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*DOI:*  
[10.57938/09919517-c844-446f-a107-2ddabeab9e3f](https://doi.org/10.57938/09919517-c844-446f-a107-2ddabeab9e3f)

*Published:* 01/05/2022

*Document Version:*  
Publisher's PDF, also known as Version of record

*Document License:*  
Unspecified

[Link to publication](#)

*Citation for published version (APA):*  
Kaszab, L., Marsal, A., & Rabitsch, K. (2022). *Asset Pricing with Costly and Delayed Firm Entry*. WU Vienna University of Economics and Business. Department of Economics Working Paper Series No. 325  
<https://doi.org/10.57938/09919517-c844-446f-a107-2ddabeab9e3f>

Department of Economics  
Working Paper No. 325

# Asset Pricing with Costly and Delayed Firm Entry

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May 2022



# Asset Pricing with Costly and Delayed Firm Entry\*

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## Abstract

Survey evidence tells us that stock prices reflect the risks investors associate with long-run technological change. However, there is a shortage of models that can rationalise long-run risks. Unlike the previous literature assuming a fixed number of products our model allows for new product varieties that appear in the form of new firms which face entry costs and delay in the entry process. The fixed variety model has a significant limitation in translating macroeconomic volatility into asset return volatility. Our model with growing varieties induces endogenous low-frequency fluctuations in productivity

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\*We are grateful to seminar participants at NOEG 2020, Dynare 2019, CEF 2018, EcoMod 2018, ICMAIF 2018, MMF 2017 conferences, Cardiff Business School, Hungarian Economics Society, Banco de Mexico and Central Bank of Hungary. Special thanks to Martin Andreasen, Gianluca Benigno, Tamas Briglevics, Alessia Campolmi, Michael Donadelli, Peter Gabriel, Max Gillman, Patrick Grüning, Henrik Kucsera, Robert Lieli, Lorenzo Menna, Patrick Minford, Victoria Nuguer, Panayiotis Pourpurides, Eyno Rots, David Staines, Balazs Vilagi, Abraham Vella and Mike Wickens for comments.

driving large, persistent variations in consumption growth and asset prices. It also changes the valuation of assets through the increase in the volatility of the pricing kernel (with a positive long-run component) and leads to higher excess returns. Our model is motivated with a simple recursively identified VAR model containing quarterly US data 1992Q3-2019Q4 with the following list of variables: total factor productivity, consumption, a measure of firm entry, and the excess return on stocks.

JEL: E13, E31, E43, E44, E62.

Keywords: firm entry, equity premium, Epstein-Zin, New Keynesian.

Recent macro-finance literature emphasizes that stock-market investors mainly care about risks associated with low frequency fluctuations such as long-run technological change (see e.g. Dew-Becker and Giglio (2016)). However, there is a shortage of models that can rationalise these long-run risks, and which can jointly match macroeconomic and financial data. The earlier literature with a fixed number of goods suffer from what Li and Palomino (2014) call the 'translation problem': macroeconomic volatility translates into asset return volatility to a very limited extent even when payoffs are calibrated to be as volatile as the data. We depart from the earlier literature with a fixed number of goods. In our model asset payoffs are, implicitly, based on an expanding variety of goods. A new variety is associated with a new firm which is subject to a sunk entry cost and a time-to-build lag in production as in Bilbiie et al. (2007, 2012). We show that uncertainty about variety (firm) growth leads to endogenous fluctuations in productivity creating long-run risks which are reflected in higher and more volatile excess returns. In line with the long-run risk literature agents in our model price future risks due to their preference for early resolution of uncertainty (i.e. Epstein-Zin preferences) meaning that they associate risks with variation in expected future consumption and leisure.

We motivate our model with a simple empirical exercise. In the spirit of Etro and Colciago (2010) we calculate the Solow residual as the measure of the technology shock from US data 1992Q3-2019Q4<sup>1</sup>. We use a VAR with four lags and identify the technology shock in the standard recursive way and

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<sup>1</sup>To calculate quarterly TFP we use the standard neoclassical production function (without including the variety effect) which contains the capital stock that is only available at annual frequency. We linearly extrapolate capital stock to quarterly frequency using the Denton-Cholette method which is standard in statistical packages. We assume that

variables are ordered as total factor productivity (TFP), real per capita personal consumption expenditure, an indicator of net business formation (the difference between firm birth and death), and one-year excess returns on stocks.<sup>2</sup> All variables are logged and detrended with a second-order polynomial. This ordering is motivated by our theoretical model and is also similar to the ordering in Savagar (2021) and Dixon and Savagar (2020). Unlike previous papers we have a macro-financial focus and also include the excess stock return in the VAR.

Figure (1) shows the responses of variables to one standard deviation shock in TFP. The vertical axis measures variables in percentage deviation from the trend. The excess return is annualised. Time is measured in quarters on the horizontal axis. The response of TFP, consumption and firm entry is similar to the ones in Savagar (2021). In line with our theoretical impulse responses the excess return is always positive on impact, although with considerable uncertainty—see the 95 percent confidence intervals bootstrapped with 10 000 replications. The rise in TFP induces a rise in consumption. It further leads to future profit opportunities which induces firm entry and higher stock returns. Our results are robust to alternative orderings of the variables and different number of lags. A constant is included in the regression.

We contribute to the macro-finance literature on long-run risks by showing that technology-driven uncertainty about variety growth induce persistent variation in expected consumption growth and asset prices. According to Bansal and Yaron (2004) three features are necessary for a macro-finance model to be successful at generating long-run risks: i) persistent expected consumption growth component, ii) significant variation in expected consumption growth, and iii) preference for an early resolution of uncertainty. We show that the firm entry model satisfies all the previous requirements for long-run risks to emerge. Our calibration of the model ensures that Epstein-Zin preferences lead to an early resolution of uncertainty so that requirement iii) is easily satisfied. i) and ii) are discussed below.

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capital share in production is 0.36 implying that the labour’s share is 0.64. Our measure of labour is non-farm business sector hours that are available at quarterly frequency. All variables are expressed in real terms using GDP implicit price deflator. We used US population aged 15-64 to express variables per capita.

<sup>2</sup>The TFP shock is ordered first and affects all variables contemporaneously. The usual lag length selection criteria (e.g. Akaike and Schwarz) suggest 2 or 3 lags but we include 4 lags which is more usual for quarterly data.

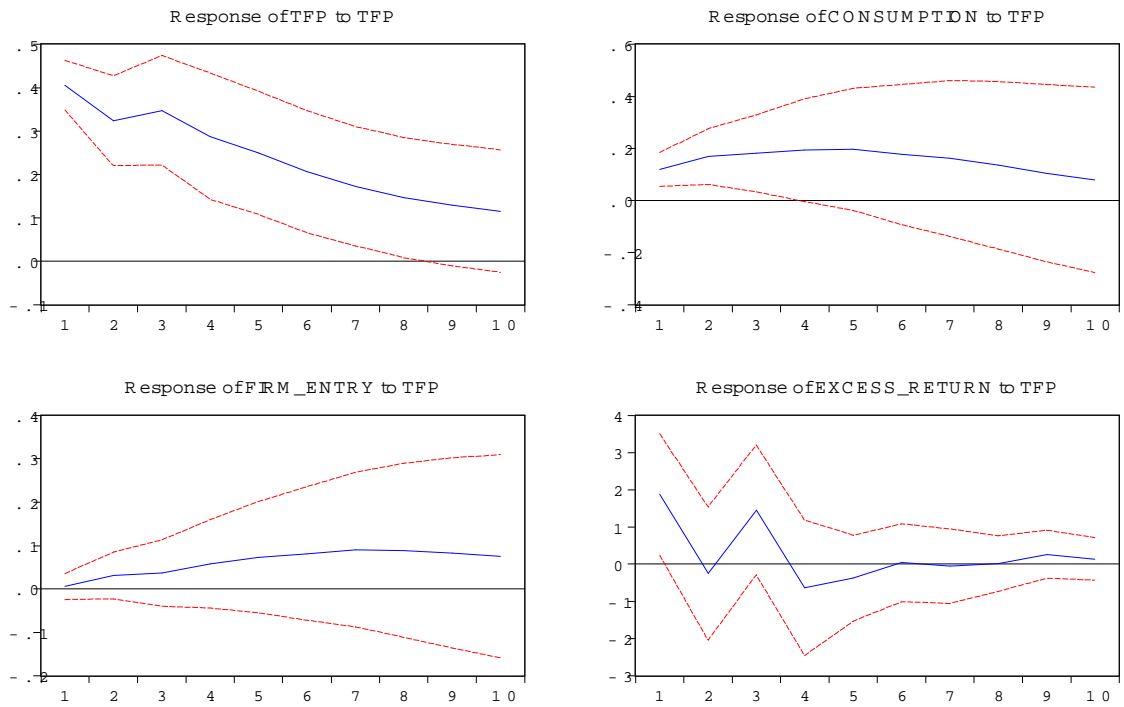


Figure 1: Impulse responses of variables to a one-standard deviation rise in technology from a recursive VAR(4). Notes: variables are measured as percentage deviation from the trend. The excess return on stocks is annualised. 95 percent confidence intervals are bootstrapped with 10000 replications.

Technology-induced uncertainty about variety growth and the fact that new products are available with delay due to barriers to firm entry drive persistent and significant variation in expected consumption growth in our model and, hence, i) is satisfied. Specifically, bad supply shocks in the entry model imply low current and future consumption. Indeed, the entry model produces positive autocorrelation in consumption growth and, hence, a positive correlation between current and expected consumption growth. Whereas the autocorrelation of consumption growth in the noentry model is virtually zero, and variables exhibit fast mean reversion following shocks. For instance, negative supply shocks in the no entry model depress current consumption but are good news for future consumption. In entry model, however, negative shocks are bad news to current and future consumption as well. To further highlight sources of long-run risks we show that expected consumption growth resulting from the entry model endogenously is similar to the exogenous process used by Bansal and Yaron (2004). Specifically, we fit an AR(1) process to expected consumption growth from the entry model, and show that its persistence and variation is similar to the exogenous growth process in Bansal and Yaron (2004). Our choice of utility implies the labour is procyclical amplifying the propagation of technology shocks and increasing the comovement of returns with output and consumption.

Uncertainty about firm entry leads to a change in asset valuations: there is extra volatility in the pricing kernel due to the comovement of its short- and long-run components<sup>3</sup>. In particular, we follow Li and Palomino (2014) and decompose the stochastic discount factor into short- and a long-term components. The short-term component is related to current consumption and labour income. The long-term component is linked to future consumption and labour income. In the entry model bad shocks to current consumption (labour income) is also bad news for future consumption (labour income). Unlike the noentry model where the short- and long-run components move in opposite directions firm entry induces them to comove positively making the pricing kernel more volatile, and leading to a higher price of risk on consumption and labour income claims.

Statistical agencies rarely adjust the consumption basket to include new products. At the beginning of the product cycle a new product is introduced

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<sup>3</sup>In an analytically solveable version of our model i.e. without entry costs, timing frictions and using separable preferences in consumption and labour one can show that the growth rate of varieties appear in the pricing kernel after removing the variety effect from consumption.

at a high price but cheaper and better versions appear through time putting downward pressure on the model-based price index that contains the variety effect (see also Scanlon (2019) for similar argument). To compare model moments to data moments we remove the variety effect from each model-based variable in line with Bilbiie et al. (2007). Importantly, our model does not generate excess variation in the consumption growth without variety. Further, the uncertainty about variety growth induces higher precautionary savings keeping the risk-free rate low and stable.

The return on the wealth portfolio (composed of the consumption and labour income claims) is unobservable and, hence, we match the excess return on the market portfolio based on S&P 500 firms. Aggregate dividends are linked to aggregate output and, therefore, exhibit a procyclical pattern<sup>4</sup>. The return on dividends can be decomposed into dividend growth and changes in the price-dividend ratio. We find that the entry model better matches the standard deviation of dividend growth and the price-dividend ratio. Overall the entry model matches sixty percent of the excess return on the aggregate dividend claim. Whereas the mean of the excess return is close to zero in noentry model. The firm entry model, thus, successfully addresses the translation problem, and captures all of the standard deviation in the excess return while the noentry model fits only a small fraction of it.

Related literature. Our model can be further motivated empirically from four directions of the literature. First, the paper by Broda and Weinstein (2010) uses product bar codes and confirms the findings of Bernard et al. (2010) on the importance of the high share of new products in total production. Further, they document that product creation is strongly procyclical at a business cycle frequency. Second, studies in industrial organisation point to the importance of entry costs and other barriers for firms to enter an industry (see e.g. Corhay et al. (2015)). Third, Scanlon (2019) shows evidence on the procyclical nature of brand growth and calibrates an endowment economy featuring brand growth to US data. He also uses the endowment model in the long-run risk setting of Bansal and Yaron (2004) and points to its merits in explaining asset prices. Fourth, Croce (2014) provides empirical evidence on the positive connection between long-run productivity risk and excess returns.

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<sup>4</sup>Due to price stickiness dividends are also affected by markups and price adjustment costs. The latter react, however, in an offsetting ways in response to technology shocks and, thus, the dividends are mainly governed by output.



Different from the earlier literature where long-run risks emerge endogenously in a production model with capital accumulation (see e.g. Kaltenbrunner and Lochstoer (2010) and Croce (2014)) our model features the 'accumulation' of firms. Our model is similar to the free entry model (no entry costs and timing frictions) of Kaszab et al. (2022) in the sense that firm entry implies more volatility in the pricing kernel and leads to an endogenous component of productivity which magnifies consumption risk. The free entry model, however, amplifies short-run risks and cannot engineer long-run risk in a setup with Epstein-Zin curvature.

Our paper aligns with Corhay et al. (2020) who focus on the joint explanation of time-varying markups and excess returns. Their model features both product and process innovation. In particular, their product innovation part is based on the firm-entry model of Bilbiie et al. (2007) while their process innovation is driven by the endogenous growth model of Kung and Schmidt (2015). Different from our model where markups are time-varying due to price-setting frictions, the Corhay et al. (2020) model contains oligopolistic competition whereby the markup (and the elasticity of substitution among varieties) depends on the number of firms. We are also related to Scanlon (2019) where he considers the asset pricing implications of new products (product groups and brands). The payoff streams in his models are exogenously specified whereas they are endogenous in our models. We use the pricing kernel decomposition of Li and Palomino (2014) whose model is different from ours in many aspects. In particular, long-run risks arise in their model due permanent technology shocks and wage rigidity. Whereas long-run risks in our setup are due to temporary technology shocks and firm entry.

The paper proceeds as follows. Section one contains description of the firm entry model and asset pricing. Section two explains how the model is parameterised. Section three presents impulse responses to a technology shock, discusses model features necessary to produce long-run risks, and compares moments from the model simulations to equivalent statistics from US data. Finally we conclude.

## 1 Entry-exit model with frictions

We start with the description of the production sector. We follow Bilbiie et al. (2007) in that firms pay a sunk entry cost which is expressed in consumption units. To make the mechanisms transparent we abstract from

physical capital in the model, and use labour as the only input of production as in Li and Palomino (2014).

## 1.1 The intermediary firm's problem

There is a mass of firms. Firm  $\omega$  employs labour,  $l_t(\omega)$ , in order to produce output,  $y_t(\omega)$ , using a constant-return-to-scale technology:  $y_t(\omega) = Z_t l_t(\omega)$  where  $Z_t$  is a stationary productivity shock:

$$\log Z_t = \rho_Z \log Z_{t-1} + \varepsilon_t^Z,$$

where  $\varepsilon_t^Z$  is an independently and identically distributed (iid) stochastic technology disturbance with mean zero and variance  $\sigma_Z^2$ . The unit cost of production is the real marginal cost which is given by  $w_t/Z_t$  where  $w_t \equiv W_t/P_t$  is the real wage.

There is also a mass of prospective entrants. Firms pay an entry cost of  $f_E$  in consumption units. Each period firms correctly anticipate their future profits and negative profits induce exit with probability  $\delta$ . The model features a time-to-build lag in the sense that firms entering at time  $t$  start to produce one period later. Therefore, the number of firms producing at period  $t$ ,  $N_t$ , is described by:

$$N_t = (1 - \delta)(N_{t-1} + N_{E,t-1}) \tag{1}$$

where  $N_E$  stands for new entrants and both new entrants and incumbents survive with probability  $1 - \delta$ .

The connection between relative prices and the number of firms (varieties):

$$\rho_t \equiv \frac{p_t}{P_t} = N_t^{1/(\theta-1)} \tag{2}$$

where  $1/(\theta - 1)$  is net markup and also captures love of variety.

Adjusting prices is costly for intermediary goods producing firms. Hence, nominal rigidity is introduced in the form of price adjustment costs that can be described with a quadratic function as in Rotemberg (1982):

$$pac_t(\omega) = \frac{\phi_P}{2} \left[ \frac{p_t(\omega)}{p_{t-1}(\omega)} - 1 \right]^2 p_t(\omega) y_t(\omega)$$

where  $\phi_P$  governs the strength of price adjustment costs. The real profits of firm  $\omega$  at time  $t$  (transferred back to households in the form of dividends) is

given by:

$$d_t(\omega) = \frac{p_t(\omega)}{P_t} y_t(\omega) - w_t l_t(\omega) - \frac{pac_t(\omega)}{P_t}. \quad (3)$$

Firms face a death shock occurring with probability  $\delta \in (0, 1)$  in each period. In each period firm  $\omega$  continues with probability  $1 - \delta$  and maximises profits by choosing  $p_t(\omega)$  optimally:

$$\max_{p_t(\omega)} E_0 \left\{ \sum_{t=0}^{\infty} [\beta(1 - \delta)]^t \frac{MUC_{t+1}}{MUC_t} d_t(\omega) \right\}, \quad (4)$$

with respect to equations (2), (3), and (5).  $MUC$  is the marginal utility of consumption (defined below in equation (10)). Optimal choice of the firm leads to the pricing condition:

$$\rho_t = \mu_t \frac{w_t}{Z_t}$$

where  $\mu_t$  denotes the gross markup function which is time-varying due to sticky prices:

$$\mu_t \equiv \frac{\theta}{(\theta - 1) \left(1 - \frac{\phi_P}{2} \pi_t^2\right) + \phi_P \pi_t (1 + \pi_t) - \beta E_t \{\Upsilon_{t+1}\}}$$

with  $\Upsilon_{t+1} \equiv (1 - \delta) M_{t,t+1} \phi_P \pi_{t+1} \frac{Y_{t+1}}{Y_t} \frac{N_t}{N_{t+1}} (1 + \pi_{t+1})$

where  $\pi_t \equiv \log(p_t/p_{t-1}) = \log(\Pi_t)$ . In the zero inflation steady-state ( $\pi = 0$ ) the gross markup is given by  $\mu \equiv \theta/(\theta - 1)$ .  $M$  is the stochastic discount factor (defined below in equation (9)).

## 1.2 The household's problem

The representative household consumes a continuum of goods defined over the measure  $N_t$ :  $C_t = \left( \int_0^{N_t} \tilde{c}_t(\omega)^{\frac{\theta-1}{\theta}} d\omega \right)^{\frac{\theta}{\theta-1}}$  where the elasticity of substitution among goods is constant and given by  $\theta > 1$ . The nominal price of a particular good  $\omega$  is denoted as  $p_t(\omega)$ . The aggregation of individual goods results in the welfare-based (CPI) price index:  $P_t = \left( \int_0^{N_t} p_t(\omega)^{1-\theta} d\omega \right)^{\frac{1}{1-\theta}}$ . The household chooses the basket of goods optimally for given expenditure.

The optimality condition yields the household's demand for each individual good  $\omega$ :

$$y_t(\omega) = \left( \frac{p_t(\omega)}{P_t} \right)^{-\theta} (C_t + PAC_t). \quad (5)$$

where  $PAC_t \equiv N_t pac_t$ . The representative household maximises the continuation value of its utility ( $V$ ) which has Epstein-Zin form:

$$V_t = U_t^{1-\sigma} + \beta \left[ E_t V_{t+1}^{\frac{1-\gamma}{1-\sigma}} \right]^{\frac{1-\sigma}{1-\gamma}}, \quad (6)$$

with respect to its flow budget constraint (derived below).  $\beta \in (0, 1)$  is the subjective discount factor. Utility ( $U$ ) at period  $t$  is derived from consumption ( $C_t$ ) and leisure ( $1 - L_t$ ) and is of the non-separable form proposed by Greenwood et al. (1988):

$$U(C_t, L_t) = \left[ \frac{\left( C_t - \chi \frac{L_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right)^{1-\sigma}}{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \quad (7)$$

In the expression (7)  $\sigma$  is related to the inverse of the intertemporal elasticity of substitution (IES) as  $\sigma^{-1}/(1 + \varphi)$ , where  $\varphi$  is the Frisch elasticity of labour supply to wages.  $\chi > 0$  pins down hours worked in steady-state. The connection between the coefficient of relative risk-aversion ( $CRRA$ ) and parameter  $\gamma$  of the recursive utility in equation (6) is given by (for derivations see the Online Appendix):

$$CRRA \simeq \left[ \sigma + \frac{\gamma - \sigma}{1 - \sigma} (1 - \sigma) \right] (1 + \varphi).$$

The previous expression tells us that  $CRRA$  increases which Frisch elasticity. With GHH utility the wealth effect of the technology shock on labour supply is eliminated and labour input becomes persistently procyclical. As a result, the propagation of the technology shock is stronger: consumption and dividends exhibit higher comovement and carry higher a price of risk. Hence, unlike separable preferences, labour does not have insurance property

in the case of bad shocks.<sup>5</sup> Hence, more flexible labour supply ( $\varphi$  higher) implies more risks (asset returns more procyclical) and households shows higher aversion to risks (CRRA is higher).

Households possess two types of assets: shares in a mutual fund of firms and government bonds. Let  $x_t$  denote the share in the mutual fund of firms entering period  $t$ . In each period the mutual fund pays the representative household the total profit (in units of currency) of all firms that produce in that period,  $P_t N_t d_t$ . In period  $t$  the representative household purchases  $x_{t+1}$  shares in a mutual fund of  $N_{H,t} \equiv N_t + N_{E,t}$  firms where the first term refer to firms already operating at time  $t$  while the second term stands for the new entrants. Only  $N_{t+1} = (1 - \delta)N_{H,t}$  firms will produce and pay dividends at time  $t + 1$ . As the household has no information about which  $\delta$  share of firms are induced to leave the market at the end of period  $t$ , it finances the continuing operation of all preexisting firms and all new entrants during period  $t$ . The nominal price of a claim to the future profit stream of the mutual fund of  $N_{H,t}$  firms at time  $t$  equals to  $S_t^{ex,nom} \equiv P_t S_t^{ex}$  where the superscript *ex, nom* marks the ex-dividend price in nominal terms. At time  $t$  the representative household holds nominal bonds and a share  $x_t$  in the mutual fund. It receives labour income,  $W_t L_t$ , interest income  $i_{t-1}$  on nominal bonds. The share in the mutual fund brings dividend income (in nominal terms),  $D_t^{nom} \equiv P_t d_t$ , plus the current nominal value of selling the shares,  $S_t^{ex,nom}$ . Therefore, the period budget constraint of the representative household (in units of currency) can be written as:

$$B_{N,t+1} + S_t^{ex,nom} N_{H,t} x_{t+1} + P_t C_t = (1 + i_{t-1}) B_{N,t} + (D_t^{nom} + S_t^{ex,nom}) N_t x_t + W_t L_t + T_t^L$$

where  $D_t^{nom}$  stands for the nominal value of dividends,  $D_t^{nom} \equiv P_t d_t$ ,  $1 + i_t$  is the gross nominal interest rate and  $T_t^L$  are lump-sum taxes in nominal terms.

Let PPI and CPI inflation rates be denoted, respectively, by

$$\begin{aligned} \Pi_{t+1} &= 1 + \pi_{t+1} = \frac{p_{t+1}}{p_t} \\ \Pi_{t+1}^C &= 1 + \pi_{t+1}^C = \frac{P_{t+1}}{P_t} \end{aligned}$$

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<sup>5</sup>In our baseline calibration with  $\sigma < 1$  labour would be procyclical even with separable preferences. Separable preferences would, however, lead to fast mean-reversion in labour following technology shocks reducing the overall procyclicality in the economy in the medium- and long-term mitigating long-run risks.

where  $\pi_t$  and  $\pi_t^C$  are the net while  $\Pi_t$  and  $\Pi_t^C$  are the gross PPI and CPI inflation rates, respectively. Next we list the optimality conditions of the households.

The intratemporal condition is given by:

$$w_t = \chi L_t^{1/\varphi}. \quad (8)$$

The bond Euler equation:

$$1 = E_t \left( M_{t,t+1} \frac{1 + i_t}{\Pi_{t+1}^C} \right),$$

where the stochastic discount factor is given by:

$$M_{t,t+1} = \beta \frac{MUC_{t+1}}{MUC_t} \left( \frac{V_{t+1}^{1/(1-\sigma)}}{\tilde{V}_t^{1/(1-\gamma)}} \right)^{\sigma-\gamma} \quad \text{where } \tilde{V}_t \equiv E_t V_{t+1}^{\frac{1-\gamma}{1-\sigma}} \quad (9)$$

and

$$MUC_t \equiv \left( C_t - \chi \frac{L_t^{1+\frac{1}{\varphi}}}{1 + \frac{1}{\varphi}} \right)^{-\sigma}. \quad (10)$$

The share Euler equation can be written as:

$$1 = (1 - \delta) E_t \left( M_{t,t+1} \frac{S_{t+1}^{ex} + d_{t+1}}{S_t^{ex}} \right),$$

where  $S_t^{ex} \equiv S_t^{ex,nom}/P_t$  is the real ex-dividend share price. Firm-level dividends follow:

$$d_t = \left( 1 - \frac{1}{\mu_t} - \frac{\phi_P}{2} \pi_t^2 \right) \frac{Y_t}{N_t}.$$

The value of the firm is linked to the entry cost (free entry condition) as:

$$S_t^{ex} = f_E.$$

The free entry condition helps to pin down the number of firms in equilibrium.

The connection between PPI and CPI inflation comes from the variety effect:

$$\frac{\Pi_t}{\Pi_t^C} = \left( \frac{N_t}{N_{t-1}} \right)^{1/(\theta-1)}. \quad (11)$$

### 1.3 Monetary Policy

Monetary policy is described by a simple Taylor rule of the form:

$$R_t = \frac{1}{\beta}(\Pi_t)^{\phi_\pi}, \quad (12)$$

where  $R_t \equiv 1 + i_t$  is the gross nominal interest rate and inflation is zero in the steady-state.

### 1.4 Equilibrium

In the symmetric equilibrium all firms make identical choices so that  $p_t(\omega) = p_t$ ,  $d_t(\omega) = d_t$ ,  $y_t(\omega) = y_t$ ,  $S_t^{ex}(\omega) = S_t^{ex}$ ,  $l_t(\omega) = l_t$ ,  $\mu_t(\omega) = \mu$  and  $pac_t(\omega) = pac_t$ . Net bond holdings are zero  $B_t = B_{t+1} = 0$  and  $x_t = x_{t+1} = 1$ .

The labour market clearing is given by:

$$L_t = N_t l_t. \quad (13)$$

The aggregate production function can be derived as:

$$\begin{aligned} C_t + PAC_t &= Y_t = N_t \rho_t y_t \\ &= N_t \rho_t Z_t l_t \\ &= \rho_t Z_t L_t \end{aligned}$$

The last row contains the aggregate production function, and the term  $\rho_t$  can be interpreted as an endogenous component of productivity. Derivations and the full list of equilibrium conditions can be found in the online appendix.

### 1.5 Asset prices

In this subsection we show how we price claims on various cash-flows such as consumption, output, dividends and labour income.

The real cum-dividend price of a cash flow  $\{\mathcal{Q}_{t+s}\}_{s=0}^\infty$  is given by

$$S_t^{\mathcal{Q}} \equiv E_t \left[ \sum_{s=1}^{\infty} M_{t,t+s} \mathcal{Q}_{t+s} \right]$$

Note that we price a claim on consumption ( $\mathcal{Q}_t = C_t$ ), labour income ( $\mathcal{Q}_t = LI_t = \frac{W_t}{P_t} L_t$ ), etc. The stochastic discount factor is model-specific, and is used to price the cash-flow of various claims.

For aggregate dividends ( $Q_t = AD_t = N_t d_t$ ) the pricing kernel need to be multiplied with  $1 - \delta$  to consider continuing firms only. Hence, the previous expression in the case of aggregate dividends can alternatively be written recursively as:

$$S_t^{AD} = AD_t + (1 - \delta)E_t\{M_{t,t+1}S_{t+1}^{AD}\},$$

where  $S_t^{AD}$  is the cum-dividend price of the claim. The previous expression can be written in Euler equation form as:

$$\begin{aligned} 1 &= E_t \left[ M_{t,t+1} \frac{(1 - \delta)S_{t+1}^{AD}}{S_t^{AD} - AD_t} \right] \\ &= E_t M_{t,t+1} R_{AD,t+1} \end{aligned}$$

where the return on the  $AD$  claim is given by:

$$R_{AD,t+1} \equiv \frac{(1 - \delta)S_{t+1}^{AD}}{S_t^{AD} - AD_t} = \frac{(1 - \delta)P_{t+1}^{AD}}{P_t^{AD} - 1} \frac{AD_{t+1}}{AD_t} \quad (14)$$

where  $P_t^{AD} \equiv S_t^{AD}/AD_t$  is the price-dividend ratio. Our measure of equity risk premium is the levered excess return based on aggregate dividends:

$$EQPR_t = \phi_{lev}(\ln R_{AD,t} - \ln R_{f,t-1}),$$

where the excess return is leveraged (see the leverage factor  $\phi_{lev}$ ) as in Croce (2014). Aggregate dividends are linked to aggregate output and, hence, follows its procyclical pattern.

## 1.6 The adjusted wealth portfolio

We follow Li and Palomino (2014) and use the so-called return representation of the pricing kernel to identify its short- and long-run components. The return representation includes the return on the adjusted wealth portfolio in the pricing kernel instead of the continuation value of utility which is difficult to interpret. In particular, the cash-flow of the adjusted wealth portfolio (containing the consumption and labour income claims) is linked to current and future consumption and labour income streams.

Let  $Q_t$  denote cash flow for the claim on the adjusted wealth portfolio:

$$Q_t \equiv C_t - \frac{LI_t}{1 + \frac{1}{\varphi}}. \quad (15)$$



The SDF in equation (9) can be written in its return form as (for derivations see the online appendix):

$$M_{t,t+1} = E_t \left[ \beta \left( \frac{MUC_{t+1}}{MUC_t} \right)^{-1} \right]^\vartheta (R_{Q,t})^{\vartheta-1}. \quad (16)$$

where  $\vartheta \equiv \frac{1-\gamma}{1-\sigma}$ . We take the natural log of equation (16) and identify the short- and long-run components as in Li and Palomino (2014):

$$m_{t,t+1} = \vartheta \ln \beta + E_t m_{t,t+1}^{SR} + E_t m_{t,t+1}^{LR} \quad (17)$$

where

$$m_{t,t+1}^{SR} \equiv -\vartheta \Delta muc_{t+1} - (1 - \vartheta) \Delta q_{t+1} \quad (18)$$

$$m_{t,t+1}^{LR} \equiv -(1 - \vartheta) \ln \left( \frac{P_{t+1}^Q}{P_t^Q - 1} \right) \quad (19)$$

where  $\Delta muc_{t+1} = \ln(\Delta MUC_{t+1})$ ,  $\Delta q_{t+1} = \ln(\Delta Q_{t+1})$ , and  $R_{Q,t}$  is decomposed into the log of price-cash-flow ratio and the log of the growth rate of  $Q$  (similar to equation (14)). Hence, the short-run component,  $m_{t,t+1}^{SR}$  contains the log of consumption and labour income growth while the long-run component is related to the log of price-cash-flow ratio on the adjusted wealth portfolio.

The price-cash-flow ratio and the return on the  $Q$  claim is, however, related to the discounted future utility from future consumption and labour income streams. In the entry model bad shocks to current consumption and labour income are also bad news for future consumption and labour income implying long-run risks. Hence, the short- and long-term components comove positively with firm entry resulting in more volatile pricing kernel, and a higher price of risk on the consumption and labour income claims.

## 1.7 Closed-form illustration

To understand determinants of the excess return on the adjusted wealth portfolio we assume that  $M_{t,t+1}$  and  $R_{Q,t+1}$  are jointly conditionally lognormally distributed and derive an analytical expression for the excess return on claim

$Q$  as follows (for derivations see the online appendix<sup>6</sup>):

$$\begin{aligned}
& \ln E_t R_{Q,t+1} - \ln R_{f,t} + \frac{1}{2} [std_t(\ln E_t R_{Q,t+1})]^2 \\
&= -cov_t(m_{t,t+1}, \ln R_{Q,t+1}) \\
&= \underbrace{-cov_t(m_{t,t+1}, \Delta q_{t+1})}_{\simeq corr_t(m_{t,t+1}, m_{t+1}^{SR}) std_t(m_{t,t+1}) std_t(m_{t+1}^{SR})} + \\
&\quad \underbrace{-cov_t(m_{t,t+1}, \ln(1 + P_{Q,t+1}))}_{\simeq corr_t(m_{t,t+1}, m_{t+1}^{LR}) std_t(m_{t,t+1}) std_t(m_{t+1}^{LR})}
\end{aligned} \tag{20}$$

where  $R_{f,t}$  is the one-period gross real risk-free rate satisfying  $1/R_{f,t} = E_t M_{t,t+1}$ ,  $m_{t,t+1} = \ln M_{t,t+1}$  and  $q_t = \ln(Q_t)$ . In the first row of equation (20)  $\frac{1}{2} [std_t(\ln E_t R_{Q,t+1})]^2$  is the usual Jensen's inequality term that pops up in lognormal approximations. The last row of equation (20) decomposes the covariances into correlations and standard deviations as well as makes use of expressions (18) and (19). As long as the correlation between  $m$  and  $m^{LR}$  (or, equivalently, the correlation between  $m^{SR}$  and  $m^{LR}$  as  $m = m^{SR} + m^{LR}$ ) is positive the excess return has a component that compensates for long-run risks. Below we show that the long-run component positively comoves with the short-run component and the whole pricing kernel in two ways: i) by presenting impulse responses, and ii) unconditional correlations of  $m^{LR}$  with  $m^{SR}$  and  $m$  as well as standard deviations.

## 2 Parametrisation

Table (1) contains the benchmark parameterisation. For a given value of  $\varphi$ ,  $\sigma$  is chosen such that it implies an  $IES > 1$  as in the long-run risk literature<sup>7</sup>. The size and persistence of the productivity shock are chosen to fit the standard deviation of consumption growth in the data and is reasonably close to that in Bilbiie et al. (2007).  $\delta$ ,  $\theta$  and  $f_E$  follows Bilbiie et al. (2007).  $\phi_{lev}$  is from Croce (2014).  $CRRA$  follows Li and Palomino (2014). Our parametrisation implies early resolution of uncertainty ( $\gamma > 1/\sigma$ ) as in Bansal and Yaron

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<sup>6</sup>To obtain closed-form solution we assume that consumption and labour are separable. However, the impulse responses and simulated data in Tables (2)-(4) are calculated with the baseline GHH specification.

<sup>7</sup>See Vissing-Jorgensen and Attanasio (2003) who estimate the IES to be higher than one for stockholders.

(2004). Our results are not sensitive to the choice of  $f_E$ .  $\varphi$  lies in between the value estimated by micro (lower than one see e.g. Pistaferri (2003)) and macro studies (typically higher than one as in Bilbiie et al. (2007)). Our choice of  $\phi_P$  implies a Calvo parameter of 0.85 as in Christiano et al. (2011)<sup>8</sup>.

Table 1: Parametrisation

Notation	Description	Entry-exit	No entry
$\sigma$	Consumption curvature	0.2	0.2
$\varphi$	Frisch elasticity	1.5	1.5
$CRRA$	Coeff. of Rel. Risk Aversion	16	16
$\beta$	Discount factor	0.99	0.99
$\bar{L}$	Steady-state hours worked	0.33	0.33
$f_E$	Sunk entry cost	1.00	–
$\delta$	Firm exit rate	0.025	0.025
$\theta$	El. of substitution	3.80	3.80
$\phi_P$	Price adj. cost	100	100
$\phi_{lev}$	Leverage factor	2.00	2.00
$\phi_\pi$	Reaction to inflation	1.50	1.50
$\rho_Z$	AR(1) of techn.	0.979	0.979
$\sigma_Z$	Std. dev. of techn. innovation	0.00367	0.00295

## 3 Results and discussion

### 3.1 Impulse responses

To explain the mechanisms in the entry model we consider a positive productivity shock which induces the entry of new firms due to positive profit opportunities. The impact of positive technology shock to macro and finance variables is shown on figure (2) and (3), respectively.

Figure (2) shows that a positive technology shock triggers firm entry leading to the growth in the log of the number of firms ( $n$ ) and a rise in the log

<sup>8</sup>We calculate  $\phi_P$  from the Calvo parameter by equating the slopes of the Phillips curves from the linearised Calvo and the Rotemberg models, respectively:  $(1 - calvo)(1 - calvo\beta)/calvo = (\theta - 1)/\phi_P$  where  $calvo$  is the probability of not resetting the price.

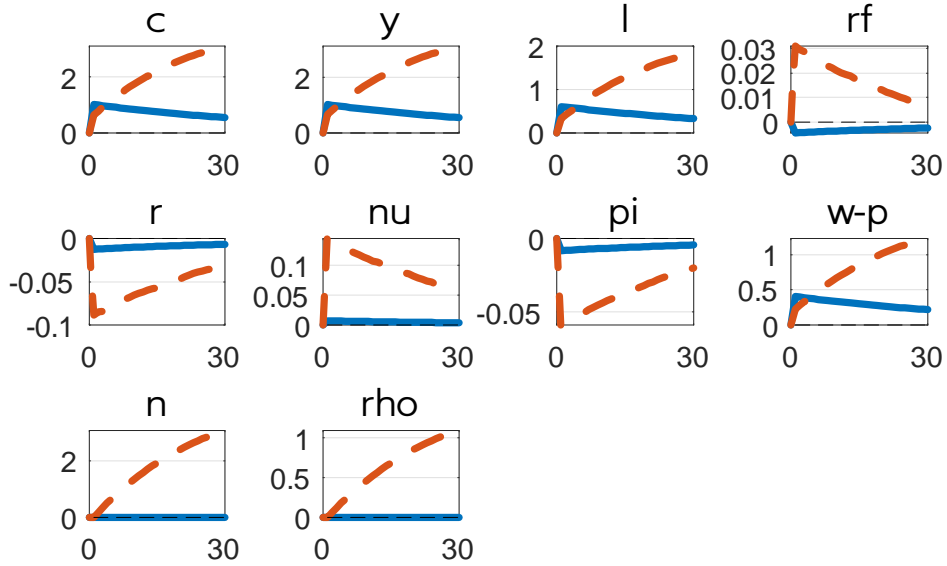


Figure 2: The response of macroeconomic variables to a positive technology shock. Notes: the responses from the no entry (solid) and entry (dashed) models are displayed. Vertical axes measure percentage deviation of the variables from the steady-state. The log of nominal and risk-free real interest rates ( $r$  and  $r_f$ ), and the inflation ( $\pi$ ) are annualised.

of relative prices ( $\rho$ )<sup>9</sup>. The positive wealth effect of the productivity boosts consumption ( $c$ ), output ( $y$ ) and labour ( $l$ ) as well with similar rising pattern over time. Figure (3) presents that the log of labour income ( $li$ ) and aggregate dividends ( $ad$ ) both expand after a positive technology shock. Investors require higher return on consumption and labour income claims as well as the adjusted wealth portfolio (see variables  $r_C$ ,  $r_{LI}$ , and  $r_Q$ , respectively, on Figure (3)) also react more in the entry model. Further, the persistent rise in expected consumption, labour income and aggregate dividends imply long-run risks. Next we explain how long-run risk emerges in the entry model.

### 3.2 Long-run risks

We show the response of the log of the pricing kernel ( $m$ ) as well as its short-run and long-run components (see  $m^{SR}$  and  $m^{LR}$ , respectively) to a positive

<sup>9</sup>Note that these two variables are not present in the no entry model.

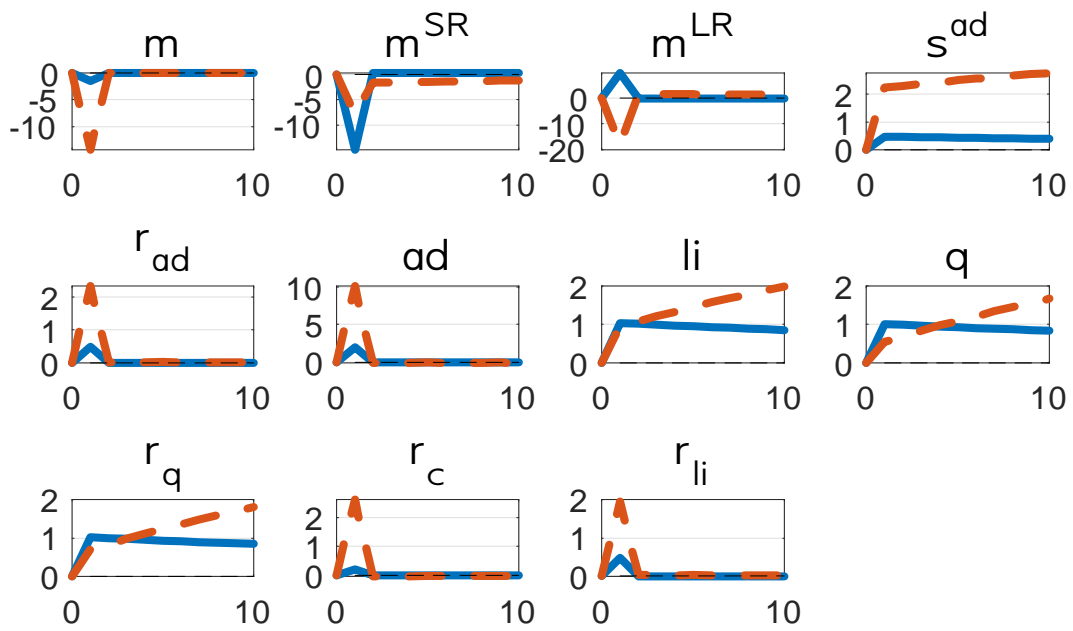


Figure 3: The response of financial variables to a positive technology shock. Notes: the responses from the no entry (solid) and entry (dashed) models are displayed. Vertical axes measure percentage deviation of the variables from the steady-state. Log-returns ( $r_q$ ,  $r_c$ ,  $r_{li}$ , and  $r_{ad}$ ) are annualised.

	Entry-exit	No entry
$\sigma(m)$	0.14	0.02
$\sigma(m^{SR})$	0.09	0.07
$\sigma(m^{LR})$	0.15	0.04
$corr(m^{SR}, m^{LR})$	0.51	-1.00
$corr(m, m^{LR})$	0.93	-0.99

Table 2: The Decomposition of the Pricing Kernel

Notes: This table contains unconditional correlations and standard deviations from the log of the pricing kernel ( $m$ ) which is decomposed into short- and long-run parts (see  $m^{SR}$  and  $m^{LR}$ , respectively) in the spirit of Li and Palomino (2014).

TFP shock on figure (3). The sign of the  $m^{LR}$  component differs across the entry and noentry models. The  $m^{SR}$  and  $m^{LR}$  components move in the same direction in the entry model generating excess movement in  $m$  as well as the returns. Hence, the long-term component of the excess return in equation (20) is positive only in the entry model. This can be explained as follows.

In endowment economy models such as Bansal and Yaron (2004) with long-run risks the short- and long-run components are driven by separate shocks. In our production models technology shocks contribute to both the short- and long-run components. Hence, the stochastic discount factor is not only driven by the volatility of each of the components but their correlation (see equation (20)) is also important to determine the volatility of the pricing kernel ( $m$ ) and the excess return. In the entry model shocks move the current and expected consumption as well as the short and long-components of the  $m$  in the same direction leading to long-run risks.

In table (2) we report the unconditional standard deviations and correlations between short- and long-run components of the pricing kernel as an approximation to the conditional ones in the decomposition above. The results confirm findings from the impulse responses: the short- and long-run components comove positively only in the case of the entry model marking the presence of long-run risks. Further, the standard deviations of  $m^{SR}$ ,  $m^{LR}$ , and  $m$  are all higher with increasing variety relative to the case of fixed variety.

Panel A			
	Data	Entry-exit	No entry
E(EQPR)	6.33	3.84	0.05
$\sigma(EQPR)$	19.42	19.46	2.85
Sharpe ratio	0.33	0.20	0.02
E(rf)	0.86	3.43	4.03
$\sigma(rf)$	0.97	0.50	0.07
Panel B			
	Data	Entry-exit	No entry
$\sigma(\Delta c)$	2.93	2.93	2.93
$AC1(\Delta c)$	0.49	0.32	0.00
$AC2(\Delta c)$	0.15	0.31	-0.00
$AC5(\Delta c)$	-0.08	0.27	-0.01
$\sigma(E[\Delta c])$	–	1.36	0.32
Panel C			
	Bansal-Yaron	Entry-exit	No entry
$\rho_x$	0.97	0.97	0.98
$\tilde{\sigma}_x$	0.50	0.35	0.06
$\sigma(E[\Delta c])/\sigma(\Delta c)$	0.34	0.46	0.11
$corr(E[\Delta c], \Delta c)$	0.34	0.67	-0.10

Table 3: Financial Moments and the Properties of Consumption Growth

Notes: In the rows E(.),  $\sigma(\cdot)$  and  $corr(\cdot, \cdot)$  refers to unconditional mean, standard deviation and correlations, respectively. AC1, 2, and 5 refer to first-, second-, and fifth-order auto-correlations, respectively. Panel A and B compare macro and finance moments based on US data 1929-1998 (taken from Bansal and Yaron (2004)) to simulated moments from the entry and no entry models. In Panel C we fit the expected consumption growth,  $E[\Delta c]$  from the entry and no entry models to an AR(1) process  $x_t = \rho_x x_{t-1} + \sigma_x \epsilon_{x,t}$ , where  $\epsilon_{x,t} \sim N(0, 1)$ , and compare to the exogenous consumption growth process in Bansal and Yaron (2004). We report the persistence parameter and the annualised volatility parameter,  $\tilde{\sigma}_x$ , from the fitted AR(1) process.

### 3.3 The fit to financial data and moments of the consumption growth

Table (3) contains selected macro and finance moments. The first column contains macroeconomic and finance data moments based on Bansal and Yaron (2004). Model moments are based on simulations for 10000 periods using third-order perturbation. The model moments are produced with the benchmark parameterisation in Table (1). Interest rates, returns, inflation, and consumption growth are all annualised percentages. Consistent with the method in Bilbiie et al. (2007) model-based macroeconomic and financial variables are taken logs, and transformed such that they do not include the variety effect and, hence, are comparable with data moments, see  $y_t \equiv \ln(P_t Y_t / p_t) = \ln(Y_t / \rho_t)$  and  $c_t \equiv \ln(C_t / \rho_t)$ , respectively. The table contains three panels. Panel A and B contain financial and macroeconomic moments, respectively<sup>10</sup>. Panel C compares statistics from the Bansal and Yaron (2004) model to our models. We start with the discussion of Panel A.

Panel A shows that the entry model generates sixty percent of empirical excess return with a volatility closely matching the data. Whereas the noentry model produces zero mean and little variation in the excess return. The Sharpe ratio from the entry model also shows improvement relative to the noentry but still lower than the data. The unconditional mean of the risk-free rate is somewhat in excess of the data while its standard deviations are lower than data. Importantly, our results are not coupled with excessively high mean and/or standard deviation of the risk-free rates so that our model is not subject to the risk-free rate puzzle.

Panel B shows that consumption growth in the entry model contains persistent predictable component unlike the no entry model. Both entry and no entry models are calibrated such that they reproduce the standard deviation of realised consumption growth ( $\sigma(\Delta c)$ ). Yet, only the entry model produces positive first-order autocorrelation of consumption growth. The latter, however, is close to zero in no entry model implying that consumption growth is i.i.d. and, hence, unpredictable. Whereas the low frequency movements in productivity induced by delayed firm entry engineers predictable movements in expected consumption growth which shows variation in the entry model (see  $\sigma(E[\Delta c])$ ) but is close to zero in the noentry model. On the downside

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<sup>10</sup>Table 5 in Appendix B contains further moments such as the standard deviation of output and labour as well as relative standard deviations and correlations.



we note, however, that the entry model overpredicts the autocorrelations at lags two and five.

Panel C<sup>11</sup> shows the coefficient and the standard error from an AR(1) process fitted to expected consumption growth which is an exogenous process in the endowment model of Bansal and Yaron (2004). While the estimated autoregressive parameters from our models,  $\rho_x$ , are close to identical to Bansal and Yaron we find that the estimated standard deviation,  $\tilde{\sigma}_x$  is somewhat larger in the entry model than their endowment model. The estimated standard deviation from the no entry model is close to zero implying that expected consumption growth exhibits zero variation in the noentry model. We further find that the relative standard deviation and correlation of expected consumption growth are also higher than the ones in Bansal and Yaron.

### 3.4 Properties of dividend growth, the price-dividend ratio and variety growth

Table (4) compares data and model moments on dividend growth, price-dividend ratio, and variety growth. Specifically, panel A compares statistics of dividend growth and the price-dividend ratio across the entry and no entry models to make the following observations. First, the standard deviation of aggregate dividend growth,  $\sigma(\Delta(ad))$ , is about half of what we see in the data. Second, the correlation between aggregate dividend and consumption growth,  $corr(\Delta(ad), \Delta c)$ , is overestimated by the entry model. Third, the entry model produces a positive autocorrelation in aggregate dividend growth,  $AC1(\Delta(ad))$ , that is slightly in excess of the data. The latter can still be interpreted as a success as the no entry model predicts that the autocorrelation of aggregate dividends is zero. Fourth, the standard deviation of the price-dividend ratio,  $\sigma(p - d)$ , is also better captured by the entry model but is still half of the data.

Panel B shows that the entry model has a persistence similar to the exogenous process in Bansal and Yaron ( $\rho_{xd}$ ) and somewhat higher standard deviation ( $\tilde{\sigma}_{xd}$ ). Further, we find that the relative standard deviation and correlation between expected and realised dividend growth is overestimated by the entry model to some extent.

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<sup>11</sup>Kung and Schmidt (2015) also explore the long-run risk properties of their model along the statistics reported in Panel C.

Finally, panel C assesses model fit to variety data from Scanlon (2019). He proxied the appearance of new products with US trademarks data 1927-2016. In particular, the standard deviation of product growth in the entry model is not distant from the data. Further, the confluence of future product growth and consumption as well as stock returns based on firm-level dividends is also well-captured by the entry model.

## 4 Conclusion

Stock market investors are mainly concerned about long-run risks. Instead of a fixed number of goods our framework assumes a growing number of products which appear in the form of new firms that face barriers to entry in the form of a sunk entry cost and timing frictions. Firm entry drives persistent movements in macroeconomic and financial variables such as output, consumption and dividends. With preferences for an early resolution of uncertainty long-risks about the future evolution in varieties are priced in the return of assets in the form of a risk-premia.

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Panel A			
	Data	Entry-exit	No entry
$\sigma(\Delta(ad))$	11.49	4.63	3.01
$AC1(\Delta(ad))$	0.21	0.34	0.00
$\sigma(E[\Delta(ad)])$	–	2.22	0.33
$corr(\Delta(ad), \Delta c)$	0.55	0.99	1.00
$E[P/D]$	3.28	4.42	4.60
$\sigma(p - d)$	0.29	0.14	0.02
$AC1(p - d)$	0.80	1.00	0.98
Panel B			
	Bansal-Yaron	Entry exit	No entry
$\rho_{xd}$	0.98	0.97	0.98
$\tilde{\sigma}_{xd}$	0.36	0.56	0.06
$\sigma(E[\Delta(ad)])/\sigma(\Delta(ad))$	0.24	0.48	0.11
$corr(E[\Delta(ad)], \Delta(ad))$	0.24	0.69	-0.10
Panel C			
	Data	Entry-exit	No entry
$\sigma(\Delta n)$	2.20	2.81	–
$corr(\Delta n, \Delta c)$	0.70	0.79	–
$corr(\Delta n, R_d)$	0.50	0.95	–

Table 4: Properties of Dividend Growth, the Price-Dividend Ratio and Variety Growth

Notes: notations are identical to Table (2). Panel A contains statistics related to aggregate dividend growth and the log of price-dividend ratio. In Panel B we fit the expected aggregate dividend growth,  $E[\Delta(ad)]$  from the entry and no entry models to an AR(1) process  $xd_t = \rho_{xd}xd_{t-1} + \sigma_{xd}\epsilon_{xd,t}$ , where  $\epsilon_{xd,t} \sim N(0, 1)$ , and compare to the exogenous dividend growth process in Bansal and Yaron (2004). We report the persistence parameter and the annualised volatility parameter,  $\tilde{\sigma}_{xd}$ , from the fitted AR(1) process. Panel C compares moments from the entry model to statistics estimated by Scanlon (2019) using US trademark data to proxy variety growth over 1927-2016.

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## 5 Appendix A—The empirical exercise

To motivate our model we run a recursively identified VAR(4) with the following ordering of the variables: TFP, consumption, stock return and firm entry. We use the following set of variables from St. Louis FRED database:

- HOANBS Nonfarm Business Sector: Hours of All Persons (index 2012)
- GDPC1 Real Gross Domestic Product. Billions of Chained 2012 Dollars, Seasonally Adjusted Annual Rate.
- GDPDEF GDP deflator base = 2012
- PCECC96 Real Personal Consumption Expenditures Billions of Chained 2012 Dollars, Seasonally Adjusted Annual Rate

Detrending variables. All data is in real terms and divided by US population. Per capital real variables are logged. In line with the cited papers we use a second-order polynomial to detrend each variables. Stock returns deliver the same results even without detrending. We check that detrended data do not feature a unit root with a ADF test. Hours worked is non-farm business hours and is available in quarterly terms. To construct TFP we need

quarterly time series for capital. We use annual capital data from BLS and use the Denton-Cholette method (linear extrapolation) to obtain quarterly data.

To construct the TFP residual from the production function we assume that the labour share is  $1 - \alpha = 0.64$ . After removing the variety effect the TFP is constructed in the usual way i.e. the residul from the production function:  $a_t = y_t - \alpha k_t - (1 - \alpha)l_t$ .

Quarterly net entry is based on BLS Business Employment Dynamics and calculated as firm births minus deaths starting in 1992. Capital is based on the Private Non-Farm capital services index 2012 downloaded from historical BLS accounts <https://www.bls.gov/mfp/mprdownload.htm#Capital%20Tables>.

To have quarterly stock returns we take a geometric average of monthly S&P 500 yields from Shiller's database. Excess return is the quarterly stock return minus the 3-month T-bill rate. Finally, our time window is restricted by the availability of net firm entry data. Our time window for the VAR exercise is then 1992Q3-2019Q4.

## 6 Appendix B—List of equilibrium conditions

### 6.1 The costly firm entry model

The entry model contains 24 variables:

$$U, V_t, \tilde{V}_t, MUC_t, M_{t,t+1}, Z_t, Y_t, C_t, L_t, N_t, N_{E,t}, d_t, AD_t, \rho_t, \mu_t, \\ MC_t, \Pi_t, \Pi_t^C, W_t/P_t, S_t^{AD}, R_t, R_{AD,t}, EQPR_t, R_{f,t}$$

and 24 equations listed below.

Recursive formulation for utility:

$$V_t = U_t^{1-\sigma} + \beta \left[ \tilde{V}_t \right]^{\frac{1-\sigma}{1-\gamma}}. \quad (21)$$

The utility function:

$$U(C_t, L_t) = \left[ \frac{\left( C_t - \chi \frac{L_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right)^{1-\sigma}}{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \quad (22)$$

Continuation value:

$$\tilde{V}_t \equiv E_t V_{t+1}^{\frac{1-\gamma}{1-\sigma}}. \quad (23)$$

The marginal utility of consumption:

$$MUC_t \equiv \left( C_t - \chi \frac{L_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right)^{-\sigma}. \quad (24)$$

The stochastic discount factor:

$$M_{t,t+1} = \beta \frac{MUC_{t+1}}{MUC_t} \left( \frac{V_{t+1}^{1/(1-\sigma)}}{\tilde{V}_t^{1/(1-\gamma)}} \right)^{\sigma-\gamma}. \quad (25)$$

Bond Euler equation:

$$1 = E_t \left[ M_{t,t+1} \frac{R_t}{\Pi_{t+1}^C} \right]. \quad (26)$$

Intratemporal condition:

$$\frac{W_t}{P_t} = \chi L_t^{1/\varphi}, \quad (27)$$

Firm level dividends:

$$d_t = \left( 1 - \frac{1}{\mu_t} - \frac{\phi}{2} \pi_t^2 Y_t \right) \frac{Y_t}{N_t}. \quad (28)$$

Aggregate dividends:

$$AD_t = N_t d_t. \quad (29)$$

Share Euler (including free entry condition  $S_t^{ex} = f_E$ ):

$$1 = (1 - \delta) E_t \left( M_{t,t+1} \frac{f_E + d_{t+1}}{f_E} \right). \quad (30)$$

Price of the aggregate real dividend claim:

$$S_t^{AD} = AD_t + (1 - \delta) E_t [M_{t,t+1} S_{t+1}^{AD}] \quad (31)$$

Return on the aggregate real dividend claim:

$$R_{AD,t} = \frac{(1 - \delta) S_t^{AD}}{S_{t-1}^{AD} - AD_{t-1}} \quad (32)$$

The risk free rate:

$$R_{f,t} = \frac{1}{E_t M_{t,t+1}} \quad (33)$$

Levered excess return as in Croce (2014):

$$EQPR_t = (1 + \phi_{lev})(\ln(R_{AD,t}) - \ln(R_{f,t-1})) \quad (34)$$

Pricing condition:

$$\rho_t = \mu_t MC_t, \quad (35)$$

where

$$\rho_t \equiv N_t^{\frac{1}{\theta-1}} \quad (36)$$

and

$$MC_t = \frac{W_t/P_t}{Z_t} \quad (37)$$

is the real marginal cost.  $\mu_t$  denotes the gross markup *function*:

$$\mu_t \equiv \frac{\theta}{(\theta - 1) \left(1 - \frac{\phi}{2} \pi_t^2\right) + \phi \pi_t (1 + \pi_t) - \beta E_t \{\Upsilon_{t+1}\}} \quad (38)$$

$$\text{with } \Upsilon_{t+1} \equiv (1 - \delta) M_{t,t+1} \phi \pi_{t+1} \frac{Y_{t+1}}{Y_t} \frac{N_t}{N_{t+1}} (1 + \pi_{t+1})$$

The connection between producer price ( $\Pi_t$ ) and consumer price ( $\Pi_t^C$ ) inflation comes from the variety effect:

$$\frac{\Pi_t}{\Pi_t^C} = \left( \frac{N_t}{N_{t-1}} \right)^{\frac{1}{\theta-1}}. \quad (39)$$

Temporary technology shock:

$$\log Z_t = \rho_z \log Z_{t-1} + \sigma_Z \varepsilon_t. \quad (40)$$

The aggregate market clearing reads as:

$$Y_t = \frac{W_t}{P_t} L_t + N_t d_t \quad (41)$$

Goods market clearing:

$$Y_t = C_t + \frac{\phi}{2} \pi_t^2 Y_t. \quad (42)$$



Monetary policy is described by a simple interest rate rule:

$$R_t = \frac{1}{\beta}(1 + \pi_t)^{\phi\pi}. \quad (43)$$

The number of firms producing at period  $t$ ,  $N_t$ , is described by:

$$N_t = (1 - \delta)(N_{t-1} + N_{E,t-1}). \quad (44)$$

The aggregate production function is given by:

$$Y_t = \rho_t Z_t L_t. \quad (45)$$

## 6.2 Model without entry

The entry model contains 18 variables ( $\rho_t$ ,  $\Pi_t^C$  and  $N_t$  drop):

$U, V_t, \tilde{V}_t, MUC_t, M_{t,t+1}, Z_t, Y_t, C_t, d_t, L_t, \mu_t, MC_t, \Pi_t, W_t/P_t, S_t^d, R_t, R_{d,t}, EQPR_t, R_{f,t}$

The aggregate production function is given by:

$$Y_t = Z_t L_t$$

The pricing condition is given by:

$$1 = \mu_t MC_t$$

where definition of the markup is given by:

$$\mu_t \equiv \frac{\theta}{(\theta - 1) \left(1 - \frac{\phi}{2}\pi_t^2\right) + \phi\pi_t(1 + \pi_t) - \beta E_t \{\Upsilon_{t+1}\}}$$

with  $\Upsilon_{t+1} \equiv M_{t,t+1}\phi\pi_{t+1}\frac{Y_{t+1}}{Y_t}(1 + \pi_{t+1})$

There is no distinction between CPI and PPI inflation so equation (11) drops.

The variety effect in equation (2) also drops.

The market clearing in equation (41) changes to:

$$Y_t = \left(1 - \frac{\phi}{2}\pi_t^2\right)^{-1} C_t$$

Dividends:

$$d_t = AD_t = Y_t - \frac{W_t}{P_t} L_t$$

The share Euler equation do not contain  $1 - \delta$  in the no entry model.

The rest of the equations are identical to those in the entry-exit model.

	US data	Entry-exit model	No entry model
$\sigma(\Delta c)$	2.71	2.93	2.93
$\sigma(\Delta y)$	5.12	3.00	2.93
$\sigma(\Delta l)$	3.73	1.96	1.74
$\sigma(\Delta c)/\sigma(\Delta y)$	0.60	0.98	1.00
$\sigma(\Delta l)/\sigma(\Delta y)$	0.78	0.65	0.59
$corr(\Delta c, \Delta y)$	0.84	1.00	1.00
$corr(\Delta c, \Delta l)$	0.41	0.99	1.00
$corr(\Delta y, \Delta l)$	0.64	0.99	1.00

Table 5: Further Macroeconomic Moments. Notes: data moments are based on Favilukis and Lin (2013), US data 1929-2013.

## 7 Further moments

We present moments related to standard deviation of output growth as well as relative standard deviations and correlations with output growth. We make it subject to future research of how the standard deviations of output and labour growth can be raised without creating excess variation in consumption growth.