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Abstract

In this article, we look at a model with (independent) system operator who faces stochastic but growing transmission demand and a penalty if frequency is not balanced. In this set up, we derive an optimal grid expansion investment strategy and analyze its value and risk implications. It turns out that the firm value is strictly concave in the level of transmission demand. Firm value, however, increases with optimal investment for any level of demand. Moreover, firm risk is decreasing in the level of demand and higher when the firm has an investment option. The risk increase corresponds to the exercise of the call option and is stronger, the closer the firm approaches its exercise trigger.

Keywords: Grid expansion; Frequency balancing (Spinning reserves); Real options.

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1 Introduction

The generation and transmission of electricity are fundamental inputs for firms in growing economies and hence have recently, received a lot of attention. While the generation of electricity is entirely determined by the employed technology (fossil energy, renewable energy, etc.) transmission of energy is often independent of the source of energy but depends heavily on the capacity of the grid. Prior to deregulation, production, transmission and distribution of electric energy have been controlled by a single, though regulated, public utility operating with a given grid. In this case, the vertically integrated public utility produced and transmitted energy on the basis of maximizing economic surplus subject to regulatory constraints, including service security for customers.

In a deregulated power system, electricity generation and transmission are separated. System operators (SO) transport electricity to locations where it is demanded by using existing grid capacity. Given technological constraints, however (the existence of Ohm's law and the necessity to balance the grid), any SO faces a delicate decision problem once demand is stochastic. It does not only need to transmit electricity by using the grid but also needs to balance the grid in order to guarantee the safety of the power system.

Consequently, global demand fluctuations or local variation in demand or supply may destroy this balance and have therefore to be avoided or balanced by the grid operator.[1] Since balanced grids are an important precondition for system security, the regulator's focus has moved towards providing a legal and economic environment to ensure the targeted level of security. One of the possibilities to do so, is to create a new additional market for balancing power or balancing reserves[2] which is often separated from the spot market, although, similar products are traded. The main difference between these two markets rests in the action timing necessary when balancing power. Hence, balancing power gives the buyer a certain flexibility that also goes along with a price difference when compared to the rigid spot market.

In this paper we model a SO with a given transmission capacity who faces stochastic demand that follows a Geometric Brownian Motion. The SO is required to balance the grid at any time the demand turns out to deviate too much from existing capacity. If demand is smaller or higher than a given percentage of existing capacity, the

[1]The grid operator in this context can be an independent system operator (ISO) or any other legal type of firm operation, as the mentioned issue is due to a technical feature and hence, independent from the legal aspects. For expository convenience of this article, we will denote the balancing firm as ISO or system operator in the following.

[2]Readers not familiar with reserve market are referred to Rebours and Kirschen (2005) who compare the definition and technical specification of reserve services in Great Britain, PJM (Pennsylvania - New Jersey - Maryland), California, Spain, the Netherlands, Germany, France, Belgium, and the UCTE as a whole. For instance, in the PJM-market, a comparable instrument is given by spinning reserves (which serve as a primary reserve among the operating reserve instruments available for tertiary control); for a detailed survey of the North American market, see e.g. Lusztig et al (2006).

SO has to step in and either buy or sell balancing power. Since these actions are costly, it is in the interest of the SO to avoid any grid imbalance. This summarizes the short run actions of the SO. In the long run as demand continuously grows and capacity constraints are frequently binding it is in the interest of the SO to invest into grid expansion. Therefore, we use a real option approach to analyze optimal investment decisions in grid expansions and determine its implications for firm value and firm risk. Additionally, we contrast the value and risk implications for the cases of no investment and that of grid expansion. It turns out that the additional costs of balancing the grid cause the firm value to be a concave function of stochastic demand with the implication that firm risk is decreasing with increasing demand levels. Firm value and risk are both higher when the firm faces a growth option to expand capacity. Both are immediate consequences of the strictly positive option value associated with grid expansions.

Transmission investments have been studied by Ramanathan and Varadan (2006). They develop a real options model that is solved using binomial tree valuation and points out all possible economic trade-offs, present in such a framework. The article by Boyle, Guthrie and Meade (2006) uses a real options framework to evaluate the investment test proposed by the regulator in New Zealand. The paper by Saphores, Gravel and Bernard (2004) analyzes a real options investment decision under the assumption that the firm must undergo a costly and time-consuming regulatory process prior to making an investment. These constraints severely influence the timing decision when to invest and might lead in some cases to earlier investment. The paper by Borenstein, Bushnell and Stoft (2000) studies the competitive effects of transmission capacity. Their model predicts, that the level of transmission capacity does neither have an impact on competition nor on the actual electricity that flows on the transmission line in equilibrium. While many of these papers use a real options approach to derive optimal investment decisions for transmission capacity neither looks at the value and/or risk consequences of these investments. Therefore, we concentrate on these questions and derive a set of new insights in grid expansion investments.

Our paper is organized as follows. In Section 2, we present our model. Section 3 studies value and risk consequences of balancing the grid in case of a growth option. Section 4 derives the value and risk implications of optimal grid investment. Using numerical techniques we derive the optimal value function, the dynamic firm betas and comparative statics results. Finally, Section 5 draws together the main findings and concludes the paper.

2 The Model

We consider a system operator (SO) who's business is to balance the network. The SO operates with a given fixed capacity, K_0. The existing capacity is irreversible and cannot be employed in alternative uses. The system operator faces stochastic demand for transmission capacity. This demand level at time t is denoted by X_t. It is further assumed that demand for transmission services follows a stochastic process specified as Geometric Brownian Motion (GBM).

$$\mathrm{d}X_t = \mu X_t \mathrm{dt} + \sigma X_t \mathrm{d}w_t, \tag{1}$$

where $\mu > 0$ is the constant growth rate of demand, $\sigma > 0$ is the constant volatility per unit of time, and dw_t is a standard Wiener process. In principle, the growth rate of demand can take positive or negative values. Nevertheless, in the existing application, we restrict it to be positive $\mu > 0$ and allow for $\mu = 0$. With positive μ, the expected demand for transmission services either grows exponentially over time or fluctuates randomly around existing capacity levels.

In an environment with stochastic demand and fixed capacity the system operator faces the following problem. In the short run, exogenous demand for transmission can either go largely beyond or below the existing capacity limit. Both cases are not desired outputs since an efficient balance of the network load requires actions by the SO and generates additional costs. In the long run, when demand continues to grow the SO faces the decision to extend his existing capacity. This requires an investment in additional transmission capacity. Given the assumption that any investment in transmission capacity is irreversible and given the flexibility of the SO to decide when to invest, investment into grid capacity forms a real option. In this paper we assume that the SO has a single growth option to expand capacity from the level K_0 to a new level K_1 (with $K_0 < K_1$). The investment costs for this expansion are denoted by IC.

Recall, from an SO's point of view over- and underruns cannot directly be controlled because of exogenous stochastic transmission demand but he can balance the system by buying or selling balancing energy which is costly.

In this model we assume that the system operator receives a constant unit price for transmission services, p, whenever network transmissions are balanced and demand X_t is within the bounds $(1-\beta)K_0 \leq X_t \leq (1+\alpha)K_0$. If demand exceeds the upper bound $b_0 \equiv (1+\alpha)K_0$, the SO has to buy balancing energy from an open market at a premium price. We assume that this premium price is constant and given by C_1. Therefore, revenues for the SO in case of $X_t > b$ are given by $bp - (X_t - b)C_1$. Symmetrically, in the case when demand is below the lower bound $X_t < a_0 \equiv (1-\beta)K_0$, the SO has to sell energy which incurs per unit costs equal to C_2 so that revenues become $pX_t - (a - X_t)C_2$.[3] Putting all these constraints together, the SO's revenue function is given by the following piecewise linear function

$$\Pi_0(X_t) = \begin{cases} b_0 p - (X_t - b_0)C_1 & \text{if } X_t \geq (1+\alpha)K_0 \\ pX_t & \text{if } (1-\beta)K_0 < X_t < (1+\alpha)K_0 \\ pX_t - (a_0 - X_t)C_2 & \text{if } X_t \leq (1-\beta)K_0. \end{cases} \tag{2}$$

[3] To be more specific, the system operator does not sell energy but has to buy negative quantities at the price C_2.

This specification demonstrates that as long as demand for transmission services lies in the interval $a_0 \leq X_t \leq b_0$, the SO will earn regular revenues equal to pX_t. In case if demand leads to an unbalanced grid the SO has to jump in and stabilize the system by buying or selling energy at the extra costs specified.
As long as the SO operates with given capacity K_0, the firm value corresponds to the expected discounted present value of future revenues, i.e.,

$$V(X) = \mathbb{E}\left\{\int_t^{\infty} e^{-r(\tau-t)}\Pi_0(X_\tau)d\tau \mid X = X_t\right\}, \tag{3}$$

where $r > 0$ is the constant discount rate that satisfies $r > \mu$.
In the case that no growth option exists, the value of the firm is the sum of the values of the assets in place. Therefore, any firm value has to satisfy a simple no arbitrage condition. Herein, total return during a given period of time of an investment in the firm's assets must be equal to the dividend payments plus the expected capital gains of the assets, i.e.,

$$rV(X) = \Pi_0(X) + \mathbb{E}\left\{dV(X)\right\}.$$

Applying Ito's Lemma to this no arbitrage condition results in the traditional Bellman equation given by

$$\frac{1}{2}\sigma^2 X^2 V''(X) + \mu X V'(X) + \Pi(X) - rV(X) = 0. \tag{4}$$

A general solution to this Bellman equation results in an explicit expression for the firm value in case the SO needs to balance the network load, and hence, faces revenue constraints if demand overruns the capacity limits. If there is no need to balance the network load, the revenues are given by $\Pi(X) = pX$ for any level of $X > 0$, which corresponds to a firm value of

$$V(X) = \frac{pX}{r-\mu}.$$

This last equation highlights an interesting relationship. The firm value corresponds to the present value of future revenues discounted with a risk adjusted rate of return. Hence, it is identical to what the simple Gordon growth model would predict.
In the case that the SO holds a growth option, management needs to decide when to expand capacity from K_0 to K_1 at investment cost of IC. The corresponding decision problem is a classical real options problem with single investment option and constant investment costs equal to IC. Hence, the decision problem of the operator becomes

$$\max_{\tau^*}\left\{V(X) \equiv \int_0^{\tau^*} e^{-rt}\Pi_0(X)dt + \int_{\tau^*}^{\infty} e^{-r(t-\tau^*)}\Pi_1(X)dt - e^{-r\tau^*}IC\right\},$$

where the profit function $\Pi_1(X)$ is related to the new capacity level K_1 and therefore is defined as

$$\Pi_1(X_t) = \begin{cases} b_1 p - (X_t - b_1)C_1 & \text{if } X_t \geq (1+\alpha)K_1 \\ pX_t & \text{if } (1-\beta)K_0 < X_t < (1+\alpha)K_1 \\ pX_t - (a_1 - X_t)C_2 & \text{if } X_t \leq (1-\beta)K_1, \end{cases} \tag{5}$$

with $b_1 \equiv (1+\alpha)K_1$ and $a_1 \equiv (1-\beta)K_1$.

In the next Section we will derive the value and risk characteristics of the SO in case of no growth option (no capacity expansion). Given the piecewise linear revenue function, the valuation of firm's assets needs to take into account the cash flow reductions associated with unbalanced demand. Hence, firm value consists of the value of the assets in place and a non-linear adjustment necessary to account for changes in the revenue function.

3 Value and Risk Implications Without Investment Option

The firm value $V(X)$ defined in (3) can be derived using the Bellman equation (4). The value function is a smooth function that consist of two parts: (i) the value of the assets in place, and (ii) the value adjustment necessary to account for the revenue constraint. Applying standard techniques and given the piecewise linearity of the revenue function given in equation (2) allows us to derive explicit formulas for the firm value.

Proposition 1. *In case the system operator has no growth option to adjust capacity levels, the value of the firm is given by*

$$V(X) = \begin{cases} V_U(X) = A_U^0 X^{\lambda_1} - \left(\frac{C_1}{r-\mu}\right) X + \frac{b_0(p+C_1)}{r} & \text{if } X_t \geq b_0 \\ V_M(X) = A_M^0 X^{\lambda_1} + B_M^0 X^{\lambda_2} + \left(\frac{p}{r-\mu}\right) X & \text{if } a_0 < X_t < b_0 \\ V_L(X) = B_L^0 X^{\lambda_2} + \left(\frac{p+C_2}{r-\mu}\right) X - \frac{a_0 C_2}{r} & \text{if } X_t \leq a_0, \end{cases} \tag{6}$$

where the constant parameters are given by

$$A_M^0 = \frac{a_0^{1-\lambda_1} C_2(\mu\lambda_2 - r)}{(\lambda_2 - \lambda_1)(r-\mu)r} < 0,$$

$$B_M^0 = \frac{b_0^{1-\lambda_2}(p + C_1)(\mu\lambda_1 - r)}{(\lambda_2 - \lambda_1)(r-\mu)r} < 0,$$

$$A_U^0 = \frac{(a_0 b_0)^{1-\lambda_1}[a_0^{\lambda_1 - 1}(p + C_1) + b_0^{\lambda_1 - 1} C_2](\mu\lambda_2 - r)}{(\lambda_2 - \lambda_1)(r-\mu)r} < 0,$$

$$B_L^0 = \frac{(a_0 b_0)^{1-\lambda_2}[a_0^{\lambda_2-1}(p+C_1) + b_0^{\lambda_2-1}C_2](\mu\lambda_1 - r)}{(\lambda_2-\lambda_1)(r-\mu)r} < 0.$$

λ_1, λ_2 correspond to the two roots of the characteristic equation of the Bellman equation and are given by

$$\begin{aligned}
\lambda_1 &= \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0 \\
\lambda_2 &= \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1.
\end{aligned}$$

Proof: See Appendix 1.
As already pointed out firm value consists of the value of the assets in place as well as a value adjustment necessary to account for the negative impact of balancing costs on revenues. All the nonlinear adjustments decrease firm value as extra costs are triggered when exogenous demand violates either the lower or upper constraints. The valuation equation clearly points out that not balancing the system can be very costly to the system operator and can even result in a negative firm value.

Corollary 1. *The firm value $V(X)$ given in (6) is strictly concave in X.*

Proof: Follows immediately by differentiating (6) with respect to X. □
While many of the existing papers that evaluate grid investments have looked at the value implications of regulatory and economic constraints, none has ever looked at the risk an operator faces. We are the first to explicitly derive the firm risk. Characterizing firm risk requires a risk measure that eventually can be quantified. Here, we follow standard finance theory and identify firm risk as the systematic risk arising from a systematic risk factor. Based on our partial equilibrium model we identify demand uncertainties as the systematic risk factor. Since the value of the firm changes as the risk factor X changes, we can apply standard arguments to show (see Berk, Green and Naik [2004]) that the firm's beta is given by

$$\beta_i(X) = \frac{\frac{\partial V_i(X)}{\partial X} X}{V_i(X)}, i = U, M, L. \tag{7}$$

Using this concept of firm risk, we are able to derive the firm's systematic risk.

Proposition 2. *The system operator's dynamic beta is given by*

$$\beta(X) = \begin{cases} \beta_U(X) = 1 + \frac{A_U^0 X^{\lambda_1}}{V_U(X)}(\lambda_1 - 1) - \frac{\frac{b(p+C_1)}{r}}{V_U(X)} & \text{if } X \geq b_0 \\ \beta_M(X) = 1 + \frac{A_M^0 X^{\lambda_1}}{V_M(X)}(\lambda_1 - 1) + \frac{B_M X^{\lambda_2}}{V_M(X)}(\lambda_2 - 1) & \text{if } a_0 < X < b_0 \\ \beta_L(X) = 1 + \frac{B_L^0 X^{\lambda_2}}{V_L(X)}(\lambda_2 - 1) + \frac{\frac{aC_2}{r}}{V_L(x)} & \text{if } X \leq a_0. \end{cases} \tag{8}$$

Proof: The proof follows immediately by differentiating (6) with respect to X and applying the definition of beta. $\square$

The system operator's beta is the sum of the asset beta, normalized to 1, and the risk adjustment arising from the cash flow changes associated with the boundaries of the balanced network load. These adjustments can either be risk increasing or decreasing. As an example let us look at $\beta_U(X)$. The fact that X exceeds the upper bound b_0 and, hence, firm value is reduced results in an increase in risk (relative to the asset beta) given by $\frac{A_U^0 X^{\lambda_1}}{V_U(X)}(\lambda_1 - 1) > 0$. While constant payments equal to $\frac{b(p+C_1)}{r}$ result in a reduction of risk equal to $-\frac{\frac{b(p+C_1)}{r}}{V_U(X)} < 0$.

To evaluate the change in firm beta associated with an increase in transmission demand X, let us refer to definition (7) and differentiate it with respect to X. This implies

$$\beta'(X) = \frac{(V''X + V')\,V - (V')^2 X}{V^2}$$

Defining the demand elasticity of marginal firm value (the percentage change of the marginal firm value associated with a one percent change in demand X) as

$$\epsilon \equiv \frac{V''X}{V'}$$

yields the following expression for the change in beta resulting from an increase in X.

$$\beta'(X) = \frac{V'}{V}\left[\epsilon + 1 - \beta\right]. \tag{9}$$

The relationship of changes in a firm's beta holds independent of its risk structure. Moreover, it can be shown that

$$\frac{\beta'(X)X}{\beta(X)} = 1 + \epsilon - \beta$$

holds. This implies that the demand elasticity of beta (the changes in firm beta arising from a one percent change in demand X) is equal to 1 minus beta plus demand elasticity of marginal firm value. Hence, for our model with a concave value function $V(X)$ it is to be expected that risk is decreasing with an increase in demand X.

Corollary 2. *Firm risk is decreasing [increasing] with an increase in demand X if and only if*

$$1 < \beta - \epsilon \quad [1 > \beta - \epsilon].$$

Proof: Follows immediately from the definition of the demand elasticity of beta and the fact that $\beta > 0$. $\square$

The concave value function, resulting from the penalty structure of non-balanced transmissions implies that firm risk decreases with an increase in demand. The

economic intuition of this result is as follows. From the firm's point of view, it is optimal to have a balanced network load since revenues are highest in this case. If demand turns out to be out of bounds, the corresponding penalty structure implies that revenues do not change too much. This implies reduced risk.
Its sign, however, depends on the curvature of the value function. With a concave value function, the following property holds, $sign(\epsilon) = -sign(V')$, i.e., demand elasticity of marginal firm value increases with a decrease in the value of the firm and decreases with an increase in firm value. Hence, dynamic beta over the entire range of X (values) can take different shapes, depending on the specification of the parameters.

4 Value and Risk Implications With Investment Option

The specification of transmission demand as a geometric Brownian motion implies that expected demand increases exponentially. This makes an investment in additional capacity attractive for the SO as it either reduces or avoids the costs arising from demand overruns given low levels of capacity. We assume that the investment problem to install additional capacity corresponds to a real option in which the operator decides when to exercise the growth option. With demand being stochastic, it is optimal to exercise the growth option only if demand for transmission services exceeds a certain threshold level, X^*. Therefore, this section analyzes the structure and the characteristics of the optimal investment trigger level.
In order to determine this optimal trigger level, (and hence the optimal timing of the investment) three cases need to be distinguished depending on the actual value of demand for capacity. It is obvious that in case actual demand is lower than the boundary a_0, the firm does not have an incentive to invest in additional capacity since it amplifies the scenario in which penalties have to be paid. On the contrary, if actual demand is higher than the upper bound b_0, there is a strong incentive for the firm to invest in additional capacity. Hence, we can conclude that the demand level X^* that triggers an investment must be above the lower bound a_0, which is a necessary condition. It might be the case, however, that it is optimal for the firm to already invest if the trigger level is in the interval $[a_0, b_0]$. This can only happen in extreme cases with specific parameter settings. Here we rule out those cases such that we only concentrate on the case in which $X^* > b_0$.
After the investment takes place the firm faces new boundaries that are relevant for network load balancing. These boundaries are $[a_1, b_1]$. In terms of this new boundaries we can distinguish three possible cases for the relative sizes of X^*, a_1 and b_1: (i) $X^* < a_1$, (ii) $a_1 < X^* < b_1$, and (iii) $X^* > b_1$. In any case, the value function of the firm prior to investment in new capacity will consist of the value of the assets in place, the value adjustment for penalties in case demand turns out to be unbalanced and the option value to invest in new capacity. After the investment,

the value function is identical to (6) but the boundaries a_0, b_0 have to be replaced by a_1, b_1 with corresponding adjustments in the integration constants, A^1_U, A^1_M, B^1_M and B^1_L.

Proposition 3. *Under the assumption that the optimal investment trigger level satisfies $b_0 < X^* < a_1$ the optimal value function of the SO is given by*

$$V(X) = \begin{cases} \begin{cases} V^0_L(X) = B^0_L X^{\lambda_2} + \left(\frac{p+C_2}{r-\mu}\right) X - \frac{a_0 C_2}{r} + K X^{\lambda_2} & \text{if } X_t \le a_0 \\ V^0_M(X) = A^0_M X^{\lambda_1} + B^0_M X^{\lambda_2} + \left(\frac{p}{r-\mu}\right) X + K X^{\lambda_2} & \text{if } a_0 < X_t < b_0 \\ V^0_U(X) = A^0_U X^{\lambda_1} - \left(\frac{C_1}{r-\mu}\right) X + \frac{b_0(p+C_1)}{r} + K X^{\lambda_2} & \text{if } b_0 < X \le X^* \end{cases} \\ \begin{cases} V^1_L(X) = B^1_L X^{\lambda_2} + \left(\frac{p+C_2}{r-\mu}\right) X - \frac{a_1 C_2}{r} & \text{if } X^* \le X \le a_1 \\ V^1_M(X) = A^1_M X^{\lambda_1} + B^1_M X^{\lambda_2} + \left(\frac{p}{r-\mu}\right) X & \text{if } a_1 < X < b_1 \\ V^1_U(X) = A^1_U X^{\lambda_1} - \left(\frac{C_1}{r-\mu}\right) X + \frac{b_1(p+C_1)}{r} & \text{if } X \ge b_1, \end{cases} \end{cases} \tag{10}$$

where the constants K (form the option value KX^{λ_2}) and the trigger level X^* are positive and determined by corresponding value matching and smooth pasting conditions. The constants A^1_U, A^1_M, B^1_M and B^1_L are entirely determined by the new boundaries a_1 and b_1.

Proof: See Appendix 2. □

It is important to point out again that the assumption $X^* < a_1$ need not hold and the two other cases (i) $a_1 < X^* < b_1$, and (ii) $X^* > b_1$ might occur. In the proof of Proposition 3, we demonstrate how the trigger level and the option constant change if any of the other two cases holds.

Proposition 4. *Under the assumption that the optimal investment trigger level satisfies $b_0 < X^* < a_1$, the dynamic beta of the SO is given by*

$$\beta(X) = \begin{cases} \begin{cases} \beta^0_L(X) = 1 + \frac{(B^0_L+K)X^{\lambda_2}}{V^0_L(X)}(\lambda_2 - 1) + \frac{\frac{a_0 C_2}{r}}{V^0_L(x)} & \text{if } X_t \le a_0 \\ \beta^0_M(X) = 1 + \frac{A^0_M X^{\lambda_1}}{V^0_M(X)}(\lambda_1 - 1) + \frac{(B^0_M+K)X^{\lambda_2}}{V^0_M(X)}(\lambda_2 - 1) & \text{if } a_0 < X_t < b_0 \\ \beta^0_U(X) = 1 + \frac{A^0_U X^{\lambda_1}}{V^0_U(X)}(\lambda_1 - 1) + \frac{K X^{\lambda_2}}{V^0_U(X)}(\lambda_2 - 1) - \frac{\frac{b_0(p+C_1)}{r}}{V^0_U(X)} & \text{if } b_0 < X \le X^* \end{cases} \\ \begin{cases} \beta^1_L(X) = 1 + \frac{B^1_L X^{\lambda_2}}{V^1_L(X)}(\lambda_2 - 1) + \frac{\frac{a_1 C_2}{r}}{V^1_L(X)} & \text{if } X^* \le X \le a_1 \\ \beta^1_M(X) = 1 + \frac{A^1_M X^{\lambda_1}}{V^1_M(X)}(\lambda_1 - 1) + \frac{B^1_M X^{\lambda_2}}{V^1_M(X)}(\lambda_2 - 1) & \text{if } a_1 < X < b_1 \\ \beta^1_U(X) = 1 + \frac{A^1_U X^{\lambda_1}}{V^1_U(X)}(\lambda_1 - 1) - \frac{\frac{b_1(p+C_1)}{r}}{V^1_U(X)} & \text{if } X \ge b_1. \end{cases} \end{cases} \tag{11}$$

Proof: The proof follows immediately from differentiating the value function in Proposition 3 with respect to X and applying the definition of beta. □

4.1 Numerical Simulations for Optimal Capacity Investment

While the case of no investment allows for a complete analytical solution, the optimal investment problem in new capacity requires a numerical analysis. Our numerical procedure is divided up in four parts. In the first part we explore the characteristics of the value function. In particular we are interested if the concavity property found in the case of no investment carries over to the investment case. In the second part we determine dynamic beta risks in case of grid expansion investments. In the third part we conduct a general comparative statics analysis and in the fourth part we finish our analysis with the derivation of optimal trigger levels and explore the role of parameter values on the optimal time to invest. Here we make use of Broyden's method (see e.g.: Press et al. (1986)). For the numerical simulations, we refer to the benchmark values of the paramters included as highlighte in table (1).

Table 1: Benchmark values

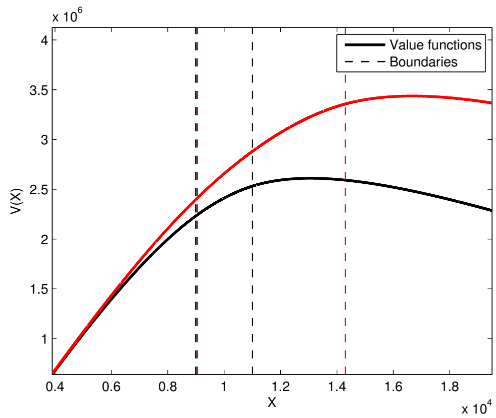
Parameter	Benchmark value
Discount rate, r	5%
Demand growth rate, μ	2%
Instantaneous standard deviation, σ	10%
Allowed upper deviation, α	10%
Allowed lower deviation, β	10%
Installed capacity level, K_0	10,000
Expanded capacity level, K_1	13,000
Price for transmission, P	5
Penalty for exceeding upper boundary, C_1	5
Penalty for demand below lower boundary, C_2	15
Unit investment cost for capacity increase, IC	100

4.2 Value function

The optimal investment strategy and the corresponding value function presented in Proposition 3 are derived under the assumption that an investment changes the bounds $[a_0, b_0]$ to the new interval $[a_1, b_1]$ with $a_0 = (1-\beta)K_0$, $b_0 = (1+\alpha)K_0$ and $a_1 = (1-\beta)K_1$, $b_1 = (1+\alpha)K_1$. Alternatively, a capacity constraint could be modeled in such a way that it only increases b_0 to b_1 and leaves a_0 unchanged. In such a case, the interval in which the SO does not need to jump in and balance the transmission becomes larger. The theoretical analysis carried out at beginning of this section can directly be applied to this case as well and does not change the results. What does change, however, is that the trigger level at which investment occurs either satisfies $X^* < b_1$ or $X^* > b_1$. In some of the numerical analysis we use the version of the model in which the grid investment only changes b_0 to b_1.

In order to numerically derive the value function, we set the demand growth rate (μ) equal to zero, i.e. we let demand fluctuate around the installed capacity level (the rest of the parameters in the benchmark parametrization is kept unchanged). The value function as a function of demand levels is shown in figure 1. The two value functions correspond to the cases without and with capacity investments. Both value functions are strictly concave with a maximum within the boundaries $[a_i, b_i]$. The concave shape of the optimal investment value function is surprising since the option to expand corresponds to a call option with strictly convex and increasing shape. Its concave curvature seems to be an immediate consequence of the boundaries and the penalties associated with unbalanced network load. What we can additionally infer from the figure is the value increase associated with the investment. More capacity increases the firm value for any level of demand.

Figure 1: Value function



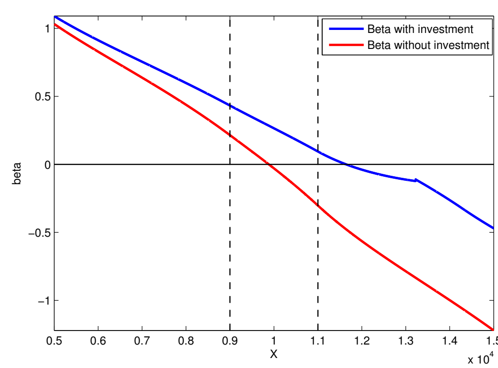
This figure shows the value functions for the original and the expanded capacity level and its boundaries. The black value function belongs to the capacity level K_0 and the dashed lines are its lower and upper boundaries. The red curve is the value function for expanded capacity level K_1. The lower boundary for this capacity level is identical with the lower boundary for the original level.

The dominance of the optimal investment value function relative to the original one is driven by the boundaries. While the lower boundaries for both capacity levels K_0 and K_1 coincide, the upper boundary of the expanded capacity level lies above the upper boundary of the original capacity level. Therefore, when demand is smaller than the lower boundaries, the operator has to pay identical penalties in both cases. With larger bounds in case of capacity investment the firm benefits from a larger range of demand levels for which no penalty needs to be paid resulting in a higher firm value. Note, however, that the lower the demand, the closer the two value functions are. This is, because the probability to reach higher demand levels becomes lower, i.e. the additional value from the no penalty operation becomes lower.

4.3 Dynamic beta

Figure 2 sheds light on the evolution of the firm's systematic risk for both cases, the one with investment and the one without it. As pointed out in Proposition 3, three areas have to be distinguished which are in line with the three partial solutions to the system operator's dynamic beta. We see that both beta-values are decreasing with increasing demand, but risk in case of the investment option is systematically larger than without the expansion option. This is an immediate consequence of the call option. Risk associated with the call option is highest when the option is close to being exercised.
Decreasing systematic firm risk with increasing levels of demand is the immediate consequence of the concave value functions. As the level of demand increases, changes in firm values become smaller and hence firm risk decreases.

Figure 2: Dynamic beta for the case without investment



This figure shows the evolution of the dynamic beta for both cases, when the firm has no investment option with installed capacity level K_0 and when it has an expansion option. The demand growth rate is chosen at $\mu = 0$, the rest of the parameters are at benchmark parametrization level.

4.4 Comparative Statics

The model presented in this paper suggests that uncertainty arising from the underlying demand process has an effect on the value function, optimal investment timing and firm risk. Moreover, a change in the transmission price or a change in demand growth rate should also have significant value and risk implications. This section carries out an illustrative sensitivity analysis based on the benchmark parametrization introduced in table 1. Clearly, this analysis can only highlight a limited amount of possible parameter values and their consequences.

Both benchmark levels that determine the transmission boundaries, α and β, are set at 10%, however a range up to 20% is examined. Both penalty levels for violations

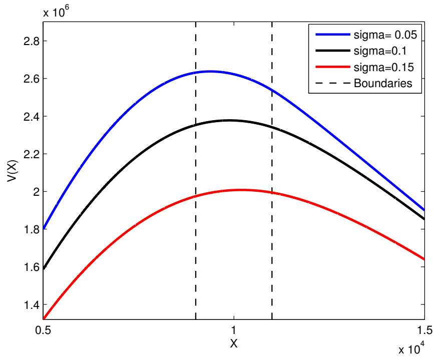


Figure 3: Change of the demand volatility

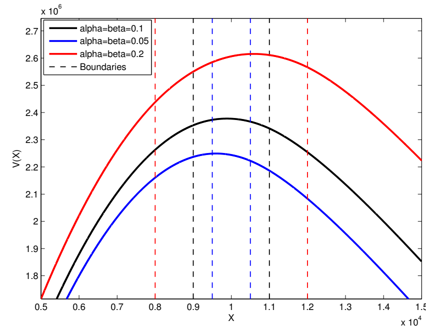


Figure 4: Change of the boundaries

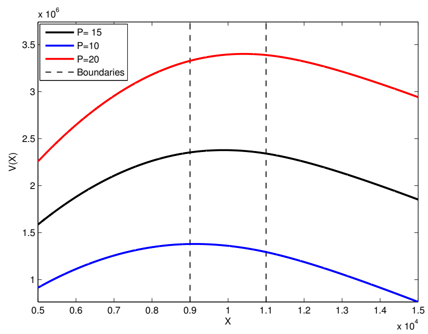


Figure 5: Change of the price

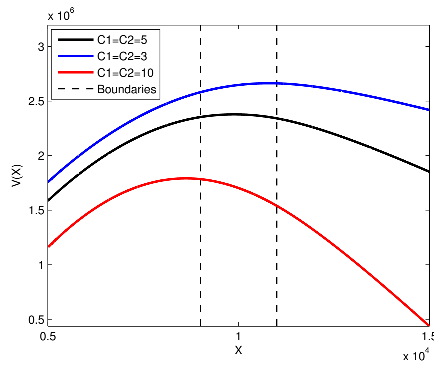


Figure 6: Change of the penalties

of the lower or the upper boundaries, C_1 and C_2 are set at a numerical value equal 5 with a range from 3 to 10. The benchmark transmission price level, P equals 15, but the range between 5 and 15 is examined. The volatility of the demand, σ is set at the level of 10%, while a range of 5-20% is studied; the unit value for investment expansion is at 100, with a range from 50 to 150.

In figures 3-6 the implications of parameter variations for the value function at K_0 capacity level is demonstrated. As expected, the uncertainty parameter and the demand growth rate have the biggest impact on the value function. With a decreasing volatility of demand, the value function shifts up and vice versa. As in the case of different demand levels, going further from the boundaries the value functions converge to each other, since the probability to reach the no penalty area decreases. With the increasing demand growth rate the value function shifts down, but also the peak of the value function changes; it shifts to the left. The reason for this change is again the additional value coming from the no penalty area. The higher the demand growth rate, the more probable it is to reach the no penalty area with increasing demand when we start out at low demand levels. The contrary holds for demand levels above the upper boundary. The sensitivity of the value function with respect

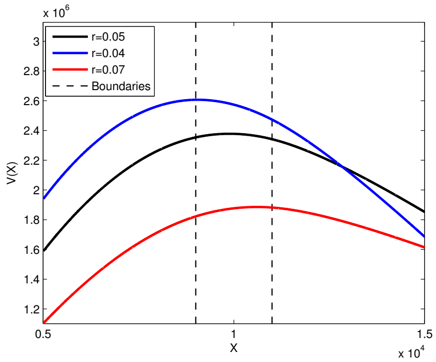


Figure 7: Change of the risk discount rate

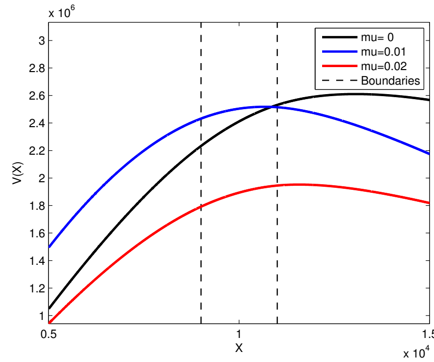


Figure 8: Change of the demand drift

to changes of remaining parameter values also works in the expected direction. An increase in the allowed deviation levels, an increase in price level and a decrease in penalty level causes an upward movement of the value function.

Table 2 gives the consequences of varying parameters of the model on the optimal investment timing. The table shows that the optimal investment timing is increasing with decreasing penalty level for violating the upper boundary and with decreasing transmission price. The penalty for violating the lower boundary has no effect on investment timing. The reason is the identical lower boundaries for the original and the expanded capacity levels. Since the identical boundaries, the firm has to pay the same penalty in both cases, therefore its level pays no role in investment decision. For the same reason, the allowed lower deviation from the capacity level has also no effect on investment timing. The allowed upper deviation influences the investment decision in the opposite way as the transmission price, i.e. the investment is postponed with the increasing level of transmission price.

The optimal investment timing reacts in the same way on penalty level for violating the upper boundary and on the transmission price level. The two effects can extinct each other when they are introduced in the opposite way. This can be seen in the last two rows of table 2, when the penalty for violating the upper level is increased with the same amount as the transmission price is decreased, and vice versa. The optimal timing of the investment is the same as for the basic parametrization case. This result is the key when a regulator plans to preserve the timing of the grid expansion. When a new, cheaper source of balancing power is introduced, the regulator has to increase the transmission price one to one to the change in the balancing power market.

As expected, the demand uncertainty and the investment cost remarkably influence the optimal timing of the investment. A decrease in demand uncertainty makes it profitable to do the investment earlier. Uncertainty of electricity demand can be

Table 2: Comparative Statics

α	β	C_1	C_2	P	σ	Unit IC	Calculated threshold
0.1	**0.1**	**5**	**5**	**15**	**0.1**	**100**	**13223.7955**
0.1	0.1	3	5	15	0.1	100	13370.9664
0.1	0.1	10	5	15	0.1	100	12945.4636
0.1	0.1	5	3	15	0.1	100	13223.7955
0.1	0.1	5	10	15	0.1	100	13223.7955
0.1	0.1	5	5	10	0.1	100	13653.1774
0.1	0.1	5	5	20	0.1	100	12945.4636
0.05	0.1	5	5	10	0.1	100	12683.4611
0.15	0.1	5	5	10	0.1	100	13763.2881
0.1	0.05	5	5	10	0.1	100	13223.7955
0.1	0.15	5	5	10	0.1	100	13223.7955
0.1	0.1	5	5	10	0.1	50	12479.3540
0.1	0.1	5	5	10	0.1	150	13856.3468
0.1	0.1	5	5	10	0.05	100	12342.1606
0.1	0.1	5	5	10	0.15	100	14054.3559
0.1	0.1	5	5	10	0.20	100	14915.0265
0.1	0.1	3	5	17	0.1	100	13223.7955
0.1	0.1	10	5	10	0.1	100	13223.7955

* Benchmark parametrization

The values are calculated for capacity increase from K_0 to K_1 level, for parameters specified in benchmark parametrization.

reduced with investments into storage capacity. Therefore, the regulator, who's objective is to assure the security of the power system intends to give incentives for electricity storage investment or to its future research.

5 Conclusion

In this paper we analyze optimal investment in grid expansion of a SO and characterize its value and risk implications. We use a simple real option framework to derive optimal timing of grid investment and its consequences for firm value. Our model builds on the assumption that the main objectives of the SO is to balance the grid. This is a formidable task when demand is stochastic and exogenous. Given installed capacity determines an interval of demand at which no balancing of the SO is necessary. If demand, however, is either larger or smaller than the boundaries of this interval, the SO needs to jump in and balance the energy. This results in extra costs for the SO and hence has both value and risk implications. We study those for the case without grid expansion and with grid expansion. We find that the existence of an expansion option increases both, the value of the firm and its systematic risk.

6 Appendix

Proof of Proposition 1. Due to the piecewise linear profit function, we can distinguish 3 different cases. First, suppose the demand for capacity exceeds the installed capacity level. Substituting the upper case in (2) into (4) yields to the following second-order differential equation

$$\frac{1}{2}\sigma^2 X^2 V_U^{''}(X) + \mu X V_U^{'}(X) - rV_U(X) - C_1 X + b(p + C_1) = 0. \tag{12}$$

Solution to this equation can be stated as a sum of the solutions to the homogenous equation and the particular solution. The solution of the homogenous part can be written as

$$V_U^h(X) = A_U^0 X^{\lambda_1} + B_U^0 X^{\lambda_2},$$

where λ_1 and λ_2 are the negative and the positive root of the fundamental quadratic equation[4], respectively. One can simply show that $V_U(X) = -\frac{C_1 X}{r-\mu} + \frac{b(p+C_1)}{r}$ satisfies equation (12) as particular solution. Therefore, the general solution becomes

$$V_U(X) = V_U^h(X) + V_U^p(x) = A_U^0 X^{\lambda_1} + B_U^0 X^{\lambda_2} - \frac{C_1 X}{r - \mu} + \frac{b(p + C_1)}{r}. \tag{13}$$

A similar procedure is used to determine the medium case $V_M(X)$ and the case of hitting the lower boundary $V_L(X)$.
In order to determine the parameter values, we follow Dixit (1993). The no bubbles condition requires that the term $B_U X^{\lambda_2}$ must be set equal to zero, or the solution explodes. A similar line of reasoning implies that when demand is below the lower limit, the term $A_L X^{\lambda_1}$ has to equal zero. To determine the remaining constants, we impose the value matching and smooth pasting conditions[5]

$$V_L[(1-\beta)K_0] = V[(1-\beta)K_0]$$

$$V[(1+\alpha)K_0] = V_U[(1+\alpha)K_0]$$

$$V_L^{'}[(1-\beta)K_0] = V^{'}[(1-\beta)K_0]$$

[4]The roots are defined as

$$\lambda_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0$$

$$\lambda_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1$$

[5]see Dixit (1993)

$$V'[(1+\alpha)K_0] = V_U'[(1+\alpha)K_0].$$

Substituting the above determined value functions we obtain the following system of equations

$$A_M^0 a^{\lambda_1} + (B_M^0 - B_L^0)a^{\lambda_2} + \frac{aC_2}{r} - \frac{aC_2}{r-\mu} = 0 \tag{14}$$

$$(A_M^0 - A_U^0)b^{\lambda_1} + B_M^0 b^{\lambda_2} + \frac{b(p+C_1)}{r-\mu} - \frac{b(p+C_1)}{r} = 0 \tag{15}$$

$$A_M^{=}\lambda_1 a^{\lambda_1 - 1} + (B_M^0 - B_L^0)\lambda_2 a^{\lambda_2 - 1} - \frac{C_2}{r-\mu} = 0 \tag{16}$$

$$(A_M^0 - A_U^0)\lambda_1 b^{\lambda_1 - 1} + B_M^0 \lambda_2 b^{\lambda_2 - 1} + \frac{p+C_1}{r-\mu} = 0. \tag{17}$$

The solution set to this system (14) - (17) can be described by the following equations, which finishes the proof

$$A_M^0 = \frac{a^{1-\lambda_1} C_2(\mu\lambda_2 - r)}{(\lambda_2 - \lambda_1)(r-\mu)r}$$

$$B_M^0 = \frac{b^{1-\lambda_2}(p+C_1)(\mu\lambda_1 - r)}{(\lambda_2 - \lambda_1)(r-\mu)r}$$

$$A_U^0 = \frac{(ab)^{1-\lambda_1}[a^{\lambda_1 - 1}(p+C_1) + b^{\lambda_1 - 1}C_2](\mu\lambda_2 - r)}{(\lambda_2 - \lambda_1)(r-\mu)r}$$

$$B_L^0 = \frac{(ab)^{1-\lambda_2}[a^{\lambda_2 - 1}(p+C_1) + b^{\lambda_2 - 1}C_2](\mu\lambda_1 - r)}{(\lambda_2 - \lambda_1)(r-\mu)r}$$

□

Proof of Proposition 3. Under the assumption that $X^* > b_0$ the trigger level and the option value of the firm can be determined in the following way. The continuation value prior to the investment is given by the value function V_U from equation (6). Now, one of the following three cases might occur.

Case 1: $X^* < a_1$

The trigger value of demand and the option value to invest can be determined by setting the continuation value equal to the stopping value, i.e. by applying the value matching and smooth pasting conditions.

$$V_U^{K_0}(X^*) + K(X^*)^{\lambda_2} = V_L^{K_1}(X^*) - IC,$$

$$(V_U^{K_0})'(X^*) + K\lambda_2(X^*)^{\lambda_2 - 1} = V_L^{K_1'}(X^*).$$

Applying these conditions to the case $X^* < a_1$ results in

$$A_U^0 X^{\lambda_1} - \left(\frac{C_1}{r-\mu}\right)X + \frac{b^0(p+C_1)}{r} + KX^{\lambda_2} = B_L^1 X^{\lambda_2} + \left(\frac{p+C_2}{r-\mu}\right)X - \frac{a^1 C_2}{r} - IC \tag{18}$$

$$A_U^0 \lambda_1 X^{\lambda_1 - 1} - \frac{C_1}{r-\mu} + K\lambda_2 X^{\lambda_2 - 1} = B_L^1 \lambda_2 X^{\lambda_2 - 1} + \frac{p + C_2}{r-\mu}. \tag{19}$$

These two conditions can only be solved numerically but they result in the trigger level and the option value constant K.

Case 2:$a_1 < X^* < b_1$

In case if the trigger demand lies in between the new boundaries, the value of the trigger demand and the option value to invest have to satisfy the following conditions

$$A_U^0 X^{\lambda_1} - \left(\frac{C_1}{r-\mu}\right) X + \frac{b^0(p+C_1)}{r} + KX^{\lambda_2} = A^1 X^{\lambda_1} + B^1 X^{\lambda_2} + \left(\frac{p}{r-\mu}\right) X - IC$$

$$A_U^0 \lambda_1 X^{\lambda_1 - 1} - \frac{C_1}{r-\mu} + K\lambda_2 X^{\lambda_2 - 1} = A^1 \lambda_1 X^{\lambda_1 - 1} + B^1 \lambda_2 X^{\lambda_2 - 1} + \frac{p}{r-\mu}.$$

Case 3:$X^* > b_1$

Finally, the case when the trigger value is above the new upper boundary it has to hold

$$A_U^0 X^{\lambda_1} + \frac{b^0(p+C_1)}{r} + KX^{\lambda_2} = A_U^1 X^{\lambda_1} + \frac{b^1(p+C_1)}{r} - IC \tag{20}$$

$$A_U^0 \lambda_1 X^{\lambda_1 - 1} + K\lambda_2 X^{\lambda_2 - 1} = A_U^1 \lambda_1 X^{\lambda_1 - 1}. \tag{21}$$

□

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