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Conjoint Analysis Using Mixed Effect Models

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1 Introduction

Conjoint analysis, introduced as a methodology for marketing research by Green and Rao (1971), has since gained widespread popularity (Wittink and Cattin, 1989). The procedure is focused on obtaining the importance of certain product attributes in motivating a consumer toward purchase from a holistic appraisal of attribute combinations called profiles. Profiles are usually evaluated on rating scales. Each consumer evaluates every profile from a prescribed set that is constructed according to a suitable experimental design. To date most applications and commercial conjoint software (i.e. SPSS, 1997) estimate the unknown parameters by relating attribute level variation to observed changes in profile evaluation for each consumer, separately.

OLS estimation at the individual consumer's level accounts for arbitrary parameter heterogeneity between consumers. Exploration of parameter and consumers' preference heterogeneity, respectively, possibly aiming at market segmentation represents the major advantage of conjoint-analysis (i.e. Hair *et al.*, 1992, p.384). Unfortunately the number of data points available at the individual consumer's level is generally very close to the number of unknown model parameters due to limited time and attention of interviewed consumers. Thus statistically based model comparison at the individual consumer's level is nearly impossible as more complex models do not leave any degrees of freedom. The resulting lack of smoothness of parameter estimates causes various annoying peculiarities:

First, as Hagerty (1986) demonstrates analytically, simpler models including for instance only main effects predict individual consumer's preferences significantly better than the true model including interactions. Reanalysis of previously published data confirmed the practical relevance of this effect. Second, any estimate of heterogeneity in a given consumer population based on the empirical distribution of parameters estimated at the individual consumer's level overstates the true amount. Third, individual level estimates as independent variables in a different model yield biased coefficients. And finally, individual level estimates as dependent variables in another different model hinder the statistical detection of variables explaining parameter heterogeneity (Otter and Strebinger, 1998).

Following the pioneering work of Allenby and Ginter (1995) and Lenk *et al.* (1994), we propose in Section 2 a mixed effect model allowing for fixed and random effects as possible statistical solution to the problems mentioned above. Parameter estimation using a new, efficient variant of a Markov Chain Monte Carlo method will be discussed in Section 3 together with problems of model comparison techniques in the context of random effect models. Section 4 presents an application of the former to a brand-price trade-off study from the Austrian mineral water market.

2 The Mixed Effect Model

The data are described by the mixed effect model:

$$y_i = Z_i\alpha + W_i b_i + \varepsilon_i, \quad \varepsilon_i \sim \mathrm{N}(0, R_i), \tag{1}$$

$$b_i \sim \mathrm{N}(0, Q_i), \tag{2}$$

where y_i is the vector of T responses for each consumer i. Z_i is the design matrix for the fixed effects α and W_i is the design matrix for the random effects b_i. A special case of this model is the *random coefficient model* $y_i = X_i\beta_i + \varepsilon_i$, $\varepsilon_i \sim \mathrm{N}(0, R_i)$, $\beta_i \sim \mathrm{N}(\beta, Q_i)$, which is a mixed effect model with $\alpha = \beta$, $Z_i = X_i$, $b_i = \beta_i - \beta$ and $W_i = X_i$.

In the present paper we estimate the covariances Q_i and R_i simultaneously with the random and the fixed effects from the data by a Bayesian approach. We assume that for each consumer Q_i is equal to an abitrary, but unknown covariance matrix Q, and that $R_i \equiv \sigma_\varepsilon^2 I$, where I is equal to the identity matrix. The amount of heterogeneity is influenced by Q: if Q tends to infinity, no restrictions are posed on the random effects b_i and we end up with individual estimation for each consumer. If, on the other hand, Q tends to 0, the random effects are restricted to be the same for all consumers and we end up with aggregate estimation for all consumer, where all heterogeneity is lost. By taking Q to be somewhere between these extremes, we put only some slight restrictions on the individual effects and preserve much of the heterogeneity among the consumers.

3 Bayesian Analysis of The Mixed Effect Model

3.1 MCMC Estimation

Bayesian estimation aims at estimating the covariances Q_i and R_i simultaneously with all random and fixed effects $b^N = (b_1, \dots, b_N)$ and α from their joint posterior density $\pi(b_1, \dots, b_N, \alpha, \theta | y^N)$ given the data $y^N = (y_1, \dots, y_N)$. θ denotes the various unknown parameters appearing in the definition of Q_i and R_i. Bayesian estimation of the mixed effect model (1) is carried out using Markov Chain Monte Carlo (MCMC) methods (see e.g. Gelfand *et al.*, 1996). Here, we introduce a new multimove Gibbs sampler, which extends the ideas of Frühwirth-Schnatter (1994) to mixed effect models. Our method consists of two blocks, the first one is sampling the parameters θ of the covariances from the conditional posterior $\pi(\theta|\alpha, b^N, y^N)$ of θ given (α, b^N) and the data and the second one is joint multimove sampling both of all random effects b^N and all fixed effects α from the conditional distribution $\pi(b_1, \dots, b_N, \alpha | y^N, \theta)$ given all observations and the covariance parameters θ.

The first step depends on the structure chosen for the covariances Q_i and R_i and in many cases turns out to be a standard Bayesian exercise. If $Q_i \equiv Q$ and $R_i \equiv \sigma_\varepsilon^2 I$, Q and σ_ε^2 are conditionally independent and this step involves sampling σ_ε^2 from an inverse gamma posterior, and sampling Q from an inverted Wishart posterior:

$$\sigma_\varepsilon^2|\alpha, b^N, y^N \sim IG(\nu_{\varepsilon,0} + NT/2, S_{\varepsilon,0} + 1/2 \textstyle\sum_{i=1}^N ||y_i - Z_i\alpha - W_i b_i||_2^2),$$
$$Q|\alpha, b^N, y^N \sim IW(\nu_{Q,0} + N/2, S_{Q,0} + 1/2 \textstyle\sum_{i=1}^N b_i b_i').$$

$IG(\nu_{\varepsilon,0}, S_{\varepsilon,0})$ and $IW(\nu_{Q,0}, S_{Q,0})$ are the priors for σ_ε^2 and Q, respectively. To carry out the second step in a multi-move manner is a real challenge, as we sample from a very high-dimensional density. Note that the joint posterior of (b^N, α) spells as: $\pi(b_1, \dots, b_N, \alpha|y^N, \theta) = \pi(b_1, \dots, b_N|\alpha, y^N, \theta) \cdot \pi(\alpha|y^N, \theta) \propto \prod_{i=1}^N \pi(b_i|y_i, \alpha, \theta)\ \pi(\alpha|y^N, \theta)$. First, we sample the fixed effects α from the marginal posterior $\pi(\alpha|y^N, \theta)$ which is derived from the marginal heteroskedastic regression model $y_i = Z_i\alpha + \varepsilon_i^\star$, $\varepsilon_i^\star \sim \mathrm{N}(0, V_i)$, $V_i = W_i Q_i W_i' + R_i$. Based on the normal prior $\mathrm{N}(\hat{\alpha}_0, A_0)$ the marginal posterior $\pi(\alpha|y^N, \theta)$ is a normal density $\mathrm{N}(\hat{\alpha}_N, A_N)$, where

$$A_N = (\textstyle\sum_{i=1}^N Z_i' V_i^{-1} Z_i + A_0^{-1})^{-1},$$
$$\hat{\alpha}_N = A_N (\textstyle\sum_{i=1}^N Z_i' V_i^{-1} y_i + A_0^{-1}\hat{\alpha}_0).$$

Conditional on α the random effects b_i are independent and are sampled from the conditional posterior $\pi(b_i|y_i, \alpha, \theta)$. These densities are given by $\pi(b_i|y_i, \alpha, \theta) \propto p(y_i|b_i, \alpha, \theta)\pi(b_i|\theta)$ and simplify to $\pi(b_i|y_i, \alpha, \theta) \sim \mathrm{N}(\hat{b}_i, B_i)$ with

$$\hat{b}_i = K_i(y_i - Z_i\alpha), \quad K_i = Q_i W_i' V_i^{-1}, \tag{3}$$
$$B_i = (I - K_i W_i) Q_i.$$

3.2 Model Comparison

Assume that various mixed effect models $\mathcal{M}_1, \ldots, \mathcal{M}_L$ are possible candidates for a model of the data. A well-known Bayesian procedure for model comparison is to compute posterior probabilities $P(\mathcal{M}_l|y^N)$ for all models $\mathcal{M}_l$ given the data y^N and the prior probabilities $P(\mathcal{M}_l)$ by Bayes' theorem: $P(\mathcal{M}_l|y^N) \propto f(y_1, \ldots, y_N|\mathcal{M}_l)P(\mathcal{M}_l)$. The factor $L(y^N|\mathcal{M}_l) := f(y_1, \ldots, y_N|\mathcal{M}_l)$ is called model likelihood. If all models have the same prior probability we select the model with the highest model likelihood.
The computation of the model likelihood from the MCMC output is far from trivial (for a recent review see DiCiccio *et al.*, 1997). Among the possible methods of computing the model likelihood we apply the candidate's formula (Chib, 1995). For a mixed effect model it is based on the following formula which holds for any θ:

$$L(y^N) = \frac{L(y^N|\theta)\pi(\theta)}{\pi(\theta|y^N)}, \tag{4}$$

where the marginal likelihood $L(y^N|\theta) = L(y_1, \ldots, y_N|\theta)$ may be computed by one run of the Kalman-Filter. Thus the model likelihood would be easy to compute, if the functional value of the marginal posterior $\pi(\theta|y^N)$ where known for some θ. An estimate of the ordinate of the marginal posterior $\pi(\theta|y^N)$ is obtained from the MCMC output $(\alpha, b^N)^{(1)}, \ldots, (\alpha, b^N)^{(M)}$. For the covariance model $Q_i \equiv Q$ and $R_i \equiv \sigma_\varepsilon^2 \cdot I$, we use:

$$\hat{\pi}(Q, \sigma_\varepsilon^2|y^N) = \frac{1}{M} \sum_{m=1}^{M} \pi(Q|(\alpha, b^N)^{(m)}, y^N)\pi(\sigma_\varepsilon^2|(\alpha, b^N)^{(m)}, y^N),$$

where the conditional densities appearing in the mixture approximation take exactly the form given in Subsection 3.1.

4 Case Study - The Austrian Mineral Water Market

4.1 Data

The data come from a brand-price trade-off study in the mineral-water category. Each of 213 Austrian consumers stated their likelihood of purchasing 15 different product-profiles offering five brands of mineral water (Römerquelle, Vöslauer, Juvina, Waldquelle, and one brand not available in Austria, Kronsteiner) at 3 different prices (2.80, 4.80, and 6.80 [all prices in ATS]) on 20 point rating scales (higher values indicate greater likelihood of purchasing). In an attempt to make the full brand by price factorial less obvious to consumers, the price levels varied in the range of $\pm$ 0.1 ATS around the respective design levels such that mean prices of brands in the design were not affected (Elrod *et al.*, 1992).

j	Effect	Unrestricted Model		Restricted Model	
		$E(\alpha_j\|y^N)$	95%-Interval	$E(\alpha_j\|y^N)$	95%-Interval
1	Const	13.65	12.63 14.54	13.98	13.04 14.84
2	RÖ	5.800	4.684 6.826	5.232	4.191 6.148
3	VÖ	5.298	4.152 6.472	4.454	3.463 5.418
4	JU	1.000	-0.134 2.160	-0.008	-0.838 0.832
5	WA	1.781	0.725 2.858	1.387	0.457 2.277
6	p	-1.901	-2.120 -1.698	-1.934	-2.140 -1.736
7	p^2	0.022	-0.092 0.134	-0.028	-0.114 0.062
8	RÖ $\cdot p$	-0.468	-0.685 -0.235	-0.366	-0.531 -0.171
9	VÖ $\cdot p$	-0.474	-0.693 -0.239	-0.348	-0.550 -0.154
10	JU $\cdot p$	-0.197	-0.416 0.039	0	0
11	WA $\cdot p$	-0.376	-0.565 -0.183	-0.299	-0.463 -0.103
12	RÖ $\cdot p^2$	-0.237	-0.373 -0.084	-0.160	-0.275 -0.038
13	VÖ $\cdot p^2$	-0.169	-0.322 -0.015	0	0
14	JU $\cdot p^2$	-0.109	-0.261 0.040	0	0
15	WA $\cdot p^2$	-0.043	-0.188 0.106	0	0

TABLE 1. Bayesian estimation of α for the unrestricted and the best restricted model (RÖ, VÖ, etc are the various brands, p and p^2 denote the linear and the quadratic price effect; RÖ $\cdot p$, etc are interaction effects)

4.2 Results

We used a fully paramaterized matrix X_i with 15 columns corresponding to the constant, four brand contrasts, a linear and a quadratic price effect, four brand by linear price and four brand by quadratic price interaction effects, respectively. The constant corresponds to the mean purchase likelihood of Kronsteiner at the lowest price level. We used dummy-coding for the remaining brands, subtracted the smallest price from the linear price column in matrix X_i, and computed the quadratic price contrast from the centered linear contrast. Theory did not suggest excluding any effect for all consumers.

At the level of an individual consumer the model would be saturated since only 15 data points are available to estimate 15 parameters leaving zero degrees of freedom. First a possibly overfitted random effect model without restrictions, neither on the fixed effects α nor on Q was estimated by the multimove sampler described in Subsection 3.1. The priors were chosen to be non-informative. After a burn-in-phase of 2000 simulations we used a stationary MCMC sample of size $M = 3000$ for estimation.

Table 1 summarizes the posterior estimates of the fixed effects α and the respective Bayesian confidence intervals. All parameters except the main effect of Juvina, the quadratic price effect, the linear price by Juvina interaction and the interactions of Juvina and Waldquelle with the quadratic

price component are significantly different from zero at the 95% level. The Römerquelle and Vöslauer main effects clearly indicate that these two brands on average are preferred to Kronsteiner when offered at the lowest price level. According to expectation the linear price effect is negative. The negative brand by linear price and brand by quadratic price interactions for Römerquelle und Vöslauer indicate that the advantage of those brands compared to the baseline Kronsteiner diminishes when prices of all three brands increase above the lowest price level. For all fixed effects the inefficiency factors ranged between 0.85 and 1.05 indicating the high efficiency of the multimove sampler.
Figure 1 plots the marginal density of selected parameters corresponding to different interactions between brand and price. The densities are close to densities of normal shape. The direct accessibility of parameter densities presents a major advantage of MCMC estimation. Irregular densities immediately highlight serious violations of distributional assumptions that might go undetected otherwise. In the case of smaller irregularities, Bayesian confidence intervals from the densities ensure that the nominal significance level holds. Relying on asymptotic distributions and standard errors from the information matrix as classical alternatives might give misleading results in such cases.

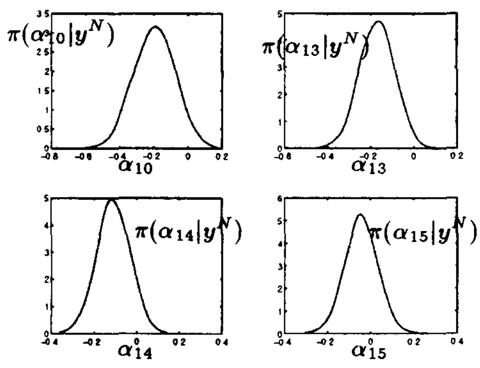

FIGURE 1. Marginal posterior of various interaction effects

Table 2 presents the diagonal elements of Q quantifying the amount of parameter heterogeneity and the respective Bayesian confidence intervals. The inefficiency of the estimated variances ranges between 1.55 and 4. All estimated parameter variances are significantly different from zero. Clearly heterogenous brand preferences are the major source for parameter heterogeneity. The marginal densities of Q_{jj} (not reproduced here) indicate regular densities for all variances. Note that these densities will not be regular if the true variance equals zero. In such cases model comparisons using likelihood ratio tests or other criteria assuming regular parameter densities, for instance the Schwarz Criterion, will give misleading results.

j	Effect	Unrestricted Model		Restricted Model	
		$E(Q_{jj}\|y^N)$	95%-Interval	$E(Q_{jj}\|y^N)$	95%-Interval
1	Const	43.98	34.41 53.89	43.69	34.39 53.58
2	RÖ	43.89	34.36 54.37	44.82	34.34 55.67
3	VÖ	54.36	41.49 65.77	58.79	44.97 73.44
4	JU	55.30	42.18 68.12	59.74	46.20 73.88
5	WA	42.78	33.13 52.60	41.98	32.64 52.02
6	p	1.978	1.544 2.455	2.127	1.662 2.672
7	p^2	0.233	0.170 0.302	0.203	0.139 0.264
8	RÖ $\cdot p$	1.232	0.872 1.599	1.189	0.850 1.513
9	VÖ $\cdot p$	1.247	0.898 1.632	1.880	1.336 2.444
10	JU $\cdot p$	1.347	0.939 1.810	1.278	0.837 1.786
11	WA $\cdot p$	0.407	0.319 0.505	0.657	0.437 0.889
12	RÖ $\cdot p^2$	0.101	0.076 0.125	0.230	0.152 0.310
13	VÖ $\cdot p^2$	0.179	0.140 0.222	0.295	0.213 0.377
14	JU $\cdot p^2$	0.058	0.043 0.074	0.162	0.112 0.213
15	WA $\cdot p^2$	0.091	0.069 0.114	0.081	0.059 0.108

TABLE 2. Bayesian estimation of Q for the unrestricted and the best restricted model

4.3 Simplifying the Model Using Exact Bayes Factors

When simplifying mixed effects models the researcher faces two independent decisions with respect to every random effect in the unrestricted model: 1. Is the mean level of the effect in the population significantly different from zero or not? 2. Is the effect random or constant in the population? All two-by-two combinations of answers to these questions might be reasonable choices where only the combination of "mean level not different from zero" together with the assertion "constant effect" indicates that a variable has no influence on the outcome.

We inspected marginal posterior densities from our base model to identify heuristically sensible starting points for model simplification. From the marginal posterior of Q_{jj} it seems that all effects are random. Some of the fixed effects α_j exhibit marginal posterior densities which cover the value 0 suggesting that the mean levels are not significantly different from zero (see Figure 1). For theoretical reasons we were reluctant to eliminate any of the main effects regarding brand and price. Therefore we chose the mean levels of the following interactions as possible candidates to be fixed to zero: Juvina by linear price, Vöslauer by quadratic price, Juvina by quadratic price, and Waldquelle by quadratic price. To test whether $\alpha_j = 0$ for some or all of these effects, we applied the Bayesian procedure of model comparison discussed in Subsection 3.2. Table 3 compares the log of the model likelihood computed from the MCMC output by the candidate's formula. We selected the model with the highest model likelihood which is the one excluding all tested effects.

Interactions Effects								
RÖ·p	VÖ·p	JU·p	WA·p	RÖ·p^2	VÖ·p^2	JU·p^2	WA·p^2	$\log L(y^N)$
+	+	+	+	+	+	+	+	-9088.7
+	+	-	+	+	+	+	+	-9081.2
+	+	+	+	+	+	-	+	-9086.4
+	+	+	+	+	+	+	-	-9083.1
+	+	-	+	+	+	-	+	-9088.7
+	+	-	+	+	+	+	-	-9084.1
+	+	+	+	+	+	-	-	-9075.5
+	+	-	+	+	+	-	-	-9078.9
+	+	-	+	+	-	-	-	-9070.6

TABLE 3. Model Comparison ('+' denotes inclusion, '-' denotes exclusion)

The right hand sides of Table 1 and Table 2, respectively, summarize the estimation results for the remaining elements of α and the diagonal elements of Q for the restricted model . From a substantive point of view interpretation does not change very much. However, two differences are noteworthy: Again Juvina does not seem to be preferred to the Kronsteiner brand on average when both brands are offered at the same price. This lack of differentiation occurs independently of the actual price level. Furthermore the advantage of Römerquelle over the Kronsteiner brand diminishes decisively faster as price increases than the advantage of Vöslauer over the Kronsteiner brand, since only Römerquelle "suffers" from a negative interaction with the quadratic price effect. From a different angle the advantage of Römerquelle over Vöslauer vanishes when price for both brands reaches the highest level.

Exploration of consumer heterogeneity with respect to the effects in the model is easily accomplished using the mean values over the simulations of b_i obtained from (3). They represent estimates for consumer i's deviation from the mean parameter α. One could plot the univariate distributions of these deviations from the constant effect for each component over all 213 consumers and compare them to the distribution of the respective OLS estimates. Furthermore, bivariate and multivariate aspects of consumer heterogeneity can be explored. Figure 2, for instance, shows the scatterplots of the Römerquelle main effect against the Vöslauer main effect resulting from individual OLS estimation, aggregate OLS estimation, and the mixed effects model, respectively. Clearly, aggregate estimation discards most of the information in the data. Comparing OLS estimation at the individual consumer's level to the mixed model results, the shrinkage effect becomes obvious. Individual consumers tend to be attracted by regions with higher consumer density. Since the Römerquelle and Vöslauer main effects are positively correlated individual consumers' OLS estimates with the combination „high preference for Römerquelle/low preference for Vöslauer" and vice versa tend to be shrunk towards the diagonal.

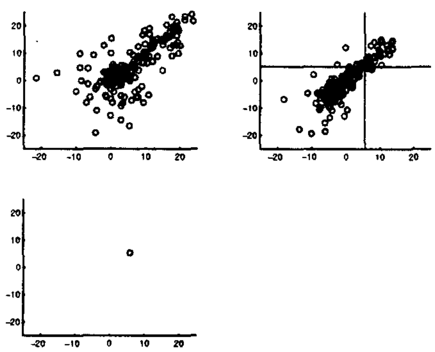

FIGURE 2. Scatter plot of the individual main effects of Römerquelle against Vöslauer estimated by individual OLS (top left), aggregated OLS (bottom left), and the mixed restricted effect model (top right),

5 Discussion and Future Research

We applied a mixed effects model to conjoint data from a brand-price tradeoff study introducing a new multimove Gibbs sampler for efficient estimation. To avoid the possible pitfalls of approximate model selection criteria we used the candidate's formula and MCMC simulations to compute exact model likelihoods. What else is gained by applying MCMC estimation instead of e.g. classical maximum likelihood techniques? Clearly, point estimates of model variances could be obtained by maximizing the marginal likelihood. Subsequently fixed and random effects and their respective standard errors are obtained from one run of the Kalman filter and the Kalman smoother, conditional on the model variances derived from maximum likelihood estimation. Therefore the uncertainty associated with estimating the model variances is not reflected in the standard errors of fixed and random effects yielding unreliable confidence intervals. Furthermore, using MCMC methods it is straightforward to obtain correct standard errors for nonlinear combinations of any parameters in the model. This feature might prove to be of great value when estimating choice probabilities from the conjoint parameters using BTL or similar models.

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