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# The Economic Role of Jumps and Recovery Rates in the Market for Corporate Default Risk\*

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## **Abstract**

Using an extensive cross-section of US corporate CDS this paper offers an economic understanding of implied loss given default (LGD) and jumps in default risk. We formulate and underpin empirical stylized facts about CDS spreads, which are then reproduced in our affine intensity-based jump-diffusion model. Implied LGD is well identified, with obligors possessing substantial tangible assets expected to recover more. Sudden increases in the default risk of investment-grade obligors are higher relative to speculative grade. The probability of structural migration to default is low for investment-grade and heavily regulated obligors because investors fear distress rather through rare but devastating events.

**Keywords:** credit default swaps, loss given default, stochastic intensity, jump-diffusion, Markov chain Monte Carlo estimation

**JEL:** C11, C15, C51, C52, E43, G13

# I. Introduction

Credit default swaps (CDS) are derivative contracts aimed at transferring default risk of an underlying reference entity from one market participant to another. The protection seller assumes the credit risk of the underlying entity by committing to compensate the protection buyer for the loss suffered in case of a default of the entity on its outstanding debt, in return for a regular protection fee paid by the CDS buyer. After a credit event, and assuming physical settlement, the seller makes a payment to the buyer equal to the notional value of the contract, and in turn receives defaulted obligations of equivalent notional value.

The present paper analyzes two issues concerning CDS pricing in the reduced-form framework which have been neglected to date but are presently gaining both academic and practitioner attention as a global downturn has evolved since mid-2007: sudden jumps in the default intensity of an obligor and expected recovery rates at default. In modeling the credit risk inherent to an obligor, we embed the specification of the stochastic default intensity into the multi-factor affine jump-diffusion framework. The model is estimated on CDS panels for an extensive cross-section of US corporate obligors, offering us an intuitive economic understanding of *how* the market perceives jump risk and loss given default (LGD) prospects in the single-name CDS market, segmented with respect to rating and industry. To the best of our knowledge we are both the first to estimate LGD and to estimate jumps from single-name corporate CDS premia using the complete CDS panel to this end (i.e., a time series of term structures).

The first contribution of our paper is identification of implied LGD and insight into its cross-sectional distribution. Namely, both in academic studies and industry reports it has been common practice to fix corporate LGD at its historical average of 60% (cf. [Houweling and Vorst \(2005\)](#), [Chen, Cheng, and Wu \(2005\)](#), [Longstaff, Mithal, and Neis \(2005\)](#), [Chen, Cheng,](#)

Fabozzi, and Liu (2008)). In contrast, our resulting LGD estimate is a model- and market-implied quantity. Taking the pre-2008 low-default environment into account, our estimates are plausible in level and their dispersion compares to that of realized LGD (cf. Altman and Kishore (1996), Altman, Brady, Resti, and Sironi (2005), Altman, Resti, and Sironi (2004), Acharya, Bharath, and Srinivasan (2007)). We investigate the relationship between loss given default and probability of default by looking at LGD across ratings as a crude proxy for credit quality. The investment grade of an obligor turns out to be an important determinant for the behavior of LGD. Furthermore, industry affiliation strongly affects LGD level as well: Obligors in sectors with substantial tangible assets are expected to recover more in default than other sectors. Exploiting the cross-section of implied LGD we provide indicative LGD values for pricing and hedging purposes consistent with rating and industry affiliation. Our indicative LGD values are particularly relevant when dealing with recovery derivatives and credit portfolios.

The second contribution of our paper relates to the dynamics employed for the default intensity of the latent Cox process governing default and survival of an obligor. We infer several *empirical stylized facts* about CDS premia from our cross-section of US corporate obligors which we reflect in our intensity specification. One feature refers to peculiar behavior of short-maturity spreads; two features relate to discontinuities (jumps) in the time series of CDS premia:

- A** A discontinuity mostly affects broad ranges of the CDS maturity spectrum.
- B** The change in CDS premia at a discontinuity is mostly positive.
- C** The 1-year CDS premium exhibits time-series variation higher-maturity premia do not share.

These observations motivate our jump-diffusion specification with two stochastic factors,

economically interpreted as default intensity and its long-run mean in the spirit of [Bakshi, Madan, and Zhang \(2006a\)](#). Since predecessor papers predominantly employ diffusion specifications, empirically investigating jumps in the default state variables is novel to the single-name CDS literature. Our findings, based on data comprising the onset of the 2007/08 credit crunch, show that substantial jumps in creditworthiness are expected both in the short and in the long term. Our paper closes this gap in the literature since related research focuses on topics adjacent to, but different from ours, such as default event risk ([Driessen \(2005\)](#)), liquidity ([Longstaff, Mithal, and Neis \(2005\)](#)), connection between equity and CDS markets ([Carr and Wu \(2006\)](#)), explicit CDS pricing ([Chen, Cheng, Fabozzi, and Liu \(2008\)](#)), sovereign CDS markets ([Pan and Singleton \(2008\)](#)), and the swap market ([Feldhütter and Lando \(2008\)](#)).

We employ Markov chain Monte Carlo (MCMC) methods to simultaneously estimate both parameters and latent processes of our multivariate jump-diffusion model. The results are manifold: The default intensity process turns out to be closely related to short-term CDS spreads, while its long-run mean captures long-term CDS spreads; this finding suggests that the two processes reflect different issues of concern to CDS markets: short-term liquidity difficulties and long-term prospects of the obligor. We do not find support for a linear relation between the risk-free and defaultable term structures. Further, the impact of sudden shocks is more damaging to the creditworthiness of investment-grade obligors. The relative risk premium on the extent of bad news affecting the long run is significantly higher for upper investment-grade obligors as well, which corresponds to findings in [Cremers, Driessen, and Meanhout \(2008\)](#). Within investment grade, though, no distinction can be made regarding frequency and effect of shocks on an obligor's default probability. Moreover, persistence in the observed CDS time series is high and carries over to the driving default state variables, suggesting that the market does not frequently revise its opinion of an obligor and tends to forget slowly after a credit-related event. Lastly, according to the [Bates \(1991\)](#) and [Bakshi,](#)

Cao, and Chen (1997) criterion our estimated risk premia imply reasonable risk preferences.

The paper proceeds as follows: The next section describes the data and discusses important features of CDS contracts with respect to which data are selected for our study. It additionally provides an economic discussion of the empirical stylized facts supported by a non-parametric analysis of the data. Section III introduces our model specification and elaborates on its capabilities to reproduce the empirical stylized facts observed in CDS panels. The main issues concerning our MCMC estimation methodology are outlined in Section IV. Section V presents a detailed economic analysis of our cross-sectional estimation results. Finally, Section VI concludes.

## II. CDS Data

### A. Data Description

CDS histories are taken from a comprehensive dataset obtained from Markit Group Ltd, which comprises daily CDS premia for a global cross-section of obligors between January 2, 2001 and May 30, 2008. By collecting indicative CDS premia from a broad range of dealers and aggregating them into a composite value, Markit ensures reasonably continuous and accurate time series. For this reason their data are increasingly employed in academic studies.

For our results to be comparable to other studies the focus is exclusively on corporate obligors based in the United States (totaling 1583 names). At the same time, by restricting the analysis to US obligors only, we reduce the effect of the delivery option on CDS spreads: Since the US corporate bond market is globally the most developed, it is more probable for US obligors – than for those based, e.g., in Europe – not to issue debt in foreign currencies.<sup>1</sup>

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<sup>1</sup> Physical delivery is the predominant form of settlement in the CDS market, accounting for approx.

We therefore restrict our analysis to USD-denominated CDS contracts and do not take into account foreign interest-rate risk or foreign exchange risk. To maintain uniformity in contracts, we only keep CDS premia for senior unsecured debt with the US standard Modified Restructuring clause. Obligors taking part in M&A activity during the sample period are excluded.

For each obligor we have a panel  $\bar{s} = \{\bar{s}_t\}_{t=1}^T$  of CDS premia, where  $\bar{s}_t = (\bar{s}_t(1y), \bar{s}_t(3y), \bar{s}_t(5y), \bar{s}_t(7y), \bar{s}_t(10y))^\top$ . Only the five canonical CDS maturities (cf. [Brigo and Mercurio \(2006\)](#), p. 719) of 1, 3, 5, 7 and 10 years are used in our analysis since these are most frequently quoted and traded. Though Markit's data aggregation mitigates the problem to a large extent, there still exists a non-negligible proportion of missing and stale spreads. We set high thresholds on data quality to guarantee enough data points for estimation and to ensure that missing data points are not clustered. First, the overall percentage of missing spreads per panel (which in our definition includes stale spreads) must not exceed 15%, and second, the length of the longest series of consecutive missing spreads must be 10 days or less. There is a clear trade-off between the length of the time period and the number of maturities available per day – the further one reaches into the past, the less term structure exists per day. For this reason we trade off the length of the time series against the number of obligors

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73% of volume according to the 2006 Credit Derivatives Report by the British Bankers' Association (BBA). Contracts admit a basket of deliverable obligations defined by a set of characteristics, the most relevant of which are currency and seniority. At default, the protection buyer thus has the option to deliver the cheapest obligation from this basket. In conversations with practitioners we have learned that generally a higher value is attached to the delivery option if the reference entity has debt outstanding in several currencies because of the foreign exchange risk and foreign interest rate risk induced. Our model implicitly treats the delivery option value as a constant subsumed under the estimate of implied LGD.



satisfying our quality criteria and work with a dataset containing 278 obligors and spanning approx. 4.5 years from January 1, 2004 to May 30, 2008 (totaling 1146 days).

[TABLE 1 about here.]

The processed set of obligors is classified into industry sectors according to the ICB scheme. The ICB classification system consists of four layers, first of which is the Industry layer comprising ten categories: *Basic Materials*, *Consumer Goods*, *Consumer Services*, *Financials*, *Health Care*, *Industrials*, *Oil&Gas*, *Technology*, *Telecommunications*, and *Utilities*. The rating distribution within each individual industry sector is presented in Table 1. Our sample of obligors is slightly tilted towards investment-grade entities.

## B. Empirical Stylized Facts

The Introduction states several empirical stylized facts about corporate CDS premia which suggest themselves when CDS premium panels are visually inspected firm by firm (Figure 1 depicts the CDS panel for Honeywell Int'l Inc. as an example). In this section we underpin these observations with economic reasoning and statistical inference, summarized in Table 2.

[FIGURE 1 about here.]

Observation **C** has already been made by [Pan and Singleton \(2008\)](#) in the context of sovereign CDS spreads. Since investment funds primarily use 1-year CDS to express views on the creditworthiness of an obligor, the economic driver behind the unique pattern in 1-year spreads is in all likelihood a supply-and-demand premium induced by such large trades. We perform a principal components analysis to statistically investigate observation **C** and gain insight on the number of latent default factors necessary to describe CDS spreads. Panel **A** confirms our claim: Even though for half of the obligors more than 95% of the variance in their

CDS panels can be explained by means of a one-factor model (87% of the variance for 90% of the obligors), there exist CDS panels for which a one-factor setting is by far insufficient, with only 69% explained. The minimum variance explained by two factors amounts to 97%. Subsequently we regress time series of the first principal component on time series of individual CDS maturities revealing that for 90% of obligors the 1-year CDS premium exhibits the lowest degree of co-movement with the first principal component. A meaningful analysis of the entire cross-section of US corporate obligors therefore calls for a two-factor model.

[TABLE 2 about here.]

Stylized fact **A** is economically motivated by noting that expectations of an imminent unfavorable event inevitably affect contracts of both short and long maturities. Similarly, when expectations about overall credit quality suddenly change, the entire CDS maturity spectrum reacts. Observation **B** reflects the fact that financial distress causes sudden upward moves in CDS premia because protection sellers demand a higher compensation for bearing the risk they perceive. In comparison, good news rather tends to propagate gradually. Statistical tests given below confirm our economic reasoning:

Neither levels nor first differences of CDS premia are normally distributed; a Jarque-Bera test strongly rejects the null hypothesis of normality for all CDS panels in our sample ( $p$ -values do not exceed even 0.01%). All maturities exhibit significant excess kurtosis on average (see Panel **D**). Furthermore, kurtosis in the levels decreases with maturity, with 1-year CDS premia exhibiting the heaviest tails. Positive unconditional skewness in CDS spreads (see Panel **C**), together with high excess kurtosis, supports our claim that large jumps are mostly positive. Consequently, if one modeled CDS premia by a diffusion process, it would hardly be possible to reproduce their time-series properties with plausible parameter values.

First differences in CDS spreads highlight a further aspect: If innovations were normal, first differences in CDS spreads would be conditionally normally distributed with skewness and excess kurtosis of zero. However, Panel F indicates extreme values of excess kurtosis for all maturities, implying additional, non-Gaussian innovations in the background driving process. The differences of short-term CDS premia exhibit both the highest skewness and the highest excess kurtosis, with both statistics decreasing for longer maturities. This finding could be due to new information relevant for the creditworthiness of an obligor being incorporated into the short-term spreads first and with a stronger impact on the spread, while the same news is incorporated into spreads of the longer maturities slightly less drastically. This result can also be considered supportive of the above-mentioned fact that fund managers tend to use the 1-year CDS to express views on an obligor, making it one of the more liquidly traded maturities.

To collect further evidence we perform two non-parametric tests for jumps: the [Lee and Mykland \(2008\)](#) volatility test<sup>2</sup> and the [Barndorff-Nielsen and Shephard \(2006\)](#) bipower variation test. Both tests are designed for high-frequency data, however. Since there is only one spread of each maturity available per day, we perform the bipower variation test on buckets of 21 daily spreads (approx. one month), which is a low observation frequency compared to [Barndorff-Nielsen and Shephard \(2006\)](#). In a simulation study (cf. [Appendix A](#)) we investigate the performance on low-frequency data for the two tests and conclude that both perform well in detecting a wrong null hypothesis of no jumps. Though a type-one error (rejecting a true null hypothesis of no jumps) is committed too often, the number of jumps detected in our sample is beyond the critical region.

[TABLE 3 about here.]

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<sup>2</sup> The [Lee and Mykland \(2008\)](#) test is based on the relationship between realized and estimated volatility.

The test output is presented in Table 3. At a 5% significance level, the bipower variation test infers a minimum of 2.0, 1.5, 1.1, 1.1 and 1.3 jumps per year for the respective CDS maturities. The nonzero-drift version of the volatility test estimates a minimum of 2.9, 1.8, 2.9, 3.1 and 2.6 jumps per year for the respective CDS maturities, also at a 5% significance level. Since the volatility test allows arbitrarily many jumps within a time period and the bipower variation test only one, the [Lee and Mykland \(2008\)](#) test infers more jumps per year compared to the [Barndorff-Nielsen and Shephard \(2006\)](#) approach. We thus conclude that there is strong evidence of jumps in the time series. In addition, both tests show that CDS spreads of different maturities tend to jump simultaneously (i.e., during the same month).

### III. Model Specification

A filtered probability space  $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbb{Q})$  is given, with  $\mathbb{Q}$  being a risk-neutral pricing measure. The standard intensity-based framework postulates a latent Cox process whose first jump time  $\tau$  determines the default time of the obligor. Let  $\eta$  denote the non-negative stochastic process for the default intensity, whose own filtration is assumed conditionally independent from the default event itself. There also exists a risk-free short rate  $r$  such that the time- $t$  price of a risk-free zero-coupon bond maturing at time  $T$  equals

$$(1) \quad P(t, T) = \mathbb{E}_t^{\mathbb{Q}} \left[ \exp \left\{ - \int_t^T r_s ds \right\} \right],$$

where the expectation is taken under the measure  $\mathbb{Q}$  and conditional on information available at time  $t$ . The price of a defaultable zero-coupon bond with zero recovery is given by

$$(2) \quad \tilde{P}(t, T) = \mathbb{E}_t^{\mathbb{Q}} \left[ \exp \left\{ - \int_t^T \tilde{r}_s ds \right\} \right],$$

where the riskless short rate is replaced by a default-adjusted short rate  $\tilde{r} := r + \eta$ . To price CDS contracts (cf. Appendix B.1) one therefore needs to model the short rate  $r$ , the default intensity  $\eta$ , as well as possible dependencies between these two processes.

For the riskless short rate we employ a two-factor affine model where one factor is interpreted as the riskless short rate and the other as its stochastic long-run mean in the spirit of [Bakshi, Madan, and Zhang \(2006a\)](#). The model dynamics under the pricing measure  $\mathbb{Q}$  are thus given by:

$$(3) \quad \begin{aligned} dr_t &= \kappa_r^{\mathbb{Q}}(m_t - r_t)dt + \sigma_r \sqrt{r_t} dW_r^{\mathbb{Q}}(t) \\ dm_t &= (\zeta_m - \kappa_m^{\mathbb{Q}} m_t)dt + \sigma_m \sqrt{m_t} dW_m^{\mathbb{Q}}(t). \end{aligned}$$

The panel of zero-yields is subject to diffusive risk generated by uncorrelated Brownian motions  $W_r^{\mathbb{Q}}$  and  $W_m^{\mathbb{Q}}$ . To accommodate the extreme persistence in riskless yields and to maintain flexibility we employ essentially affine market prices of risk (cf. [Duffee \(2002\)](#)), so that the speed of mean reversion is estimated separately under each measure to account for the risk premium.<sup>3</sup> This preserves the structure and interpretation of the model above under both probability measures.

Our specification of the stochastic default intensity is also embedded in the affine frame-

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<sup>3</sup> Model parameters under both measures satisfy the admissibility conditions from [Duffie, Filipović, and Schachermayer \(2003\)](#); additionally we constrain the mean-reversion parameters so that the processes are non-explosive:

$$(4) \quad \kappa_r^{\mathbb{Q}}, \kappa_m^{\mathbb{Q}}, \kappa_r^{\mathbb{P}}, \kappa_m^{\mathbb{P}} > 0 \quad \text{and} \quad \zeta_m \geq 0.$$

work. Academic literature testing alternative specifications of stochastic intensity-based models is rather scarce and the majority of these studies does not use CDS, but corporate bond data. Most papers employ a square-root diffusion for the default intensity (e.g., Longstaff, Mithal, and Neis (2005), Driessen (2005)), though there are some (e.g., Chen, Cheng, and Wu (2005)) which use a Gaussian OU process. Brigo and Cousot (2006) find that it is difficult to produce large enough implied default-swaption volatilities or historical volatilities of CDS spreads when fitting the shifted square-root diffusion (SSRD) model to a one-day term structure of CDS spreads only. Subsequently, Brigo and El-Bachir (2006) investigate the pricing performance of an extended SSRD model (only on a one-day term structure of CDS premia as well), which results in an improved fit through the addition of exponential jumps in the stochastic default intensity.

Stylized fact **C**, supported by our non-parametric study in Section II.B and empirical evidence in Chen, Cheng, and Wu (2005), Driessen (2005) and Pan and Singleton (2008), suggests the use of a two-factor model for a firm’s default intensity. The first factor, default intensity  $\eta$ , is meant to capture short-term behavior, while the second factor, its stochastic long-run mean  $\gamma$ , is supposed to reflect long-term behavior. The diffusive parts of both state variables are specified in the square-root form to maintain a strictly positive default intensity, thereby forgoing stochastic volatility though. To model stylized fact **A** we assume that jumps occur to the default intensity and its long-run mean simultaneously. Jump times are governed by a Poisson process  $Z^{\mathbb{Q}}$  with jump frequency  $l^{\mathbb{Q}}$ . To accommodate stylized fact **B** and retain positivity, we employ exponential jump sizes  $J_{\eta}^{\mathbb{Q}}$  and  $J_{\gamma}^{\mathbb{Q}}$  in the default state variables  $\eta$  and  $\gamma$ , with expected jump sizes  $\mu_{\eta}^{\mathbb{Q}}$  and  $\mu_{\gamma}^{\mathbb{Q}}$ , respectively. For parsimony the jump sizes are assumed

independent. The model dynamics under the pricing measure  $\mathbb{Q}$  are thus given by:

$$(5) \quad \begin{aligned} d\eta_t &= \kappa_\eta^\mathbb{Q}(\gamma_t - \eta_t) dt + \sigma_\eta \sqrt{\eta_t} dW_\eta^\mathbb{Q}(t) + J_\eta^\mathbb{Q}(t) dZ^\mathbb{Q}(t) \\ d\gamma_t &= (\zeta_\gamma - \kappa_\gamma^\mathbb{Q} \gamma_t) dt + \sigma_\gamma \sqrt{\gamma_t} dW_\gamma^\mathbb{Q}(t) + J_\gamma^\mathbb{Q}(t) dZ^\mathbb{Q}(t). \end{aligned}$$

We take into account the risk premium on fluctuations in the default state variables: Following [Duffee \(2002\)](#) our market prices of diffusive risk take on the essentially affine form, so that the speed of mean reversion is estimated separately under each measure to account for the risk premium. The jump-related change of measure preserves the compound Poisson property, i.e., both the Poisson occurrence and size distribution of jumps, while allowing for risk premia on all jump parameters. Model dynamics are again of the same form under both probability measures.<sup>4</sup> Details about the change of measure are given in [Appendix C](#).

Evidence for dependence between riskless and defaultable rates is rather mixed (cf. [Chen, Cheng, Fabozzi, and Liu \(2008\)](#)). [Brigo and Alfonsi \(2005\)](#) report that CDS are insensitive to correlation between default intensity and interest rate processes.<sup>5</sup> Other studies find a negative relation (e.g., [Driessen \(2005\)](#)), though with a negligible impact on hazard rates (cf. [Duffie, Saita, and Wang \(2007\)](#), Fig. 5). [Feldhütter and Lando \(2008\)](#) provide a discussion of evidence from recent literature. We therefore abstain from modeling dependence between

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<sup>4</sup> Model parameters under both measures satisfy constraints analogous to (4).

<sup>5</sup> To empirically investigate this issue we perform estimations with a default intensity of the form  $\eta + cr + em$ , where  $c, e \geq 0$  and  $r, m$  are the riskless factors from (3). This does not cause an additional computational burden since the above formulation preserves affinity. Estimates of  $c$  and  $e$  are very close to zero and exhibit high variability (similarly to the results in [Feldhütter and Lando \(2008\)](#) for the short rate), which indicates only weak evidence for dependence.

riskless and default state variables.<sup>6</sup>

The debt acceleration clause motivates the recovery of face value (also called recovery of par) formulation, which coincides with the definition of the default payment in CDS contracts. As opposed to the recovery of market value formulation, recovery of face value allows separate identification of default and recovery risks when daily term-structure information is taken into account, as shown in [Zhang \(2003\)](#), [Bakshi, Madan, and Zhang \(2006b\)](#) and [Pan and Singleton \(2008\)](#). [Pan and Singleton \(2008\)](#) illustrate that long-maturity CDS are essential for identification since the impact of changes in the recovery rate on short-maturity CDS is low. [Bakshi, Madan, and Zhang \(2006b\)](#) report that, while corporate bond data support the recovery of Treasury formulation, the CDS market prefers recovery of face value, which in comparison also yields less volatile expected recovery rates. We therefore employ the recovery of face value formulation.

In light of our full-scale specification of the default intensity we keep the recovery parsimonious: As common both in academic studies and industry practice, we assume the *expected* recovery of face value at default constant in time, over maturities, and under change of mea-

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<sup>6</sup> There is strong evidence, though, for the presence of common factors in CDS premia. For one, several papers ([Berndt, Douglas, Duffie, Ferguson, and Schranz \(2008\)](#), [Carr and Wu \(2006\)](#), [Pan and Singleton \(2008\)](#)) establish a strong relation between default arrival rates and equity market volatility (as measured by the VIX index). Further, in a study of CDS liquidity, [Tang and Yan \(2007\)](#) find that premia of actively traded CDS contracts are rather exposed to systematic (cross-sectional) liquidity risk than to idiosyncratic liquidity characteristics. Since we filter obligors on the basis of the percentage of missing spreads in their panels, our selected sample consists of obligors with high CDS trading activity. Since the aim of the present paper is more accurate modeling on the individual obligor level, a model-based investigation of commonalities in CDS premia is left for future research.



sure. However, in contrast to a vast majority of empirical literature we do *not* restrict the recovery to an exogenously specified, mostly arbitrary level,<sup>7</sup> but rather estimate it as an additional model parameter and investigate its determinants.

## IV. Estimation Methodology

Our specifications of the risk-free and default-risk models in systems (3) and (5) result in a four-factor model for the default-adjusted short rate. Estimation of the respective models is performed in two stages: First, the riskless model (3) is estimated once on a panel of riskless zero-yields. Subsequently the default-risk model (5) is estimated issuer by issuer conditionally on the (same) parameters and state variables of the riskless model. We employ a Bayesian estimation methodology with Markov chain Monte Carlo (MCMC) techniques to cope with the involved, high-dimensional posterior distribution.

Let the vector  $\chi$  comprise the parameters under  $\mathbb{Q}$  and  $\mathbb{P}$  of both the riskless and default-risk models, the LGD parameter, as well as the parameters in the error-covariance matrices  $\Sigma_\rho$  and  $\Sigma_\varepsilon$  defined below. Similarly let  $\chi_r$  and  $\chi_d$  be subvectors of  $\chi$  comprising only parameters of the riskless resp. default-risk models.

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<sup>7</sup> The reasons put forward are that CDS prices reportedly do not react to the recovery parameter (Houweling and Vorst (2005)), or that it is market practice to fix the recovery at some particular level (e.g., 25% for sovereigns in Pan and Singleton (2008), and 40% for US corporates in Chen, Cheng, and Wu (2005)). Another reason is that papers focus on the 5-year CDS maturity and thus have no means to disentangle recovery from default risk (e.g., Berndt, Douglas, Duffie, Ferguson, and Schranz (2008), Longstaff, Mithal, and Neis (2005), Chen, Cheng, Fabozzi, and Liu (2008)).

**Stage 1: Estimation of the Riskless Model.** Riskless zero-coupon yields are obtained from swap rates, which are available on a daily basis for standard maturities between 1 and 10 years (1, 2, 3, 4, 5, 7 and 10 years). For maturities shorter than one year we use money-market (Libor) rates for 1, 3, 6 and 9 months. [Feldhütter and Lando \(2008\)](#) find that swap rates are the best parsimonious proxy for riskless rates. The estimation is performed on ten years of daily zero-yields bootstrapped from USD swap rates between June 1, 1998 and May 30, 2008. We choose the extended ten-year sample for better parameter identification. All riskless data are obtained from Datastream.

We take a panel  $\bar{y} = \{\bar{y}_t\}$  of seven zero-yields, where  $\bar{y}_t = (\bar{y}_t(1m), \bar{y}_t(6m), \bar{y}_t(1y), \bar{y}_t(2y), \bar{y}_t(5y), \bar{y}_t(7y), \bar{y}_t(10y))^\top$ , and assume that these are observed with cross-sectionally and intertemporally i.i.d. errors  $\varrho_t \sim \text{MVN}(0, \Sigma_\varrho)$  which additively enter the observation equation:

$$(6) \quad \bar{y}_t = y(r_t, m_t, \chi_r) + \varrho_t.$$

The affine pricing function (model yield)  $y$  arises from the zero-bond price (1) by solving the system of ODEs from [Duffie, Pan, and Singleton \(2000\)](#) for our risk-free model (3). The covariance matrix  $\Sigma_\varrho$  is assumed to be a 7-dim. diagonal matrix with diagonal entries given by  $[\Sigma_\varrho]_{ii} = \exp\{a_0 + a_1 M_i + a_2 M_i^2\}$ ,  $i = 1, \dots, 7$ , with  $M_i$  the respective maturity. The time step  $\Delta$  between observations is constant and equal to 1/250, which is short enough to discretize system (3) under the objective measure  $\mathbb{P}$  for likelihood inference:

$$(7) \quad p(r, m, \chi_r | \bar{y}) \propto p(\bar{y} | r, m, \chi_r) p(r, m | \chi_r) \pi(\chi_r).$$

Density  $p(\bar{y} | r, m, \chi_r)$  is determined by the observation equation (6), while transition density  $p(r, m | \chi_r)$  arises from our risk-free term structure model. Prior distributions  $\pi(\chi_r)$  for

parameters sampled by the Metropolis-Hastings algorithm are normal if supported on the whole real line, or gamma if supported on the positive real line, in both cases with high variances. Parameters sampled by Gibbs steps are endowed with conjugate priors. The MCMC estimation procedure is standard; an extensive reference is [Johannes and Polson \(2006\)](#). Table 4 contains parameter estimates of our risk-free term structure model.

[TABLE 4 about here.]

**Stage 2: Estimation of the Default-Risk Model.** Since the default-risk model (5) under  $\mathbb{Q}$  may only be used for pricing, risk premia must be considered for parameter inference under  $\mathbb{P}$  from the time series. Appendix C describes how the parameters reflect the change of measure. For time-series inference we therefore discretize system (5) under the objective measure  $\mathbb{P}$ .<sup>8</sup>

$$(8) \quad \begin{aligned} \eta_{t+1} &= \eta_t + \kappa_\eta^\mathbb{P}(\gamma_t - \eta_t)\Delta + \sigma_\eta\sqrt{\eta_t\Delta}\epsilon_\eta(t+1) + J_\eta^\mathbb{P}(t+1)\tilde{Z}^\mathbb{P}(t+1) \\ \gamma_{t+1} &= \gamma_t + (\zeta_\gamma - \kappa_\gamma^\mathbb{P}\gamma_t)\Delta + \sigma_\gamma\sqrt{\gamma_t\Delta}\epsilon_\gamma(t+1) + J_\gamma^\mathbb{P}(t+1)\tilde{Z}^\mathbb{P}(t+1). \end{aligned}$$

Innovations  $\epsilon_\eta(t)$  and  $\epsilon_\gamma(t)$  are  $N(0,1)$ -distributed random variables. The jump indicator  $\tilde{Z}^\mathbb{P}(t)$  has a Bernoulli distribution with daily jump probability  $1 - \exp\{l^\mathbb{P}/250\}$ , whereas the jump sizes  $J_\eta^\mathbb{P}(t)$  and  $J_\gamma^\mathbb{P}(t)$  are independent and exponentially distributed with expected jump sizes  $\mu_\eta^\mathbb{P}$  resp.  $\mu_\gamma^\mathbb{P}$ .

Using the already estimated riskless results, model-implied CDS premia  $s = \{s_t\}_{t=1}^{1146}$ , where  $s_t = (s_t(1y), s_t(3y), s_t(5y), s_t(7y), s_t(10y))^\top$ , are expressed in terms of the latent state

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<sup>8</sup> The bias introduced by a discretization at a daily time step is negligible (cf. [Johannes and Polson \(2006\)](#), [Johannes \(2004\)](#)). [Eraker, Johannes, and Polson \(2003\)](#) also employ daily discretization for a bivariate jump-diffusion and find no evidence for bias.

variables  $\eta$  and  $\gamma$ , and the (constant, yet unknown) loss given default as  $s_t = s(\eta_t, \gamma_t, \chi_d)$  (cf. Appendix B). We assume that the panel of CDS premia  $\bar{s} = \{\bar{s}_t\}_{t=1}^{1146}$  is observed with an additive i.i.d. error  $\varepsilon_t \sim \text{MVN}(0, \Sigma_\varepsilon)$ :

$$(9) \quad \bar{s}_t = s(\eta_t, \gamma_t, \chi_d) + \varepsilon_t.$$

The covariance matrix  $\Sigma_\varepsilon$  is assumed to be a 5-dim. diagonal matrix with diagonal entries given by  $[\Sigma_\varepsilon]_{ii} = \exp\{b_0 + b_1 M_i + b_2 M_i^2\}$ ,  $i = 1, \dots, 5$ , with  $M_i$  the respective maturity.

Several existing empirical studies employing affine jump-diffusions establish that jump intensities under the pricing measure  $\mathbb{Q}$  are difficult to identify; the papers therefore proceed to set equal the jump intensities under  $\mathbb{P}$  and  $\mathbb{Q}$  or to fix them to some particular value.<sup>9</sup> To the best of our knowledge there does not exist a suitable reference estimate for an average number of jumps in the default intensity of an obligor, so we rely on the co-movement observed between the CDS and equity markets (as evidenced, e.g., in [Berndt, Douglas, Duffie, Ferguson, and Schranz \(2008\)](#), [Carr and Wu \(2006\)](#)) and draw on equity-based estimates in our parameter choice. [Chernov, Gallant, Ghysels, and Tauchen \(2003\)](#), [Eraker, Johannes, and Polson \(2003\)](#), and [Eraker \(2004\)](#) estimate jump intensities for equity (index) time series in the range of approx. 1 to 1.7 jumps per year. Thus, for reasons of parsimony and identification we fix the jump intensity  $l^{\mathbb{Q}}$  to 1. Note that the estimated jump size parameters  $\mu_\eta^{\mathbb{Q}}$  and  $\mu_\gamma^{\mathbb{Q}}$  are therefore relative to the fixed jump intensity under  $\mathbb{Q}$ . Jump intensity  $l^{\mathbb{P}}$  is estimated from the data.

Furthermore, in each data panel of CDS premia  $\bar{s}$  there are both observed values  $\bar{s}^o$  and missing values  $\bar{s}^m$ ; it holds that  $\{\bar{s}^o, \bar{s}^m\} = \bar{s}$ . The joint posterior density of the parameters

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<sup>9</sup> [Broadie, Chernov, and Johannes \(2007\)](#), for example, employ a two-stage procedure: They estimate the jump intensity from time-series information and then use it to price options. This is not feasible in our setting because none of the state variables is directly observed.

with the latent state variables and missing CDS prices is given by

$$(10) \quad p(\bar{s}^m, \eta, \gamma, \chi_d \mid \bar{s}^o, r, m, \chi_r) \propto p(\bar{s} \mid r, m, \eta, \gamma, \chi) p(\eta, \gamma \mid \chi_d) \pi(\chi_d).$$

Density  $p(\bar{s} \mid r, m, \eta, \gamma, \chi)$  represents a multivariate normal distribution arising from the observation equation (9), while transition density  $p(\eta, \gamma \mid \chi_d)$  is defined by our specification of the default state variables  $\eta$  and  $\gamma$  in (8). The posterior distribution of the parameters is therefore by construction compatible with both time series and term structure. Prior distributions  $\pi(\chi_d)$  of default-risk parameters are normal for parameters with support on the whole real line and gamma for parameters with support on the positive real line, both with high variances. Parameters sampled with Gibbs steps are endowed with conjugate priors. The exception is an informative gamma prior on the jump intensity under  $\mathbb{P}$ , ensuring that jumps are rare events (for an economic justification cf. Johannes and Polson (2006) and Eraker, Johannes, and Polson (2003)). Proposal densities for the Metropolis steps are random-walk. All further details concerning parameter sampling and conditional densities are found in Appendix D.

## V. Empirical Results

This section presents and discusses parameter estimates of the defaultable model (5). Since 278 obligors are considered and parameters are estimated firm by firm, Table 5 displays a cross-sectional overview of the point estimates for each of the defaultable model parameters. The point estimates are taken to be the multivariate median from the posterior distribution of the parameters conditional on the data (cf. Collin-Dufresne, Goldstein, and Jones (2009)). Out of 5,000,000 draws from the MCMC sampler only every 1,000th draw is recorded to remedy high autocorrelation in the parameter paths. The first 3,000 samples are discarded

and the remaining 2,000 used in the computation of the point estimates.

[TABLE 5 about here.]

Table 6 displays aggregate summary statistics of the posterior observation errors for our model specification. Judging from the median values of mean observation errors in Panel A, the overall fit to the data is tight – the median errors range from -0.64 bp to 0.76 bp.<sup>10</sup> The distribution of mean observation errors for the short 1-year and 3-year maturities is negatively skewed, though, which can be discerned from the means being smaller than the medians, as well as from the deeply negative minimum mean error in relation to the maximum. Such pronounced skewness is probably due to rare negative movements in the short-maturity CDS spreads. In comparison, the distributions of the mean observation errors for the remaining, longer maturities are almost symmetric. We conclude that the idiosyncratic behavior of the 1-year premium is difficult to fit even with our two-factor jump-diffusion specification. Nevertheless, the default intensity process  $\eta$  seems to approximately capture the evolution of the short-maturity (1-year) CDS premia, while its stochastic long-run mean process  $\gamma$  resembles the longer-maturity (10-year) CDS premia. Additionally to 1-year and 10-year median observation errors being the lowest, support for this claim is provided by the first-order autocorrelations of observation errors in Panel C, which show that there is less autocorrelation left in the errors for the 1-year and 10-year contracts (mean of 59% resp. 70%) than in contracts of other maturities, meaning that the two default components absorb the autocorrelation inherent in these CDS premia to a large extent.

[TABLE 6 about here.]

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<sup>10</sup> For comparison, Longstaff, Mithal, and Neis (2005) report root mean squared errors of 10 to 17 bp using a square-root diffusion for the default intensity and an OU process as “liquidity”, while the pricing errors in Chen, Cheng, and Wu (2005) exhibit properties similar to ours.

Our default model employs mean-reverting processes for both state variables. Estimates of parameters determining mean reversion in the two processes ( $\kappa_{\eta}^{\mathbb{Q}}$  and  $\kappa_{\eta}^{\mathbb{P}}$  resp.  $\kappa_{\gamma}^{\mathbb{Q}}$  and  $\kappa_{\gamma}^{\mathbb{P}}$ ) are close in value to estimates from similar specifications for the risk-free term structure, both processes exhibiting slow mean reversion. Persistence is especially pronounced under the pricing measure  $\mathbb{Q}$  with median cross-sectional mean-reversion parameter estimates of 0.0378 for the default intensity  $\eta$  and 0.0004 for its long-run mean  $\gamma$ ; even the highest cross-sectional estimates under  $\mathbb{Q}$  are still low at 1.25 for  $\eta$  resp. 0.3 for  $\gamma$ . In comparison, historical mean reversion is higher: Cross-sectionally, the  $\mathbb{P}$  estimates of the speed of mean reversion have a median of 0.02 (maximum of 12) for the default intensity, and a median of 1 (maximum of 10) for its long-run mean. This discrepancy suggests that while CDS spreads are slowly mean-reverting, the market rather expects spreads to persist at one level.

Furthermore, the long-run mean  $\gamma$  exhibits higher persistence than the default intensity  $\eta$ , which is not surprising in light of our finding above that the long-run mean is related to long-term CDS spreads, whereas the default intensity captures short-term behavior. While short-term prospects of an obligor do fluctuate, his long-term outlook is less frequently revised and thus remains at a certain level for longer periods, resulting in higher persistence. High persistence is noticeable in Figure 2, which displays posterior estimates for the trajectory of the default intensity and its long-run mean for Honeywell Int'l Inc. as an example.<sup>11</sup> Our mean-reversion estimates under  $\mathbb{P}$  are in line with one-factor results by [Chen, Cheng, Fabozzi, and Liu \(2008\)](#): They report averages of 0.973 for their *Financials* sector and 0.904 for *Industrials* (Reuters, not ICB sectors), which agree with the median estimate of 0.9957 for our  $\gamma$  process. Moreover, our long-run level of  $\gamma$  confirms their observation that due to regulation *Financials*

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<sup>11</sup> Note that the default intensity never actually touches the zero boundary though it attains very low values due to the exceptionally low credit spreads during the credit bubble of 2005 and 2006.

are more homogeneous than *Industrials* in their long-term expected returns.

[FIGURE 2 about here.]

## A. Loss Given Default

As discussed in Section III, the recovery of face value formulation implied by the CDS contract structure permits separate identification of default risk and loss given default (LGD) if the estimation is performed on the full term structure of CDS spreads. Our estimation results confirm that it is possible to disentangle LGD from default risk in this case – the parameters are well identified, as inferred from tight confidence bands in their posterior distributions: The 2.5% and 97.5% quantiles averaged over the cross-section are only 5% lower resp. higher than the point estimates. The cross-sectional distribution of implied LGD is presented in Figure 3 as a histogram; Table 7 reports descriptive statistics.<sup>12</sup> Studies find a pronouncedly positive

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<sup>12</sup> To check the plausibility of our LGD estimates we perform a supplemental analysis: Based on the arguments provided by Pan and Singleton (2008) on the estimation of implied LGD from CDS data we slightly expand the naïve model  $\bar{s}_t(T) = \text{LGD} \cdot \text{PD}_t(T)$  by specifying a state-space model for the probability of default  $\text{PD}_t(T)$ :

$$\text{PD}_t(T) = \lambda_t + c_1(\lambda_t - \mu_\lambda)^2 + c_2(T - 5) + c_3(T - 5)^2 + c_4(T - 5)(\lambda_t - \mu_\lambda),$$

where  $\mu_\lambda$  is the unconditional mean of the first-order autoregressive process  $\lambda$ . The state-space model (A-8) is estimated firm by firm with a first-order autoregressive state-space equation. The  $(T - 5)$  variables are included in the set of prediction variables to account for the term structure of observed spreads. The coefficient of the term structure,  $c_2$ , is positive. The LGD estimates are even lower than in our model. Of course this analysis does not prove that implied LGD is generally low, but it does show that our implied LGD estimates are neither an artifact of our model specification nor a by-product of our estimation methodology.



dependence between the corporate default rate and realized LGD over the business cycle (cf. academic papers such as [Altman, Brady, Resti, and Sironi \(2005\)](#), [Bruche and González-Aguado \(2008\)](#), reports by US rating agencies such as [Verde, Rosenthal, Oline, and Tutterow \(2008\)](#), [Emery, Ou, Tennant, Kim, and Cantor \(2008\)](#), and practitioners' publications, e.g., [Singh \(2004\)](#)). A period with rare defaults such as the one investigated here will therefore tend to exhibit low realized LGD rates.<sup>13</sup> Our mean implied LGD of 21%, as well as the right (positive) skew of the distribution (as reflected in a median of 10%) are in all likelihood due to the low-default environment during the years 2004 to 2007 covered by our sample.

[FIGURE 3 about here.]

[TABLE 7 about here.]

Existing studies of (through-the-business-cycle) realized LGD such as [Altman and Kishore \(1996\)](#), [Altman, Brady, Resti, and Sironi \(2005\)](#), [Altman, Resti, and Sironi \(2004\)](#), [Acharya, Bharath, and Srinivasan \(2007\)](#), and [Altman \(2006\)](#), which analyze historical LGD from US defaults through the 1970s, 1980s and 1990s, point out debt seniority, macroeconomic

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<sup>13</sup> The following statistics exist for our sample period from 2004 to 2007: [Emery, Ou, Tennant, Kim, and Cantor \(2008\)](#) of Moody's report speculative-grade default rates between 0.9% in 2007 and 2.4% in 2004, much lower than Moody's long-term average of roughly 4.4% (since 1983), along with realized LGD rates between 45% in 2006 and 49% in 2007, which are far below Moody's long-term estimate of 63% (since 1982). In comparison, [Verde, Rosenthal, Oline, and Tutterow \(2008\)](#) of Fitch report US high-yield default rates between 0.5% in 2007 and 3.1% in 2005, much lower than their long-term average of roughly 5%, along with realized LGD rates between 33.6% in 2007 and 42.4% in 2005, which are as well far below the long-term estimate of around 60%. Numerous academic studies on the same topic (cited in the text) put forth very similar figures.

variables, and industry characteristics (such as competitive structure, leverage, nature of assets, or regulation) as the main determinants of realized LGD rates. The CDS market being dominated by contracts written on senior unsecured debt, we next concentrate on analyzing the effect of sector affiliation and rating class on implied LGD.

Denote by  $S = (S_{\text{BM}}, \dots, S_{\text{Utl}})$  the matrix containing sector dummy vectors  $S_i$ , and by  $R = (R_{\text{AAA}}, \dots, R_{\text{BB}})$  the matrix containing rating dummy vectors  $R_j$ . Each rating dummy vector  $R_j$  contains firms rated  $j$  or better in order to facilitate interpretation. The following set of linear regressions of the implied LGD cross-section on the sector and rating matrices is performed:

$$(11) \quad \text{LGD} = \beta_{S_{\setminus i}}^\top S_{\setminus i} + \beta_R^\top R + \varepsilon, \quad i = 1, \dots, 10,$$

where  $S_{\setminus i}$  denotes the matrix  $S$  without its  $i$ -th column, and  $\beta_{S_{\setminus i}} \in \mathbb{R}^9$  resp.  $\beta_R \in \mathbb{R}^5$  the corresponding coefficient vectors. Excluding one sector at a time amounts to setting this sector as the base case and investigating whether implied LGD of other sectors significantly differs from the base sector's. Analogously, the base rating is always B or worse. Table 8 displays relative differences in implied LGD between the ICB Industry categories (Panel A) and ratings (Panel B). LGD implied by the panel of CDS written on an obligor of deep speculative-grade rating – B or worse – belonging to the individual sectors is given on the diagonal. Each figure listed is significant at the 10% level. Approximately 50% of variance in implied LGD is explained by sector and rating, with roughly 40% due to rating and 10% due to sector information.

[TABLE 8 about here.]

Considering only Panel B, rating improvements within speculative grade (up to BB) as well as into investment grade (from BB to BBB) entail reductions in the market's perceived

LGD by roughly 20% each. In particular, a clear-cut distinction is observed in the level of implied LGD between investment-grade and speculative-grade issuers. A further upgrade to rating A gives rise to a deduction of almost 8% in expected LGD. Our findings are in line with historical statistics of realized LGD from [Emery, Ou, Tennant, Kim, and Cantor \(2008\)](#), as well as empirical results by [Bakshi, Madan, and Zhang \(2006b\)](#), who show that a worsening of risk-neutral hazard rates is associated with a decline in risk-neutral recovery rates in a sample of BBB-rated bonds.

The findings on sector affiliations shown in Panel A support the intuition that obligors with substantial tangible assets are expected to recover more in default, as already documented for realized LGD, e.g., in [Varma and Cantor \(2004\)](#): Sectors *Technology* and *Utilities* exhibit implied LGD rates which are significantly lower than those of most other sectors, while the *Consumer Goods* and *Financials* sectors exhibit implied LGD significantly higher than many other sectors. Means of implied LGD within each sector, reported in the second to last row of Panel A, confirm these relations between sectors, which are again comparable to those of realized LGD reported in [Varma, Cantor, and Hamilton \(2003\)](#). In particular, the *Financials* sector shows markedly high implied LGD rates; this is probably an effect of the credit crunch which began in mid-2007, causing investors to anticipate higher LGD rates for the *Financials* sector in the ensuing economic downturn. In the last row of Panel A we also observe substantial dispersion of estimates within individual sectors. These dispersion figures are in line with the values reported for historically realized LGD in the academic studies and agency reports cited above. Summing up, our analysis thus indicates that realized and implied LGD share common determinants with similar sensitivities, but that knowledge of specific issuer characteristics is still indispensable due to high variability.

## B. Jumps

Motivated by observations **A** and **B** and supported by our analysis of descriptive statistics for the corporate CDS panels in Section II.B, our model accommodates jumps in the state variables driving default risk. We now turn to the discussion of parameter estimates for the jump size, the jump frequency, and the associated risk premia.

Examining the estimated jump frequency  $l^{\mathbb{P}}$  in the cross-section (cf. Table 5), jumps arrive at an average frequency of 4 to 5 jumps per year. The jump frequencies are thus in line with the number of jumps inferred by the non-parametric tests from Section II.B. Note also the lower quantile implying that 90% of CDS panels exhibit more than 1.7 jumps per year, with 10% of our cross-section exhibiting more than 9.43 jumps per year. The minimum frequency amounts to 0.58, meaning that in our cross-section the default factors jump at least once in two years.

Jump sizes in the default state variables are assumed independent and exponentially distributed under both measures, and the expected jump sizes are affected by the measure change from  $\mathbb{Q}$  to  $\mathbb{P}$ . Since high CDS spreads imply high default state variables with high jumps, we analyze relative jump sizes, i.e., absolute jump sizes normalized by the average spread of the corresponding maturity: As found previously, the evolution of short-term CDS premia is approximately captured by the default intensity process  $\eta$ , while its stochastic long-run mean process  $\gamma$  resembles long-term CDS premia. We therefore normalize expected jump size in the default intensity  $\eta$  for each obligor by its average 1-year premium  $\bar{s}_{\text{avg}}(1y)$ , and expected jump size in the stochastic long-run mean  $\gamma$  by its average 10-year premium  $\bar{s}_{\text{avg}}(10y)$ . The functional form of the market price of jump risk (cf. Appendix C and Broadie, Chernov, and

Johannes (2007)) suggests defining market risk premia on (relative) jump sizes as:

$$(12) \quad \frac{\mu_{\eta}^{\mathbb{P}} - \mu_{\eta}^{\mathbb{Q}}}{\bar{s}_{\text{avg}}(1y)} \quad \text{and} \quad \frac{\mu_{\gamma}^{\mathbb{P}} - \mu_{\gamma}^{\mathbb{Q}}}{\bar{s}_{\text{avg}}(10y)} .$$

Table 5 shows the cross-sectional distribution of (absolute) expected jump sizes (scaled to represent basis points), whereas Table 9 contains the cross-sectional distribution of relative jump sizes and their corresponding risk premia. The behavior of expected jump sizes is similar relative to the 1-year and 10-year CDS spreads: Under the empirical measure  $\mathbb{P}$ , in the median, the expected jump in the default intensity  $\eta$  amounts to almost 60%, whereas the expected jump in its long-run mean  $\gamma$  accounts for 10 times the corresponding spread. Under the pricing measure  $\mathbb{Q}$  the expected jump in  $\eta$  constitutes 3% of the spread, while the expected jump in  $\gamma$  makes 9 times the corresponding spread. The cross-sectional distributions in all four cases, i.e., both in absolute and relative magnitude, exhibit heavy right tails: The highest relative jump size in the default intensity  $\eta$  is 4 times the corresponding spread under  $\mathbb{Q}$ , resp. 50 times under  $\mathbb{P}$ , and 64 times the spread under  $\mathbb{Q}$ , resp. 59 times under  $\mathbb{P}$  in the long-run mean  $\gamma$ .

[TABLE 9 about here.]

Model-implied risk premia allow us to assess the plausibility of our parameter estimates comparing the  $\mathbb{P}$  and  $\mathbb{Q}$  measures: Bates (1991) and Bakshi, Cao, and Chen (1997) formulate a simple criterion to check whether a model implies reasonable risk preferences in its general-equilibrium translation – the estimated risk premia should be close to zero. Though there exist large extreme premia of -4 and 50 times the corresponding spread for jumps in  $\eta$  and  $\pm 55$  times the spread for jumps in  $\gamma$ , the cross-sectional distribution of relative jump-size premia is pronouncedly peaked around zero, with median values of 46% for the default intensity and

30% for its long-run mean. Analogously defined risk premia on persistence in the default intensity are close to zero as well. These near-zero risk premia for the average obligor thus confirm the adequacy of our jump-diffusion default model.

Next, we perform an analysis of the jump frequency  $l^{\mathbb{P}}$ , the relative jump sizes and the corresponding risk premia analogous to the one performed for implied LGD and described by regression (11) (detailed results upon request). The coefficient of determination amounts to 6% for relative jump sizes in the default intensity and 33% in its long-run mean. Two patterns emerge regarding the rating dependence of relative jump sizes.

First, an upgrade from speculative to investment grade entails significantly higher relative jumps under both measures both in short and long maturities, meaning that jumps constitute a larger proportion of the corresponding spread for better ratings. The relative risk premium on jump sizes in  $\gamma$  is also significantly higher for upper investment-grade obligors, which is in line with findings in [Cremers, Driessen, and Meanhout \(2008\)](#), where larger jump-size premia are observed for highly rated corporates as well. Creditworthy obligors exhibit low long-term default rates – the probability of structural migration to default is considered low – so the only way these obligors can reach distress is through a damaging event surprisingly happening. Such an interpretation is additionally supported by findings related to investment-grade obligors regarding their LGD (cp. Section [V.A](#)).

Second, there is no distinction among the individual investment-grade ratings: The frequency of sudden shocks, the relative increases in the short- and long-term default state variables triggered by shocks, as well as the corresponding jump size premia do not significantly differ across investment-grade ratings under either measure. As above, creditworthy obligors get into distress primarily due to sudden, unanticipated events (fraud, global crash), and such events tend to do similar damage to all investment-grade firms.

Relative jump sizes in the long-run mean  $\gamma$  are highly distinguishable between sectors as

well. Though the *Financials*, *Telecommunication*, and *Utilities* sectors exhibit significantly lower relative jump sizes than others under both measures, their jump risk premia are inconspicuous. In contrast, relative risk premia on jump sizes in  $\gamma$  for the *Consumer Goods*, *Industrials*, and *Oil&Gas* sectors are significantly lower than for most other sectors.

Relative jump sizes in the default intensity  $\eta$  display a less distinctive pattern: The *Oil&Gas* and *Utilities* sectors exhibit significantly higher relative jump sizes, but only under the pricing measure  $\mathbb{Q}$ , meaning that the market is wary of sudden unfavorable news affecting their short-term prospects. The risk premium on jump sizes in  $\eta$  is significantly lower only for the *Utilities* sector; since this sector is heavily regulated,<sup>14</sup> the probability of a structural default is low compared to other sectors. Endogenously these obligors most probably run into distress due to mismanagement or fraud (a recent example is the energy provider Enron), whereas a major exogenous source of risk for *Utilities* are short-term violent fluctuations in the oil price. This economic interpretation is additionally supported by results regarding our mean-reversion parameters, which yield higher persistence for *Oil&Gas* and *Utilities* in the long run under  $\mathbb{P}$ , along with a lower risk premium on mean reversion (with an  $R^2$  of 25%), both relative to other sectors.

## VI. Conclusion

From an extensive cross-section of US corporate CDS panels we draw inference about implied loss given default (LGD) as well as risk premia attached to sudden jumps in CDS spreads. For this purpose we estimate an affine two-factor intensity-based model, where one factor is interpreted as the default intensity, and the other as its stochastic long-run mean. The two

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<sup>14</sup> The ICB *Utilities* sector contains the energy-providing subsectors *Electricity*, *Gas Distribution*, *Water*, and *Multiutilities*.

processes reflect different issues of concern to CDS markets: short-term liquidity difficulties resp. long-term prospects of the obligor. To investigate effects of unpredictable events we allow discontinuities in both processes. Our data comprise approx. 4.5 years of daily observations for CDS spreads on a cross-section of 278 US corporate obligors in the five most liquid maturities (1, 3, 5, 7 and 10 years). Estimation is engineered by means of Bayesian simulation methods, which reliably provide a distribution of parameters conditional on the data even in relatively small samples such as ours.

We obtain a tight fit of our model to the data. The stochastic long-run mean process exhibits higher persistence than the default intensity itself, suggesting that investors often update short-term prospects of individual obligors, while revising long-term outlooks less frequently. An upgrade from speculative to investment grade entails higher relative jumps in both the short- and long-term state variable under both the pricing and objective measure. If a sudden unanticipated shock occurs, investment-grade obligors are therefore exposed to higher relative increases in their default probability than speculative-grade ones. Among the individual investment-grade ratings no distinction shows with respect to any of the jump parameters: The frequency of sudden shocks as well as increases in both short- and long-term default probability are homogenous across investment-grade ratings. In the onset of the credit crunch investors apparently do not consider the rating within investment grade to be relevant for the likelihood of default.

Estimates of LGD are well identified, confirming that it is possible to disentangle recovery from default risk when using the recovery of face value formulation. However, the point estimates themselves are widely dispersed within sectors, similarly to realized LGD from earlier studies. Nevertheless, rating and industry explain 50% of the variance in implied LGD, with obligors possessing substantial tangible assets expected to recover significantly more in default. We also find a clear-cut distinction in implied LGD between investment-grade and



speculative-grade issuers. Our results show that industry practice to fix LGD regardless of rating and industry is not compatible with market data. Using our cross-section of implied LGD we provide figures which are, on average, consistent with rating and industry affiliation.

Many questions are left for future research. We observe jump clustering from the second half of 2007, preceding the credit crunch, which suggests a state-dependent default intensity. Many more years of data would be necessary to ensure identification of these parameters, however. Stochastic volatility is another empirical trait which cannot be reflected in an affine model like ours for a positive process such as the default intensity. Recent advances in the theory of Wishart processes promise improved modeling along this line. Another interesting topic is investigating non-linear dependencies between (stochastic) LGD, defaultable and riskless state variables. Finally, portfolio credit risk calls for identification of idiosyncratic and market factors to model contagion effects.

# Appendix

## A. High-Frequency Tests on Low-Frequency Data

Using our parameter estimates for each obligor we simulate 1,000 paths of CDS premia for all five maturities using our specifications (3) and (5) of the risk-free and default models. Both tests infer a mean number of 2.5 to 6 jumps per year for the simulated panels at 1%, 5% and 10% significance levels. A null hypothesis of no jumps is rejected in the [Barndorff-Nielsen and Shephard \(2006\)](#) test in almost all test versions.<sup>15</sup> At significance levels of 1% (5%) the [Lee and Mykland \(2008\)](#) test does not reject the null hypothesis in 2.2% (1%) of paths. Both tests therefore perform well (i.e., have enough discriminatory power) in detecting a false null hypothesis.

We additionally consider paths without jumps to investigate properties of both tests when the null hypothesis is true, i.e., using our default model (5) without the compound Poisson parts. Panel C of Table 3 displays the distribution of the number of jumps inferred when the data-generating process does not jump. A type-one error is committed too often in this case: Running the [Lee and Mykland \(2008\)](#) test at a 5% significance level shows that less than 1.14 jumps per year are inferred for half of the paths and less than 2 jumps for 90% of the paths, while the maximum is below 3 jumps per year. Similar behavior is observed in the [Barndorff-Nielsen and Shephard \(2006\)](#) test. Since the estimated number of jumps for our actual data is larger than the 90% or 95% quantile in Panel C, we conclude that there is strong evidence of jumps in our sample.

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<sup>15</sup> [Barndorff-Nielsen and Shephard \(2006\)](#) construct three different test statistics. Only the results of their adjusted ratio test are reported. The other two test statistics yield similar results (cf. Panel C of Table 3).

## B. Credit Default Swaps

### 1. Valuation

There are two sides to a CDS contract: the fixed leg, comprising the fee payments by the protection buyer, and the default leg, containing the contingent payment by the protection seller. The exact cash flow structure of the fixed leg in a standard contract, as laid down in the 2003 ISDA Credit Derivatives Definitions, is specified as follows: Premium payment dates are fixed and do not depend on the specific contract date. Payments are made quarterly on the 20<sup>th</sup> of March, June, September and December. Thus, if a CDS is contracted between those dates, the first period is not a full quarter and the first premium payment is adjusted accordingly. In addition, we account for the variable maturity of CDS contracts: As a result of fixing the premium payment dates, the length of the protection period varies and depends on the contract date since the quoted CDS maturity begins on the first premium payment date. Furthermore, the accrued premium in case of default must be taken into account. We assume absence of any transaction costs and other market imperfections, and ignore counterparty risk on both sides of the contract.<sup>16</sup> Since we price CDS contracts at initiation only, time- $t$  values of the fixed and floating legs below correspond to contracts initiated at time  $t$ .

Consider a CDS with outstanding premium payments at times  $T_1 < T_2 < \dots < T_N$ , with  $T_1 \geq T_0 = t$  and maturity at  $T = T_N$ , where both premium (p.a.) and notional amounts are normalized to 1. If default happens within the protection period, the protection buyer has

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<sup>16</sup> More precisely, the assumption is that during the life of the contract the counterparties either maintain the credit rating underlying generic (e.g., A-rated) CDS or have symmetric default probabilities (credit quality), cf. [Duffie and Singleton \(1997\)](#). We presume that this aspect has a low impact on the spreads of typical CDS contracts.

made  $I(\tau) = \max\{0 \leq n < N : T_n \leq \tau\}$  premium payments, the possibly remaining ones being no longer due, except for an accrued premium payment of  $\tau - T_{I(\tau)}$  at the default time. Hence, the value of the fixed leg of a CDS contract initiated at time  $t$  with maturity  $T$  is given by

$$(A-1) \quad L_t^{\text{fix}}(T) = \sum_{n=1}^N (T_n - T_{n-1}) \tilde{P}(t, T_n) + \mathbb{E}_t^{\mathbb{Q}} \left[ \int_t^T (u - T_{I(u)}) \eta_u \exp \left\{ - \int_t^u \tilde{r}_v dv \right\} du \right].$$

Employing the recovery of face value formulation and normalizing the default payment to 1 as well, the time- $t$  value of the default leg is given by

$$(A-2) \quad L_t^{\text{def}}(T) = \mathbb{E}_t^{\mathbb{Q}} \left[ \int_t^T \eta_u \exp \left\{ - \int_t^u \tilde{r}_v dv \right\} du \right].$$

At initiation of a CDS the premium is chosen such that the contract value to both parties is zero. Since the value of the fixed leg is homogeneous of degree 1 in the premium amount, it follows that the premium  $s_t(T)$  on a CDS contract initiated at time  $t$  with maturity  $T$  equals

$$(A-3) \quad s_t(T) = \text{LGD} \frac{L_t^{\text{def}}(T)}{L_t^{\text{fix}}(T)}.$$

## 2. Computation

To compute model-implied CDS premia we reverse the order of integration when evaluating the default leg (A-2) and the accrued premium in the fixed leg (A-1). The resulting integrands are of the form  $(A(u-t) + B(u-t) \eta_t) \exp\{\alpha(u-t) + \beta(u-t) \eta_t\} P(t, u)$  and  $(u - T_{I(u)}) (A(u-t) + B(u-t) \eta_t) \exp\{\alpha(u-t) + \beta(u-t) \eta_t\} P(t, u)$ , respectively, where  $\alpha$ ,  $\beta$ ,  $A$  and  $B$  are deterministic functions of model parameters which arise as solutions to a system of generalized Riccati ODEs (cf. [Duffie, Pan, and Singleton \(2000\)](#)). These ODEs also account for jumps

in the default state variables  $\eta$  and  $\gamma$ . The jump sizes in the default state variables are independent and exponentially distributed (cf. Section III). The moment generating function of an exponential distribution with parameter  $\mu$  is of the form  $\Psi(u) = 1/(1 - \mu u)$ . To engineer the joint jump transform we specify  $\Omega(u, v) = \Psi_\eta(u)\Psi_\gamma(v)$ .

## C. Default Risk Premia

The structure of possible risk premia in an intensity-based model for an individual obligor allows compensation for fluctuations in the default intensity, as well as compensation for the default event itself (cf. Jarrow, Lando, and Yu (2005)). The latter is relevant, e.g., when one is dealing with a severe default event directly affecting the economy (the portfolio) and thus carrying a non-zero risk premium.<sup>17</sup> On passing from the risk-neutral measure  $\mathbb{Q}$  to the historical measure  $\mathbb{P}$  for tractability we assume diversifiable default event risk in the sense of Jarrow, Lando, and Yu (2005), implying equivalence between the  $\mathbb{P}$  and  $\mathbb{Q}$  intensity functions. In this case it suffices taking into account only the usual risk premium on fluctuations in the default intensity.

We parameterize the change of measure by specifying the Radon-Nikodým density process  $L = L^D L^J$ , with  $L^D$  denoting the diffusion-related part and  $L^J$  the jump-related part. Following Duffee (2002) our market price of diffusive risk  $\Lambda = (\Lambda_\eta, \Lambda_\gamma)$  takes on the essentially affine form, meaning that  $L^D = \mathcal{E}(\Lambda \cdot W^\mathbb{Q})$ ,<sup>18</sup> where  $W^\mathbb{Q} = (W_\eta^\mathbb{Q}, W_\gamma^\mathbb{Q})$ . The jump-related

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<sup>17</sup> In this case the default event is grave enough to entail macroeconomic consequences *at* the default time, e.g., on endowments or consumption, meaning that the market's perception of default risk changes at and due to a specific default event. Such setups arise, e.g., in the presence of counterparty risk (as in Kusuoka (1999) and Jarrow and Yu (2001)) or contagion (as in Collin-Dufresne, Goldstein, and Helwege (2003)).

<sup>18</sup>  $\mathcal{E}(X)$  denotes the stochastic exponential of a process  $X$  (cf. §I.4f and §II.8a in Jacod and Shiryaev (2003))

part preserving the compound Poisson property equals  $L^J = \mathcal{E}((y-1) \star (\nu - \nu_c^{\mathbb{Q}}))$ , where  $\nu$  is our jump measure, and  $\nu_c^{\mathbb{Q}}(dt, dx) = dt \nu_c^{\mathbb{Q}}(x) dx$  its compensator. The function  $y : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$  embodying the market price of jump risk must be deterministic and time-independent to preserve the compound Poisson property,<sup>19</sup> and in addition it must take on the following form to preserve the distribution of jump sizes:

$$(A-4) \quad y(x_\eta, x_\gamma) = \frac{l^{\mathbb{P}}}{l^{\mathbb{Q}}} \frac{\mu_\eta^{\mathbb{Q}}}{\mu_\eta^{\mathbb{P}}} \frac{\mu_\gamma^{\mathbb{Q}}}{\mu_\gamma^{\mathbb{P}}} \exp \left\{ \frac{\mu_\eta^{\mathbb{P}} - \mu_\eta^{\mathbb{Q}}}{\mu_\eta^{\mathbb{P}} \mu_\eta^{\mathbb{Q}}} x_\eta + \frac{\mu_\gamma^{\mathbb{P}} - \mu_\gamma^{\mathbb{Q}}}{\mu_\gamma^{\mathbb{P}} \mu_\gamma^{\mathbb{Q}}} x_\gamma \right\},$$

where  $l^{\mathbb{P}}$  resp.  $l^{\mathbb{Q}}$  is the jump frequency in the default state variables. Jump sizes under the historical measure  $\mathbb{P}$  are again independent and exponentially distributed with expected jump sizes  $\mu_\eta^{\mathbb{P}}$  and  $\mu_\gamma^{\mathbb{P}}$ , and the jump frequency is affected by the measure change as well. The Lévy density is then transformed into  $\nu_c^{\mathbb{P}}(x) = y(x) \nu_c^{\mathbb{Q}}(x)$ . Mathematically it is admissible to change both the jump frequency and the jump size distribution of a compound Poisson process on a finite time interval when passing to an equivalent measure, as long as the support of the jumps remains the same (cf. [Cont and Tankov \(2004\)](#), Prop. 9.6). Employing the above parameterization of the measure change, the model dynamics under  $\mathbb{P}$  are given by:

$$(A-5) \quad \begin{aligned} d\eta_t &= \kappa_\eta^{\mathbb{P}}(\gamma_t - \eta_t) dt + \sigma_\eta \sqrt{\eta_t} dW_\eta^{\mathbb{P}}(t) + J_\eta^{\mathbb{P}}(t) dZ^{\mathbb{P}}(t) \\ d\gamma_t &= (\zeta_\gamma - \kappa_\gamma^{\mathbb{P}} \gamma_t) dt + \sigma_\gamma \sqrt{\gamma_t} dW_\gamma^{\mathbb{P}}(t) + J_\gamma^{\mathbb{P}}(t) dZ^{\mathbb{P}}(t). \end{aligned}$$

As a consequence of modeling within the Cox process framework, our model specification satisfies the no-jump condition from [Collin-Dufresne, Goldstein, and Hugonnier \(2004\)](#):<sup>20</sup> Due

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for definition and properties, and §III.3c and §III.5a for the connection to change of measure).

<sup>19</sup> It must also satisfy the technical condition  $\int_{\mathbb{R}^2} (\sqrt{y(x)} - 1)^2 \nu_c(dx) < \infty$ .

<sup>20</sup> Cases in which the doubly stochastic setting no longer holds are studied in [Duffie, Schroder, and Skiadas \(1996\)](#), [Kusuoka \(1999\)](#), [Jarrow and Yu \(2001\)](#), [Collin-Dufresne, Goldstein, and Helwege \(2003\)](#) and

to the assumption inherent in the Cox process setup that the filtration generated by the default components is conditionally independent of the default event, there do not exist feedback effects from the default time into the default components to be considered in the valuation. The default event  $\tau$  of the obligor does not directly affect economic factors influencing the corresponding defaultable zero-coupon bond price  $\tilde{P}(\cdot, T)$ ; in particular it does not affect the default components  $\eta$  and  $\gamma$  and the risk-free interest rate  $r$ . Concretely, both under the  $\mathbb{Q}$  and  $\mathbb{P}$  measures we maintain the doubly stochastic assumption so that contemporaneous jumps in the default intensity and the point process driving the default event are excluded. This fact implies that the pricing kernel (the Radon-Nikodým density process) does not jump at the default time, meaning that valuation methods developed, e.g., in [Collin-Dufresne, Goldstein, and Hugonnier \(2004\)](#) need not be applied.

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[Collin-Dufresne, Goldstein, and Hugonnier \(2004\)](#). The no-jump condition as formulated in [Collin-Dufresne, Goldstein, and Hugonnier \(2004\)](#) reads as follows: Let  $V_t := \mathbb{E}_t^{\mathbb{Q}}[\exp\{-\int_t^T (r_s + \eta_s) ds\} X]$  be the pseudo-value process of a defaultable security paying a random amount  $X$  at maturity  $T$  if no default happens, and zero otherwise. If no specific assumptions are made on the composition of the information filtration, the ex-dividend value of the security is given by

$$S_t = \mathbb{1}_{\{t < T\}} \mathbb{1}_{\{\tau > t\}} \left( V_t - \mathbb{E}_t^{\mathbb{Q}} \left[ e^{-\int_t^{\tau} r_s ds} \Delta V_{\tau} \right] \right),$$

where  $\Delta V_{\tau}$  denotes the jump in the pseudo-value process at the default time. If the process  $V$  is predictable and thus  $\Delta V_{\tau} = 0$ , then  $S_t = \mathbb{1}_{\{t < T\}} \mathbb{1}_{\{\tau > t\}} V_t$ , i.e., the valuation of the security reduces to the computation of its expected discounted cash flows using the risk-adjusted discount rate.

## D. MCMC Estimation

Most of the full conditional densities in this appendix can be found in [Jones \(1998\)](#) or [Johannes and Polson \(2006\)](#) and are stated here for completeness.

### 1. Drawing Jump Times and Sizes

We employ a time discretization of jump processes as in [Johannes \(2004\)](#) or [Johannes and Polson \(2006\)](#). Using this approximation jumps can only occur at discrete grid points. Jump times are represented by a discrete latent indicator process  $\tilde{Z}$ . Each component of  $\tilde{Z}$  is a Bernoulli random variable with daily jump probability  $1 - \exp(-l/250)$ , where  $l$  denotes the corresponding jump frequency per year. To avoid extensive use of the computationally expensive exponential function while still maintaining high precision we approximate  $1 - \exp(-l/250)$  by  $l/250$ . Due to the simple structure of  $\tilde{Z}_t$  for each  $t$  (0 or 1) we can sample it from a Bernoulli density with parameter

$$(A-6) \quad \frac{p(\eta_t, \gamma_t \mid \eta_{t-1}, \gamma_{t-1}, J_t, \tilde{Z}_t = 1, \chi_d)}{p(\eta_t, \gamma_t \mid \eta_{t-1}, \gamma_{t-1}, J_t, \tilde{Z}_t = 1, \chi_d) + p(\eta_t, \gamma_t \mid \eta_{t-1}, \gamma_{t-1}, J_t, \tilde{Z}_t = 0, \chi_d)},$$

where  $J_t := (J_\eta(t), J_\gamma(t))$  are sizes of the time- $t$  jumps in  $\eta$  and  $\gamma$ , respectively. The jump sizes in the default components are exponentially distributed with expected jump size  $\mu_\eta$  and  $\mu_\gamma$ . Updates of  $J_i(t)$ ,  $i = \eta, \gamma$ , are obtained with Metropolis steps in which jump size  $J_i^{(g+1)}(t)$  is accepted with probability

$$(A-7) \quad \min \left\{ \frac{p(J_i^{(g+1)}(t) \mid \mu_i) p(\eta_t, \gamma_t \mid \eta_{t-1}, \gamma_{t-1}, \tilde{Z}_t, J_t^{(g+1)}, \chi_d)}{p(J_i^{(g)}(t) \mid \mu_i) p(\eta_t, \gamma_t \mid \eta_{t-1}, \gamma_{t-1}, \tilde{Z}_t, J_t^{(g)}, \chi_d)}, 1 \right\},$$



where  $p(J_i^{(\cdot)}(t) \mid \mu_i)$  is an exponential density given the respective jump size parameter. The transition density  $p(\eta_t, \gamma_t \mid \eta_{t-1}, \gamma_{t-1}, \tilde{Z}_t, J_t^{(\cdot)}, \chi_d)$  conditions on jump times and jump sizes and is therefore normal by the Euler approximation. Samples from the posterior distribution of  $l^{\mathbb{P}}$  are obtained with Gibbs steps.

## 2. Drawing LGD

Model-implied CDS premia  $s_t = (s_t(1y), s_t(3y), s_t(5y), s_t(7y), s_t(10y))^{\top}$ , are expressed in terms of latent state variables and LGD as (cf. Appendix B.1):

$$(A-8) \quad s_t = s(\eta_t, \gamma_t, \chi_d) = \text{LGD} \cdot \left( \frac{L_t^{\text{def}}(1y)}{L_t^{\text{fix}}(1y)}, \frac{L_t^{\text{def}}(3y)}{L_t^{\text{fix}}(3y)}, \frac{L_t^{\text{def}}(5y)}{L_t^{\text{fix}}(5y)}, \frac{L_t^{\text{def}}(7y)}{L_t^{\text{fix}}(7y)}, \frac{L_t^{\text{def}}(10y)}{L_t^{\text{fix}}(10y)} \right)^{\top}.$$

Note that LGD is thus a coefficient in a panel regression conditional on other parameters and all state variables, as well as the data, missing and observed. We assume a truncated normal prior with  $\mathbb{1}_{\{\text{LGD} \in [0,1]\}}$  as truncation function. A standard Gibbs sampler is employed to sample from the posterior distribution.

## 3. Drawing Missing CDS Premia

The target density is

$$(A-9) \quad p(\bar{s}^m \mid \bar{s}^o, r, m, \eta, \gamma, \chi) \propto p(\bar{s}^m, \bar{s}^o \mid r, m, \eta, \gamma, \chi) = p(\bar{s} \mid r, m, \eta, \gamma, \chi),$$

which is a normal density by the observation equation (9). Proposing from a random walk we draw missing prices  $\bar{s}_{(g+1)}^m$  one by one, accepting with probability

$$(A-10) \quad \min \left\{ \frac{p(\bar{s}^o, \bar{s}_{(g+1)}^m \mid r, m, \eta, \gamma, \chi)}{p(\bar{s}^o, \bar{s}_{(g)}^m \mid r, m, \eta, \gamma, \chi)}, 1 \right\}$$

since the proposal densities cancel out.

## 4. Drawing Intensity Realizations

Defaultable state variables cannot be directly inverted from CDS prices as a consequence of the observation error in equation (9); we therefore draw realizations from the latent time series with a Metropolis step conditional on the parameters, CDS prices, risk-free state variables, jump times, jump sizes and LGD. The target density is given by:

(A-11)

$$\begin{aligned}
p(\eta_t, \gamma_t \mid \eta_{\setminus t}, \gamma_{\setminus t}, r, m, \bar{s}, J, \tilde{Z}, \chi) &\propto p(\eta_t, \gamma_t, \bar{s}_t \mid \eta_{\setminus t}, \gamma_{\setminus t}, \bar{s}_{\setminus t}, r, m, J, \tilde{Z}, \chi) \\
&\propto p(\bar{s}_t \mid \eta_t, \gamma_t, r, m, \chi) p(\eta_t, \gamma_t \mid \eta_{t+1}, \gamma_{t+1}, \eta_{t-1}, \gamma_{t-1}, r, m, J, \tilde{Z}, \chi) \\
&\propto p(\bar{s}_t \mid \eta_t, \gamma_t, r, m, \chi) p(\eta_{t+1}, \gamma_{t+1} \mid \eta_t, \gamma_t, r, m, J_{t+1}, \tilde{Z}_{t+1}, \chi) \times \\
&\quad \times p(\eta_t, \gamma_t \mid \eta_{t-1}, \gamma_{t-1}, r, m, J_t, \tilde{Z}_t, \chi).
\end{aligned}$$

Due to the Euler discretization the above density is a product of normal densities and thus easily sampled. We employ random-walk proposals (again the proposal densities cancel out) and accept  $(\eta_t^{(g+1)}, \gamma_t^{(g+1)})$  with probability

$$\begin{aligned}
\min \left\{ 1, \frac{p(\bar{s}_t \mid \eta_t^{(g+1)}, \gamma_t^{(g+1)}, r, m, \chi) p(\eta_{t+1}, \gamma_{t+1} \mid \eta_t^{(g+1)}, \gamma_t^{(g+1)}, r, m, J_{t+1}, \tilde{Z}_{t+1}, \chi)}{p(\bar{s}_t \mid \eta_t^{(g)}, \gamma_t^{(g)}, r, m, \chi) p(\eta_{t+1}, \gamma_{t+1} \mid \eta_t^{(g)}, \gamma_t^{(g)}, r, m, J_{t+1}, \tilde{Z}_{t+1}, \chi)} \times \right. \\
\left. \times \frac{p(\eta_t^{(g+1)}, \gamma_t^{(g+1)} \mid \eta_{t-1}, \gamma_{t-1}, r, m, J_t, \tilde{Z}_t, \chi)}{p(\eta_t^{(g)}, \gamma_t^{(g)} \mid \eta_{t-1}, \gamma_{t-1}, r, m, J_t, \tilde{Z}_t, \chi)} \right\}.
\end{aligned}$$

(A-12)

Since the first CDS premia are observed at  $t = 1$ , the target density for the first realization of the default intensity  $(\eta_0, \gamma_0)$  is

$$(A-13) \quad p(\eta_0, \gamma_0 \mid \eta_{\lambda_0}, \gamma_{\lambda_0}, r, m, \bar{s}, J, \tilde{Z}, \chi) \propto p(\eta_1, \gamma_1 \mid \eta_0, \gamma_0, J_1, \tilde{Z}_1, \chi) p(\eta_0, \gamma_0 \mid \chi).$$

We employ random-walk proposals and accept the  $(g + 1)$ -st Metropolis step with probability

$$(A-14) \quad \min \left\{ \frac{p(\eta_1, \gamma_1 \mid \eta_0^{(g+1)}, \gamma_0^{(g+1)}, J_1, \tilde{Z}_1, \chi) p(\eta_0^{(g+1)}, \gamma_0^{(g+1)} \mid \chi)}{p(\eta_1, \gamma_1 \mid \eta_0^{(g)}, \gamma_0^{(g)}, J_1, \tilde{Z}_1, \chi) p(\eta_0^{(g)}, \gamma_0^{(g)} \mid \chi)}, 1 \right\}.$$

The target density of  $(\eta_T, \gamma_T)$  is given by

$$(A-15) \quad \begin{aligned} p(\eta_T, \gamma_T \mid r, m, \bar{s}, \eta_{\lambda_T}, \gamma_{\lambda_T}, J, \tilde{Z}, \chi) &\propto p(\eta_T, \gamma_T, \bar{s}_T \mid \eta_{\lambda_T}, \gamma_{\lambda_T}, r, m, \bar{s}_{\lambda_T}, \chi) \\ &= p(\bar{s}_T \mid \eta_T, \gamma_T, r_T, m_T, \chi) p(\eta_T, \gamma_T \mid \eta_{T-1}, \gamma_{T-1}, J_T, \tilde{Z}_T, \chi). \end{aligned}$$

Proposing from  $p(\eta_T, \gamma_T \mid \eta_{T-1}, \gamma_{T-1}, J_T, \tilde{Z}_T, \chi)$ , we accept the  $(g + 1)$ -th Metropolis step with probability

$$(A-16) \quad \min \left\{ \frac{p(\bar{s}_T \mid \eta_T^{(g+1)}, \gamma_T^{(g+1)}, r_T, m_T, \chi)}{p(\bar{s}_T \mid \eta_T^{(g)}, \gamma_T^{(g)}, r_T, m_T, \chi)}, 1 \right\}.$$

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	BM	CG	CS	Fin	HC	Ind	OG	Tech	TC	Util	Total
AAA				4		1					5
AA		1	1	7	3						12
A	6	6	9	23	5	14	7	3	3	2	78
BBB	8	16	24	9	3	18	11	4	3	18	114
BB	3	9	11	3	1	3	2	4	4	3	43
B	3	7	7		2	1	1	1		1	23
CCC		2	1								3
Total	20	41	53	46	14	37	21	12	10	24	278

*TABLE 1. Distribution of Obligors across Sectors and Ratings:* The table presents the distribution of our cross-section of 278 obligors into S&P whole-letter rating classes and ICB Industry (Layer 1) categories.

*Panel A. Principal Components Analysis (in %)*

cumulative $R^2$	min	10%	median	mean	90%	max
1st factor	69.28	87.58	95.06	93.65	98.66	99.73
2nd factor	97.23	98.59	99.43	99.31	99.86	99.98

*Panel B. Mean CDS Spreads (in bp)*

	min	10%	median	mean	90%	max
$\bar{s}(1y)$	4	9	20	56	141	952
$\bar{s}(3y)$	8	17	35	83	194	1160
$\bar{s}(5y)$	11	25	52	106	251	1209
$\bar{s}(7y)$	15	31	61	116	278	1141
$\bar{s}(10y)$	19	38	72	127	300	1155

*Panel C. Skewness in CDS Spreads*

	min	10%	median	mean	90%	max
$\bar{s}(1y)$	0.0836	1.2080	2.0116	2.1142	3.0666	5.0384
$\bar{s}(3y)$	0.0823	0.9372	1.9567	1.9746	3.0329	4.5605
$\bar{s}(5y)$	-0.2632	0.5172	1.8408	1.7977	2.8819	4.3191
$\bar{s}(7y)$	-0.3983	0.4217	1.6257	1.6718	2.8658	4.2838
$\bar{s}(10y)$	-0.5085	0.2641	1.3876	1.4755	2.7912	4.3565

*Panel D. Excess Kurtosis in CDS Spreads*

	min	10%	median	mean	90%	max
$\bar{s}(1y)$	-1.1202	0.4169	3.5266	4.9661	10.3202	35.2089
$\bar{s}(3y)$	-1.1803	-0.0291	3.9053	4.6426	9.5669	28.9064
$\bar{s}(5y)$	-1.2020	-0.4969	3.8426	4.1608	9.2159	22.8371
$\bar{s}(7y)$	-1.3026	-0.5086	2.9902	3.8964	8.8494	24.0456
$\bar{s}(10y)$	-1.4448	-0.6400	2.1130	3.4023	8.6097	26.6116

*Panel E. Skewness in CDS Spread Differences*

	min	10%	median	mean	90%	max
$\Delta\bar{s}(1y)$	-14.8295	-1.7980	0.8253	1.2173	5.7185	15.2134
$\Delta\bar{s}(3y)$	-15.3646	-1.0702	0.8754	1.3952	4.8532	17.1937
$\Delta\bar{s}(5y)$	-13.9045	-1.9777	0.6141	1.2876	5.1401	21.3230
$\Delta\bar{s}(7y)$	-16.1772	-1.7252	0.5598	1.0501	4.3950	19.4012
$\Delta\bar{s}(10y)$	-15.5408	-1.5145	0.4504	0.8918	4.2415	18.2259

*Panel F. Excess Kurtosis in CDS Spread Differences*

	min	10%	median	mean	90%	max
$\Delta\bar{s}(1y)$	8.4357	14.0820	43.9864	73.2589	168.8019	499.5324
$\Delta\bar{s}(3y)$	5.3071	12.7933	32.3053	61.8412	142.3526	486.1720
$\Delta\bar{s}(5y)$	5.2405	14.4141	34.9244	67.9026	157.6387	574.6914
$\Delta\bar{s}(7y)$	5.1215	10.9772	29.5209	58.3178	131.6520	515.3002
$\Delta\bar{s}(10y)$	4.6999	8.9164	22.6595	53.1533	122.8871	482.8054

TABLE 2. *Summary Statistics of CDS Data:* The table displays summary statistics for levels and first differences of 278 US corporate CDS premium panels with maturities 1y, 3y, 5y, 7y, and 10y.

*Panel A. Barndorff-Nielsen and Shephard (2006) Test Results*

	min	10%	median	mean	90%	max
$\bar{s}(1y)$	2.0	3.3	4.2	4.2	5.3	6.6
$\bar{s}(3y)$	1.5	2.4	3.5	3.6	4.8	6.2
$\bar{s}(5y)$	1.1	2.2	3.5	3.4	4.6	5.7
$\bar{s}(7y)$	1.1	2.2	3.3	3.4	4.6	6.2
$\bar{s}(10y)$	1.3	2.2	3.3	3.4	4.4	6.2

*Panel B. Lee and Mykland (2008) Test Results*

	min	10%	median	mean	90%	max
$\bar{s}(1y)$	2.9	7.5	9.9	9.9	12.3	16.3
$\bar{s}(3y)$	1.8	5.1	7.7	7.7	10.3	16.9
$\bar{s}(5y)$	2.9	5.2	7.5	7.6	10.1	14.1
$\bar{s}(7y)$	3.1	4.6	6.8	7.0	9.5	14.1
$\bar{s}(10y)$	2.6	4.6	6.6	7.0	9.9	16.3

*Panel C. Test Results on Simulated Paths*

	min	10%	median	mean	90%	95%	99%	max
BNS <sub>1</sub>	0.7	1.5	2.4	2.3	3.1	3.3	4.2	4.6
BNS <sub>2</sub>	0.2	0.9	1.8	1.7	2.4	2.6	3.2	3.3
BNS <sub>3</sub>	0.2	0.7	1.1	1.2	1.8	2.0	2.4	2.4
LM	0.0	0.6	1.1	1.2	2.0	2.2	2.5	2.9

*TABLE 3. Summary Statistics for Jump Tests:* Panels A and B display the number of jumps per year inferred by the Barndorff-Nielsen and Shephard (2006) and Lee and Mykland (2008) tests at a 5% significance level. Panel C presents the number of jumps inferred from spreads simulated without jumps at a 5% significance level as well. BNS<sub>1</sub> is the linear setting from Barndorff-Nielsen and Shephard (2006), BNS<sub>2</sub> is their ratio test, and BNS<sub>3</sub> their augmented ratio test; LM stands for the Lee and Mykland (2008) test.

Parameter	2.5%	median	97.5%
$\kappa_r^{\mathbb{P}}$	0.2562717	5.760154	15.89878
$\kappa_r^{\mathbb{Q}}$	1.290559	1.342053	1.393930
$\kappa_m^{\mathbb{P}}$	0.2227588	5.839969	16.45602
$\kappa_m^{\mathbb{Q}}$	0.3456469	0.3650545	0.3844102
$\zeta_m$	0.02212169	0.0231497	0.02417191
$\sigma_m$	0.0698724	0.09385755	0.1196880
$\sigma_r$	0.02698494	0.032301	0.03784421
$a_0$	-14.9868	-14.92510	-14.86260
$a_1$	0.9902467	1.031170	1.071350
$a_2$	-0.0634382	-0.05980287	-0.05611948

TABLE 4. *Posterior Estimates for the Riskless Model:* The table shows point estimates for the riskless model, taken to be the multivariate median, as well as quantiles from the posterior distribution of the parameters conditional on the data (see Collin-Dufresne, Goldstein, and Jones (2009) for a definition of the multivariate posterior median). Estimates are based on 10 years of daily panel data of zero-coupon yields bootstrapped from US swap rates. Out of 5,000,000 draws from the Gibbs-Metropolis sampler only every 1,000th draw is recorded to remedy high autocorrelation in the parameter paths. From the remaining 5,000 draws only the last 3,000 are taken into the computation.

Parameter	min	10%	median	mean	90%	max
$\kappa_\eta^{\mathbb{P}}$	0.0002	0.0062	0.0206	0.1683	0.1704	12.7784
$\kappa_\eta^{\mathbb{Q}}$	0.0103	0.0199	0.0378	0.0763	0.1761	1.2502
$\kappa_\gamma^{\mathbb{P}}$	0.0012	0.1034	0.9957	1.3032	2.7469	10.3533
$\kappa_\gamma^{\mathbb{Q}}$	0.0000	0.0001	0.0004	0.0063	0.0039	0.3002
$\zeta_\gamma$	0.0000	0.0005	0.0043	0.0206	0.0682	0.2313
$\sigma_\eta$	0.0520	0.1076	0.1636	0.1865	0.2752	0.8991
$\sigma_\gamma$	0.0670	0.1512	0.3928	0.4122	0.6580	1.2315
$l^{\mathbb{P}}$	0.5821	1.6930	4.3155	5.3972	9.3572	47.6121
$\mu_\eta^{\mathbb{P}} \times 10^4$	0.0083	0.0996	27.1053	44.5526	110.235	544.348
$\mu_\eta^{\mathbb{Q}} \times 10^4$	0.0101	0.0810	0.6056	6.7545	22.0913	111.659
$\mu_\gamma^{\mathbb{P}} \times 10^4$	0.1316	73.1032	655.171	932.253	2359.52	4640.43
$\mu_\gamma^{\mathbb{Q}} \times 10^4$	0.0824	2.3233	749.679	1053.89	2740.74	7398.19
$b_2$	-0.2505	-0.1084	0.0373	0.0213	0.0903	0.2169
$b_1$	-3.0349	-1.3453	-0.3482	-0.3120	0.8732	2.4224
$b_0$	-22.2945	-18.5978	-16.0353	-15.2886	-10.2588	-6.1592

TABLE 5. *Cross-Sectional Summary Statistics for the Posterior Estimates of the Defaultable Model:* This table presents the cross-sectional distribution of parameters (i.e., across obligors) for our default model. On the single-obligor level point estimates (taken to be the multivariate median from the posterior distribution of the parameters conditional on the data) are based on approx. 4.5 years of daily CDS spreads for a panel of 1y, 3y, 5y, 7y, and 10y maturities. Out of 5,000,000 draws from the Gibbs-Metropolis sampler only every 1,000th draw is recorded to remedy high autocorrelation in the parameter paths. From the remaining 5,000 draws only the last 3,000 are taken into the computation.

*Panel A. Mean Observation Errors (in bp)*

	min	10%	median	mean	90%	max
$s(1y)$	-99.1618	-11.2176	-0.2111	-4.6628	0.1581	1.2117
$s(3y)$	-31.9530	-6.2127	-0.6486	-2.2542	-0.0848	0.8633
$s(5y)$	-3.9847	0.1008	0.7632	1.5649	4.4333	10.3461
$s(7y)$	-7.9430	-1.5773	-0.3626	-0.4431	0.5631	4.3787
$s(10y)$	-3.5782	-0.6160	0.0503	0.0552	0.7339	2.5506

*Panel B. Standard Deviation of Observation Errors (in bp)*

	min	10%	median	mean	90%	max
$s(1y)$	0.1329	0.8207	2.2682	10.4307	27.5869	134.6981
$s(3y)$	0.4454	1.0036	2.4449	5.5660	13.3320	69.0095
$s(5y)$	0.4932	0.8265	1.8891	3.5837	7.8686	35.0435
$s(7y)$	0.4657	0.8080	2.0349	2.8732	6.0526	30.8616
$s(10y)$	0.0200	0.1788	2.8762	4.3414	10.2212	48.1333

*Panel C. Autocorrelation of Observation Errors*

	min	10%	median	mean	90%	max
$s(1y)$	-0.034	0.339	0.574	0.590	0.908	0.986
$s(3y)$	0.382	0.640	0.798	0.787	0.926	0.983
$s(5y)$	0.136	0.652	0.809	0.792	0.924	0.989
$s(7y)$	0.138	0.435	0.722	0.696	0.925	0.968
$s(10y)$	-0.280	0.162	0.831	0.699	0.958	0.984

*TABLE 6. Summary Statistics of Observation Errors:* Panel A displays the distribution of mean observation errors, and Panel B the distribution of standard deviations of observation errors. Panel C shows statistics of first-order autocorrelations of the observation errors.

Statistic	Value
Minimum	3.36%
10% Quantile	4.91%
Median	10.15%
Mean	20.99%
90% Quantile	58.75%
Maximum	99.84%
Std. Deviation	21.93%
Skewness	1.5902
Kurtosis	4.6817

*TABLE 7. Descriptive Statistics for the Cross-Section of Implied LGD:* The table describes the cross-sectional distribution (i.e., across obligors) of the implied LGD parameters.



Panel A. Sector Differences in LGD (in %)

Sector	BM	CG	CS	Fin	HC	Ind	OG	Tech	TC	Util
Basic Materials	<i>52.17</i>	8.51		21.47						
Consumer Goods	-8.51	<i>60.68</i>		12.97		-7.63	-6.99	-12.00		-12.40
Consumer Services			<i>57.65</i>	16.00				-8.96		-9.37
Financials	-21.47	-12.97	-16.00	<i>73.64</i>	-18.08	-20.60	-19.96	-24.96	-14.30	-25.37
Health Care				18.08	<i>55.57</i>			-6.88		-7.29
Industrials		7.63		20.60		<i>53.04</i>				-4.77
Oil & Gas		6.99		19.96			<i>53.69</i>			-5.41
Technology		12.00	8.96	24.96	6.88			<i>48.68</i>	10.66	
Telecomm.				14.30				-10.66	<i>59.34</i>	-11.06
Utilities		12.40	9.37	25.37	7.29	4.77	5.41		11.06	<i>48.28</i>
Mean	17.40	31.27	24.98	28.13	17.10	11.05	13.26	15.27	23.42	10.14
Std. Deviation	19.55	25.80	26.05	25.09	23.49	11.76	12.58	11.35	14.85	5.78

Panel B. Rating Differences in LGD (in %)

Rating	Markdown
AAA	insignificant
AA	insignificant
A	-7.70
BBB	-20.47
BB	-21.32

TABLE 8. *Relative Differences in Implied LGD between Sectors and Ratings:* The table shows relative differences in implied LGD between the ICB Industry classes (Panel A) and ratings (Panel B). The last two rows in Panel A additionally show means and standard deviations of implied LGD within each of the sectors. Each row in Panel A contains regression coefficients resulting from the cross-sectional regression (11) of implied LGD on sector and rating matrices. Excluding one sector at a time amounts to setting this sector as the base case and investigating whether implied LGD of other sectors significantly differs from the base sector's. The base rating is always B or worse. The implied LGD for each sector-and-rating base case is given italicized on the diagonal. Each figure listed is significant at the 10% level. The adjusted  $R^2$  of the regressions is 50.61%. For example, suppose one wants to determine the relative difference in implied LGD between a B-rated obligor in the *Industrials* sector and an A-rated bank: The difference in LGD is created by a 3-letter rating difference and differing sectors. The rating improvements contribute by lowering the LGD by 49.49% in sum (cp. Panel B). On the other hand, the bank belonging to the *Financials* sector increases its LGD by 20.60% (cp. Panel A, row marked *Industrials*). The net outcome is a decrease in LGD by 28.89%, meaning that the implied LGD of the A-rated bank would amount to approximately  $53.04\% - 28.89\% = 24.15\%$ .

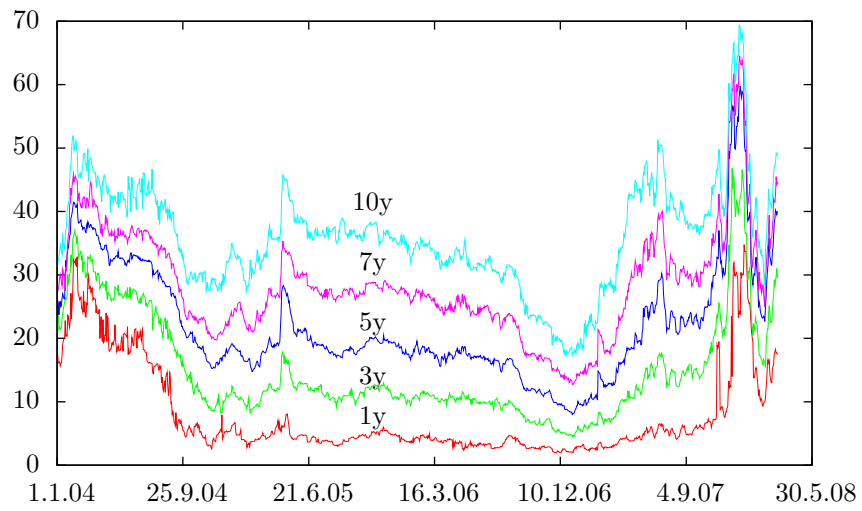
*Panel A. Relative Jump Sizes*

Parameter	min	10%	median	mean	90%	max
$\mu_{\eta}^{\mathbb{P}}/\bar{s}_{\text{avg}}(1y)$	0.0007	0.0050	0.5947	2.0354	5.4275	50.5116
$\mu_{\eta}^{\mathbb{Q}}/\bar{s}_{\text{avg}}(1y)$	0.0001	0.0028	0.0286	0.2232	0.4666	4.0794
$\mu_{\gamma}^{\mathbb{P}}/\bar{s}_{\text{avg}}(10y)$	0.0004	0.3154	10.1315	12.6702	28.9615	58.9757
$\mu_{\gamma}^{\mathbb{Q}}/\bar{s}_{\text{avg}}(10y)$	0.0003	0.0136	8.7066	14.0991	37.2871	64.0820

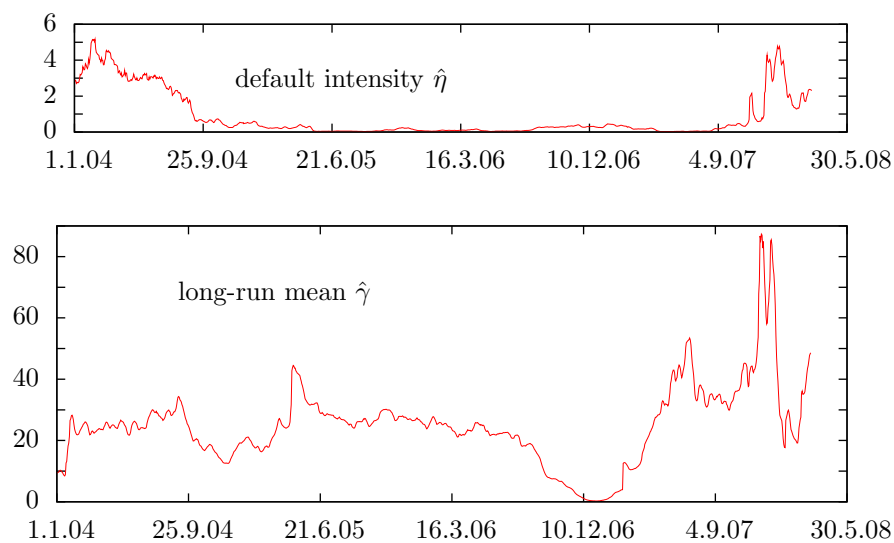
*Panel B. Relative Risk Premia on Jump Sizes*

	min	10%	median	mean	90%	max
$(\mu_{\eta}^{\mathbb{P}} - \mu_{\eta}^{\mathbb{Q}})/\bar{s}_{\text{avg}}(1y)$	-4.0648	-0.1105	0.4588	1.8123	5.1025	50.5065
BSD $_{\eta}$	1.3514	2.0444	2.7251	3.5884	7.6520	9.5456
$(\mu_{\gamma}^{\mathbb{P}} - \mu_{\gamma}^{\mathbb{Q}})/\bar{s}_{\text{avg}}(10y)$	-51.6545	-16.9627	0.3003	-1.4288	13.6192	54.3209
BSD $_{\gamma}$	7.8559	11.8453	13.4987	13.7296	16.3305	19.5878

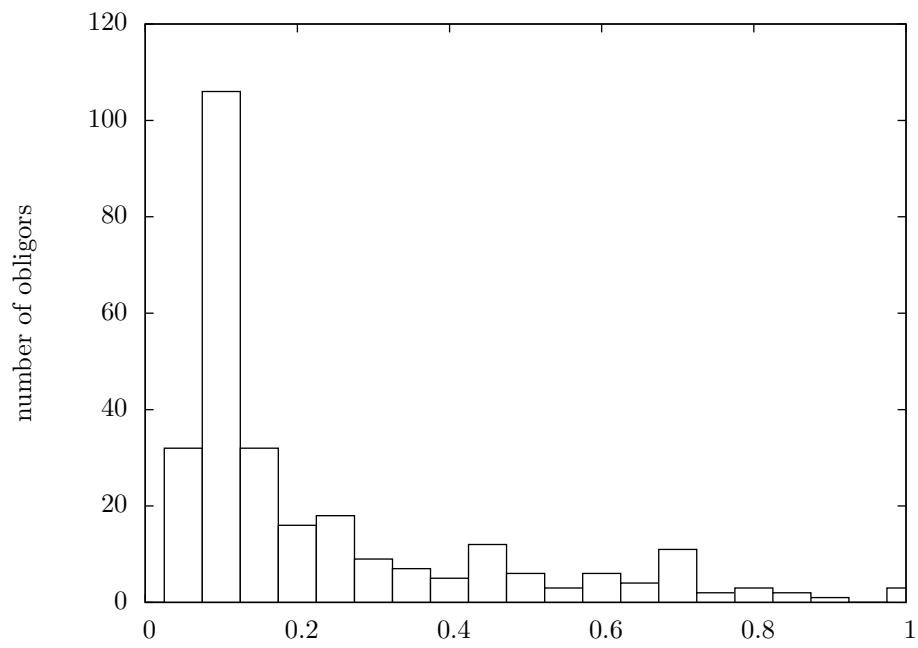
*TABLE 9. Relative Jump Sizes and Corresponding Risk Premia:* The table presents the cross-sectional distribution of relative jump sizes, i.e., (absolute) expected jump sizes normalized by the average spread of the corresponding maturity, as well as the cross-sectional distribution of the corresponding risk premia, including the distribution of their respective bootstrap standard deviations (BSD).



*FIGURE 1. Time Series of Observed CDS Premia:* The figure displays time series of observed CDS premia (in bp) for Honeywell Int'l Inc. (Markit ticker HON).



*FIGURE 2. Posterior Trajectories of Latent Default State Variables:* The figure displays posterior realizations of the default components  $\eta$  and  $\gamma$  (in %) for Honeywell Int'l Inc. (Markit ticker HON) conditional on HON premia, missing values, and our model specification. Out of 5,000,000 draws from the Metropolis-Gibbs sampler only every 1,000th draw is recorded to remedy high autocorrelation in the parameter paths. From the remaining 5,000 draws only the last 3,000 draws from the time series are used for computation.



*FIGURE 3. Cross-Sectional Histogram of Implied LGD:* The figure plots the cross-sectional distribution (i.e., across obligors) of the implied LGD parameters.