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On the Robustness of the Rank-Based CUSUM Chart against Autocorrelation

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Abstract: Even a modest positive autocorrelation results in a considerable increase in the number of false alarms that are produced when applying a CUSUM chart. Knowledge of the process to be controlled allows for suitable adaptation of the CUSUM procedure. If one has to suspect the normality assumption, nonparametric control procedures such as the rank-based CUSUM chart are a practical alternative. The paper reports the results of a simulation study on the robustness (in terms of sensitivity of the ARL) of the rank-based CUSUM chart against serial correlation of the control variable. The results indicate that the rank-based CUSUM chart is less affected by correlation than the observation-based chart: The rank-based CUSUM chart shows a smaller increase in the number of false alarms and a higher decrease in the ARL in the out-of-control case than the the observation-based chart.

Key words: process control, nonparametric procedure, ranks, robustness, average run length (ARL), autocorrelation.

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1 Introduction

Control charts allow to detect whether a process $\{X_t\}$ stays in the state of statistical control, e.g., they tell us whether a parameter of the distribution of X_t such as the mean remains at a specified level. The properties of control charts such as the average run length (ARL) curve are determined by the chart parameters. In a specific situation, the proper choice of these parameters depends on the distribution of X_t. The control limits of most charts for measurement data such as the Shewhart or the CUSUM chart are derived under the assumption that the X_t are, at least in the in-control case, independently and normally distributed.

The assumption of independence of the control variable is violated in many types of todays manufacturing processes. This is especially true for continuous process industries, for computer-integrated manufacturing, and for laboratory measurements; see, e.g., Berthouex et al. (1978), Harris and Ross (1991), Hunter (1990) and other discussants of Lucas and Saccucci (1990). It is well-known that autocorrelation causes a decrease of the ARL for the Shewhart chart or an increase of false alarms; see, e.g., Crowder (1987), Magarah and Woodall (1991), and Böhm and Hackl (1996). For the CUSUM chart we also have to expect an increasing number of false alarms if the control variable is positively correlated; see Böhm and Hackl (1996), and Hawkins and Olwell (1998). In both cases, even modest autocorrelation results in considerable changes in the ARL and in the number of false alarms.

Given these effects, adjustment of existing techniques needs to be considered, e.g., by adapting the parameters such that intended ARL is achieved. Johnson and Bagshaw (1974) have shown in an asymptotic analysis that rescaling of the CUSUM statistic is a suitable approach if the control variable follows a Markov or a moving average process. A method that has been suggested by Montgomery and Mastrangelo (1991) assumes the in-control process to follow a similar time-series process; the approach entails two charts, viz. (i) a time plot of fitted values (common cause chart) and (ii) a control chart of the residuals (special cause chart); ARIMA models are used to get the fitted values. The idea goes back to Berthouex et al. (1978) and to Alwan and Roberts (1988).

All these approaches are based on the assumption that the control variable follows normality which is reasonable in many situations, especially if the control procedure is based on the sample average of a few observations that are taken at each control time, so that the central limit theorem applies. However, technical or economic reasons may cause that only single observations can be taken at each control time. Typical cases are manufacturing processes with a slow production rate or with an automatic on-line measurement technology where every production item is analyzed. In this case one cannot rely on the central limit theorem for normality.

Nonparametric control chart procedures are a practical alternative if one must base the procedure on individual observations or has to suspect for another reason the normality assumption. Nonparametric control techniques are discussed by Bakir and Reynolds (1979), Park and Reynolds (1987), Bhattacharyya and Frierson (1981), McDonald (1986), Hackl and Ledolter (1991, 1992). The common feature of all these techniques is the use of ranks instead of observations of X_t. E.g., Bhattacharyya and Frierson (1981) and McDonald (1986) discuss cumulative sum procedures where the rank of the current observation is based on all previous measurements; this corresponds to Parent's (1965) definition of sequential ranks. Hackl and Ledolter (1991, 1992) use a definition of sequential rank where the actual observation is ranked among the most recent g observations or a set of g in-control observations and introduce rank-based exponentially weighted moving average (EWMA) charts. The properties of rank-based or nonparametric procedures are independent of the distributional assumptions, at least for processes that are in control. Furthermore, these procedures will be less affected by distributional contamination or single extreme observations than the usual control chart techniques. Corresponding results are reported in the mentioned papers.

An important question is whether the superiority of rank-based control procedures is maintained if the control variable is autocorrelated. This paper presents results of an investigation of the robustness of rank-based or nonparametric procedures with respect to autocorrelation of the control variable. Given the complexity of the ARL distribution, particularly in the out-of-control case, we use Monte Carlo experiments for investigating the effects of interest. Section 2 defines the rank-based CUSUM procedure which is the object of our investigation. In Sections 3 and 4 we describe the design of the simulation study and report the results, respectively. The robustness aspects are discussed in Section 5. We conclude with recommendations and some comments in the final section.

2 The Rank-Based CUSUM Procedure

As it is mentioned in Section 1, the properties of rank-based procedures are independent of the distributional assumptions, at least for processes that are in control. Several concepts of ranks have been used by various authors; we prefer the so-called sequential ranks as they imply convenient stochastic properties of process control procedures such as CUSUM- or EWMA-charts that are based on them.

2.1 Ranking Procedure

We assume that X_t, $t = 1, 2, \ldots$, is a sequence of independent random variables from a continuous probability distribution $F(x)$. The process control problem is that of quickly detecting changes in $F(x)$. The process is considered under control if $F(x) = F_0(x)$. We base the standardized rank R_t of the observation X_t on the in-control distribution $F_0(x)$ and define

$$R_t = 2\left[F_0(X_t) - \frac{1}{2}\right] . \tag{1}$$

Properties of R_t are easily derived. We can show that R_t is uniformly distributed between -1 and 1, with mean zero and variance 1/3, and that the time series of ranks is independent.

If the in-control distribution is not specified, then one can use a historic sample of size $g-1$, $\{Y_1, Y_2, \ldots, Y_{g-1}\}$ taken from the in-control process, as a reference and define the standardized rank of X_t as

$$R_t = \frac{2}{g}\left(R_t^* - \frac{g+1}{2}\right) \tag{2}$$

R_t^* is the rank of X_t with respect to $Y_1, \ldots, Y_{g-1}$. That is, $R_t^* = 1 + \sum_{i=1}^{g-1} I(X_t > Y_i)$, where the indicator function $I(X_t > Y_i) = 1$ if $X_t > Y_i$, and 0 otherwise. Taking the historic in-control sample as fixed, one can show that the ranks in (2), exactly like those in (1), are independent and uniformly distributed. The only difference is that the ranks in (2) follow a discrete uniform distribution on the g points

$$\frac{1}{g} - 1, \frac{3}{g} - 1, \ldots, 1 - \frac{1}{g},$$

with mean zero and variance $(g^2 - 1)/3g^2$.

A special case of historic sample is the set of $g-1$ observations immediately prior to the actual time t. This sample shifts like a window over the time-scale with every new observation. This mode of constructing explains the name "sequential ranks" that is used for such ranks. All what is said above about the stochastic properties of R_t is still valid. However, it should be mentioned that control charts that are based on sequential ranks might be less effective in indicating slowly growing out-of-control situations than charts that are based on rankings with respect to a fixed historic sample of in-control observations.

2.2 The CUSUM Procedure

The main purpose of process control procedures is to detect a change of the distribution of the control variable from the in-control distribution as soon as possible after its onset. Whereas the Shewhart chart plots the observations of the control variable as they are obtained at successive control points, more general control techniques are based on weighted sums of the observations. The exponentially weighted moving average (EWMA) technique, suggested by Roberts (1959) and further discussed by Hunter (1986) and by Lucas and Saccucci (1990), is defined by geometrically decreasing weights. The cumulative sum (CUSUM) chart goes back to Page (1954) and Barnard (1959); it gives unit weights to each of the past observations. For a general survey see Hawkins and Olwell (1998).

These statistics can also be based on sequential ranks as defined in (1) and (2). A rank-based EWMA-technique is discussed by Hackl and Ledolter (1991, 1992). In a similar way, a control technique based on cumulative sums of sequential ranks can be defined as will be shown in the following.

The rank-based CUSUM statistics are defined as

$$c_t = \max(c_{t-1} + R_t - k, 0)\,, \; t = 1, 2, \ldots, \tag{3}$$

where R_t are the standardized ranks. The parameters of the CUSUM technique are k, the reference value, and the control limit h. The statistics c_t can be analyzed in the usual chart form: As long as $c_t \leq h$, the process is considered in control. The reference value k can be interpreted as the tolerated deviation in the sense that c_t remains at zero as long as k is not exceeded. These fact can help to chose k in agreement with the practical needs. The pair of parameters (k, h) determines the properties of the CUSUM technique such as the ARL of the in-control process.

3 The Simulation Study

The aim of our simulation study is to investigate the effects of serial correlation on the ARL of the one-sided rank-based CUSUM technique. Valuable insight can be gained from comparing the rank-based with the observation-based CUSUM technique. This will be done on the basis of the same sets of data. The question of interest is which procedure performs better, given the same ARL in the case of the in-control process. To allow to distinguish the effect of serial correlation from that of non-normality in the rank- and observation-based procedures, we extend our study by generating the data with varying degrees of skewness.

3.1 The Generation of Data for the Control Variable

For our simulations, we generate data of the univariate control variable X_t according to the AR(1) process

$$X_t = \alpha X_{t-1} + Z_t\,, \; t = 1, 2, \ldots, \tag{4}$$

with $X_0 = 0$ and $|\alpha| < 1$. For Z_t we generate data so that they are identically and independently distributed. For cases of symmetric control variables we generate Z_t from $N(0, \sigma^2)$. Skewed control variables are generated from the Beta-distribution $B(p, q)$ with parameters p and q that is defined on the interval (a, b): For Z_t we find

$$\begin{aligned} \mathrm{E}\{Z_t\} &= a + (b-a)\frac{p}{p+q}, \\ \mathrm{Var}\{Z_t\} &= \frac{(b-a)^2 pq}{(p+q)^2(p+q+1)}, \end{aligned}$$

for all t. Given p and q so that the distribution of Z_t has a certain form, we choose a and b so that $a = bc/(1-c)$ with $c = p/(p+q)$ which implies $\mathrm{E}\{Z_t\} = 0$. The skewness γ_z also depends on p, q, a, and b; numerical values are given in the corresponding tables below. For the control variables, we obtain for all t

$$\begin{aligned} \mathrm{E}\{X_t\} &= 0 \\ \mathrm{Var}\{X_t\} &= \frac{\mathrm{Var}\{Z_t\}}{1-\alpha^2} = \sigma_x^2 \\ \gamma_x &= \frac{(1-\alpha^2)^{2/3}}{1-\alpha^3}\,\gamma_z \end{aligned}$$

if $|\alpha| < 1$. Numerical values for the skewness γ_x are given in the respective tables below together with the results of our simulation study.

A historic sample is used to get standardized ranks R_t from (2). As this sample a set of 1000 observations was generated from the in-control distribution ($g = 1001$). For small and moderate values of g, the statistical properties of rank-based procedures must be expected to depend on g (cf. Hackl and Ledolter, 1991, 1992). The rather large value for g was chosen because this dependence on g is not a major point of interest in our study, and the large value of g simplifies presentation and discussion of the results. The ranks R_t are obtained by ranking the generated values of the control variable X_t with respect to the historic sample. The standardized ranks of observations of the in-control process follow the discrete uniform distribution with mean zero and variance 1/3.

3.2 The CUSUM Procedure

We denote by L_δ the ARL of a process $\{X_t\}$ if its level is shifted by $\delta\sigma_x$ from the level when the process is in control.

The observation-based CUSUM statistics C_t, $t = 1, 2, \ldots$, are obtained from

$$C_t = \max(C_{t-1} + X_t - k, 0)\,. \tag{5}$$

The reference value k and the control limit h are chosen so that L_0 is about (but not less) 100 for the case that the control variable follows the normal distribution. For deriving the ARL of the observation-based CUSUM procedure, the integral equation

$$L(s) = 1 + L(0)F(k-s) + \int_0^h f(x+k-s)L(x)dx \tag{6}$$

can be used; here, s is the starting value of the CUSUM statistic C_t ($0 \leq s \leq h$) and f and F are the density and distribution function, respectively, of the control variable (cf. Van Dobben de Bruyn, 1968). An approximate solution of this equation can be found by substituting the integral by a sum where the range of integration is split up in sufficiently narrow intervals. Among all pairs (k, h) that pair was chosen for which L_1, the ARL for the process with a level that is shifted by one σ_x, is minimal. This approach resulted in a CUSUM procedure with $k = 0.5$ and $h = 3\sigma_x$; the numerical approximation of the ARL of this procedure results in $L_0 = 121.4$.

The parameters of the corresponding rank-based CUSUM procedure are again so chosen that L_1 is minimal among all pairs (k, h) for which the in-control ARL close to L_0 as chosen for the observation-based CUSUM procedure. For that purpose, Monte Carlo (MC-)estimates of the ARL were obtained from 10.000 replications for various pairs (k, h). The rank-based CUSUM procedure chosen has the parameters $k = 0.3$ and $h = 1.6$ with an MC-estimate $L_0 = 115.7$, the standard error being 1.21.

The observation-based and rank-based CUSUM procedures with $k = 0.5$, $h = 3\sigma_x$ and $k = 0.3$, $h = 1.6$, respectively, are basis for all results that are given below.

3.3 The Design of the Simulation Study

Data of the control variable X_t were generated by using the AR(1) process (4). For Z_t, the distributions were chosen to be

- $N(0, 1)$,
- $B(1, 100)$, i.e., $p = 1$ and $q = 100$, with $a = -1$ and $b = 100$, resulting in $\gamma_z = 1.94$, and
- $B(100, 1)$, with $a = -100$ and $b = 1$, so that $\gamma_z = -1.94$.

The autoregressive parameter was set to be $\alpha = -0.9(0.3)0.9$. For each case, first a historic sample of 1000 in-control values was generated. Given this sample, time series of the control variable were generated that were used to estimate the ARL of the control procedures of interest.

For each time-series, CUSUM statistics C_t were calculated according to (5) by use of the reference value $k = 0.5$, and the run length determined as the smallest index t of C_t where C_t exceeds the control limit $h = 3\sigma_x$. The same set of data was used to get a value of the run length for the rank-based CUSUM statistics. The observations of the control variable X_t were ranked with respect to the historic sample of in-control values according to (2), and CUSUM statistics C_t of the standardized ranks R_t were calculated according to (3) with $k = 0.3$ and $h = 1.6$. The in-control ARLs were estimated by repeating this process 1000 times. Analogous results were obtained for two out-of-control situations where the level of X_t has been shifted by $\delta\sigma_x$ with $\delta = 0.5$ and 1.

The resulting estimates and their standard deviations for the in-control situation are shown in *Table 1*. *Tables 2* and *3* show the results for the out-of-control situation with $\delta = 0.5$ and 1, respectively. *Figure 1* shows the ARL as a function of the autoregressive parameter α on a logarithmic scale for the three values of δ. The ARL-curves over δ are shown for three values of α in *Figure 2*.

4 Robustness of the Rank-based CUSUM Chart

We first discuss the situation of the in-control process (see *Table 1* and *Figure 1(a)*). In the case of the uncorrelated control variable, the rank-based CUSUM procedure is nearly uneffected by skewness. This is in contrast to the observation-based CUSUM chart that shows a decreasing ARL (too frequent alarms) in the presence of a right-skewed control variable ($\gamma_x = \gamma_z = 1.94$) and a tremendously increasing ARL for negative skewness.

The robustness of the rank-based CUSUM procedure against non-symmetry can also be observed in the case of the positively correlated ($\alpha > 0$) control variable, where the pattern of changes of the ARL is very similar for the three cases of skewness: The ARL decreases with increasing $\alpha < 0$, resulting in an increased number of alarms. For negative correlation ($\alpha < 0$), the ARL increases. The extent of this increase depends on γ_x. For large negative values of α, only lower limits are given as the runs were censored at 10^6. For the observation-based CUSUM chart the increase of the ARL for $\alpha < 0$ depends on the skewness and is in all cases not as large as for the rank-based CUSUM procedure; for larger values of negative correlation and $\gamma_x < 0$ the ARL decreases again.

In the out-of-control situation, the ARL-curves for the rank-based CUSUM procedure become flatter with growing δ (see *Tables 2* and *3* and *Figures 1(b)* and *1(c)*). The effect of serial correlation deminishes; for $\delta = 1$ and the symmetric distribution, the ARL-curve is so flat that its maximum is only about 1.25 times its minimum. For skewed distributions, the ARL-curves coincide with that for the symmetric distribution for highly correlated control variables ($|\alpha|$ close to 1) and shows even smaller values for small and moderate values of $|\alpha|$. These findings for the out-of-control situation can be summarized by saying that the rank-based CUSUM procedure is to a high degree robust against serial correlation, to a good degree robust against skewness.

Also for the observation-based CUSUM chart, the effect of skewness on the ARL-curve decreases in the situation of an out-of-control process. The general pattern over the range of α is a parabola with a maximum ARL at about $\alpha = -0.3$ and $\alpha = 0$ for $\delta = 0.5$ and $\delta = 1$, respectively, for the cases of skewness under study. The observation-based CUSUM procedure is less robust against both serial correlation and skewness than the rank-based CUSUM procedure.

A comparison of the ability to give a quick alarm in the out-of-control case shows a rather similar behavior for both techniques except for $|\alpha|$ close to 1. *Figure 2* reveals that the rank-based procedure shows slightly smaller ARL values than the observation-based chart for small and moderate values of α. However, this comparison is distorted by differences in the ARL values in the in-control case (except for $\alpha = \gamma_x = 0$). Looking at the relative changes in the ARL with increasing δ indicates that the rank-based CUSUM chart is advantageous, i.e., shows a lower ARL-curve, when the control variable is positively skewed. In the practically less important situation of a negatively skewed control variable the observation-based CUSUM chart is preferable.

5 Concluding Remarks

In the design stage of a process control procedure, distributional properties of a sample from the control variable will result in a suitable choice of the control chart parameters. In the situation of an autocorrelated control variable, the parameters of a CUSUM chart can be chosen so that intended properties such as the ARL of the in-control process is achieved.

Nonparametric control charts, among them the rank-based CUSUM technique, are suitable alternatives when the assumption of normality of the control variable is violated. The robustness of rank-based procedures against distributional contamination or single extreme observations is well-known. Corresponding results are reported by Hackl and Ledolter (1991, 1992) who compare the rank-based versus the observation-based EWMA chart.

The results of our simulation study similarly indicate that the rank-based CUSUM chart is - in contrast to the observation-based CUSUM procedure - robust against deviations in the form of the distribution: The rate of false alarms does practically not depend on the skewness. This superiority gradually diminishes for increasing positive autocorrelation. In the out-of-control case, the differences in the respective ARL-curves of the two type of charts are not substantial although here again, the rank-based CUSUM chart are advantageous in practically relevant situations: For the rank-based CUSUM chart a larger decrease in the ARL as the consequence of a shift in the level of the control variable is observed than for the observation-based CUSUM chart.

A Tables

Table 1: Monte Carlo estimates $\hat{L}_0$ and estimated standard errors of ARL for the observation- and rank-based CUSUM procedures applied to the same in-control process $X_t = \alpha X_{t-1} + Z_t$ for various distributions of Z_t and values of α; each estimate is based on 1000 replications.

			rCUSUM		oCUSUM	
distribution of Z_t	α	γ_x	$\hat{L}_0$	st.err.	$\hat{L}_0$	st.err.
$B(100, 1)$	-0.9	-0.09	$> 10^6$	$> 10^3$	611.0	28.9
$\gamma_z = -1.94$	-0.6	-0.82	$> 10^6$	$> 10^3$	1312.8	42.4
	-0.3	-1.64	20023.1	1249.3	104657.4	3264.0
	0.0	-1.94	118.7	3.75	8226.0	256.9
	0.3	-1.73	43.5	1.34	173.9	5.25
	0.6	-1.27	26.0	0.89	36.1	2.24
	0.9	-0.60	5.7	0.09	13.9	0.73
$N(0, 1)$	-0.9	0.0	$> 10^6$	$> 10^3$	1549.1	68.4
$\gamma_z = 0$	-0.6	0.0	24706.0	951.4	1707.0	564.8
	-0.3	0.0	586.9	19.3	407.4	12.3
	0.0	0.0	119.7	3.79	115.1	3.56
	0.3	0.0	50.0	1.61	54.2	1.69
	0.6	0.0	29.1	0.96	26.0	0.94
	0.9	0.0	18.0	1.01	16.1	0.95
$B(1, 100)$	-0.9	0.09	$> 10^6$	$> 10^3$	359.5	16.4
$\gamma_z = 1.94$	-0.6	0.82	3317.5	110.5	117.6	3.87
	-0.3	1.64	453.8	14.8	71.1	2.15
	0.0	1.94	119.7	3.67	52.1	1.63
	0.3	1.73	46.0	1.43	36.6	1.20
	0.6	1.27	29.2	0.99	22.9	0.84
	0.9	0.60	19.2	1.02	16.6	0.99

Table 2: Monte Carlo estimates $\hat{L}_{0.5\sigma_x}$ and estimated standard errors of ARL for the observation- and rank-based CUSUM procedures applied to the same out-of-control process $X_t = \alpha X_{t-1} + Z_t$ with $\delta = 0.5$ for various distributions of Z_t and values of α; each estimate is based on 1000 replications.

			rCUSUM		oCUSUM	
distribution of Z_t	α	γ_x	$\hat{L}_{0.5}$	st.err.	$\hat{L}_{0.5}$	st.err.
$B(100, 1)$	−0.9	−0.09	50.0	1.76	9.2	0.09
$\gamma_z = -1.94$	−0.6	−0.82	18.7	0.43	17.8	0.30
	−0.3	−1.64	11.0	0.24	20.9	0.46
	0.0	−1.94	8.6	0.19	18.7	0.46
	0.3	−1.73	8.6	0.23	13.9	0.36
	0.6	−1.27	8.7	0.26	10.2	0.32
	0.9	−0.60	9.1	0.48	6.8	0.44
$N(0, 1)$	−0.9	0.0	130.4	26.3	9.4	0.09
$\gamma_z = 0$	−0.6	0.0	38.5	1.06	17.7	0.40
	−0.3	0.0	23.5	0.62	20.1	0.49
	0.0	0.0	17.6	0.43	16.8	0.40
	0.3	0.0	14.2	0.38	13.3	0.39
	0.6	0.0	11.6	0.36	9.6	0.33
	0.9	0.0	9.6	0.47	7.4	0.39
$B(1, 100)$	−0.9	0.09	95.2	15.1	9.4	0.10
$\gamma_z = 1.94$	−0.6	0.82	29.7	0.77	19.0	0.47
	−0.3	1.64	19.0	0.43	20.3	0.56
	0.0	1.94	15.0	0.33	18.6	0.51
	0.3	1.73	13.5	0.34	15.1	0.47
	0.6	1.27	11.9	0.34	10.7	0.41
	0.9	0.60	9.8	0.52	6.8	0.41

Table 3: Monte Carlo estimates $\hat{L}_1$ and estimated standard errors of ARL for the observation- and rank-based CUSUM procedures applied to the same out-of-control process $X_t = \alpha X_{t-1} + Z_t$ with $\delta = 1$ for various distributions of Z_t and values of α; each estimate is based on 1000 replications.

			rCUSUM		oCUSUM	
distribution of Z_t	α	γ_x	$\hat{L}_1$	st.err.	$\hat{L}_1$	st.err.
$B(100, 1)$	−0.9	−0.09	6.7	0.13	4.2	0.03
$\gamma_z = -1.94$	−0.6	−0.82	5.8	0.09	5.2	0.05
	−0.3	−1.64	5.3	0.09	5.8	0.07
	0.0	−1.94	5.0	0.08	5.8	0.09
	0.3	−1.73	5.1	0.10	5.6	0.12
	0.6	−1.27	5.5	0.15	4.9	0.13
	0.9	−0.60	5.5	0.24	4.3	0.18
$N(0, 1)$	−0.9	0.0	7.3	0.13	4.2	0.03
$\gamma_z = 0$	−0.6	0.0	7.3	0.10	5.3	0.05
	−0.3	0.0	7.0	0.10	6.2	0.09
	0.0	0.0	6.8	0.11	6.4	0.12
	0.3	0.0	6.5	0.13	6.2	0.15
	0.6	0.0	6.3	0.17	5.5	0.20
	0.9	0.0	5.8	0.26	4.2	0.16
$B(1, 100)$	−0.9	0.09	7.3	0.14	4.2	0.03
$\gamma_z = 1.94$	−0.6	0.82	6.1	0.06	5.8	0.06
	−0.3	1.64	5.7	0.04	6.7	0.11
	0.0	1.94	5.6	0.05	7.4	0.15
	0.3	1.73	5.9	0.09	7.3	0.18
	0.6	1.27	6.2	0.14	5.9	0.18
	0.9	0.60	6.4	0.27	4.2	0.20

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List of Figures

Figure 1: ARL of rank-based (rCUSUM, black) and observation-based CUSUM procedure (oCUSUM, gray) as a function of the autoregressive parameter α for (a) the in-control situation and two out-of-control situations: (b) $\delta = 0.5$ and (c) $\delta = 1$. The skewness of the distribution of the control variable is zero (gamma=0), positive (gamma>0), or negative (gamma<0).

Figure 2: ARL of rank-based (rCUSUM, black) and observation-based CUSUM procedure (oCUSUM, gray) as a function of δ for (b) uncorrelated ($\alpha = 0$) and two cases of correlated control variable: (a) $\alpha = -0.3$ and (c) $\alpha = 0.3$. The skewness of the distribution of the control variable is zero (gamma=0), positive (gamma>0), or negative (gamma<0).

figure 1

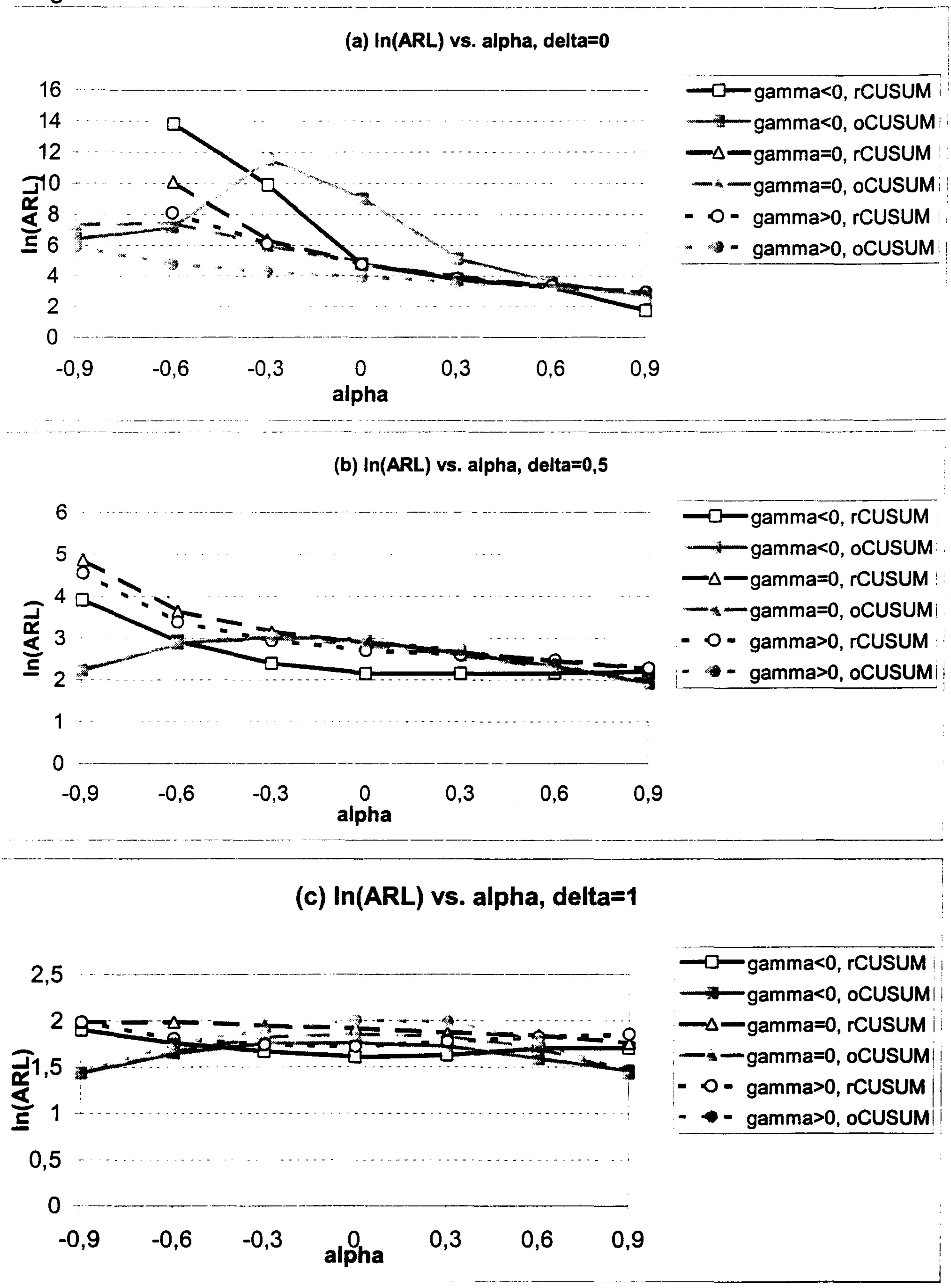
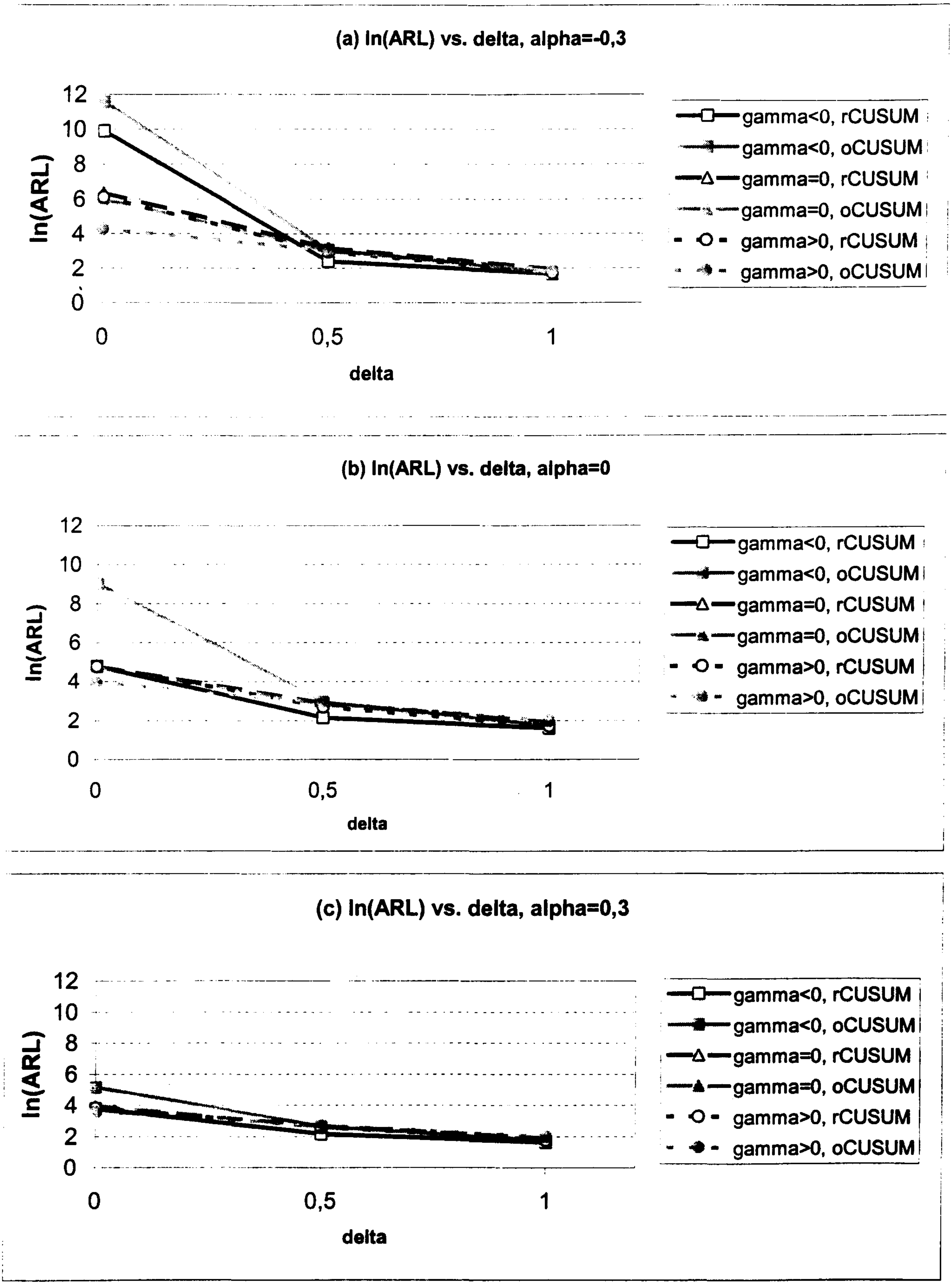

(a) ln(ARL) vs. alpha, delta=0
16
14
12
10
8
6
4
2
0
ln(ARL)
-0,9
-0,6
-0,3
0
0,3
0,6
0,9
alpha
gamma<0, rCUSUM
gamma<0, oCUSUM
gamma=0, rCUSUM
gamma=0, oCUSUM
gamma>0, rCUSUM
gamma>0, oCUSUM
(b) ln(ARL) vs. alpha, delta=0,5
6
5
4
3
2
1
0
ln(ARL)
-0,9
-0,6
-0,3
0
0,3
0,6
0,9
alpha
gamma<0, rCUSUM
gamma<0, oCUSUM
gamma=0, rCUSUM
gamma=0, oCUSUM
gamma>0, rCUSUM
gamma>0, oCUSUM
(c) ln(ARL) vs. alpha, delta=1
2,5
2
1,5
1
0,5
0
ln(ARL)
-0,9
-0,6
-0,3
0
0,3
0,6
0,9
alpha
gamma<0, rCUSUM
gamma<0, oCUSUM
gamma=0, rCUSUM
gamma=0, oCUSUM
gamma>0, rCUSUM
gamma>0, oCUSUM

figure 2


(a) ln(ARL) vs. delta, alpha=-0,3
ln(ARL)
12
10
8
6
4
2
0
0
0,5
1
delta
gamma<0, rCUSUM
gamma<0, oCUSUM
gamma=0, rCUSUM
gamma=0, oCUSUM
gamma>0, rCUSUM
gamma>0, oCUSUM
(b) ln(ARL) vs. delta, alpha=0
ln(ARL)
12
10
8
6
4
2
0
0
0,5
1
delta
gamma<0, rCUSUM
gamma<0, oCUSUM
gamma=0, rCUSUM
gamma=0, oCUSUM
gamma>0, rCUSUM
gamma>0, oCUSUM
(c) ln(ARL) vs. delta, alpha=0,3
ln(ARL)
12
10
8
6
4
2
0
0
0,5
1
delta
gamma<0, rCUSUM
gamma<0, oCUSUM
gamma=0, rCUSUM
gamma=0, oCUSUM
gamma>0, rCUSUM
gamma>0, oCUSUM