introduced by Blouin, Core, and Guay (2010) that explicitly takes care of mean reversion of corporate income. We merge the MTR database with COMPUSTAT firm characteristics to calculate total-asset-weighted average marginal tax rates after interest expense over the available horizon from 1994 to 2012, which yields 30.6%. Average marginal tax rates peak in 1993 (33.0%) and are lowest in 2010 (22.0%).

Second, as a robustness check, we analyze John Graham's file of simulated tax rates.[19] The average marginal tax rate after interest expense over the last 20 years, that is, from 1994 to 2013, is estimated to be 25.9%. Again, average simulated marginal tax rates in the sample period are lowest in 2010 (18.7%) and highest in 1995 (30.7%). The total-asset-weighted average marginal tax rate before interest expenses over the stated period is 33.1%. This is close to the asset-weighted mean value from COMPUSTAT MTR reported above. In the base case of our numerical analysis, we therefore use a corporate tax rate of 30.6%. We also provide comparative statics for our tax parameters to illustrate how sensitive our results are to changing tax rates.

Recent empirical estimates of corporate bankruptcy costs have considerably changed the academic community's view of their magnitude. Early papers estimated bankruptcy costs by investigating sets of defaulted firms and estimating these costs to be only a few percent of the firm's asset value.[20] More recently, researchers have accounted for the fact that a subset of defaulted firms is likely to produce a biased bankruptcy cost estimate for the entire population of firms. They argue that low-destress cost firms are overrepresented in this sample and, thus, existing estimates of bankruptcy costs might be significantly downward biased. Reindl, Stoughton, and Zechner (2017) infer implied distress costs from market prices of equity and prices of put options employing a dynamic capital structure model. They show that estimated bankruptcy costs vary considerably across industries from below 10% to well over 60% with typical values in the range between 20% to 30%. In our calibration we refer to Glover (2016), who estimates parameters of a structural trade-off model of the firm with time-varying macroeconomic conditions by employing simulated methods of moments. He estimates the mean firm's cost of default with 45% and

[19] We thank John Graham for providing us with his comprehensive set of simulated marginal tax rates covering the period from 1980 to 2013 and for his advice on calibrating our model to the U.S. tax code. Please see Graham (1996a) and Graham (1996b) for details about the applied simulation procedure. Graham and Mills (2008) use the federal government's tax return data to show that simulated marginal tax rates provided in the file are close approximations.

[20] See, for example, the following papers for studies on default costs (estimated averages are in parentheses): Warner (1977) (5.3%), Ang, Chua, and McConnell (1982) (mean 7.5%, median 1.7%), Weiss (1990) (3.1%), Altman (1984) (6.0%), and Andrade and Kaplan (1998) (10% to max. 23%).

Table 2
Base case parameters

Parameter	
Riskless rate of interest r	5%
Personal tax rate τ_p	19.6%
Corporate tax rate τ_c	30.6%
Volatility of the cash flow process σ	13%
Risk-adjusted drift $\hat{\mu}$	0%
Bankruptcy cost g	34.39%
Transaction costs for rolling over debt k_i	0.5%
Transaction costs after recapitalization k_r	1%
Call premium λ	0%

the median firm's cost with 37% of asset value. Our model specifies bankruptcy costs as a fraction g of the face value of debt. Thus, aiming for a base case parametrization that resembles median bankruptcy costs, we select g such that a firm with optimally chosen debt maturity experiences bankruptcy costs of 37% of its asset value. This leads to a base case parameter of $g = 34.39\%$. Below, we provide comparative statics to estimate the effect of varying bankruptcy costs (e.g., across industries).

4. Debt Maturity, Capital Structure Dynamics, and Firm Value

We start by analyzing the effect of average debt maturity on the firm's optimal refinancing decision. If not otherwise mentioned, we use the base case parameters listed in Table 2. First, we explore firms that have issued debt with long maturity. Panel A of Figure 1 illustrates the optimal rollover rate nomalized by the retirement rate, δ/m, over the inverse leverage ratio y for a bond with long maturity, $T=30$ years, $m=0.0\dot{3}$. Since the state variable y is proportional to the firm's cash flow level, y serves as a proxy of the firm's profitability. We can see that equityholders optimally choose to rollover all expiring debt ($\delta/m=1$) over a large range of firm states, especially in bad states, that is, low y. Only immediately before calling the bonds to subsequently issue more debt, that is, in a region near $\overline{y}$, does it become optimal for equityholders to use equity to repay maturing debt. The intuition for this latter result is straightforward. In this leverage region, it is optimal to use retained earnings to finance principal repayments since it would be inefficient to incur transaction costs for a new bond issue, knowing that the bond will be called in the near future with high probability.

Panel B of Figure 1 shows the optimal rollover rate, δ/m, for a bond with a $T=10$-year maturity, $m=0.1$. In contrast to the 30-year bond, debt with a shorter maturity is not fully rolled over if the firm's state deteriorates. There is a region of y below the initial inverse leverage $\acute{y}$ near the bankruptcy threshold $\underline{y}$, where rollover is an interior optimum as derived in Propositions 2 and 3. Intuitively, under full rollover, prices of new bonds would be too low, since they reflect high leverage and future costs of financial distress. It is in equityholders' own interest to partly use equity to refund maturing debt, despite the fact that it

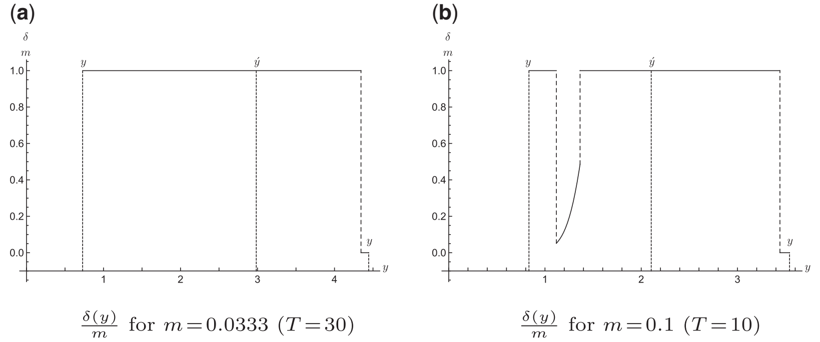

Figure 1
Optimal debt rollover for long and short maturities
Panel A: Optimal rollover rate for a long-maturity bond, $T=30$ years, as a function of the inverse leverage ratio y under base case parametrization (see Table 2). Low and high levels of y correspond to states of low and high profitability of the firm. Equityholders will not engage in debt reduction in bad states of the firm.
Panel B: A maturity of $T=10$ years serves as a commitment that equityholders will not rollover all expiring debt in bad states, that is, engage in debt reduction.

implicitly also benefits the remaining bondholders. In equilibrium, bondholders anticipate this and price bonds more attractively.

When increasing debt maturity above a critical value, this region of debt reduction vanishes. Under base case parametrization, the critical maturity at which this happens is $T=23.86$ years, $m=0.04192$. This is the shortest maturity without debt reduction. Debt with maturity shorter than this critical value induces debt reduction, maturities longer than this critical value do not induce debt reductions in bad states.

Interestingly, the firm's willingness to use equity to repay debt is nonmonotonic in the inverse leverage ratio, y. When the firm approaches bankruptcy, that is, for y near $\underline{y}$, equityholders terminate their effort to reduce debt. In this region, they once again fully rollover debt and dilute existing debtholders by reissuing the expired debt. This is in accordance with Proposition 5, which proves that when loss-given-default is less than 100%, firms engage in full rollover when close to bankruptcy. Thus, when pushed very close to bankruptcy, equityholders are no longer willing to make additional voluntary equity investments in the firm. On the contrary, they would rather issue new debt at the maximum rate allowed by debt covenants, even if this can only be done at unfavorable prices, that is, when high credit spreads are charged by investors.

Figure 1 also reveals an additional range in which the firm does not fully rollover maturing debt. This occurs when the inverse leverage ratio approaches $\bar{y}$, that is, the threshold, at which all outstanding debt is repurchased and replaced by a new, larger debt issue. The intuition for this is straightforward. In this range it would not be worthwhile to incur the transaction costs associated with debt rollover, since the firm anticipates that the repurchase of the entire outstanding debt is imminent. Thus, transaction costs associated with debt rollover are not expected to be "amortized" via the present value of the additional tax shields. In this region, repaying maturing debt with retained earnings is optimal.[21]

To summarize, the above numerical analysis generates four main insights. First, for sufficiently long maturities, equityholders never use retained earnings or equity issues to repay maturing debt, except immediately before a discrete leverage increase. This result changes if the average debt maturity is shortened. In this case, there exists a range of leverage ratios strictly above the initial optimum for which equityholders find it optimal to partly use retained earnings of equity to repay maturing debt. This is in accordance with the evidence presented in Section 5 and with the findings of Hovakimian, Opler, and Titman (2001), who report that long-term debt is an impediment to movements toward the target leverage ratio.

Second, at the initial leverage ratio, $\acute{y}$, the firm always holds its debt level constant and fully rolls over maturing debt, $\delta = m$. This follows directly from the optimality of the initial leverage ratio.

Third, near the restructuring threshold, $\overline{y}$, the firm entirely refrains from issuing debt. In this range, incurring the transaction costs associated with debt rollover would not be worthwhile, since the firm anticipates that the repurchase of the entire outstanding debt is imminent.

Fourth, near the bankruptcy threshold, $\underline{y}$, the firm fully rolls over all expiring debt, that is, $\delta = m$. Thus, even with short-term debt outstanding, equityholders resume issuing debt if the leverage ratio becomes sufficiently high. In this case, equityholders are no longer willing to invest in debt reductions to keep their equity option alive. This latter result follows from Proposition 5.[22]

Bankruptcy costs, corporate taxes, and critical debt maturity: We find that bankruptcy costs as well as the magnitude of the tax shield of debt financing represent the main determinants for the critical average maturity that triggers voluntary debt reductions. The lower the bankruptcy costs the shorter the maturity required to provide incentives for voluntary debt reductions. Figure 2 plots the critical average maturity over bankruptcy costs for two different levels of corporate tax, τ_c. The lower line represents our base case, where the corporate tax rate is calibrated to the average marginal tax rate provided by COMPUSTAT MTR database (τ_c = 30.6%, see Section 3).

21 This region at which debt is not rolled over in states close to $\overline{y}$ has a negligible effect on total firm value, as we will discuss below when we calibrate the model. This is so since firms spend only a small fraction of time in this region and because there are no significant agency problems associated with the rollover behavior of the firm in this region, since debt is essentially riskless. In contrast, the effect of debt reduction in bad states of the firm has a large effect on firm value, as we argue throughout the paper. We quantify the valuation effect of the former region below, when discussing optimal maturity choice and the associated tax benefits of debt.

22 In the Internet Appendix A.7, we derive a condition under which firms never have an incentive to increase the debt retirement rate m above the originally contracted level. This condition holds in all of our numerical analyses and is a manifestation of the leverage ratchet effect in Admati et al. (2018).

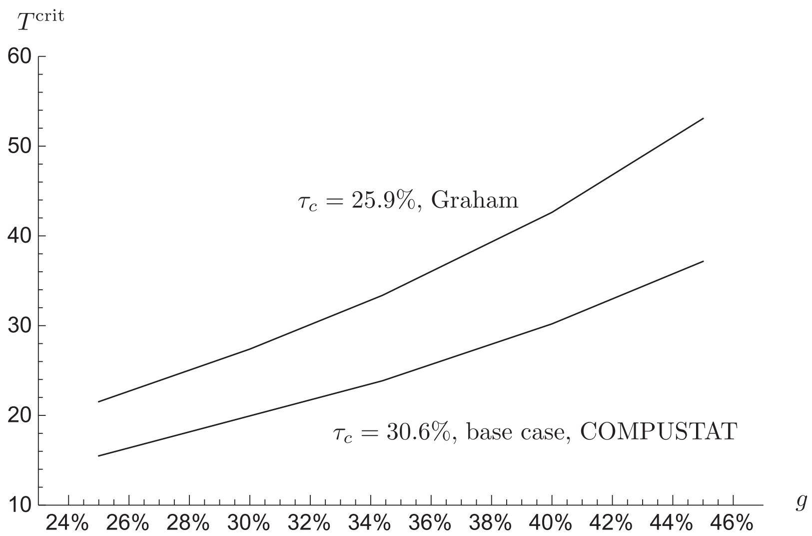

Figure 2
Critical debt maturity that induces debt reduction
Critical average debt maturity below which the commitment to debt reductions in bad times is credible as a function of bankruptcy costs. Critical maturities are plotted for the base case parameterization $\tau_c = 30.6\%$, which is the average marginal tax rate estimated from COMPUSTAT MTR database, applying the approach of Blouin, Core, and Guay (2010). Additionally, also plotted is critical maturity for the average tax rate from John Graham's database, that is, $\tau_c = 25.9\%$. See Section 3 for more details.

The upper line represents critical debt maturities over bankruptcy costs when using the lower average effective corporate tax rate implied by the marginal tax rate data provided by John Graham ($\tau_c = 25.9\%$,). It is evident, that in the case of higher tax shields it requires shorter debt maturities to induce sufficient incentive for equityholders to engage in active debt reduction when the firm's cash flows deteriorate. This result is quite intuitive, since actively replacing retired debt with equity reduces the firms tax shields and, hence, providing larger tax shields reduces the incentive to substitute debt with equity.

With base case parameterization, that is, $\tau_c = 30.6\%$, $g = 34.39\%$, debt maturity below the critical maturity of 23.86 years induces debt reductions in bad times. With lower tax shields, using $\tau_c = 25.9\%$ the critical debt maturity at $g = 34.39\%$ is 33.4 years. Bankruptcy costs as low as $g = 25\%$ require average maturities of 15.49 years and 21.53 years when using corporate tax rates of 30.6% and 25.8%, respectively. Bankruptcy costs as high as $g = 45\%$ induce debt reduction for average maturities below 37.16 and 53.07 years, respectively. Thus, with lower bankruptcy costs, it needs shorter-term debt to induce debt reductions.

Debt maturity and firm value: Next, we consider the effect of debt maturity on firm value and illustrate the potential benefit of a short-term debt maturity with base case parameters. Results for different parameterizations are reported

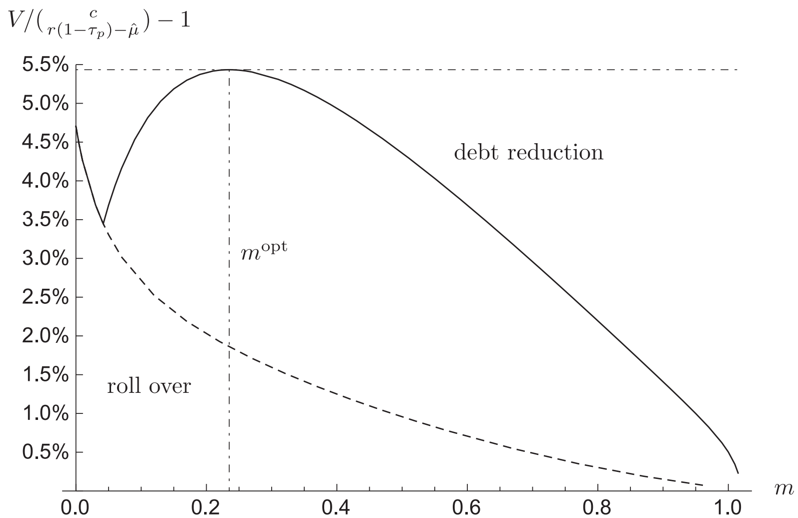

Figure 3
Tax advantage and debt maturity
The tax advantage of debt at the optimal initial leverage for the base case firm plotted against the retirement rate m. The dotted line represents the corresponding tax advantage for a firm that has to keep the debt level constant and, therefore, rolls over all expiring debt. The relation between the maturity structure of debt and firm value is nonmonotonous. Firm value is maximized at a maturity of $\approx$4.26 years.

below. Figure 3 displays the tax advantage, that is, the extent to which the value of the optimally levered firm exceeds the value of the unlevered productive assets as a function of the retirement rate of debt, m.

The figure also displays the relative value of a reference firm (dotted line), which is assumed to always fully rollover maturing debt with new debt issues.[23] For the reference firm, total firm value is maximized by choosing the longest possible maturity for its debt, as reported in Leland (1994b) and Leland and Toft (1996). By contrast, if the firm can engage in debt reductions, the relationship between total firm value and the maturity structure of debt is not monotonic.[24] This is so because debt with sufficiently short maturity induces more efficient capital structure adjustments by equityholders when the firm's cash flows decrease, thereby lowering probability of default and, hence, expected bankruptcy costs. This result is driven by the fact that the firm with the higher fraction of maturing debt (shorter maturity) essentially has greater flexibility of managing, that is, reducing, its leverage in the relatively bad states.

[23] This is modeled as in Leland (1994b). In addition, we also allow the firm to increase its debt by repurchasing all debt outstanding and to issue a higher amount of debt.

[24] Evidence for this nonmonotonicity is provided by Guedes and Opler (1996), who report that investment grade firms seem to be indifferent between issuing debt at the long end of the maturity spectrum and issuing debt at the short end of the spectrum.

This flexibility increases the value of the firm since it can operate with higher debt ratios, thereby shielding its taxable income more effectively.[25]

As illustrated in Figure 3, the beneficial effect of shorter debt maturity on future capital structure dynamics outweighs the disadvantage due to higher transactions costs from rolling over maturing debt. In the base case, overall firm value is maximized at a debt maturity of $\approx$4.26 years.[26,27]

Debt capacity: The commitment effect of debt maturity also has a significant effect on the optimal initial leverage ratio. In contrast to existing results in the finance literature, we find that shorter debt maturities lead to higher debt capacities.

Figure 4 illustrates this effect. The figure plots the initial optimal leverage as a function of m for the base case firm. Unlike firms that must rollover all maturing debt, firms that choose the rollover rate optimally actually *increase* their debt capacity as they shorten their debt maturities. The optimal initial leverage increases from approximately 39% for perpetual bonds and reaches its maximum with 80% at an average debt maturity of approximately 1.5 years. For very short maturities, debt capacity decreases, due to the transaction costs incurred when rolling over debt. At the firm-maximizing debt maturity of 4.26 years, the firm's debt capacity is approx. 65%.

Compared to the models of Leland (1994b), Leland and Toft (1996), or Leland (1998), our model generates higher initial target leverage ratios. However, over the lifetime of the firm, average leverage ratios are comparable. The relatively high initial leverage in our model is due to the fact that investors rationally anticipate low bankruptcy risk due to less than full debt rollover in bad states. This commitment effect of short debt maturities leads to higher initial but similar average leverage ratios due to subsequent leverage reductions. Lower initial leverage ratios would obtain if debt maturities were lumpy. In this case the leverage reducing effect of maturing debt would only take place at discrete points in time.[28]

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[25] We are grateful to an anonymous referee for providing this intuition.

[26] We simulate 50,000 firms over 100 years and find that the optimally levered firm (optimal maturity $\approx$4.26 years) spends on average 8.9% of the time in states of debt reduction. Debt reduction decreases the risk-neutral probability of default to 0.3% per year compared to a 1.3% default probability per year of a firm with an optimal amount of infinite horizon debt, which is the optimal maturity choice when debt must always be fully rolled over.

[27] It is part of the optimal strategy that equityholders stop debt rollover close to the restructuring boundary $\overline{y}$ to save transaction costs. In contrast to the region at which the firm does not fully rollover debt in bad states, the effects of stopping rollover in good states on firm value are small. In our calibrated base case, a firm following the optimal rollover restructures at a leverage ratio $1/\overline{y}=35.16\%$, while a firm that does not stop rollover near the restructuring boundary chooses to restructure at a leverage ratio of 34.59%. The effect on initial firm value is less than 1 basis point.

[28] As discussed in the introduction, Geelen (2016) and Chen, Xu, and Yang (2020) analyze the polar opposite case, in which only a single debt maturity is outstanding. This indeed leads to lower initial target leverage and higher expected bankruptcy probabilities, compared to our setup. Analyzing the effect of different degrees of maturity granularity appears to be a fruitful avenue for future research.

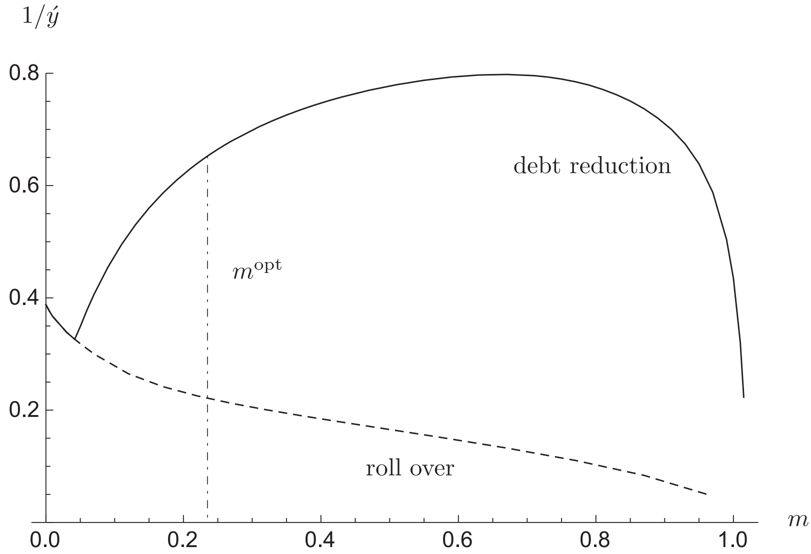

Figure 4
Optimal initial leverage and debt maturity
Optimal initial leverage ratios, $1/\acute{y}$, plotted over the retirement rate, m. Without allowing for downward restructuring, debt capacity decreases when moving from long- to short-term debt. For firms that explicitly consider debt reduction, debt capacity increases once maturity is sufficiently short in order to commit to debt reductions to avoid financial distress. Only at the very short end do transaction costs lead to a deterioration in debt capacity.

4.1 Comparative statics

In this section, we explore the effect of various model parameters on firm value, optimal debt maturity and dynamic capital structure policy.

Bankruptcy costs: We first focus on the role of bankruptcy costs. The key role of bankruptcy costs for the commitment to debt reductions was already discussed above. Figure 5 plots the tax advantage of debt, that is, the extent to which the initial firm value exceeds the unlevered firm value, for different levels of bankruptcy costs. Several effects can be seen: (a) lower bankruptcy costs require a shorter debt maturity in order to induce voluntary debt reductions; (b) lower bankruptcy costs reduce the maximum attainable tax advantage of debt; (c) lowering bankruptcy costs moves the optimal finite maturity toward shorter maturities, and (d) for very low bankruptcy costs, it becomes relatively more advantageous to issue console bonds.

The most surprising effect is that higher bankruptcy costs imply higher firm values. Higher bankruptcy costs make it easier for equityholders to credibly commit to debt reductions. The resultant decrease in the expected probability of bankruptcy more than offsets the effect of the increased costs given a default.

Transaction costs: The costs associated with rolling over debt are another key determinant of firm value when finite maturity debt is issued. Figure 6 illustrates the effect on firm value for different values of k_i. When moving

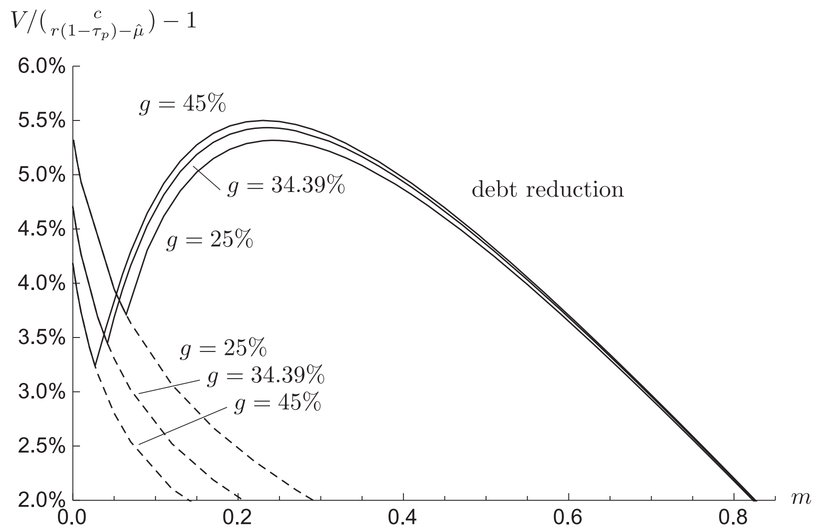

Figure 5
Tax advantage and bankruptcy costs
Tax advantage plotted over the retirement rate, *m*, for different levels of bankruptcy costs. Lower bankruptcy costs require a shorter debt maturity to induce voluntary debt reductions and reduce the maximum attainable tax advantage of debt. Lowering bankruptcy costs reduces the optimal maturity.

to lower values of k_i, we observe that (a) firms with shorter-term debt gain relatively more and (b) the local maximum of total firm value for finite debt maturity moves toward shorter maturities. In our model, transaction costs, k_i, are assumed to be proportional to the notional amount of debt that is rolled over. Thus, shorter bond maturity is, ceteris paribus, associated with higher costs per unit of time.[29]

Cash flow characteristics: Figure 7 shows how changes in cash flow characteristics affect total firm value. Panel 7a plots the initial tax advantage as a function of the retirement rate for several values of cash flow volatility, σ. Moving to higher volatilities (a) results in lower firm value, (b) requires shorter debt maturity to induce debt reductions, and (c) moves the local maximum of total firm value toward shorter maturity debt. High cash flow volatilities reduce the firm's debt capacity but increase the option value for equityholders and thus make them more reluctant to default on their debt obligations. This makes the commitment effect of short-term debt relatively less advantageous and requires shorter debt maturities to induce voluntary debt reductions.

29 One could argue that the burden of the higher transaction costs for lower maturities actually reduces bond prices and, in turn, incentivizes equityholders to engage in debt reductions, rather than short debt maturity by itself. To ensure this is not the case, we solve a model that features constant transaction costs per unit of time, $mk_i=$const. Thus, we assume that rolling over short-term debt is cheaper to do than rolling over long-term debt, so that the transaction costs per period are the same for different maturities. Under this alternative assumption, debt reduction still occurs only when debt maturity is below some threshold.

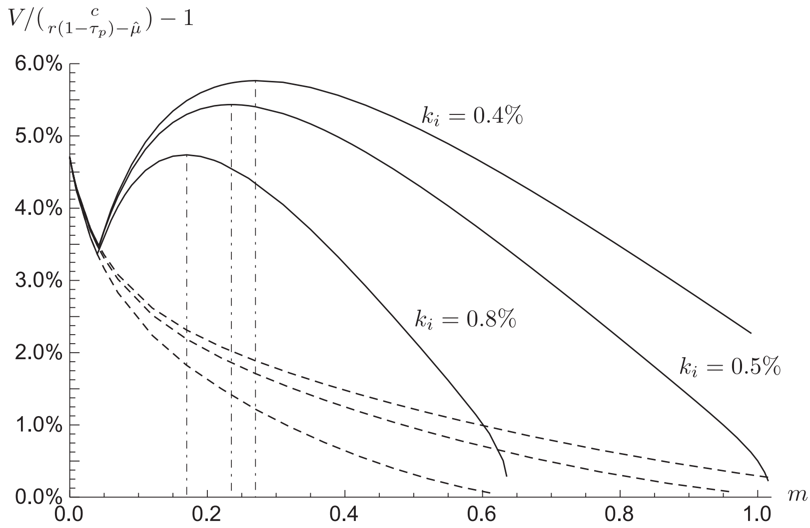

Figure 6
Tax advantage and rollover costs
Tax advantage for different costs, k_i, associated with rolling over debt plotted over the retirement rate, m. With lower k_i, the optimal maturity moves toward shorter-term debt.

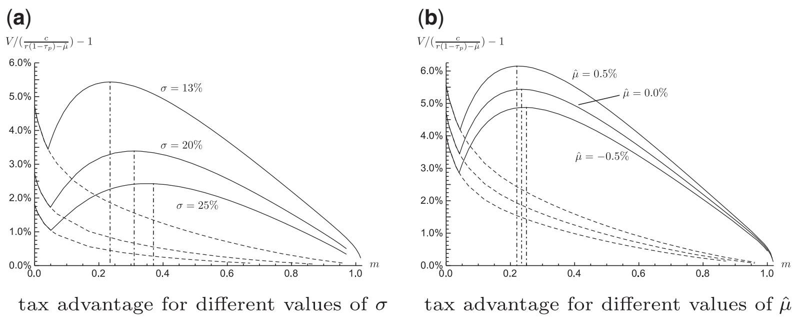

Figure 7
Tax advantage and cash flow dynamics
Tax advantage plotted over the retirement rate, m, for different values of σ and $\hat{\mu}$.

Panel 7b displays the tax advantage of debt as a function of the retirement rate for several values of the risk-adjusted cash flow growth rate $\hat{\mu}$. Moving to higher growth (a) increases firm value and (b) shifts optimal maturity toward long-term debt. This is so since the commitment to decrease leverage in response to decreasing cash flows is less valuable for firms with high expected cash flow growth rates.

Bankruptcy costs and debt capacity: Next, Figure 8 plots the firm's optimal initial leverage ratio, which we refer to as the firm's debt capacity, for different

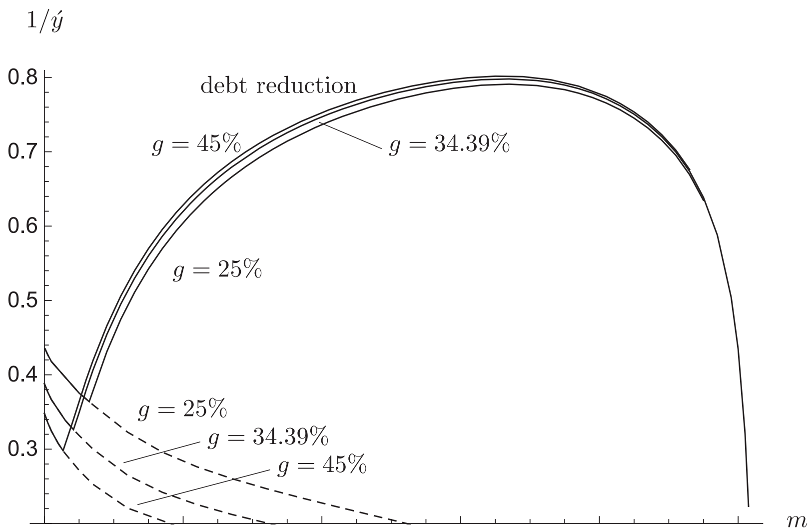

Figure 8
Optimal leverage and bankruptcy costs
Optimal initial leverage ratios, $1/\acute{y}$, plotted over the retirement rate, m. High bankruptcy costs lead to high debt capacity if using short-term debt.

debt maturities and for different levels of bankruptcy costs. Consistent with the findings reported above, higher bankruptcy costs are associated with a higher debt capacity since equityholders can commit to debt reductions when cash flows decrease. This results in a reduced bankruptcy probability, which more than offsets the higher bankruptcy costs conditional on default.

Firm value and corporate tax rates: Finally, Figure 9 shows the tax advantage of debt plotted against the retirement rate m for the base case firm with $\tau_c = 30.6\%$. As a comparison we plot the tax advantage when corporate taxes are estimated from John Graham's marginal tax rate data, $\tau_c = 25.9\%$. Higher tax shields caused by higher corporate tax rates lead to (a) lower optimal debt maturity and (b) a higher tax advantage at the optimal debt maturity. While a higher corporate tax rate intuitively leads to a higher tax advantage if debt is used optimally, the result that higher tax rates reduce optimal maturity is less obvious. As a direct consequence, higher tax rates make debt reduction less desirable, because reducing debt diminishes the associated tax shield. A secondary effect is that debt capacity increases with shorter debt, and higher debt capacity ex ante allows the firm to use debt more aggressively, which increases the debt tax shield. From Figure 9, we see that the latter effect dominates the direct effect, and, overall, higher corporate tax rates reduce optimal debt maturity, from an optimal average maturity of 6.25 years for $\tau_c = 25.9\%$ to 4.26 years in the base case with an effective corporate tax rate of $\tau_c = 30.6\%$.

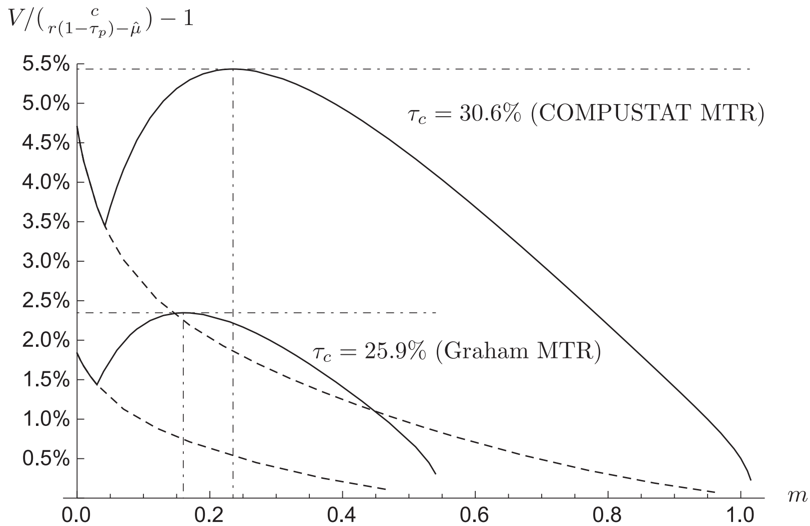

Figure 9
Tax advantage and corporate tax
Tax advantage of debt for different levels of corporate tax, τ_c, plotted over the retirement rate, m. High corporate tax rates lead to higher tax advantages and lower optimal maturity.

5. Evidence

Our analysis generates several predictions linking capital structure choices to various firm characteristics. Most importantly, the analysis implies a distinct relation between debt maturity and leverage dynamics, that is, firms with shorter debt maturities exhibit more pronounced debt reductions when firm value drops. In addition, the model implies relations between debt maturities and firm characteristics, such as cash flow volatilities, corporate tax rates, or bankruptcy costs. This section discusses evidence on these model predictions. Although not intended to represent a fully-fledged empirical test, it documents that even simple analyses reveal patterns that accord well with our theory.

5.1 Sample construction

To relate our theoretical predictions to evidence, we use annual Compustat data for the period from April 1962 to March 2017. As in Graham and Leary (2011), and DeAngelo, Gonçalves, and Stulz (2018), we exclude firms not incorporated in the United States, utilities (SIC codes in the range 4900 to 4949), financials (SIC codes 6000 to 6999), and firm-years in which the book value of total assets is below US\$10 million. We select only firm-years for which either *long-term debt* or debt in *short-term liabilities* are available (as in DeAngelo, Gonçalves, and Stulz 2018). We exclude observations with missing *total assets*, *cash*, or *market capitalization* (i.e., *share price at the end of fiscal year* times *common shares outstanding*), with negative *net-debt ratio* and with negative

Table 3
Number of observations in leverage buckets

	Market leverage (quantile)					
Short-term debt (quant.)	0%–20%	20%–40%	40%–60%	60%–80%	80%–100%	Sum
0%–20%	4,115	3,318	2,945	2,448	1,892	14,718
20%–40%	3,644	3,363	3,213	2,698	1,899	14,817
40%–60%	3,137	3,161	3,062	3,022	2,512	14,894
60%–80%	2,740	2,970	3,019	3,074	3,016	14,819
80%–100%	1,372	2,031	2,503	3,444	5,042	14,392
Sum	15,008	14,843	14,742	14,686	14,361	73,640

Number of observations in the buckets created by the double sort on *short-term debt*, that is, debt due within 3 years, normalized by total debt, and *market leverage*, that is, long-term debt plus debt in short liabilities over the book value of total liabilities plus market capitalization of equity

cash. Finally, we truncate the data at the 97.5% EBIT volatility quantile to exclude implausibly high values. This results in a sample of 73,640 firm-years.

To measure the extent to which a firm uses short-term debt in its capital structure, we calculate the fraction of *debt due within 3 years*, normalized by total debt.[30] Market leverage is measured as long-term debt plus debt in short-term liabilities over the total market value of the firm's assets (total liabilities plus market capitalization). We use the *net debt ratio* (i.e., *book leverage* minus *cash ratio*) to calculate changes in leverage over consecutive years.

5.2 Empirical findings

In a first step we analyze whether firms' leverage changes are associated with the fraction of short-term debt. To do this we calculate transition probabilities in the spirit of DeAngelo, Gonçalves, and Stulz (2018), who show that firms with high market leverage tend to reduce their net debt ratio. We build on their study and apply a double sort. First, with respect to market leverage, and second, with respect to short-term leverage, that is, debt due within 3 years as a fraction of total debt. Along both dimensions, we sort firms into quintiles. Break points for short-term debt sorts are 7.40%, 15.89%, 26.64%, and 42.91% of debt due within 3 years. Break points for market leverage sorts are 7.05%, 14.95%, 25.13%, and 41.34%.[31]

Table 3 states the number of observations in each bucket. We see that buckets along the main diagonal are most densely populated, that is, firms' market leverage and their fraction of short-term debt are positively correlated, with a correlation coefficient of 28.85%.

[30] Our findings are qualitatively robust to alternative definitions of short-term debt. For example, we have also used debt due within 2 years to total assets as a measure of short-term debt usage and find a very similar result.

[31] In comparing these break points to our calibrated numerical examples, one needs to consider that the above empirical values are based on a leverage definition that only considers long-term debt plus debt in short liabilities. When focusing on firms' total liabilities, which include, for example, accounts receivables and pension obligations, one obtains substantially higher leverage ratios. In this case, the firm at the 80th percentile exhibits a market leverage of 71.4%.

Table 4
Short-term debt and subsequent changes in net debt

	Short-term debt (quant.)	Market leverage (quantile) 0%–20%	20%–40%	40%–60%	60%–80%	80%–100%
1	0%–20%	1.0260	1.2082	0.6781	0.4753	2.0100
2	20%–40%	1.4422	0.6560	0.0931	−0.3104	−0.2827
3	40%–60%	0.9005	0.4733	−0.3224	−1.4850	−0.9754
4	60%–80%	0.2006	−0.0677	−0.7645	−1.5709	−1.6737
5	80%–100%	−1.8954	−0.8083	−1.0250	−2.1890	−2.0350
	1-5	2.9214***	2.0165***	1.7031***	2.6643***	4.0449***
	p	5.5049×10^{-6}	9.4398×10^{-6}	7.1051×10^{-6}	1.2320×10^{-12}	6.1937×10^{-15}

Average changes in net debt in the year following the double quintile sort on *short-term debt*, that is, debt due within 3 years, normalized by total debt, and *market leverage*, that is, long-term debt plus debt in short liabilities over book value of total liabilities plus market capitalization of equity. Changes are measured as averages of $(\textit{net debt ratio})_{t+1} - (\textit{net debt ratio})_t$ as a percentage. Short-term debt is defined as debt expiring within 3 years over total assets. Row *1-5* exhibits the differences between changes in leverage of the average first quintile firm and that of the average fifth quintile firm. The p-value of the corresponding t-test is given in the last row.

Our capital structure theory predicts that firms with higher levels of short-term debt have stronger incentives to reduce debt than firms with longer debt maturities. Furthermore, our results also imply that the incentive to delever is nonmonotonic in market leverage. Moving from low market leverage to high market leverage, the incentive to delever first increases, but for very high market leverage, the incentive to reduce net debt vanishes.

Table 4 explores evidence for these predictions. It reports the average changes in net debt ratios, stated as a percentage and measured over the year that follows the sort.[32] In each of the market leverage buckets (columns of Table 4), firms with more short-term debt delever significantly more (or lever less) than firms with low levels of short-term debt.

For example, consider the column for market leverage 20%–40%, which displays the change in the net debt ratio in the subsequent year for firms in this market leverage bucket. It shows that firms in the bottom quintile of usage of short-term debt, that is, row 0% to 20%, on average *increase* their net-debt ratio by approximately 1%. By contrast, firms that are in the top quintile, that is, row 80%–100%, *decrease* their net-debt ratio by approximately 1.9% in the subsequent year. In fact, as can be seen from the first row of Table 4, the average firm that almost entirely relies on long-term debt does not engage in leverage reductions, regardless of its initial market leverage, consistent with our model. Moving from low to high fractions of short-term debt, one observes that firms increasingly engage in deleveraging. For example, row 5 shows that the average firm that almost exclusively uses short-term debt and that is in a market leverage bucket between 60% and 80% reduces its market leverage in the subsequent year by more than 2%. These findings are also in accordance with Hovakimian, Opler, and Titman (2001), who also conclude that long-term

32 We have also performed this analysis using changes in net debt ratios of the median firms, rather than the means. We have obtained qualitatively equivalent results.

Table 5
Short-term debt and subsequent exclusion from the data set

	Market leverage (quantile)					
Short-term debt (quant.)	0%–20%	20%–40%	40%–60%	60%–80%	80%–100%	Avg
0%–20%	3.5480	5.0934	4.0747	4.1258	5.7082	4.3756
20%–40%	3.8419	3.8061	4.7619	4.3365	4.9500	4.2654
40%–60%	3.2196	4.5872	5.3560	4.5996	3.8217	4.3306
60%–80%	2.8832	4.3098	5.5979	5.6604	4.4761	4.6224
80%–100%	2.9155	3.4466	4.0751	5.8362	7.9730	5.6629
Avg	3.3715	4.3118	4.8026	4.9843	5.8144	4.6456

Fraction of firms (expressed as a percentage) exiting from the data set in the year following the sort

debt hinders leverage reductions. Thus, the first prediction of our theory seems to be empirically relevant and strongly supported by firm behavior.

Next, we turn to the nonmonotonicity of leverage adjustments when conditioned on initial market leverage. Recall that in our model firms increase leverage when they reach a lower leverage threshold. Then, for increasing initial market leverage ratios, firms do not adjust the face value of debt over a range. Eventually they start reducing debt, by rolling over less than 100% of the expiring debt before they start gambling for resurrection by fully rolling over expiring debt again as they get close to their bankruptcy thresholds. This predicted nonmonotonicity along increasing initial market leverage is also apparent in Table 4. The change in net debt is generally U shaped when moving from low to high market leverage buckets. Consider row 3, for example. While firms in the market leverage bucket 60% to 80% reduce their leverage in the following period by approximately 1.5%, firms in the highest market leverage bucket, that is, 80%–100% reduce leverage by only less than 1%. Consistent with our model, firms with intermediate market leverage have the highest incentive to reduce their leverage.

We believe that the results in Table 4 understate the true nonmonotonicity of debt reductions, since our data set does not fully capture the leverage effects of firm defaults. Compustat has no flag that indicates the reason a firm drops out of the database. However, Table 5 shows that exits are more frequent for firms with high market leverage. For example, Table 5 reveals that almost 6% of those firms in the highest market leverage bucket exit the sample in the subsequent year, whereas less than 3.4% of firms do so in the lowest market leverage bucket. Hence, we conclude that part of this increase is due to more frequent bankruptcies of firms in high leverage buckets. Since these firms will generally exhibit increasing market leverage until they default, the nonmonotonicity of debt reductions is likely to be even more pronounced than what is illustrated in Table 4.

In addition to the main result that links debt maturity to firms' propensities to delever, our model also delivers comparative statics for debt maturities and firms' corporate tax rates, their cash flow volatilities, and their bankruptcy costs. To explore these predictions, we use a simple linear regression framework, with the fraction of short-term debt in a firm's capital structure over total liabilities as the dependent variable. Cash flow volatility is proxied by the standard deviation of firms' EBIT over total assets. We hereby require at least eight annual observations. As a proxy for the corporate tax rate we use past income taxes over EBIT averaged over the last 8 years. We use this definition since a forward-looking measure of marginal tax rates, as analyzed in Graham (1996a,b), Blouin, Core, and Guay (2010), would induce a severe endogeneity problem. This is so since leverage changes directly affect future effective corporate tax rates. Since bankruptcy costs are not directly observable at the firm level, we use estimates provided by Reindl, Stoughton, and Zechner (2017), aggregated for the Fama and French 17 industries.

Columns 1 to 3 of Table 6 show univariate regressions of the fraction of short-term debt over total liabilities on our proxies for bankruptcy costs, cash flow volatility and corporate tax rates. Column 4 displays the multivariate regression results for the three explanatory variables. Column 5 also includes the additional controls suggested in table 1 of Graham and Leary (2011).

Focusing first on columns 1 to 4, we can see that the coefficients for the model-based explanatory variables all have the right sign in the univariate regressions as well as in the multivariate regression. Furthermore, RSZ_MeanBC and ebitVola are both significant at the 1% level in all four regression specifications. Thus, higher bankruptcy costs are associated with longer debt maturities, whereas higher cash flow volatilities are associated with shorter debt maturities, consistent with the model.

In column 5, we control for additional firm characteristics, such as size, leverage, asset tangibility, profitability, and the market-to-book ratio of equity. We see that the coefficient for ebitVola still remains positive at the 1% significance level. The negative relation between short-term debt usage and bankruptcy costs is also obtained with the extra control variables. However, this latter result is no longer statistically significant. When interpreting this, one needs to consider that the bankruptcy cost proxies are necessarily noisy estimates of the firms' true bankruptcy costs, as they are aggregated at industry levels. For example, we know that firms within industries display a significant degree of heterogeneity, especially in size, which is usually highly right skewed. Therefore, the bankruptcy costs of very large firms, expressed as a percentage of total assets, are likely different from those of our proxies. We therefore also run the regression with the control variables when we truncate observations at the 95th percentile. As can be seen in column 6, for this sample the coefficient for bankruptcy costs is indeed statistically significant and has the predicted sign. We also note that the coefficients of the additional control variables in columns 5 and 6 all have intuitive signs and are highly significant, except for market-to-book ratio of equity, which is insignificant.

Thus, although the empirical analysis provided in this subsection does not represent a full empirical test of our dynamic capital structure theory, it produces evidence that supports the model predictions. Firms with high ratios

Table 6
Debt maturity and firm characteristics

	Dependent variable:					
	Fraction of short-term debt over total liabilities					
	(1)	(2)	(3)	(4)	(5)	(6)
RSZ_MeanBC	−0.070***			−0.051***	−0.004	−0.030**
	(0.013)			(0.013)	(0.012)	(0.014)
EbitVola		0.442***		0.440***	0.102***	0.078***
		(0.014)		(0.014)	(0.014)	(0.015)
PastIncomeTaxes			−0.001	−0.0002	0.001*	0.001
			(0.001)	(0.001)	(0.001)	(0.001)
Assets					−0.030***	−0.044***
					(0.0005)	(0.001)
Leverage					0.246***	0.249***
					(0.006)	(0.007)
Tangible					−0.038***	−0.029***
					(0.004)	(0.005)
Profitability					−0.060***	−0.051***
					(0.009)	(0.010)
Market-to-book equity					−0.00000	−0.00000
					(0.00003)	(0.00003)
Constant	0.220***	0.179***	0.211***	0.186***	0.342***	0.411***
	(0.002)	(0.001)	(0.001)	(0.002)	(0.005)	(0.006)
Observations	27,466	27,466	27,466	27,466	27,466	22,572
R^2	.001	.036	.0001	.036	.205	.214
Adjusted R^2	.001	.035	.000	.036	.205	.213
Residual SE	0.177 (dof = 27,464)	0.174 (dof = 27,464)	0.178 (dof = 27,464)	0.174 (dof = 27,462)	0.158 (dof = 27,457)	0.165 (dof = 22,563)
F-statistic	28.059***	1,011.257***	1.606	342.412***	886.699***	766.653***
	(dof = 1; 27,464)	(dof = 1; 27,464)	(dof = 1; 27,464)	(dof = 3; 27,462)	(dof = 8; 27,457)	(dof = 8; 22,563)

This table presents the results from linear regressions of the fraction of short-term debt over total liabilities on bankruptcy costs, cash flow volatility and the corporate tax rate. Bankruptcy cost estimates, denoted by *RSZ_MeanBC*, are provided by Reindl, Stoughton, and Zechner (2017), aggregated for the Fama and French 17 industries. Cash flow volatility, denoted by *ebitVola*, is proxied by the standard deviation of firms' EBIT over total assets. Finally, tax rates, denoted by *pastIncomeTaxes*, are proxied by past income taxes over EBIT, averaged over the past 8 years. The additional control variables in columns 5 and 6 are assets, defined by the log of total book assets; leverage, defined by the sum of short-term and long-term debt over total book assets; tangible, defined by property plants and equipment over total book assets; profitability, defined as operating income over total book assets; and market-to-book equity, defined by the ratio of market value of stocks outstanding over book value of equity. *p <.1; **p <.05; ***p <.01.

of short-term debt reduce leverage more aggressively than firms with more long-term debt, and we provide support for the predicted nonmonotonicity between leverage and subsequent debt reductions. Furthermore, firms' cash flow volatilities are positively related to their usage of short-term debt, whereas their bankruptcy costs are associated with longer debt maturities. While these results support our model, some of the findings also may be consistent with other debt maturity theories. We therefore believe that a detailed and fully-fledged empirical analysis of the documented strong link between debt maturity, delevering, and firm characteristics is a promising direction for future research.

6. Conclusions

This paper explores the effects of debt maturity on subsequent dynamic capital structure adjustments. We find that long debt maturities create agency costs in states where the firm's profitability is low, since equityholders have no incentive to reduce debt. Thus, equityholders of firms with long-term debt maturities âŁ˜âŁ˜underinvest" in leverage reductions in low-profitability states. By contrast, short debt maturities imply a higher fraction of maturing debt, thereby creating more flexibility to reduce leverage in bad states without transferring value to debtholders and thereby mitigating the debt overhang problem. This value-enhancing effect of short debt maturities must be traded off against increased transaction costs associated with refinancing a larger fraction of debt in any given period. This trade-off generates a new theory of optimal debt maturity.

We find that the equityholders' incentives to engage in debt reductions is nonmonotonic in the firm's leverage. For moderate drops in the firm's profitability, equityholders find it in their own best interest to repay maturing debt at least partly with equity, thereby mitigating the leverage increasing effect of decreasing cash flows. However, if the firm's cash flows continue to drop until it is pushed toward bankruptcy, then equityholders resume issuing new debt and gamble for resurrection.

Ex ante, the debt capacity of the firm increases if it uses debt with sufficiently short maturity. We find that high costs of bankruptcy induce a stronger incentive to use short-term debt since this reduces the expected probability of bankruptcy for given debt level. Higher tax shields caused by a higher corporate tax rate also makes shorter-term debt more advantageous, since increased debt capacity associated with short-term debt allows for a better utilization of debt tax shields. Since long-term debt is stickier in downturns, it has a particularly adverse effect on the probability of bankruptcy for higher risk firms. Firms with higher cash flow risk therefore prefer shorter debt maturities.

In our numerical examples, the leverage-reducing effect of short debt maturities in bad states leads to relatively high initial leverage ratios and low bankruptcy probabilities. This is a direct consequence of the perfectly granular maturity structure that we assume in our model. Any lumpiness in firms' debt maturities would limit the leverage and reduce the effects that we identify to discrete points in time, whereas leverage would still remain sticky in between maturity dates. Thus, exploring the effects of lumpy debt maturities on initial leverage, leverage dynamics, and bankruptcy probabilities appears to be a promising direction for future research.

Our main findings and comparative statics accord well with empirical studies and with our own evidence presented in this paper. Most importantly, firms with high ratios of short-term debt reduce leverage more aggressively in downturns; some evidence points to the predicted nonmonotonicity between leverage and subsequent debt reductions; and firms' cash flow volatilities are positively related to their use of short-term debt, whereas bankruptcy costs are associated with longer debt maturities. Other empirical predictions of our theory, such as the effects of growth or transaction costs on maturity, remain to be tested. Also, a fully-fledged empirical test of how our theory of corporate debt maturity stands up against alternative explanations would be interesting.

A. Appendix

A.1 Derivation of Equation (4)

The inverse leverage ratio with respect to the unlevered firm value, y_t, depends on two state variables, the cash flow of the firm's productive assets, c_t, and the current face value of debt, B_t. Thus one can write $y_t = y(c_t, B_t)$. If the debt level is adjusted by repurchasing all existing debt with face value B_t and issuing new debt with face value B_t^*, the leverage ratio immediately jumps to the new value; that is, in this case we have

$$dy_t = \left(\frac{1}{B_t^*} - \frac{1}{B_t}\right) \frac{c_t}{r(1-\tau_p)-\hat{\mu}}, \tag{A1}$$

and therefore

$$\frac{dy_t}{y_t} = \frac{B_t}{B_t^*} - 1. \tag{A2}$$

In the absence of a discrete adjustment, the inverse leverage ratio, y_t, follows a diffusion and its dynamics can be determined using a Taylor series expansion and Itô's lemma

$$dy_t = \frac{\partial y}{\partial c} dc + \frac{\partial y}{\partial B} dB + \frac{1}{2}\left(\frac{\partial^2 y}{\partial c^2}(dc)^2 + \frac{\partial^2 y}{\partial B^2}(dB)^2\right) + \frac{\partial^2 y}{\partial c \partial B} dc dB. \tag{A3}$$

Neglecting all terms that are $o(dt)$ gives

$$\begin{aligned} dy_t &= \frac{1}{B_t} \frac{1}{r(1-\tau_p)-\hat{\mu}} c_t(\mu dt + \sigma dW_t) \\ &\quad - \frac{1}{B_t^2} \frac{c_t}{r(1-\tau_p)-\hat{\mu}} (-(m-\delta) B_t dt) \\ &= y_t\big((\hat{\mu} + (m-\delta))dt + \sigma dW_t\big) \end{aligned} \tag{A4}$$

for $0 \leq \delta \leq m$.

A.2 Proof of Proposition 1

For a given time-invariant, B-homogeneous capital structure strategy S of the form (7), we can apply the theorem of Feynman and Kač (see, e.g., Øksendal 2003) and transfer valuation equations (5) and (6) into partial differential equations that the value functions of equity and debt must satisfy for $y \in [\underline{y}, \overline{y}]$,

$$\begin{aligned} r(1-\tau_p)E \;=\; & \underbrace{\frac{1}{2}\sigma^2 y^2 \frac{\partial^2 E}{\partial y^2} + (\hat{\mu}+(m-\delta))y\frac{\partial E}{\partial y}}_{\text{exp. instantaneous growth in } E \text{ driven by the dynamics of } y} + \underbrace{\frac{\partial E}{\partial t}}_{\text{explicit time dependence}=0} \\ & + \underbrace{\frac{\partial E}{\partial B}(\delta-m)B}_{\text{inst. growth driven by the change in } B} - \underbrace{(i(1-\tau_c)+m)B}_{\text{expenses for coupons and amortization}} \\ & + \underbrace{(1-k_i)\delta D}_{\text{proceeds from issuing new debt}} + \underbrace{(r(1-\tau_p)-\hat{\mu})yB}_{\text{after-tax cash flow from operations}}, \end{aligned} \tag{A5}$$

$$\begin{aligned} r(1-\tau_p)D \;=\; & \underbrace{\frac{1}{2}\sigma^2 y^2 \frac{\partial^2 D}{\partial y^2} + (\hat{\mu}+(m-\delta))y\frac{\partial D}{\partial y}}_{\text{exp. instantaneous growth in } D \text{ driven by the dynamics of } y} \\ & + \underbrace{\frac{\partial D}{\partial t}}_{\text{explicit time dependence}=0} + \underbrace{\frac{\partial D}{\partial B}(\delta-m)B}_{\text{inst. growth driven by the change in } B} \\ & + \underbrace{B_t(i(1-\tau_p)+m)}_{\text{cash flow from coupon and amortization}} - \underbrace{\delta D}_{\text{expenses for buying new issues}}. \end{aligned} \tag{A6}$$

At the reorganization thresholds $\underline{y}$ (default) and $\overline{y}$ (debt repurchase), boundary conditions (9), (10), (12), and (13) must be satisfied. Assume, all market participants correctly anticipate the firm's capital structure strategy S. To avoid arbitrage, the expected return of equity under the risk-neutral probability measure (right-hand side of Equation (A5)) must equal the riskless (after personal tax) interest. The expected return (under the risk-neutral measure) consists of the instantaneous expected growth in the value of the equity claim (terms one to three) plus the net cash flow to equity (remaining terms). For given capital structure strategy, the firm's debt level changes at a rate $-(m-\delta(y))$, with m the contracted rate of amortization and $\delta(y)$ the chosen reissuance schedule. Consequently, the drift rate of the inverse leverage ratio y_t is $\hat{\mu}+(m-\delta_t)$, which runs into the first term on the right-hand side of (A5). The second term vanishes since there is no explicit time dependence of E. The third term constitutes the expected change in the value of equity driven by changes in the debt level B. The remaining terms on the left-hand side of (A5) characterize the flow of cash to equity, expenses for coupon payment and debt amortization, proceeds from reissuing new debt, and the after-tax cash flow from operations, c, which is written as function of the state variables B and y according to Equation (2), $c=r(1-\tau_p)-\hat{\mu})yB$. Equation (A6) follows the same logic for the value of debt, D. The net cash flows to debtholders are characterized by the last two terms on the right-hand side and consists of coupons and debt amortization as well as of expenditures of debtholders for the purchase of new debt issues (at market price). We confirm in Proposition 1, 3, and 4 that a B-homogeneous capital structure strategy S (see Equation (7)) results in value functions that are linear-homogeneous in the debt level B. Thus, we substitute

$E(B,y)=B\tilde{E}(y)$ and $D(B,y)=B\tilde{D}(y)$ into Equations (A5) and (A6). This yields Hamilton Jacobi Bellman Equations (8) and (11) for equity value and debt value per unit of face value of debt, $\tilde{E}(y)$ and $\tilde{D}(y)$, respectively.

A.3 Proof of Proposition 2

In this section we derive a Markov perfect Nash equilibrium capital structure strategy where investors price debt based on rational beliefs about the firm's debt rollover, default and recapitalization decisions and, given these prices, firms have no incentive to deviate from the conjectured capital structure strategy. There is no precommitment, except that firms must repurchase the existing debt (i.e., eliminate a debt covenant) before they can issue new debt with a higher face value.Proposition 1 states the system of valuation equations for a given strategy S. From there we proceed in two steps. First, we determine the optimal equilibrium rollover rate δ when bankruptcy and recapitalication thresholds $\underline{y}$ and $\overline{y}$ are given. In a second step, we develop optimality conditions for the optimal choice of these thresholds. The choice of the initial leverage $\acute{y}$ and the maturity amortization m is done from the perspective of total firm owners. Optimality conditions are discussed in Sections 2.2 and 2.3. In the first step, we formulate the dynamic game in which investors price debt and equity based on rational beliefs about the firm's optimal capital structure. In the resultant Markov perfect Nash equilibrium, the conjectured optimal strategy results in market values of debt and equity that do not create incentives for the firm to deviate. See, for example, the chapter on optimal stochastic control in Björk (2004) or the chapter on stochastic differential games in Dockner et al. (2000). Let us define the differential operator as

$$\mathcal{A}^{\delta}=\frac{1}{2}\sigma^2 y^2\frac{\partial^2}{\partial y^2}+(\hat{\mu}+[m-\delta])y\frac{\partial}{\partial y}.$$

Conjecturing that equityholders apply the optimal rollover δ^* for given restructuring thresholds $\underline{y}$ and $\overline{y}$, we determine the resultant values of debt and equity must satisfy the following simultaneous system of equations.

$$\begin{aligned}0 &= \sup_{\delta(y)}\Big\{\mathcal{A}^{\delta}\tilde{E}(y)-(r(1-\tau_p)+[m-\delta(y)])\tilde{E}(y)+(1-k_i)\delta(y)\tilde{D}(y)\\ &\qquad -(i(1-\tau_c)+m)+(r(1-\tau_p)-\hat{\mu})y\Big\}\\ &\qquad\qquad \text{(equity valuation / strategy optimization)} \qquad \text{(A7)}\end{aligned}$$

$$\begin{aligned}\text{s.t.}\quad & (9),(10),\\ \delta^* &\in \operatorname{argsup}_{\delta(y)}\Big\{\mathcal{A}^{\delta}\tilde{E}(y)-(r(1-\tau_p)+[m-\delta(y)])\tilde{E}(y)+(1-k_i)\delta(y)\tilde{D}(y)\\ &\qquad -(i(1-\tau_c)+m)+(r(1-\tau_p)-\hat{\mu})y\Big\}\\ &\qquad \ldots\text{sup in (A7) is attained at } \delta^*,\text{ pointwise} \quad \text{(verification)} \qquad \text{(A8)}\end{aligned}$$

$$0 = \mathcal{A}^{\delta}\tilde{D}(y)+(i(1-\tau_p)+m)-(r(1-\tau_p)+m)\tilde{D}(y) \quad \text{(debt valuation)} \qquad \text{(A9)}$$

s.t. (12), (13).

Thus, if $\tilde{E}$ and $\tilde{D}$ derived under anticipation of the optimal rollover schedule $\delta^*(y)$ satisfy verification (A8), then $\tilde{E}$ and $\tilde{D}$ are the value functions of equity and debt in equilibrium, see Björk (2004). The first-order condition of optimal choice of δ in (A7) is derived by differentiating the expression in the brackets with respect to δ, which yields $-y\frac{\partial\tilde{E}}{\partial y}(y)+\tilde{E}(y)-(1-k_i)\tilde{D}(y)$.

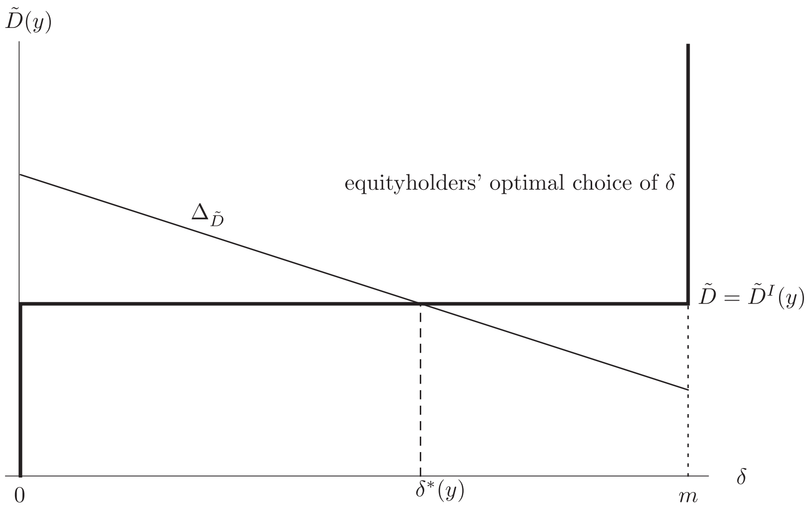

Figure A.1
Stable interior equilibrium
The bold line plots equityholders' optimal rollover choice from Equation (15). The downward-sloping line illustrates the marginal response $\Delta_{\tilde{D}}$. With downward sloping $\Delta_{\tilde{D}}$, the equilibrium is robust against perturbations in δ.

Hence, the derivative of δ is positive if and only if $\tilde{D}(y)$ exceeds the critical threshold $\tilde{D}^I(y)$ stated in Proposition 2. Consequently, $\delta(y)=m$ is the optimal choice of the rollover rate in this case. If $\tilde{D}(y)<\tilde{D}^I(y)$, the derivative is negative and, thus, $\delta(y)=0$ is the optimal choice. Only if $\tilde{D}(y)=\tilde{D}(y)^I$, equityholders are indifferent in their choice of $\delta(y)$. This completes the proof of Proposition 2.

A.4 Proof of Proposition 3

While for $\tilde{D}(y)<\tilde{D}^I(y)$ and $\tilde{D}(y)>\tilde{D}^I(y)$ the choices $\delta(y)=0$ and $\delta(y)=m$ are stable equilibria (robust to perturbations in δ) we have to determine conditions under which an interior choice of δ is a stable Nash equilibrium in the sense that small perturbations in δ will not push the equilibrium to one of the boundary points. Figure A.1 illustrates a stable interior equilibrium. The bold line indicates equityholders' optimal choice of δ contingent on the value of debt relative to the critical threshold $\tilde{D}^I$, as stated by Proposition 2. Interior optima arise at states y, where neither $\delta=0$ nor $\delta=m$ are feasible equilibrium solutions. The marginal reaction of the market value of debt to perturbations in the rollover rate can be derived as follows. Consider a state y with $0<\delta^*<m$ and increase debt rollover from the optimum equilibrium rollover δ^* to some δ for a time span of length dt. The resultant change in state y equals $\Delta_y=-(\delta-\delta^*)y\,dt$, according to the dynamics of y in (4). And consequently, the marginal impact on the debt value (through controlling y by perturbing δ) is

$$\Delta_{\tilde{D}}=-\frac{\partial \tilde{D}}{\partial y}(\delta-\delta^*)dt. \tag{A10}$$

That is, a positive perturbation in δ results in a marginal decrease in $\tilde{D}$ if and only if $\frac{\partial \tilde{D}}{\partial y}>0$, which serves as a necessary condition for a stable interior equilibrium.

One essential result of analyzing the interior equilibrium is that equityholders are indifferent to the particular choice of δ, and their indifference allows for an analytical solution of the entire problem. Since equityholders are indifferent, equity value in a region with interior choice of δ^*

can be determined by simply setting $\delta = 0$ in (8). Value of debt follows directly from $\tilde{D}(y) = \tilde{D}^I(y)$ and the particular value for δ^* is determined by substitution of $\tilde{D}(y) = \tilde{D}^I(y)$ into Equation (11) and solving for δ. Finally we conclude that in an interior equilibrium equity must be convex. Since $\tilde{D}(y) = \tilde{D}^I(y)$, we have $\frac{1-k_i}{y}(\partial \tilde{D}/\partial y) = (\partial^2 \tilde{E}/\partial y^2) > 0$, since stability of the equilibrium requires $(\partial \tilde{D}/\partial y) > 0$.

A.5 Proof of Proposition 4

In regions at which $\delta^* = m$ or $\delta^* = 0$, the value function for $\tilde{E}$ and $\tilde{D}$ are the general solutions of the second-order ordinary differential equations (8) and (11) after substituting $\delta^* = m$ or $\delta^* = 0$. In a region of an interior equilibrium $0 < \delta^* < m$ we know from Proposition 3 that $\tilde{D} = \tilde{D}^I$. The value of equity must be the solution of the Hamilton-Jacobi-Bellman equation (8) with the optimal δ^* from Proposition 3 substituted for δ. After substitution, the valuation equation for equity becomes independent of δ and exactly equal to the valuation equation for δ st to zero. Therefore, the solution corresponds to the value function in regions with $\delta = 0$. This is so because in an internal equilibrium the value of equity is invariant to the choice of δ. The constants $E_{1,2}$ and $D_{1,2}$ are determined by boundary conditions (9), (10), (12), and (13), which apply at $\underline{y}$ and $\overline{y}$ and by value matching and smooth-pasting conditions at the interior boundary between the regions of full rollover, no rollover, and partial rollover (see, e.g., Dixit (1993)). These conditions state that if such a boundary, $\tilde{y}$, is transitory (i.e., the state variable y can freely move back and forth across the boundary along both directions), then the value functions of debt and equity must be continuous and smooth at $\tilde{y}$,

$$
\begin{aligned}
\lim_{y \to \tilde{y}^-} E(y) &= \lim_{y \to \tilde{y}^+} E(y), \\
\lim_{y \to \tilde{y}^-} \frac{\partial E(y)}{\partial y} &= \lim_{y \to \tilde{y}^+} \frac{\partial E(y)}{\partial y}, \\
\lim_{y \to \tilde{y}^-} D(y) &= \lim_{y \to \tilde{y}^+} D(y), \\
\lim_{y \to \tilde{y}^-} \frac{\partial D(y)}{\partial y} &= \lim_{y \to \tilde{y}^+} \frac{\partial D(y)}{\partial y}.
\end{aligned}
$$

These conditions yield a set of linear equations that determines all constants E and D in the value functions stated in Proposition 4.

A.6 Proof of Proposition 5

Suppose loss-given-default is less than 100%. This is the case if $g < 1$ and optimal bankruptcy occurs at a level $\underline{y}$ such that the value of the remaining assets exceeds bankruptcy costs. Then it follows from boundary condition (12) that $\tilde{D}(\underline{y}) > 0$. However, for the value of equity and its partial derivative of the inverse leverage ratio, it follows from boundary condition (9) and optimality condition (18) that

$$\lim_{y \to \underline{y}} \tilde{E}(y) = \lim_{y \to \underline{y}} \frac{\partial \tilde{E}(y)}{\partial y} = 0.$$

Therefore, in a neighborhood of $\underline{y}$ it is true that

$$\tilde{D}(y) > \frac{1}{1-k_i}\left(y \frac{\partial \tilde{E}}{\partial y}(y) - \tilde{E}(y)\right).$$

According to Proposition 2, Equation (15) this implies that $\delta = m$ is the optimal strategy.

References

Acharya, V. V., D. Gale, and T. Yorulmazer. 2011. Rollover risk and market freezes. *Journal of Finance* 66:1177–209.

Admati, A. R., P. M. DeMarzo, M. F. Hellwig, and P. Pfleiderer. 2018. The leverage ratchet effect. *Journal of Finance* 73:145–98.

Altman, E. I. 1984. A further empirical investigation of the bankruptcy cost question. *Journal of Finance* 39:1067–89.

Anderson, R. W., and S. Sundaresan. 1996. Design and valuation of debt contracts. *Review of Financial Studies* 9:37–68.

Andrade, G., and S. N. Kaplan. 1998. How costly is financial (not economic) distress? evidence from highly leveraged transactions that became distressed. *Journal of Finance* 53:1443–93.

Ang, J. S., J. H. Chua, and J. J. McConnell. 1982. The administrative costs of corporate bankruptcy: A note. *Journal of Finance* 37:219–26.

Benzoni, L., L. Garlappi, R. S. Goldstein, J. Hugonnier, and C. Ying. 2020. Optimal debt dynamics, issuance costs, and commitment. Working paper, Federal Reserve Bank of Chicago.

Björk, T. 2004. *Arbitrage theory in continuous time*. 2 ed. New York: Oxford University Press.

Blouin, J., J. E. Core, and W. Guay. 2010. Have the tax benefits of debt been overestimated? *Journal of Financial Economics* 98:195–213.

Brunnermeier, M. K., and M. Oehmke. 2013. The maturity rat race. *Journal of Finance* 68:483–521.

Calomiris, C. W., and C. M. Kahn. 1991. The role of demandable debt in structuring optimal banking arrangements. *American Economic Review* 81:497–513.

Chaderina, M. 2018. Rollover risk and the dynamics of debt. Working paper, University of Oregon.

Chen, H., R. Cui, Z. He, and K. Milbradt. 2017. Quantifying liquidity and default risks of corporate bonds over the business cycle. *Review of Financial Studies* 3:852–97.

Chen, H., Y. Xu, and J. Yang. 2020. Systematic risk, debt maturity, and the term structure of credit spreads. *Journal of Financial Economics*. Advance Access published September 18, 2020, 10.1016/j.jfineco.2020.09.002.

Cheng, I.-H., and K. Milbradt. 2012. The hazard of debt: Rollover freezes, incentives, and bailouts. *Review of Financial Studies* 25:1070–110.

Childs, P. D., D. C. Mauer, and S. H. Ott. 2005. Interactions of corporate financing and investment decisions: The effect of agency conflicts. *Journal of Financial Economic* 79:667–90.

Choi, J., D. Hackbarth, and J. Zechner. 2018. Corporate debt matruity profiles. *Journal of Financial Economics* 130:484–502.

Christensen, P. O., C. R. Flor, D. Lando, and K. R. Miltersen. 2014. Dynamic capital structure with callable debt and debt renegotiation. *Journal of Corporate Finance* 29:644–61.

Dangl, T., and J. Zechner. 2004. Credit risk and dynamic capital structure choice. *Journal of Financial Intermediation* 13:183–204.

DeAngelo, H., A. S. Gonçalves, and R. M. Stulz. 2018. Corporate deleveraging and financial flexibility. *Review of Financial Studies,* 31:3122–74.

DeMarzo, P., and Z. He. Forthcoming. Leverage dynamics without commitment. *Journal of Finance*.

Diamond, D. W. 1991. Debt maturity structure and liquidity risk. *Quarterly Journal of Economics* 106:709–37.

———. 1993. Seniority and maturity of debt contracts. *Journal of Financial Economics* 33:341–68.

Diamond, D. W., and Z. He. 2014. A theory of debt maturity: The long and short of debt overhang. *Journal of Finance* 69:719–62.

Diamond, D. W., and R. G. Rajan. 2001. Liquidity risk, liquidity creation, and financial fragility: A theory of banking. *Journal of Political Economy* 109:287–327.

Dixit, A. 1993. *The art of smooth pasting*. Reading; UK: Harwood Academic Publishers.

Dockner, E. J., S. Jørgensen, N. Van Long, and G. Sorger. 2000. *Differential games in economics and management science*. Cambridge, UK: Cambridge University Press.

Dumas, B. 1991. Super contact and related optimality conditions. *Journal of Economic Dynamics and Control* 15:675–85.

Ericsson, J. 2000. Asset substitution, debt pricing, optimal leverage and optimal maturity. *Finance* 21:39–69.

Fischer, E. O., R. Heinkel, and J. Zechner. 1989. Dynamic capital structure choice: Theory and tests. *Journal of Finance* 44:19–40.

Flannery, M. J. 1986. Asymmetric information and risky debt maturity choice. *Journal of Finance* 41:19–37.

———. 1994. Debt maturity and the deadweight cost of leverage: Optimally financing banking firms. *American Economic Review* 84:320–31.

Geelen, T. A. 2016. Debt maturity and lumpy debt. Working paper, Copenhagen Business School.

Glover, B. 2016. The expected cost of default. *Journal of Financial Economics* 119:284–99.

Goldstein, R., N. Ju, and H. Leland. 2001. An EBIT-based model of dynamic capital structure. *Journal of Business* 74:483–512.

Gomes, J., U. Jermann, and L. Schmid. 2016. Sticky leverage. *American Economic Review* 106:3800–28.

Graham, J. R. 1996a. Debt and the marginal tax rate. *Journal of Financial Economics* 41:41–73.

———. 1996b. Proxies for the corporate marginal tax rate. *Journal of Financial Economics* 42:187–221.

Graham, J. R., and C. R. Harvey. 2001. The theory and practice of corporate finance: evidence from the field. *Journal of Financial Economics* 60:187–243.

Graham, J. R., and M. T. Leary. 2011. A review of empirical capital structure research and directions for the future. *Annual Review of Financial Economics* 3:309–45.

Graham, J. R., and L. F. Mills. 2008. Using tax return data to simulate corporate marginal tax rates. *Journal of Accounting and Economics* 46:366–88.

Guedes, J., and T. Opler. 1996. The determinants of the maturity of corporate debt issues. *Journal of Finance* 51:1809–33.

He, Z., and K. Milbradt. 2014. Endogenous liquidity and defaultable bonds. *Econometrica* 82:1443–508.

———. 2016. Dynamic debt maturity. *Review of Financial Studies* 29:2677–736.

He, Z., and W. Xiong. 2012a. Dynamic debt runs. *Review of Financial Studies* 25:1799–843.

———. 2012b. Rollover risk and credit risk. *Journal of Finance* 67:391–429.

Hovakimian, A., T. Opler, and S. Titman. 2001. The debt-equity choice. *Journal of Financial and Quantitative Analysis* 36:1–24.

Ju, N., R. Parrino, A. M. Poteshman, and M. S. Weisbach. 2005. Horses and rabbits? optimal dynamic capital structure from shareholders and manager perspectives. *Journal of Financial and Quantitative Analysis* 40:1129–65.

Jungherr, J., and I. Schott. 2020a. Optimal debt maturity and firm investment. Working Paper, University of Bonn.

———. 2020b. Slow debt, deep recessions. Working Paper, University of Bonn.

Lehar, A. 2018. Restructuring failure and optimal capital structure. Working Paper, University of Calgary.

Leland, H., and D. Hackbarth. 2019. Debt maturity and the leverage ratchet effect. *Finance* 40:13–44.

Leland, H. E. 1994a. Corporate debt value, bond covenants, and optimal capital structure. *Journal of Finance* 49:1213–52.

———. 1994b. Bond prices, yield spreads, and optimal capital structure with default risk. Working Paper, University of California, Berkeley.

———. 1998. Agency costs, risk management, and capital structure. *Journal of Finance* 53:1213–43.

Leland, H. E., and K. B. Toft. 1996. Optimal capital structure, endogeneous bankruptcy, and the term structure of credit spreads. *Journal of Finance* 51:987–1019.

Mao, L., and Y. Tserlukevich. 2015. Repurchasing debt. *Management Science* 61:1648–62.

Mella-Barral, P. 1999. The dynamics of default and debt reorganization. *Review of Financial Studies* 12:535–78.

Mella-Barral, P., and W. Perraudin. 1997. Strategic debt service. *Journal of Finance* 52:531–56.

Myers, S. C. 1977. Determinants of corporate borrowing. *Journal of Financial Economics* 5:147–75.

Øksendal, B. 2003. *Stochastic differential equations*. 6 ed. Berlin, Germany: Springer.

Reindl, J., N. Stoughton, and J. Zechner. 2017. Market implied costs of bankruptcy. Working Paper, BI Norwegian Business School.

Strebulaev, I. 2007. Do tests of capital structure theory mean what they say? *Journal of Finance* 62:1747–87.

Titman, S., and S. Tsyplakov. 2007. A dynamic model of optimal capital structure. *Review of Finance* 11:401–51.

Warner, J. B. 1977. Bankruptcy, absolute priority, and the pricing of risky debt claims. *Journal of Financial Economics* 4:239–76.

Weiss, L. A. 1990. Bankruptcy resolution. *Journal of Financial Economics* 27:285–314.

Welch, I. 2004. Capital structure and stock returns. *Journal of Political Economy* 112:106–30.