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Assessing the effects of quality regulation in Norway with a quality regulated version of dynamic DEA

Philipp von Geymueller   Anton Burger*

1st March 2007

Abstract

In order to find out why energy-not-supplied in Norway - the most important indicator for the quality of service in the quality-regulation regime there - decreased more pronounced before the introduction of quality-regulation in 2001 than after it, we develop a dynamic quality-DEA-model and apply it to a representative sample of distribution-net operators. Our model enables us to calculate a counter-factual and thus to tentatively answer the question: What would have happened, had there been no quality-regulation? This way we find strong evidence that the quality-regulation in Norway did not have an effect on the behavior of the firms.

Keywords: DEA, dynamic DEA, efficiency, quality regulation, Norway, counter-factual

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1 Introduction

In the wake of major blackouts and plummeting customer satisfaction with the quality of service in the liberalized electricity markets in Europe, it has become a pressing question whether and in how far quality-regulation can counter such developments.

When trying to answer this difficult question in a positive - instead of a normative - fashion, the focus of attention naturally shifts to Norway as it was one the very first countries where quality of service was explicitly and rigorously included in the regulatory regime. Specifically, the electricity suppliers have to pay a penalty according to the expected cost that the undelivered energy causes with the respective customers. This regime was introduced in 2001, revised in 2003 and 2005 and led to the following development of *energy-not-supplied (ENS)* in Norway:

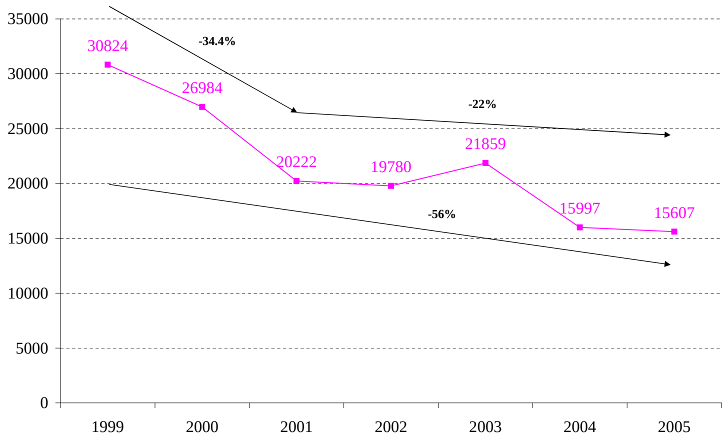

Figure 1: Development of ENS (MWh) in Norway

Source: Norwegian Water Resources and Energy Directorate (NVE) - `http://www.nve.no`

As can be seen, between 1999 (pre quality-regulation) and 2005 the amount of ENS decreased by more than 50%. What can also be seen, however, is, that the average annual rate of reduction was much higher before (1999-2001: -9.8%) than after the introduction of quality-regulation (2001-2005: -6.27%).

As this development of ENS raises the question whether quality-regulation actually made a difference, we develop a dynamic DEA-model that incorporates quality-regulation as an explicit constraint. This way we are able to calculate a counter-factual, i. e. we can tentatively answer the question: What would have happened between 2001 and 2005, had there been no quality-regulation.

In what follows, we first develop the model and explain its theoretic properties and then apply it to a representative sample of Norwegian distribution-net providers. Finally we draw conclusions, as to why or why not quality-regulation did in fact make a difference.

2 The Model

2.1 Background

Data Envelopment Analysis (DEA) as pioneered by Charnes et al. (1979) is a non-parametric method to estimate the efficiency of enterprises. Its popularity stems from the fact that it can specify an efficient frontier without the need for the definition of a production function by laying a convex hull around the empirically available input-output combinations of the players in the sample. Following Farrell's pioneering approach (Farrell, 1957), efficiency of the respective enterprise can then, for example, be measured by the distance between the observation and the estimated ideal on the efficient frontier. Mathematically this is accomplished by formulating for each enterprise a linear program in which the objective function represents the efficiency of the respective enterprise and the constraints stand for the restrictions of the production possibility set. This basic setting can and has been modified in many different ways in order to better resemble the specific empirical situation. In this manner Fare and Logan (1992) have shown what effects the explicit inclusion of the rate-of-return regulation (RoR) as a constraint can have on the measured efficiency of the enterprises.

When trying to do the same with the regulation of quality the following aspects have to be considered:

First, the quality of electricity supply can to a great extend directly be attributed to the level of so called “quasi-fixed” capital inputs, like transformer stations and transmission cables/lines, that cannot be adjusted to their optimal levels instantaneously such that decisions about their level in one period have important implications not only for the efficiency in that period but also for that of subsequent ones. In other words: The characteristics of the provision of quality call for a dynamic perspective that captures the intertemporal aspects of investment in quasi-fixed inputs more accurately than the conventional static view. Amongst the first ones to realize this necessity were Nemoto and Goto and they thus augmented conventional DEA by treating quasi-fixed inputs at the end of one period as if they were outputs in that period and essential inputs in the subsequent one (Nemoto and Goto, 1999). In this setting the firm faces installation costs: the more resources are consumed in installing quasi-fixed inputs, the less there are left over for producing outputs[1]. On the other hand, more quasi-fixed inputs in the next period mean greater production possibilities and therefore profits in that and subsequent periods. This is the basic trade-off the firm faces: Either maximize output myopically in this period or invest in quasi-fixed inputs to increase output in subsequent ones. To sum up: Measures for quality (like ENS) should be treated like quasi-fixed inputs in Nemoto and Goto's framework when incorporating it in the DEA-framework.

This finding directly leads us to the second important point, that has to be considered when adapting the DEA-framework for quality, namely that, when taking a measure like ENS to represent the level of quality, we cannot directly use it as a "normal" quasi-fixed input as Nemoto and Goto have done since it has the characteristics of an "undesirable" output. Several approaches on how to amend DEA for such a situation have been suggested. Dyckhoff and Allen (2001) for example mention Knox et al. (1995) who take the reciprocals of the undesirable outputs and treat them as normal outputs. Instead of taking the reciprocal, undesirable outputs can also be transformed by a translation (for example by multiplying them with -1) (c.f. Dyckhoff and Allen, 2001, p. 314). Fare et al. (1996) use the "weak disposability assumption" to model the undesirable output, i. e. they assume that reducing the undesirable output requires increased quantities of inputs or decreased quantities of desirable outputs. Courcelle et al. (1998), on the other hand, assess the economic and environmental performance of munic-

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[1] Nemoto and Goto were also able to relate their approach seamlessly to the adjustment-cost theory of investment, so that it provides a non-parametric alternative to the econometric Euler equation approach (Nemoto and Goto, 2003, Appendix).

ipal waste collection and sorting programs by defining the undesired output as the ratio between the amount of material sent to final disposal and the total amount of material leaving the processing plant. This so-called "residue ratio" is then treated like a normal input in DEA (c.f. Dyckhoff and Allen, 2001, p. 314). Korhonen and Luptacik (2004) show that these approaches have nearly identical mathematical properties so that practical considerations should lead the way when deciding on which one to choose.

We therefore choose Knox et al.'s approach, as it allows for an easy and intuitive integration of a measure of quality as an undesired quasi-fixed input in the concept of dynamic DEA.

How this is accomplished shall be shown in the following section.

2.2 Formulation

2.2.1 The production possibility set[2]

Let x_t denote a $l \times 1$ vector of variable inputs used in the period t, k_t a $m \times 1$ vector of quasi-fixed inputs at the end of period t , and y_t a $n \times 1$ vector of outputs produced in period t. The firm (or "decision making unit" - DMU) puts x_t and k_{t-1} into the production process P_t in order to supply y_t to the market and to hold k_t at the end of that period. This basic setting shall be illustrated by figure 2. All combinations of $(x_t, k_{t-1}) \in \mathbb{R}^+_{l+m}$ and $(k_t, y_t) \in \mathbb{R}^+_{l+m}$, where the latter is producible from the former, constitute the

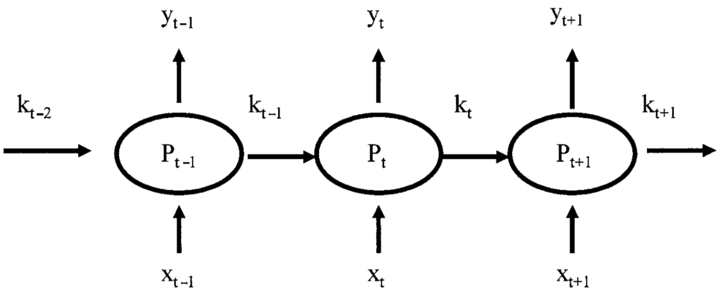

Figure 2: The technology of dynamic DEA

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[2] The mathematical description of the production possibility set is taken from Nemoto and Goto (2003)

production possibility set in period t:

$$\Phi_t = \{(x_t, k_{t-1}, k_t, y_t) \in \mathbb{R}_{l+m} \times \mathbb{R}_{m+n} | (x_t, k_{t-1}) \text{ can yield } (k_t, y_t)\} \tag{1}$$

It is required that Φ_t satisfies the regularity conditions:

(i) if $(\tilde{x}_t, \tilde{k}_{t-1}, k_t, y_t) \in \Phi_t$ and $(\tilde{x}_t, \tilde{k}_{t-1}) \leq (k_t, y_t)$, then $(x_t, k_{t-1}, k_t, y_t) \in \Phi_t$;

(ii) if $(x_t, k_{t-1}, \tilde{k}_t, \tilde{y}_t) \in \Phi_t$ and $(\tilde{x}_t, \tilde{k}_{t-1}) \geq (k_t, y_t)$, then $(x_t, k_{t-1}, k_t, y_t) \in \Phi_t$;

(iii) Φ_t is closed and convex

If the production technology is constant returns to scale, Φ_t becomes a convex cone:

(iv) if $(x_t, k_{t-1}, k_t, y_t) \in \Phi_t$, then $(cx_t, ck_{t-1}, k_t, y_t) \in \Phi_t$ for any $c > 0$.

As we are ultimately interested in empirical results we want to find a more accurate description of Φ_t that satisfies the above conditions than what a mere arbitrary guess of a production function á la Cobb-Douglas can yield. DEA provides a solution to this problem by constructing a polyhedral convex hull enveloping (hence the name) the observed data:

Suppose we have N observations, i. e. firms, with variable inputs $X_t = (x_{t1}, x_{t2}, \ldots, x_{tN})$ (each x_{ti} represents the input-vector of a firm), quasi-fixed inputs $K_{t-1} = (k_{t-11}, k_{t-12}, \ldots, k_{t-1N})$ at the beginning of period t and quasi-fixed inputs $K_t = (k_{t1}, k_{t2}, \ldots, k_{tN})$ at the end of period t.

Assuming variable returns to scale, the smallest set comprising these observations and satisfying (i)-(iv) takes the form:

$$\begin{aligned}\hat{\Phi}_t = &\{(x_t, k_{t-1}, k_t, y_t) \in \mathbb{R}_+^{l+m} \times \mathbb{R}_+^{m+n} | X_t\lambda_t \leq X_t, \\ &K_{t-1}\lambda_t \leq k_{t-1}, K_t\lambda_t \geq k_t, Y_t\lambda_t \geq y_t, i'\lambda_t = 1, \lambda_t \geq 0\}\end{aligned} \tag{2}$$

where λ_t is a $N \times 1$ intensity vector whose j-th element is denoted by λ_{tj} and i is a $N \times 1$ vector of ones.

2.2.2 Technical efficiency - the additive model

Technical efficiency in the DEA-context can be defined in several ways. In the original formulation of Charnes et al. (1979), referred to as CCRmodel, it was defined either as to what extend the inputs of each DMU could be reduced proportionally while remaining on the same isoquant (input-orientation) or as by how much the outputs could be increased proportionally while holding inputs constant (output-orientation). In our dynamic context this leads to problems as the quasi-fixed inputs have the character of outputs in period t and that of inputs in period $t+1$ and therefore, when trying to determine the technical efficiency of a DMU, both an input- and an output-orientation is required. The so-called additive model circumvents the above problem by combining both orientations. Here efficiency is somewhat defined the other way round: For each DMU, the maximal sum of all slacks, i. e. the distances to the efficient frontier in all inputs and outputs, is determined. A DMU is efficient, only if this sum is zero. Figure 3 is supposed to illustrate the differences of the 2 concepts.

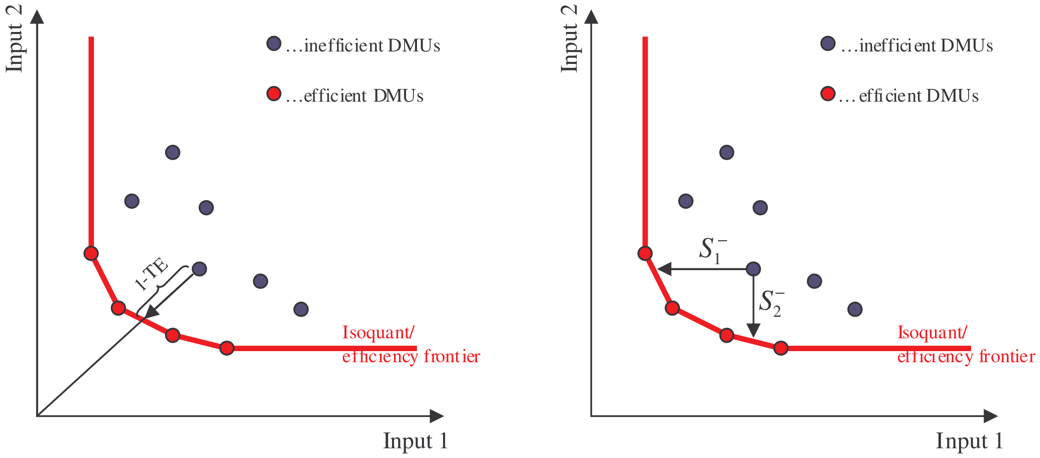

Figure 3: The CCR- (left) and the Additive model (right)

2.2.3 The intertemporal LP-problem[3]

Taking up the efficiency-concept from the additive model leads to the following intertemporal optimization problem: Maximize the sum of the slacks of all factors over the entire time-horizon subject to the restrictions of the production possibility frontier as given by (2).

---

[3] LP…linear programming

This problem is equivalent to the following linear program:

$$
\begin{aligned}
\max_{\left\{S_{k_0},S_{y_t},S_{x_t},S^+_{k_t},k_t,S^-_{k_t},\lambda_t\right\}_{t=1}^{T}} \quad & \gamma S_{k_0}+\sum_{t=1}^{T}\gamma^t\,\Big(S_{x_t}+S_{y_t} \\
& \qquad +d_{S^+_{k_t}}S^+_{k_t}+d_{S^-_{k_t}}S^-_{k_t}\Big) \\
\text{s.t.} \qquad k_{t-1}-K_{t-1}\lambda_t-S_{k_{t-1}} &= 0 \qquad t=1 \\
k_{t-1}-K_t\lambda_t-S^-_{k_t} &= 0 \qquad t=2,\dots,T \\
x_t-X_t\lambda_t-S_{x_t} &= 0 \qquad t=1,2,\dots,T \\
Y_t\lambda_t-S_{y_t}-y_t &= 0 \qquad t=1,2,\dots,T \\
K_t\lambda_t-S^+_{k_t}-k_t &= 0 \qquad t=1,2,\dots,T-1 \\
i'\lambda_t &= 1 \qquad t=1,2,\dots,T \\
S_{k_0},S_{y_t},S_{x_t},S^+_{k_t},k_t,S^-_{k_t},\lambda_t &\geq 0 \qquad t=1,2,\dots,T \\
d_{S^+_{k_t}},d_{S^-_{k_t}} &\in \{0,1\} \qquad t=1,2,\dots,T
\end{aligned}
\tag{3}
$$

where k_0 is the initial exogenous value of quasi-fixed inputs, γ is a discount factor and $d_{S^+_{k_t}}$ and $d_{S^-_{k_t}}$ are dummy-variables whose value is either 0 or 1[4]. The program determines for each DMU the maximal slack-value for each input and output category for every point in time.

The intertemporal aspect in this program is represented by the constraints 2 and 5: The program tries to find the combination of $S^+_{k_t}, S^-_{k_t}$ and k_t for each period that maximizes the total slack. In other words: Whereas the values of the variable inputs of each period are the exogenously given (but controllable by the firm) observed data, only the initial value for the quasi-fixed input is given exogenously and the subsequent optimal values are determined in the process of the optimization. This is where the basic trade-off of the firm is manifested: On the one hand it wants to close its gap to the efficient frontier concerning the outputs and thus also increase its amount of quasi-fixed inputs in period t but on the other hand such an increased amount of quasi-fixed inputs reduces its efficiency concerning the inputs in period $t+1$. This dilemma shall be illustrated by figure 4.

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[4] These dummy-variables are necessary for modeling the regulation of quality as will be explained in section 2.2.4.

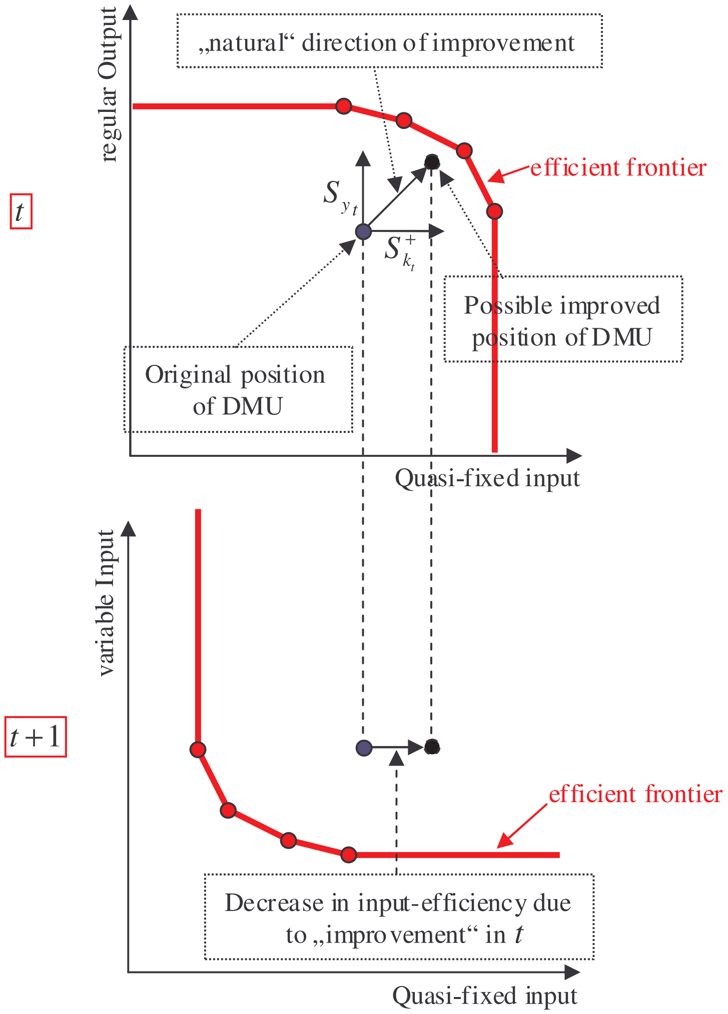

Figure 4: The trade-off between output improvement and input-efficiency deterioration

2.2.4 Modeling the regulation of quality

As already mentioned above, quality enters into our model as an undesired quasi-fixed inputs in the form of ENS. That means that the exact nature of the above described decision-problem of the firm depends on whether quality is regulated or not:

No regulation of quality In this case, the firm maximizes its regular output and minimizes its variable inputs but doesn't care about the ensuing level of the quasi-fixed input. In the context of the LP-problem (3) this means that the program has to ignore the slack of the quasi-fixed input $S_{k_t}^-$ in period $t+1$ but should seek to maximize its slack $S_{k_t}^+$ in period t[5]. This behavior can be achieved by setting $d_{S_{k_t}^+}$ to 1 and $d_{S_{k_t}^-}$ to 0 in the periods without regulation of quality.

Remark. It is a big merit of the inter-temporal formulation that it naturally sets a lower non-zero bound for k_t^*, the optimal level of k_t in t and $t+1$, so that the potential problem of obtaining a zero as the optimal value, when the undesired output enters as its reciprocal, is automatically alleviated. This is shown in the appendix.

With regulation of quality When quality is regulated like in Norway, such that there is a penalty for every MWh of energy-not-supplied, the firm will adapt its objective such that not only its regular outputs are maximized and its variable inputs are minimized but **also** such that its quasi-fixed inputs are minimized. In the context of the LP-problem (3) this means that the program has to ignore the slack of the quasi-fixed input $S_{k_t}^+$ in period t but should seek to maximize its slack $S_{k_t}^-$ in period $t+1$. This behavior can be achieved by setting $d_{S_{k_t}^+}$ to 0 and $d_{S_{k_t}^-}$ to 1 in the periods with regulation of quality.

To sum up: By setting the dummies in the objective function appropriately, we can simulate the decision problem of the firm for the case of regulation

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[5] Remember: The quasi-fixed input enters as its reciprocal in the LP-problem so that for example a bigger slack in t (i. e. a value closer to 0) means a bigger actual value of the variable.

and non-regulation of quality. Since the model then determines the respective ensuing optimal level of quality for the firm, we can calculate the respective optimal paths of quality and thus a counter-factual that will enable us to draw conclusions as to what difference the regulation in reality has made.

3 The Data

The dataset originally consisted of the fifty largest Norwegian electricity distribution firms as published by the Norwegian Water Resources and Energy Directorate (NVE). After eliminating units with insufficient data quality the model was finally applied to 29 DMUs (decision making units) in the years 1999-2005. Input- and output-factors were chosen for the following reasons:

Variable inputs As the sole variable input, total expenditures (TOTEX), which consist of operating expenditures (OPEX) and capital expenditures (CAPEX), were chosen. In our case, OPEX comprised the costs for network losses, wages and other costs. Following Korhonen and Syrjänen (2003), costs for transmission services were not included in the OPEX, as they are beyond the control of a single unit. Our capital expenditures (CAPEX) consisted of depreciation plus the value of the assets multiplied with the so called fair rate of return. The fair rate of return is set by the regulator and serves as a reasonable approximation of the actual financing costs of a firm. According to Grasto (1997) and Kinnunen (2003) the fair rate of return which is used in Norway is the return of a medium term government bond (risk free rate) plus a two percent risk premium, where debt and equity are treated equally. Having a figure for annual capital expenditures allows us to compute total expenditures (TOTEX) by adding OPEX and CAPEX.

Quasi-fixed Inputs As already indicated above, energy-not-supplied (ENS) which measures the amount of energy (in MWh) that could not be delivered due to failures of the distribution system was used as the sole quasi-fixed input. To be more precise, ENS measures how much energy customers would have used, if there had been no failure by considering the typical load curve of customers at the time of the outage.

Outputs Two outputs were chosen to account for the most important dimensions: The amount of energy delivered over the network (MWh) and the number of customers.

4 Results[6]

Figure 5 shows the actual development of the sum of ENS for our sample. As can be seen, the picture is more or less the same as with the whole of Norway (c. f. figure 1), albeit more drastic: Here too, it is striking, that the reduction in ENS was much more pronounced before the introduction of quality regulation in 2001 than after it. From the very different courses of the upper dotted ($\text{mean}_{\text{act. data}} + \text{st. dev}_{\text{act. data}}$) and lower dotted lines ($\text{mean}_{\text{act. data}} - \text{st. dev}_{\text{act. data}}$) in figure 6 we infer that the regulation affected enterprises with relatively high ENS differently than those with relatively low ENS and conjecture the following:

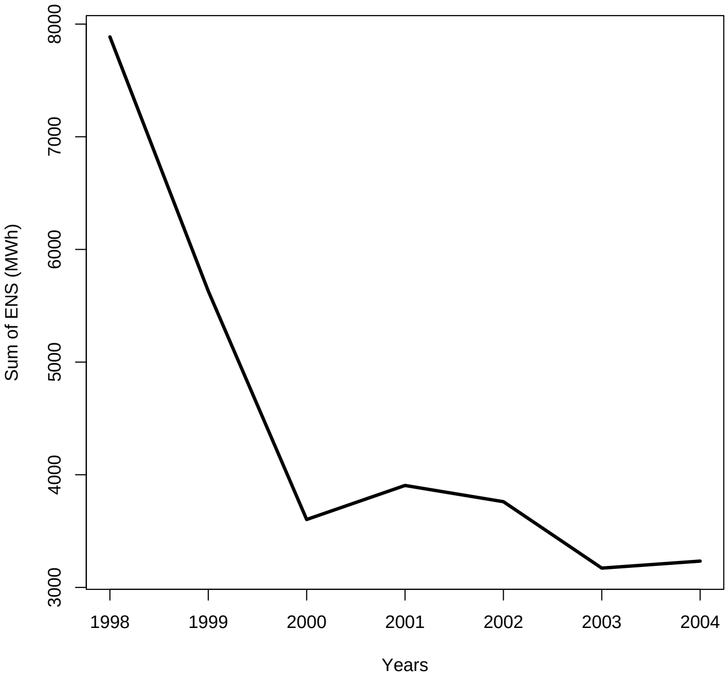

Figure 5: Development of the sum of ENS of the sample

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[6] The calculations were all done in R with the package “LP-solve” and a discount factor γ of $\left(\frac{1}{1+0.06}\right)$.

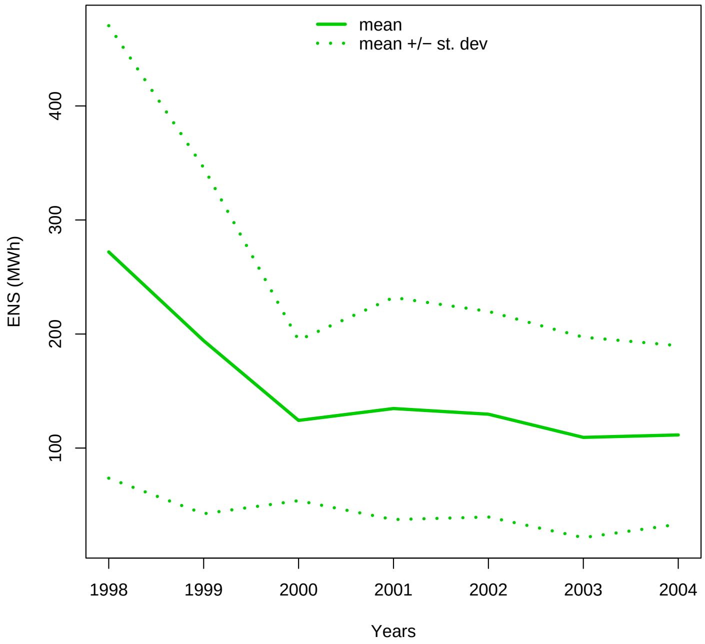

Figure 6: Development of the $sample\_mean \pm standard\_deviation$ of the actual ENS

Conjecture 1. *In the whole time-span the "upper fraction" (UF)*[7] *acted as if no regulation had been in place.*

Conjecture 2. *The "lower fraction" (LF)*[8] *acted as if quality had been regulated in the whole time-span.*

We utilize our two versions of the model - regulated and non-regulated - to test the above hypotheses. To be more precise: In the unregulated model, quality is not subject to regulation at any point, this is modeled as described in section 2.2.4. In the regulated model, quality is not regulated from 1999 until 2000 but subject to regulation from 2001 until 2005. In other words, the regulated model simulates "reality" whereas the unregulated model stands for the counter-factual.

Figure 7 shows the UF and the LF of the calculated optimal paths of the respective models together with the UF and LF of the actual values of ENS. Apparently, the optimal paths of the 2 models are identical from 1998 until

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[7] Henceforth we will call $mean + st.dev$ "upper fraction" (of the sample).

[8] Henceforth we will call $mean - st.dev$ "lower fraction" (of the sample).

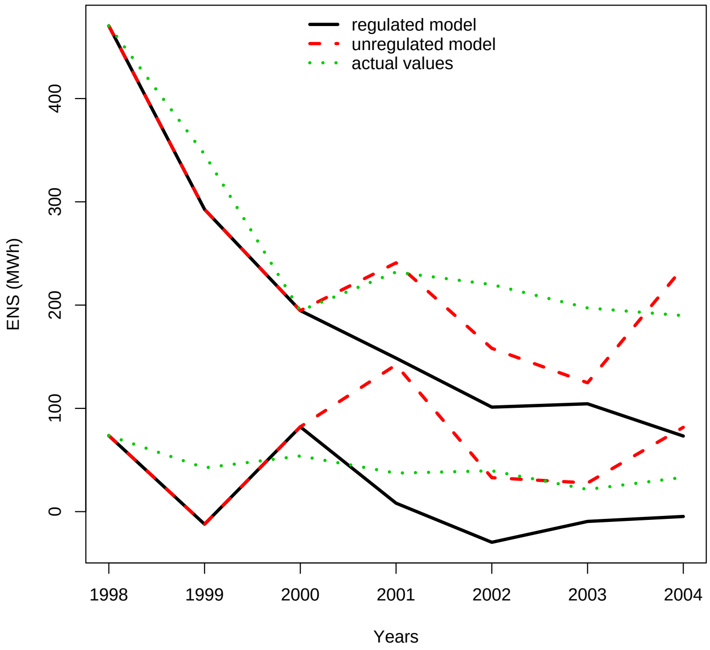

Figure 7: Development of the $sample\_mean \pm standard\_deviation^*$ of the actual ENS, the regulated model and the unregulated model

*...sample-mean omitted

2000 - this is the logical result of the identical model specification in this time-span. From 2001 on, however, the 2 paths diverge significantly.
From 1999 until 2000 both models' prediction for the UF is very similar to the UF of actual values. From 2000 onwards, however, the gap between the regulated UF and the actual UF steadily increases whereas the UF of the unregulated model fluctuates around the actual UF. Therefore, we can conclude that, if at all, the unregulated model predicts the actual UF better than the regulated model, which supports our conjecture 1.
Conjecture 2 is supported by the fact that the unregulated LF fluctuates rather erratically around the actual LF in the whole time-span whereas not only the shape of regulated LF looks very similar to the actual LF but also both - the regulated LF and the regulated UF - seem to converge towards the actual LF.

To sum up: For our sample, the regulation of quality in Norway did not have a significant effect on the amount of ENS:
Neither did it induce those firms with relatively high ENS to change their behavior, nor those with relatively low ENS.

5 Conclusion

Our investigation was led by the quest to find answers as to why ENS - the most important indicator for the quality of service in the quality-regulation regime in Norway - declined much more pronounced before the introduction of quality-regulation in 2001 than after it. In order to answer this difficult question we first developed a dynamic DEA-model in which ENS enters as a “quasi-fixed” input. This way the short run fixity of the level of quality and the thus inter-temporal aspects of investment-decisions could be treated more adequately than in traditional static DEA-models. Moreover, it enabled us, by manipulating the objective function, to explicitly model the incorporation or absence of quality in a regulatory regime.
Equipped with this model we looked at the data of 29 electricity distribution operators. This sample showed broadly the same characteristics as the whole of Norway: Apparently those operators with relatively high ENS, reduced ENS significantly before 2001 but then they followed a flat path. On the other hand, those operators with relatively low ENS showed low reduction

rates the whole time. Inferring from this observation, we conjectured first, that the firms with relatively high ENS acted as if regulation had been absent in the whole time-span and second, that the firms with relatively low ENS acted as if regulation had been present in the whole time-span.

Utilizing our dynamic quality-DEA-model in a regulated and a unregulated version in order to calculate the optimal respective ENS-paths we could find strong support for the above conjectures and we can therefore conclude that, at least for our sample, the regulation of quality in Norway did not have a significant effect.

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A Appendix

Lemma. *The solution for the undesired output when entered as its reciprocal is non-zero in the dynamic DEA-Model with variable returns to scale.*

Proof. The constraints 2 and 5 in (3) determine the value of k_t. Adding constraint 2 of period $t+1$ to constraint 5 of period t eliminates k_t and yields the following expression:

$$\underbrace{K_t\lambda_t - S^+_{k_t}}_{k_t} \underbrace{-K_t\lambda_t - S^-_{k_t}}_{-k_t} = 0$$

Suppose, now, that k_t is zero. That means that the right part of the above expression must be zero too. Since $S^-_{k_t}$ is positive by assumption, therefore $K_t\lambda_t$ either has to be zero or negative which both contradicts our assumption of variable return to scale as represented by constraint 6. $\square$