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DOI:
10.57938/c87ebc08-10b3-4321-955b-420153467ae0

Published: 01/12/2015

Document Version
Publisher's PDF, also known as Version of record

Measuring the impact of unconventional monetary policy on the US business cycle

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Abstract

The paper estimates a dynamic macroeconometric model for the US economy that captures two important features commonly observed in the study of the US business cycle, namely the strong co-movement of key macroeconomic quantities, and the distinction between expansionary and recessionary phases. The model extends the factor-augmented vector autoregressive model of Bernanke et al. (2005) by combining Markov switching with factor augmentation, modeling the Markov switching probabilities endogenously, and adopting a full Bayesian estimation approach which uses shrinkage priors for several parts of the parameter space. Exploiting a large data set for the US economy ranging from 1971:Q1 to 2014:Q2, the model is applied to measure not only the dynamic effects of unconventional monetary policy within distinct stages of the business cycle, but also the dynamic response of the recession probabilities, based on conducting counterfactual simulations. The results obtained provide new insights on the effect of monetary policy under changing business cycle phases, and highlight the importance of discriminating between expansionary and recessionary phases of the business cycle when analyzing the impact of monetary policy on the macroeconomy.

Keywords: Non-linear FAVAR, business cycle, monetary policy, structural model, US economy

JEL Codes: C30, E52, F41, E32.

December 1, 2015

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1 Introduction

In the most recent episode of financial turmoil, central banks reacted to the economic downturn by lowering short-term interest rates decidedly. When the nominal policy rate approaches the zero lower-bound, conventional monetary policy tools become ineffective, since several key transmission channels fail to operate properly. In such a low interest rate environment, central banks resorted to unconventional monetary policy instruments to stimulate aggregate demand and generate inflationary pressure. Apart from raising inflation expectations and providing stimulus to the economy, variants of unconventional monetary policy have also been designed to improve financial market conditions and provide liquidity to stumbling financial institutions (Kapetanios et al., 2012)[1]. Such non-standard policy tools, commonly termed quantitative easing, have been widely used by most major central banks across the globe during the last few years. The unconventional nature of such measures stems from the fact of lying outside the traditional conduct of monetary policy.

The literature investigating the macroeconomic effects of such monetary policy instruments is still somewhat sparse. One notable exception is the study by Lenza et al. (2010) that attempts to quantify the macroeconomic consequences of unconventional monetary policy measures adopted by the European Central Bank (ECB). Based on Bayesian vector autoregressions (VARs), this study provides evidence that the ECB's implementation of non-standard monetary policy in response to the crisis influenced output and inflation positively, but with a considerable time lag. In another study, Baumeister and Benati (2013) estimate a set of time-varying parameter VARs and analyze the effect of an unconventional monetary policy shock, measured through a decrease in the 10-year government bond spread, in the UK and the US[2]. The authors additionally assume that the central bank is constrained by the zero lower-bound, and is thus not able to lower short-term interest rates any further. They find large positive effects of unconventional monetary policy on output growth and prices at the zero lower-bound.

These studies measure the dynamic effects of unconventional monetary policy on selected macroeconomic quantities, but overlook how unconventional monetary policy affects economic conditions within different stages of the business cycle. They, moreover, utilize — as numerous other studies that rely on VARs to identify the dynamic effects of conventional monetary policy shocks — only a small set of variables to represent the economy. But small-scale models usually suffer from severe misspecification, leading to anomalies often observed in empirical research[3]. If the objective of interest,

[1] For a review of the various measures used in the recent global financial crisis, see also Aït-Sahalia et al. (2009), Lenza et al. (2010), and Baumeister and Benati (2013).

[2] The authors investigate the cases of the UK and the US since unconventional monetary policy in these countries was designed to lower the long-term government bond spread, effectively removing duration risk from investors' portfolios (Baumeister and Benati, 2013).

[3] Lack of information has been identified as the source of, for example, the so-called price puzzle, i.e. the fact that prices increase in response to a monetary tightening policy (see Christiano et al., 1999; Giannone and Reichlin, 2006; Bańbura et al., 2010).

however, is the relationship between central banks' actions and the macroeconomy, it is of prime importance to use a far larger information set, since central banks monitor – and effectively respond to – hundreds of variables that aim to provide information on the current and future state of the economy.

While the availability of macroeconomic data proves to be of great help when it comes to modeling issues, an increased information set typically yields more complex models that are heavily parameterized. As a solution to this "curse of dimensionality" problem, recent research in factor models (for a survey see Stock and Watson, 2011) suggests to summarize the information from a large panel of time series by a set of latent variables, commonly referred to as factors. This implies that a large number of time series may be effectively driven by relatively few factors, explaining a large fraction of the sample variance (see Boivin and Ng, 2005; Bernanke et al., 2005; Stock and Watson, 2005; Giannone et al., 2008; Korobilis, 2014). If most important information is embodied in a small number of factors, a natural way to solving the curse of dimensionality problem is to combine traditional structural analysis with dynamic factor models for large data sets.

This paper uses a Markov switching factor-augmented vector autoregressive (FAVAR) model with endogenously time-varying transition probabilities (henceforth MS-FAVAR-TVP model) to analyze the dynamic effects of unconventional monetary policy shocks under changing business cycle phases, and exploiting hereby a large data set for the US economy covering the period from 1971:Q1 to 2014:Q2. The model allows for discretely, but jointly changing parameters and volatilities. The transition probabilities of the Markov chain employed follow a probit specification in the spirit of Amisano and Fagan (2013). We assume the underlying hidden Markov chain that governs the state dynamics to be driven by a large set of predictors.

The modeling approach proposed allows for the possible nonlinear transmission of monetary policy and provides information on the potential driving forces of business cycle transitions. The recent monetary history during the financial crisis of 2008/2009 reveals that nonlinearities are a key property of the data and should thus be modelled explicitly[4]. Only then the analysis can reveal whether there is an important source of nonlinearity in the transmission mechanism of interest or only in parameters that leave the transmission mechanism of interest unaltered.

The new aspects of the present approach – relative to classic papers such as Sims and Zha (2006), and Bernanke et al. (2005) – are first the combination of Markov switching with factor augmentation, secondly the introduction of dependence between the transition probabilities and the explanatory variables, and thirdly a full Bayesian estimation approach. While exploiting the information contained in the large information set, the MS-FAVAR-TVP model is prone to overfitting, leading to the curse-of-dimensionality

[4]Several studies emphasized the usefulness of accounting for nonlinearities in the analysis of business cycles and monetary policy. Cogley and Sargent (2002), and Primiceri (2005) adopt nonlinear models to analyze the transmission mechanisms of monetary policy, while Korobilis (2013) extends the work of Bernanke et al. (2005) to a time-varying parameter framework. This latter approach combines the virtues of a large-dimensional model with drifting parameters and stochastic volatility.

problem. To shrink the system towards a stylized prior representation of the model, we impose the well-known Minnesota prior (Litterman, 1986; Sims and Zha, 1998).

The remainder of the paper is structured as follows. The section that follows presents the MS-FAVAR-TVP model, lays out the Bayesian approach to estimation and inference, with particular emphasis on the specification of the priors for the state and observation equations, and the probit model, the description of the Markov chain Monte Carlo algorithm and the identification of the model. In Section 3 the model is applied to draw a picture of the dynamic relationship between business cycle phases and unconventional monetary policy. Analyzing the effects of unconventional monetary policy instruments on the macroeconomy proves to be a challenging task. It calls for a dynamic analysis of how output, unemployment, consumer price inflation and other macroeconomic quantities react to unconventional monetary policy measures. In order to construct our no policy counterfactual, we assume that the macroeconomic effects of unconventional monetary policy come through the impact on the difference between 10-year government bond yields and the short-term interest rate. The key identifying assumption is that the central bank is unable to move the short-term interest rate within the first two years after the shock hits the economy.

The final section summarizes the key results and concludes. An appendix provides additional information about the posterior distributions and how to simulate them, the variables used in the study, and the out-of-sample performance of the MS-FAVAR-TVP model in comparison with FAVAR and MS-VAR model variants and their time-varying counterparts.

2 A formal framework

It is natural to begin with a brief discussion of the factor-augmented VAR model of Bernanke et al. (2005). Then, for the purpose of detecting and quantifying nonlinear effects of monetary policy shocks under changing business cycles in the US, the FAVAR model is extended to allow for Markov switching in the state equation. Finally, a full Bayesian estimation approach is outlined. Hereby the specification of the priors for the state and observation equations, and for the probit model are described in some detail along with the Markov chain Monte Carlo algorithm used.

2.1 The factor-augmented vector autoregressive model

The FAVAR model of the business cycle consists of two equations: a transition equation and a measurement equation. The transition equation describes the dynamics of observable economic variables and unobserved factors, while the factors and economic variables are related by an observation or measurement equation.

Let $\boldsymbol{x}_t$ be an $N \times 1$ vector of economic variables including measures of the monetary policy stance. These variables are assumed to be observable at time $t = 1, \ldots, T$ and to drive the dynamics of the economy. Standard practice is to use data for $\boldsymbol{x}_t$ and to estimate a VAR, a structural VAR or another multivariate time series model.

In monetary application contexts, however, it is useful to take additional economic information into account, that is not fully captured by $\boldsymbol{x}_t$. Assume that this additional information can be summarized by a $K \times 1$ vector of unobserved factors, say $\boldsymbol{f}_t$, where K is small. These factors may represent concepts such as output, prices or interest rates that economists typically use in theoretical models but which are unobserved in reality.

We assume that the joint dynamics of $(\boldsymbol{f}_t', \boldsymbol{x}_t')'$ are given by a transition equation of the form[5]

$$\begin{pmatrix} \boldsymbol{f}_t \\ \boldsymbol{x}_t \end{pmatrix} = \Phi(L) \begin{pmatrix} \boldsymbol{f}_{t-1} \\ \boldsymbol{x}_{t-1} \end{pmatrix} + \boldsymbol{u}_t \quad \text{for } t = 1, \ldots, T \tag{2.1}$$

where $\Phi(L)$ is a polynomial in the lag operator L of finite order Q, and $\boldsymbol{u}_t$ represents error terms with mean zero and variance-covariance matrix $\boldsymbol{\Sigma}_u$. Note that Eq. (2.1) is a VAR in $(\boldsymbol{f}_t', \boldsymbol{x}_t')'$ that reduces to a standard VAR in $\boldsymbol{x}_t$ if the terms of $\Phi(L)$ that relate $\boldsymbol{x}_t$ to $\boldsymbol{f}_{t-1}$ are all zero.

The unobserved factors are extracted by a large panel of M indicators, $\boldsymbol{y}_t$, providing information on several important sectors of the economy. Note that M may be much greater than the number of factors and observed variables in the VAR system so that $M \gg K + N$. We assume that the factors and variables are related by the following observation equation

$$\boldsymbol{y}_t = \boldsymbol{\Lambda}^f \boldsymbol{f}_t + \boldsymbol{\Lambda}^x \boldsymbol{x}_t + \boldsymbol{e}_t \tag{2.2}$$

with $\boldsymbol{\Lambda}^f$ and $\boldsymbol{\Lambda}^x$ representing $M \times K$ and $M \times N$ matrices of factor loadings, while $\boldsymbol{e}_t$ is an $M \times 1$ vector of normally distributed zero mean disturbances with a diagonal $K \times K$ variance-covariance matrix $\boldsymbol{\Sigma}_e$. Equation (2.2) captures the idea that both $\boldsymbol{f}_t$ and $\boldsymbol{x}_t$, which in general may be correlated, represent common forces that drive the dynamics of $\boldsymbol{y}_t$. Hence, conditional on $\boldsymbol{x}_t$, $\boldsymbol{y}_t$ contains times series that are sampled with measurement errors surrounding the underlying latent factors in $\boldsymbol{f}_t$. Note that Eq. (2.2) implies that $\boldsymbol{y}_t$ depends only on contemporaneous values of the factors. But this is not restrictive in practice since $\boldsymbol{f}_t$ may be interpreted as including arbitrary lags of the fundamental factors (Bernanke et al., 2005).

2.2 A factor-augmented vector autoregressive model with Markov switching and time-varying transition probabilities

The system given by Eqs. (2.1)-(2.2) is the FAVAR model proposed by Bernanke et al. (2005). Our main innovation relative to this model is the incorporation of regime switching with endogenous time-varying transition probabilities so that the extended model encompasses the two main characteristics of the US business cycle identified by Burns and Mitchell (1946), namely a large degree of co-movement among a broad set of economic variables, and the distinction between expansionary and recessionary phases

[5] Equation (2.1) can be interpreted as a lower dimensional empirical model where the dynamics are driven by a low-dimensional vector of (reduced-form) shocks. Boivin and Giannoni (2006) have shown that Eq. (2.1) can be related to a log-linearized dynamic stochastic general equilibrium model.

in its evolution. As opposed to time-varying parameter models that imply smoothly changing coefficients over time, our MS-FAVAR-TVP model assumes rapid shifts of the parameters[6] (and thus the underlying transmission mechanisms). Nonlinearities are typically introduced in the modeling framework by either assuming that the economy moves through a large number of possible regimes (Cogley and Sargent, 2002; Primiceri, 2005) or by postulating the dynamics of the economy being characterized by just a few distinct economic regimes (Sims and Zha, 2006; Koop et al., 2009).

Let us assume that the R-dimensional vector $\boldsymbol{z}_t = (\boldsymbol{f}'_t, \boldsymbol{x}'_t)'$ with $R = K + N$ follows a Qth-order Markov switching VAR,

$$\boldsymbol{z}_t = \sum_{q=1}^{Q} \boldsymbol{A}_{qS_t} \boldsymbol{z}_{t-q} + \boldsymbol{\varepsilon}_t. \tag{2.3}$$

S_t denotes a Markov regime switching discrete process, the coefficient matrices $\boldsymbol{A}_{qS_t}$ ($q = 1, \ldots, Q$) are regime-specific and of dimension $R \times R$, $\boldsymbol{\varepsilon}_t$ is a normally distributed zero mean error term with regime-specific variance-covariance matrix $\boldsymbol{\Sigma}_{\varepsilon S_t}$. The subscript S_t in $\boldsymbol{A}_{qS_t}$ and $\boldsymbol{\Sigma}_{\varepsilon S_t}$ indicates that all parameters are allowed to change across regimes. We assume that S_t is an unobserved binary Markov switching variable indicating whether the economy is in an expansionary ($S_t = 0$) or recessionary ($S_t = 1$) phase with transition probabilities given by

$$\boldsymbol{P}_t = \begin{pmatrix} p_{11,t} & p_{12,t} \\ p_{21,t} & p_{22,t} \end{pmatrix} \tag{2.4}$$

where $p_{ij,t} = Prob(S_t = j | S_{t-1} = i)$ with $\sum_{j=1}^{2} p_{ij,t} = 1$ for all i and t. This implies that the transition probabilities are allowed to vary over time. Note that the higher $p_{jj,t}$ is, the longer the process is expected to remain in state j.

A convenient parametrization for this mechanism is the probit specification (Amisano and Fagan, 2013)[7],

$$Prob(S_t = j | S_{t-1} = i, \boldsymbol{w}_{t-1}) = p_{ij,t} = \phi(\boldsymbol{\gamma}' \boldsymbol{w}_{t-1}) \tag{2.5}$$

with

$$\phi(\omega) = \int_{-\infty}^{\omega} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\omega^2\right\} \, d\omega. \tag{2.6}$$

Hereby $\boldsymbol{w}_{t-1}$ is a G-dimensional vector including (both endogenous and exogenous) variables that may be viewed as early warning predictors for business cycle changes.

In this way the parameter γ_g, i.e. the gth element of $\boldsymbol{\gamma}$, measures the sensitivity of probability $p_{ij,t}$ with respect to w_{gt-1}, i.e. the gth element of $\boldsymbol{w}_{t-1}$. Note that Eq. (2.5)

[6] This is in contrast to Primiceri (2005), Canova and Gambetti (2009), Koop et al. (2009), and Korobilis (2013) who use time-varying parameter models that imply smoothly evolving autoregressive coefficients and changing error variances.

[7] An alternative would be to use a logit specification that provides advantages if the number of regimes is greater than two (see, for example, Kaufmann, 2015).

resembles a standard probit model with an underlying latent variable regression given by

$$r_t = \boldsymbol{\gamma}' \boldsymbol{w}_{t-1} + \epsilon_t \tag{2.7}$$

where $r_t \in \mathbb{R}$ is a continuous latent variable, $\boldsymbol{\gamma}$ is a G-dimensional parameter vector that determines the transition probabilities, and ϵ_t denotes the error with variance normalized to unity for identification purposes.

2.3 A Bayesian approach to estimation and inference

Densely parameterized models, such as our MS-FAVAR-TVP model, are known to yield a good in-sample fit, but poor out-of sample forecasts due to parameter uncertainty. To address this issue, we use a full Bayesian approach to inference in the model described in the previous subsection, and we impose prior information to obtain reliable parameter estimates of $\boldsymbol{A}_{qS_t}(q = 1, \ldots, Q)$. Note that traditional estimation methods rely on numerical optimization, which is daunting in the presence of irregular likelihood surfaces often encountered in the estimation of Markov switching models.

To simplify prior implementation let us rewrite the Eq. (2.3) as

$$\boldsymbol{z}_t = \boldsymbol{A}'_{S_t} \boldsymbol{d}_t + \boldsymbol{\varepsilon}_t \tag{2.8}$$

where $\boldsymbol{A}_{S_t} = (\boldsymbol{A}_{1S_t}, \ldots, \boldsymbol{A}_{QS_t})'$ is a $RQ \times R$ matrix of stacked coefficients, and $\boldsymbol{d}_t = (\boldsymbol{z}'_{t-1}, \ldots, \boldsymbol{z}'_{t-Q})'$ denotes a RQ-dimensional data vector. Conditional on S_t and $\boldsymbol{f}_t$ the model can be represented as a standard regression model, which implies that standard priors can be used (Zellner, 1973). Stacking the rows of $\boldsymbol{z}_t$ and $\boldsymbol{d}_t$ yields the corresponding $T_{S_t} \times R$ and $T_{S_t} \times RQ$ regime-specific full data matrices, denoted by $\boldsymbol{Z}_{S_t}$ and $\boldsymbol{D}_{S_t}$, where T_{S_t} is the number of observations related to the regime prevailing at time t.

Prior distributions for the state equation

We impose a set of conditionally conjugate priors given by

$$vec(\boldsymbol{A}_{S_t})|\boldsymbol{\Sigma}_{S_t} \sim \mathcal{N}(vec(\underline{\boldsymbol{A}}), \boldsymbol{\Sigma}_{\varepsilon S_t} \otimes \underline{\boldsymbol{V}}_A) \tag{2.9}$$

where $\underline{\boldsymbol{A}}$ denotes the $R \times RQ$ prior mean matrix, while $\underline{\boldsymbol{V}}_A$ is a $RQ \times RQ$ prior variance-covariance matrix. The prior variance on the coefficients is governed by the Kronecker product $\boldsymbol{\Sigma}_{\varepsilon S_t} \otimes \underline{\boldsymbol{V}}_A$, which is a matrix of dimension $R^2Q \times R^2Q$.

The prior on the variance-covariance matrix is of inverted Wishart form given by

$$\boldsymbol{\Sigma}_{s_t} \sim \mathcal{IW}(\underline{\boldsymbol{C}}, \underline{v}) \tag{2.10}$$

with $\underline{\boldsymbol{C}}$ being a $R \times R$ prior scale matrix and $\underline{v}$ are the prior degrees of freedom. We specify the matrices

$$\underline{\boldsymbol{A}} \text{ such that } E\{[\boldsymbol{A}_{qS_t}]_{ij}\} = \begin{cases} \underline{a}_i & \text{for } q = 1 \text{ and } i = j \\ 0 & \text{for } q > 1 \text{ and } i \neq j \end{cases} \tag{2.11}$$

$$\underline{\boldsymbol{V}}_A \text{ such that } var\{[\boldsymbol{A}_{qS_t}]_{ij}\} = \frac{\theta^2}{q^2}\frac{\sigma_i}{\sigma_j} \tag{2.12}$$

for $q = 1, \ldots, Q$; $i = 1, \ldots, R$; $j = 1, \ldots, RQ$. The notation $[\boldsymbol{A}_{qS_t}]_{ij}$ selects the (i, j)th element of the matrix concerned. The prior mean associated with the first own lag of variable i is given by $\underline{a}_i$, whereas for higher lag orders and other lagged variables the prior mean is set equal to zero. The hyperparameter θ controls the tightness of the prior. σ_i and σ_j are standard deviations obtained by running a set of univariate autoregressions on $\boldsymbol{z}_t$[8]. They serve to account for the different variability of the data. This prior is a conjugate variant of the Minnesota prior put forward by Doan et al. (1984), and Litterman (1986). The rationale behind the Minnesota prior is that a priori a random walk proves to provide a good representation of the data. Thus it might be sensible to center the system on a (multivariate) random walk process that implies setting $\underline{a}_{ij} = 1$ for $i = j$. Between regimes we assume prior homogeneity, implying that the same set of priors is used for both regimes. This is not essential and could be relaxed quite easily. Note that assuming different regime-specific prior models could be useful in terms of shrinking the parameters towards selecting appropriate submodel specifications.

A convenient feature of the natural conjugate prior is the fact that it can be interpreted as data arising from an artificial dataset. Bańbura et al. (2010) show how the moments of the Minnesota prior can be matched through so-called "dummy"-observations. This is achieved by concatenating the following matrices to $\boldsymbol{Z}$ and $\boldsymbol{D}$,

$$\underline{\boldsymbol{Z}} = \begin{pmatrix} diag(\underline{a}_1\sigma_1, \ldots, \underline{a}_R\sigma_R)/\theta \\ \cdots\cdots\cdots\cdots\cdots \\ \mathbf{0}_{R(Q-1)\times R} \\ \cdots\cdots\cdots\cdots\cdots \\ diag(\sigma_1, \ldots, \sigma_R) \end{pmatrix} \tag{2.13}$$

$$\underline{\boldsymbol{D}} = \begin{pmatrix} \boldsymbol{J}_Q \otimes diag(\sigma_1, \ldots, \sigma_R)/\theta & \mathbf{0}_{RQ\times 1} \\ \cdots\cdots\cdots\cdots\cdots & \cdots \\ \mathbf{0}_{R\times RQ} & \mathbf{0}_{R\times 1} \end{pmatrix} \tag{2.14}$$

with $\boldsymbol{J}_Q = (1, \ldots, Q)'$. Loosely speaking, the first two blocks of the matrices in Eqs. (2.13) and (2.14) implement the prior on the coefficients associated with the lags of $\boldsymbol{z}_t$ and the final block the prior on $\boldsymbol{\Sigma}_{\varepsilon S_t}$.

[8]We obtain the standard deviations by running the autoregression using the principal components estimator for the latent factors.

Prior distributions for the probit model

We also have to specify priors on the latent regression model given by Eq. (2.7). Following George and McCulloch (1993) we impose a stochastic search variable selection prior on the elements of $\boldsymbol{\gamma}$. Specifically, the prior on the parameter associated with the gth factor in Eq. (2.7) is given by

$$\gamma_g|\delta_g \sim \mathcal{N}(0, \tau_0^2)\delta_g + \mathcal{N}(0, \tau_1^2)(1-\delta_g) \text{ for } g = 1, \dots, G \tag{2.15}$$

where δ_g is a binary random variable controlling which normal prior to use for the gth coefficient. The prior variances τ_0^2 and τ_1^2 are set such that $\tau_0^2 \gg \tau_1^2$. Thus, if δ_g equals one, the prior on the gth coefficient is effectively rendered non-influental. This captures the notion that no significant prior information for that parameter is available, centering the corresponding posterior distribution around the maximum likelihood estimate. If δ_g equals zero, we impose a dogmatic prior, shrinking γ_g towards zero. This case would lead to a posterior which is strongly centered around zero, implying that we can safely regard that coefficient to be equal to zero. Let us introduce a scalar parameter h_g set such that

$$h_g = \begin{cases} \tau_0^2 \text{ if } \delta_g = 1 \\ \tau_1^2 \text{ if } \delta_g = 0. \end{cases} \tag{2.16}$$

Storing the h_gs in a $G \times G$ matrix $\boldsymbol{H} = diag(h_1, \dots, h_G)$ permits to state the prior in terms of a multivariate normal distribution,

$$\boldsymbol{\gamma}|\boldsymbol{H} \sim \mathcal{N}(\boldsymbol{\mu}_\gamma, \boldsymbol{HH}) \tag{2.17}$$

with $\boldsymbol{\mu}_\gamma$ denoting the G-dimensional prior mean vector, assumed to equal zero.

We impose a Bernoulli prior on the elements of $\boldsymbol{\delta} = (\delta_1, \dots, \delta_G)$,

$$\delta_g \sim Bernoulli\ (\underline{p}_g) \tag{2.18}$$

where $Prob(\delta_g = 1) = \underline{p}_g$ denotes the prior inclusion probability. In this specific application context the SSVS prior allows us to investigate the relative importance of different factors on the evolution of the business cycle.

Prior distributions for the observation equation

To complete the prior setup we also have to specify a suitable set of prior distributions on the factor loadings in Eq. (2.2). To simplify prior implementation let us collect $\boldsymbol{\Lambda}^f$ and $\boldsymbol{\Lambda}^x$ in a $M \times (K+N)$ matrix $\boldsymbol{\Lambda} = (\boldsymbol{\Lambda}^f, \boldsymbol{\Lambda}^x)$. Similar to the prior choice discussed above we impose a mixture Gaussian prior on the jth element of $\boldsymbol{\lambda} = vec(\boldsymbol{\Lambda})$,

$$\boldsymbol{\lambda}_j|\iota_j \sim \mathcal{N}(0, \varrho_0^2)\iota_j + \mathcal{N}(0, \varrho_1^2)(1-\iota_j) \text{ for } j = 1, \dots, M(K+N). \tag{2.19}$$

Here, ι_j is a binary random variable, and ϱ_0, ϱ_1 are hyperparameters controlling the tightness of the prior. Conditional on $\boldsymbol{\iota} = (\iota_1, \dots, \iota_{M(K+N)})$ we can again state this prior as a multivariate Gaussian prior on $\boldsymbol{\lambda}$,

$$\boldsymbol{\lambda}|\boldsymbol{L} \sim \mathcal{N}(\underline{\boldsymbol{\mu}}_\Lambda, \boldsymbol{LL}) \tag{2.20}$$

where $\boldsymbol{L} = diag(l_1, \ldots, l_{M(K+N)})$ and l_j is defined as

$$l_j = \begin{cases} \varrho_0^2 \text{ if } \iota_j = 1 \\ \varrho_1^2 \text{ if } \iota_j = 0. \end{cases} \tag{2.21}$$

Similar to the prior on $\boldsymbol{\delta}$, we impose a set of Bernoulli priors on the elements of $\boldsymbol{\iota} = (\iota_1, \ldots, \iota_{M(K+N)})$,

$$\iota_j \sim Bernoulli(\underline{\rho}_j) \tag{2.22}$$

with $Prob(\iota_j = 1) = \underline{\rho}_j$ being the prior inclusion probability of a given variable in the observation equation.

Finally, the last ingredient still missing is the prior on the innovation variances of the state equation, where we use inverted Gamma priors on the M diagonal elements of $\boldsymbol{\Sigma_e}$, denoted by ς_j $(j = 1, \ldots, M)$,

$$\varsigma_j \sim \mathcal{IG}(\underline{\alpha}_j, \underline{\beta}_j) \tag{2.23}$$

with $\underline{\alpha}_j$ denoting a prior shape parameter and $\underline{\beta}_j$ a prior scale parameter.

The Markov chain Monte Carlo algorithm

Up to now we have remained silent on how to obtain estimates for $\boldsymbol{f}_t$. The literature suggests two routes. The first route to produce consistent estimates of the latent factors (see, for example, Bernanke et al., 2005) involves using a two-step estimation approach in which the factors are estimated by principal components, prior to estimation of the FAVAR. That is, one estimates the space spanned by the first K principal components of $\boldsymbol{y}_t$. This yields consistent (in the large T, M case) estimates of the true space spanned by $\boldsymbol{f}_t$ and $\boldsymbol{x}_t$. Conditional on the principal components one can proceed as in the standard Markov switching VAR case. This approach has the advantage to be computationally fast and easy to implement. One disadvantage, however, is that estimation based on principal components treats the factors $\boldsymbol{f}_t$ to be known, thus neglecting the noise surrounding $\hat{\boldsymbol{f}}_t$, the estimate of $\boldsymbol{f}_t$.

The second route, which we are going to follow, accounts for this fact by utilizing simulation based methods, treating the latent factors as unobserved parameters, estimating these parameters along the other model parameters in one step and using Markov chain Monte Carlo techniques (see, for example, Kim and Nelson 1999). This can be implemented by means of the well-known state-space algorithms, like those put forth in Carter and Kohn (1994), and Frühwirth-Schnatter (1994). However, while still straightforward to implement, this increases the computational burden considerably.

Conditional on the factors and the latent states in $\boldsymbol{s} = (S_1, \ldots, S_T)'$, the parameters of the transition equation (2.3) can be simulated using simple Gibbs steps, iteratively sampling from the (conditional) posterior distributions of the parameters in Eq. (2.3). In practice, under the conjugate prior this step is quite fast, implying that even if we increase the number of factors, computation does not become prohibitively slow. Sampling the latent states S_t is simplified by the fact that we face a numerical integration problem with discrete support. Several options are possible, however we employ the filter put forward by Kim and Nelson (1999), and Amisano and Fagan (2013). The implementation of these steps is described in detail in Appendix A.

2.4 Identification of the factor-augmented vector autoregressive model

The model described above is econometrically unidentified and cannot be estimated. There are three different sets of restrictions that have to be imposed on the model. The first involves a minimum set of normalization restrictions on the observation equation needed to identify the latent factors and the corresponding loadings. The second relates to the label switching problem that controls the prevailing phase of the business cycle. Finally, the identification of the structural shocks in the transition equation requires further restrictions.

Identification problems associated with the latent factors

The factors and their loadings of the MS-FAVAR model in Eqs.(2.2)-(2.3) are not separately identified. We identify the sign and the scale of the factors and the loadings by imposing a standard identification scheme commonly used in the literature on FAVAR models (see Bernanke et al., 2005). That is, we set the upper $K \times K$ block of $\mathbf{\Lambda}^f$ to an identity matrix and the upper $K \times N$ block of $\mathbf{\Lambda}^x$ to zero.

Note that this choice implies that our findings could be sensitive with respect to the ordering of the variables in $\boldsymbol{y}_t$. However, the robustness of our findings can be assessed quite easily by resorting to the two-step estimation approach mentioned in the previous subsection. This approach is order invariant and thus can be used to investigate the importance of different orderings of the variables contained in $\boldsymbol{y}_t$. In our application, the results stay similar when we use the two-step approach as opposed to the full Bayesian approach.

Label switching problem

Since the likelihood function of the model is invariant with respect to permutation of the labels of the states we have an identification problem. This problem, known as the label switching problem (Amisano and Fagan, 2013), poses no real problem for the estimation of the model, but for the economic interpretation of the estimation results. In the present application context we analyze two regimes, a recessionary and an expansionary regime. To achieve identification we impose restrictions on the main diagonal elements of $\mathbf{\Sigma}_{\varepsilon S_t}$. More specifically we assume that the $S_t = 1$ marks a "recessionary" regime if

$$[\mathbf{\Sigma}_{\varepsilon S_t=1}]_{11} > [\mathbf{\Sigma}_{\varepsilon S_t=0}]_{11}. \tag{2.24}$$

Equation (2.24) implies that the errror variance of the first element of $\boldsymbol{z}_t$ is larger in the recessionary regime.

Structural identification

Finally, Eq. (2.3) presents the reduced form of the model. The (regime-specific) structural form of the model is given by

$$\tilde{\boldsymbol{A}}_{0S_t}\boldsymbol{z}_t = \sum_{q=1}^{Q} \tilde{\boldsymbol{A}}_{qS_t}\boldsymbol{z}_{t-q} + \tilde{\boldsymbol{\varepsilon}}_t. \tag{2.25}$$

$\tilde{\boldsymbol{A}}_{0S_t}$ denotes a $R \times R$ matrix of impact coefficients, $\tilde{\boldsymbol{A}}_{qS_t}$ $(q = 1, \ldots, Q)$ are $R \times R$ matrices of lagged structural coefficients and $\tilde{\boldsymbol{\varepsilon}}_t$ are standard normally distributed structural errors. Multiplying with $\tilde{\boldsymbol{A}}_{0S_t}^{-1}$ from the left yields the reduced form of the model in Eq. (2.3). Note that the reduced form errors are given by $\tilde{\boldsymbol{A}}_{0S_t}^{-1}\tilde{\boldsymbol{\varepsilon}}_t$. Following Uhlig (2005), and Baumeister and Benati (2013), we impose a mixture of sign and zero impact restrictions to recover the structural shocks of our model. More specifically, note that we can decompose the regime-specific variance-covariance matrix $\boldsymbol{\Sigma}_{\varepsilon S_t}$ as

$$\boldsymbol{\Sigma}_{\varepsilon S_t} = \underbrace{\tilde{\boldsymbol{A}}_{0S_t}^{-1}\tilde{\boldsymbol{R}}_{S_t}}_{\overline{\boldsymbol{A}}_{0S_t}}\underbrace{\tilde{\boldsymbol{R}}_{S_t}'(\tilde{\boldsymbol{A}}_{0S_t}^{-1})'}_{\overline{\boldsymbol{A}}_{0S_t}'} = \tilde{\boldsymbol{A}}_{0S_t}^{-1}(\tilde{\boldsymbol{A}}_{0S_t}^{-1})' \tag{2.26}$$

for any $R \times R$-dimensional orthonormal rotation matrix $\tilde{\boldsymbol{R}}_{S_t}$ with $\tilde{\boldsymbol{R}}_{S_t}\tilde{\boldsymbol{R}}_{S_t}' = \boldsymbol{I}_R$. After specifying a set of sign restrictions, we search for rotation matrices until impulse responses are found that satisfy all restrictions imposed. Following the approach put forward by Rubio-Ramirez et al. (2010), we search for ten rotation matrices that meet the sign restrictions. To eliminate the resulting model uncertainty we pick the rotation matrix that yields the impulse response function closest to the median impulse response function.

Traditionally, sign restrictions are based on "conventional wisdom" stemming from macroeconomic theory (see Baumeister and Benati, 2013). In our empirical application we also impose zero impact restrictions that assume some variables to react sluggishly with respect to an unconventional monetary policy shock. Given that our focus is on the pure spread shock, theoretically possible restrictions should be imposed in order to pin down the shock of interest. The exact sign restrictions used and the construction of the zero impact restrictions are discussed in more detail in subsection 3.3.

3 The dynamic responses of the US economy to unconventional monetary policy

So far we described the MS-FAVAR-TVP model in fairly general terms[9]. In this section we apply the model to investigate the dynamic relationship between unconventional

[9]To empirically motivate our model, Appendix B provides evidence on the out-of-sample performance in comparison to (nested) competing model variants such as the FAVAR and MS-VAR model variants and their time-varying counterparts, using a simple small-scale recursive forecasting exercise.

monetary policy and the business cycle. Subsection 3.1 briefly describes the data set and the model specification used for this specific application to address three issues. First, we investigate how well this model specification detects major US recessions in subsection 3.2. Secondly, we analyze what are the dynamic effects of unconventional monetary policy within distinct stages of the business cycle on major macroeconomic aggregates (see subsection 3.3). Finally, subsection 3.4 presents some evidence on how such policy dynamically influences the recession probabilities, through the probit specification adopted for the transition probabilities.

3.1 Data and model implementation

The full data set used contains 48 quarterly time series for the United States ranging from 1971:Q1 to 2014:Q2. The series selected represent seven main categories of macroeconomic time series (number of series in parentheses), covering all important segments of the economy: real output and income (16); employment and unemployment (4); consumption expenditures (5); interest rates, spreads, and credit quantities (6); price indices (2); orders, inventories and sales (3); housing (10); and miscellaneous (2). All data series are seasonally adjusted, if applicable, and transformed to be approximately stationary according to the transformation codes outlined in Stock and Watson (2011). The list of the series is given in Appendix C. $M = 46$ variables are included in the $\boldsymbol{y}_t$ vector, while we use the term spread, measured in terms of the 10-year government bond spread, and the Fed funds rate as observable variables in $\boldsymbol{x}_t$, implying $N = 2$. Since a wide variety of commonly identified indicators are already included in $\boldsymbol{y}_t$ and $\boldsymbol{x}_t$ we set $\boldsymbol{w}_t = (\boldsymbol{y}_t', \boldsymbol{x}_t')'$. This choice is crucial because it allows us to link the responses of the observed variables in $\boldsymbol{y}_t$ and $\boldsymbol{x}_t$ (with respect to an unconventional monetary policy shock) to the transition probabilities, effectively obtaining responses of recession probabilities.

Before proceeding to the empirical results a brief word on the specification of the MS-FAVAR-TVP model is necessary. Consistent with the vast majority of papers in the literature, the lag order is set equal to two ($Q = 2$). In addition, this choice is motivated by the fact that our model is heavily parameterized, and that higher lag orders would lead to parameter draws that lie outside the stationary region, especially for the recessionary regime. Conditional on the number of lagged endogenous variables and based on using not only the deviance information criterion (Spiegelhalter et al., 2002), but also Bayesian and classical information criteria, we set the number of factors equal to four.

With regard to the prior specification, we set the tightness hyperparameter equal to $\theta = 0.1$. This setting relies on varying θ on a discrete grid of different values for $\theta \in \{0.001, 0.01, 0.1, 0.5, 2, 10^2\}$, and using both the deviance and the Bayesian information criterion to discriminate between models. It is worth noting that higher levels of θ lead to explosive responses, with eigenvalues of the corresponding companion matrices significantly exceeding unity. In addition, the prior mean is set equal to 0.2, placing considerable prior mass on stationary regions of the parameter space. This choice — motivated by the need to obtain draws from the posterior distribution — implies stable and plausible impulse response functions.

Since we standardize the variables in $\boldsymbol{w}_t$, the hyperparameters of the mixture normal priors are set equal to $\tau_0^2 = 1$ and $\tau_1^2 = 0.1$. The prior on the free elements of $\boldsymbol{\Lambda}$ is equal to $\varrho_0^2 = 10$ and $\varrho_1^2 = 0.1$. Experimenting with different choices of ϱ_0 and ϱ_1 has led to qualitatively the same results. Finally, we take $\underline{\alpha}_j = \underline{\beta}_j = 0.01$ to render this prior effectively non-influential.

Model estimation is based on the MCMC algorithm described in subsection 2.3. More specifically, we simulate a chain consisting of 70,000 draws where we discard the first 35,000 draws as burn-ins. Traditional convergence criteria suggest that the Markov chain reached its stationary distribution. Moreover, inefficiency factors tend to be between 20 and 40 for most coefficients of the model.[10]

3.2 Replicating US business cycle behavior

Before discussing the responses of some major macroeconomic aggregates to an unconventional monetary policy shock, this subsection presents the regime allocation produced by our model. Figure 1 reports the posterior mean of the filtered recession probabilities (i.e. moving into $S_t = 1$), and this figure clearly indicates that the model manages to capture most recent recessions, beginning with the recession in 1973-1975. This recession was caused by sharp increases in government spending and energy prices, most notably the price of oil, leading to a stagflationary period within the US. The two recessions in the early 1980s were a consequence of the federal reserves' pronounced regime shift, when chairman Paul Volcker started to fight inflation by increasing the policy rate dramatically. In 1990, the US experienced a relatively short period of negative growth caused by high oil prices, high debt levels and a low level of consumer confidence in the US. The period between the recession in the early 1990s and the recession following the burst of the dot-com bubble and the September 11th attacks was the longest period of sustained growth in recent US history. Note that the early 2000s recession is the only downturn our model was unable to identify. This may be due to the fact that this recession was by far the mildest one, merely resulting in an aggregate GDP loss of 0.3% from peak to trough. Finally, the last recession in our sample is the recent financial crisis, which led to sustained losses in output. Again, the model captures this period rather well, allocating high recession probabilities for the corresponding quarters.

[Fig. 1 about here.]

In conclusion, Fig. 1 clearly indicates that the model identifies all major recessions in our sample except for the recession in the early 2000s. In the subsection that follows, we direct our attention to the question whether unconventional monetary policy shocks influence the economy in an asymmetric fashion, i.e. whether responses of the selected

[10] The corresponding convergence diagnostics are available upon request.

macroeconomic quantities used to summarize the current state of the economy react differently in recessions and expansions.

3.3 Unconventional monetary policy in different business cycle regimes

Since the beginning of 2009, when the policy rate in the US reached the effective zero lower-bound, short-term nominal interest rates became ineffective as a monetary policy tool. Given the increasing relevance of such a low interest rate environment we simulate the unconventional monetary policy shock under the assumption that the central bank is unable to influence the policy rate at the zero lower-bound.

For the purpose of analyzing how unconventional monetary policy operates within the different regimes of the business cycle identified in the previous subsection, we simulate a 100 basis points (bps) unconventional monetary policy shock *conditional on the regime* and trace its effect on a subset of variables included in $\boldsymbol{y}_t$. We define an unconventional monetary policy shock as an unexpected increase of the difference between the yield on 10-year government bonds and the policy rate. In principle, several other ways of simulating unconventional monetary policy shocks would be available. However, we follow Baumeister and Benati (2013), and assume that the Fed is influencing economic activity through the long-end of the treasury yield curve. More specifically, we assume that the Fed reduces the term spread associated with 10-year government bonds to reduce market-wide duration risk and lower the general level of borrowing costs to the private sector.

We identify a "pure" spread shock by assuming that the impulse response vector associated with a contractionary unconventional monetary policy shock leads to a decrease of real GDP growth, consumer price inflation, housing starts and hours worked in manufacturing while the unemployment rate and the term spread increase on impact. In addition, we also assume that the short-term interest rate reacts sluggishly with respect to an unconventional monetary policy shock. This is achieved by constructing a $R \times R$ deterministic rotation matrix $\overline{\boldsymbol{R}}_{S_t}$,

$$\overline{\boldsymbol{R}}_{S_t} = \begin{bmatrix} \boldsymbol{I}_K & \boldsymbol{0}_{K\times N} \\ \boldsymbol{0}_{N\times K} & \boldsymbol{U}_{S_t} \end{bmatrix} \tag{3.1}$$

with

$$\boldsymbol{U}_{S_t} = \begin{pmatrix} \cos(\varphi_{S_t}) & -\sin(\varphi_{S_t}) \\ \sin(\varphi_{S_t}) & \cos(\varphi_{S_t}) \end{pmatrix}. \tag{3.2}$$

The matrix $\overline{\boldsymbol{R}}_{S_t}$ satisfies $\boldsymbol{I}_R = \overline{\boldsymbol{R}}_{S_t}\overline{\boldsymbol{R}}'_{S_t}$. We define the rotation angle φ_{S_t} as

$$\varphi_{S_t} = \tan^{-1}([\overline{\boldsymbol{A}}_{0S_t}]_{ij}/[\overline{\boldsymbol{A}}_{0S_t}]_{ii}) \tag{3.3}$$

where $[\overline{\boldsymbol{A}}_{0S_t}]_{ij}$ is the impact response of variable i (the short-term interest rate) with respect to a shock to variable j (the term spread). Post-multiplying $\overline{\boldsymbol{A}}_{0S_t}$ with $\overline{\boldsymbol{R}}_{S_t}$ yields a new impact matrix where the contemporaneous response of the short-term interest rate with respect to an unconventional monetary policy shock equals zero. In addition, we manipulate the structural coefficients related to the monetary policy rule such that the interest rate is not reacting within eight quarters after an unconventional monetary policy shock hit the economy. This is achieved by "zeroing-out" the coefficients. While manipulating structural coefficients is generally not immune to the Lucas critique (Lucas, 1976), manipulating the historical structural shocks, however, leads to results that are qualitatively similar to the ones presented in this paper.

[Fig. 2 about here.]

The US impulse response functions for a 100 bps unconventional monetary policy shock are shown in Fig. 2. All impulse response function plots include the 16th and 84th percentiles of the posterior distribution (henceforth referred to as credible sets) in addition to the median response for the next fourteen quarters after impact. Inclusion of the credible sets allows us to see when the period-by-period response becomes insignificantly different from zero. We document the amplitude of the response of twelve macroeconomic variables: real GDP in panel (a), personal income in panel (b), personal consumption expenditures in panel (c), the unemployment rate in panel (d), average weekly hours of production and nonsupervisory employees in panel (e), the ISM (Institute for Supply Management) employment index in panel (f), consumer price inflation in panel (g), 10-year treasury constant maturities minus the federal funds rate in panel (h), Fed's funds rate in panel (i), Moody's seasoned Baa corporate bond yield in panel (j), commercial and industrial loans in panel (k), and housing starts in panel (l). The responses are computed on the prevailing regime, thus providing evidence on the responses within the two distinct business cycle phases.

The amplitudes of the impulse response functions are displayed for expansionary and recessionary periods of the business cycle on the left and the right hand side of Fig. 2, respectively. The impact estimates reported show not only the simultaneous effects, but also how persistent these effects are at time horizons one to fourteen quarters that include the future period impacts arising from time dependence. Units on the vertical axis are in percentage points. The interpretation is, for example, that a 100 bps shock to the 10-year government bond spread reduces GDP growth (on impact) by 0.61 percentage points in expansionary phases of the business cycle, while by 0.92 percentage points in recessions (see panel (a)). Both responses are significant within the first quarter after the impact, and then insignificant afterwards.

The second panel, panel (b), shows rather strong immediate reactions of personal income within both cycle phases that can be traced back to adverse effects of contractionary monetary policy shocks on the labor market. As panels (e) and (f) indicate, the unemployment rate tends to increase and hours worked to fall as a reaction to an unconventional monetary policy shock, exhibiting significant downward pressures on wages.

Panel (c) presents the responses of personal consumption expenditures. The time zero effects are very similar in both stages of the business cycle, with a drop of 0.23

percentage points in expansions and 0.25 percentage points in recessions. The effects fade out after around one quarter in a recession and this is more or less in agreement with Beaudry and Koop (1993), while in expansions the effects survive longer to about seven quarters. Computing the cumulative sum of the responses within expansions reveals that the level of consumption declines by around 1.6 percent, which is roughly in line with the results obtained by Walentin (2014), who reports a 0.3 percent decline of consumption with respect to a 33 bps mortgage spread shock, and this translates into a 1.2 percent decline when the shock is normalized to 100 basis points.

The next panel, panel (d), depicts the dynamic responses of the unemployment rate. The magnitudes of the impact responses are 0.76 percentage points in the expansionary state and 0.89 percentage points in the recessionary state. In expansions the effect is statistically significant over the whole time horizon considered, but rapidly fades out after one year in recessions. It is noteworthy that the sign of the responses is consistent with standard New Keynesian dynamic stochastic general equilibrium models (see, for example, Christiano et al., 1999) which predict falling employment and rising unemployment as a direct consequence of the contraction of real activity.

In line with the increase in the unemployment rate, responses of average weekly hours of production and nonsupervisory employees in panel (e) suggest that hours worked decrease, and this in part reflects the drop in real activity observed in the panels above. Within expansions, this effect is significantly different from zero within the first five quarters, petering out afterwards. By contrast, responses in recessions die out after around two quarters.

The results for the ISM employment index (NAPMEI) are given in panel (f). Here it is noteworthy that consistent with the rise of the unemployment rate, the employment index falls in response to a monetary policy shock. The index, however, reacts faster and the responses are not significantly different from zero after around two quarters.

Panel (g) reports the dynamic responses of consumer price inflation. The reader may observe that inflation tends to decrease on impact by around 0.5 percentage points within both stages of the business cycle. After one quarter, the effect on inflation becomes insignificant, suggesting that the impact of unconventional monetary policy on prices is rather short-lived within both regimes. Note that our results provide some evidence that the large information set used alleviates the price puzzle, i.e. the common finding in the VAR-based literature that prices increase in response to a restrictive monetary policy shock. Similar to the responses of real GDP growth, and consumption, the impact on consumer price inflation in recessionary periods exhibits a larger degree of uncertainty. With respect to the impact magnitudes, this finding is grossly in concordance with the results provided in Baumeister and Benati (2013) that show a 0.5 percentage point decrease of inflation for the US from 1965 to 2007, while in the years ranging from 2008 to 2011 the impact responses are somewhat more pronounced as compared to the present study.

The response of the term spread is presented in panel (h). By construction, the term spread increases by 100 bps on impact, falling slightly over time within the expansionary regime. After fourteen quarters, the effect on the term spread is still existent, suggesting a longer lasting effect of central banks' operations to achieve a yield compression at the long-end of the yield curve. By contrast, within a recession the effects tend to level out after one year and a half, suggesting a much faster adjustment of financial markets back to previous levels of the spread. Comparing our findings with the results presented in Chen et al. (2012) — these authors report the effect of unconventional monetary policy on the term spread petering out after around fourteen quarters — reveals that a linear VAR model produces results lying between our findings for recessions and expansions.

The response of the Fed funds rate in panel (i) indicates that the central banks' operation essentially does not influence the nominal short-term interest rate within the first two years. After eight quarters, short-term interest rates are allowed to move freely according to the dynamics implied by the model. This implies that interest rates are increasing only marginally after exiting the zero lower-bound within a recession, but the effects are not significantly different from zero.

Panel (j) displays the time profile of the posterior median response of the BAAFFM spread, which serves as a measure of the risk spread (i.e. the yield difference between corporate credit and the US federal funds rate). We observe that the BAAFFM spread increases in a persistent fashion in expansions, rising by around 150 bps. In recessions, the response of the BAAFFM spread is again much more short-lived. After rising sharply to 165 bps within the first quarter, the effect on the risk spread fades out within five quarters. The relatively long lasting effect of unconventional monetary policy in expansionary periods might be due to the fact that financial market participants have little incentive to rebalance their portfolios and change positions. Within a recession, in contrast, institutional constraints might force investors to liquidate risky assets and thus to react such that the negative effect of contractionary unconventional monetary policy is damped.

The strong responses of the credit spread lend some evidence that the portfolio rebalancing and the market liquidity channel (Joyce et al., 2011) operate properly within both regimes. The increase in the credit spread makes it more difficult for companies to raise funds, effectively worsening credit conditions. The responses of credit growth, measured in terms of the growth rate of commercial and industrial loans to an unconventional monetary policy shock, depicted in panel (k), confirm this conjecture. The maximum slow down in credit growth is reached at around two quarters. While credit growth contracts statistically significant within four quarters in a recession, the impact tends to be much more persistent within an expansionary period. This provides further evidence that financial market participants tend to react in a persistent fashion within expansions, while reactions in recessions are rather short-lived.

The final panel, panel (l), shows the dynamic responses of housing starts in both regimes. We observe significant reactions of the housing market for the first quarter, leading to sharp declines in housing starts as a response to an unconventional monetary policy shock. Housing starts drop by around 0.12 percentage points at time zero in expansions and by 0.18 percentage points in recessions. This decline may be caused by worse financing conditions through the financial institutions. It is worth noting that the impulse responses in both regimes tend to be rather short-lived and exhibit a similar time profile. However, the impact magnitudes suggest that within business cycle recessions the effect on housing starts is much stronger.

3.4 Does unconventional monetary policy increase the likelihood of a recession?

The previous two subsections presented the estimated regime allocation produced by the MS-FAVAR-TVP model (see Fig. 1) and the responses of selected variables to a 100 bps unconventional monetary policy shock, within business cycle recessions and expansions (see Fig. 2). In this subsection, we link the responses of the observed variables in $\boldsymbol{y}_t$ and $\boldsymbol{x}_t$ to the transition probabilities, effectively obtaining dynamic responses of recession probabilities within both stages of the business cycle.

Recall that $\boldsymbol{w}_t = (\boldsymbol{y}_t', \boldsymbol{x}_t')'$. This enables us to compute the dynamic responses of recession probabilities within an integrated model framework. Figure 3 displays the posterior median of impulse responses of recession probabilities within distinct stages of the business cycle across time, along with the 16th and 84th credible sets at time horizons one quarter (see panel (a)), four quarters (see panel (b)) and twelve quarters (see panel (c)). Note that the nonlinear nature of the probit model implies that the specific responses depend on the current level of $\boldsymbol{w}_t$. Hence, we can compute the impulse response function at any point in time. In addition, the (regime-specific) responses of $\boldsymbol{w}_t$ allow us to compute a hypothetical scenario that provides information on how recession probabilities change as a consequence of different responses of $\boldsymbol{w}_t$ (i.e. responses in recessions and expansions) and how these depend on the current level of $\boldsymbol{w}_t$.

[Fig. 3 about here.]

Inspection of Fig. 3 reveals two features that are worth noting. First, within both stages of the business cycle, contractionary monetary policy increases the likelihood of recession probabilities at time horizon one quarter. All the point estimates lie within the plotted 16th and 84th credible sets (see panel (a)). During the nineties and mid 2000s, for instance, a 100 bps spread shock increases the likelihood of moving into a downturn by 6.2 percentage points in expansions, with 8.2 percentage points in the case of a recessionary regime.

Second, the results differ for the two business cycle phases. Panel (b) indicates that the impact of monetary policy on recession probabilities is rather short-lived in recessions, and dies out within the first year. This result appears to be a direct consequence of the result described in the previous subsection, where we found the responses of most macroeconomic aggregates being rather short-lived in recessionary phases. Under expansionary conditions, in contrast, the effect on recession probabilities still persists after one year even though at a slightly lower level, and this effect is statistically significant.

4 Closing remarks

This paper has developed a dynamic macroeconometric model for the US economy that is able to discriminate between business cycle phases, thus incorporating salient features commonly observed in the study of business cycles. The model allows for time-varying transition probabilities of the underlying Markov chain which controls the regime allocation. The transition probabilities are assumed to be governed by a probit regression model that includes the same information set as the dynamic factor model. This enabled us to compute the dynamic responses of recession probabilities within an integrated model framework, ultimately linking the consequences of unconventional monetary policy with the likelihood of falling into a downturn.

The model was used to shed some light on the dynamic relationship between unconventional monetary policy conducted by the Fed, by providing evidence on the different transmission mechanisms within expansions and recessions, and by measuring the direct effect of policy actions on the propensity of moving into a downturn. We performed a simple counterfactual simulation for this purpose and simulated a 100 bps unconventional monetary policy shock, measured in terms of an increase of the 10-year government bond spread at the zero lower-bound. The responses were computed conditional on the prevailing regime, thus providing evidence on the responses within recessions and expansions.

Our empirical analysis has yielded several interesting results. Two key findings stand out. *First*, contractionary US unconventional monetary policy exerts powerful effects on the economy within both stages of the business cycle. All variables measuring output growth drop significantly across regimes, while labor market conditions deteriorate sharply and consumer price inflation slightly decreases in the short-run. Financial market responses point towards increases of different spreads, like the credit spread and the term spread. In addition, available credits also tend to fall.

Second, we detected significant differences in the persistence of the macroeconomic effects for the two distinct regimes. Responses in recessions tend to be short-lived, while being far more persistent in expansions, though not always significantly different from zero. Evidence suggests that the recessionary pressure arising from contractionary unconventional monetary policy is higher in recessions, responses reacting stronger and less persistently[11]. Across time, the results reveal heterogeneity between expansionary times and recessions, and this finding highlights the importance of discriminating between distinct economic regimes of the business cycle when analyzing the macroeconomic impact of monetary policy on the macroeconomy.

There are several directions in which research on the MS-FAVAR-TVP model can be usefully extended to account for more general patterns of regime switching behavior. One is to generalize the model to more than two regimes, and another is to introduce

[11] Identification of the sources of such differences is beyond the scope of this paper. But we conjecture that one reason being the more transitory nature of unconventional monetary policies within recessions that can be traced back to financial market participants being more active with respect to portfolio rebalancing.

additional latent variables that would allow different sectors of the economy to be in different states.

Acknowledgment: The authors thank the Vienna University of Economics and Business for research support. This paper is a substantially revised version of a paper circulated under the title "A Markov switching factor-augmented VAR model for analyzing US business cycles and monetary policy", Department of Economics Working Paper Series 201, WU Vienna University of Economics and Business, Vienna.

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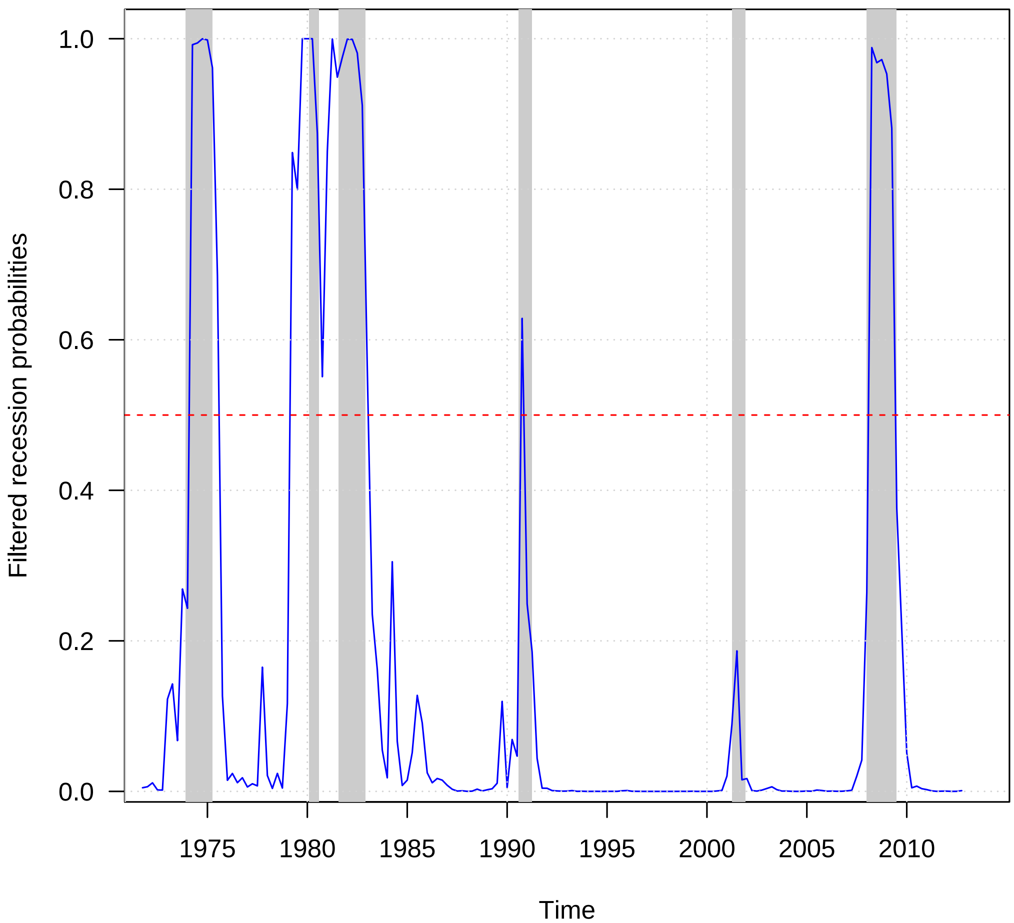
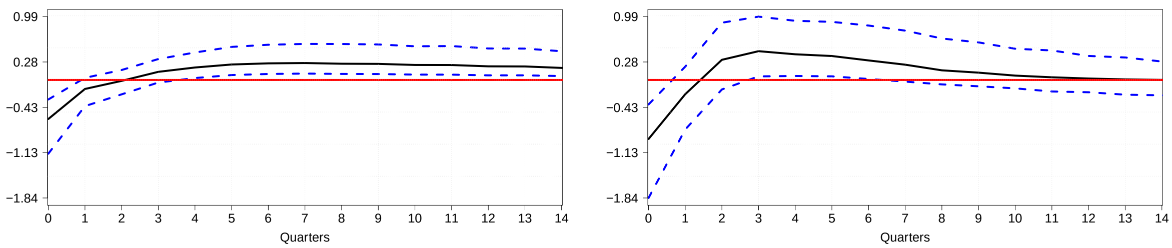


Notes: Posterior mean of the filtered probabilities to move into a downturn. The dashed red line indicates the 0.5 probability level. Grey shaded areas refer to recessions dated by the Business Cycle Dating Committee of the National Bureau of Economic Research (`www.nber.org`). Results are based on 35,000 posterior draws.

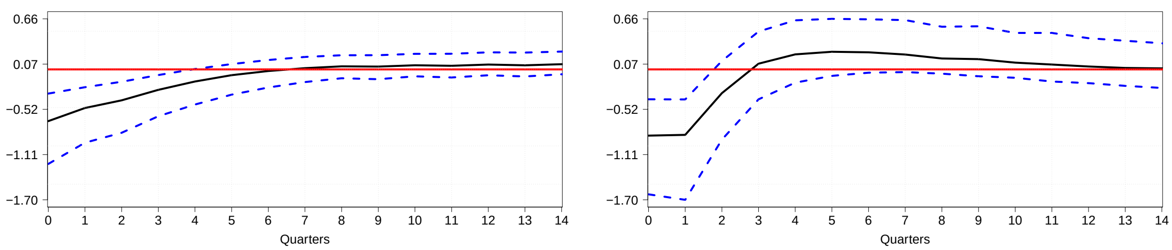
Fig. 1: Posterior mean of filtered recession probabilities

Response in ’expansionary phase’ **Response in ’recessionary phase’**

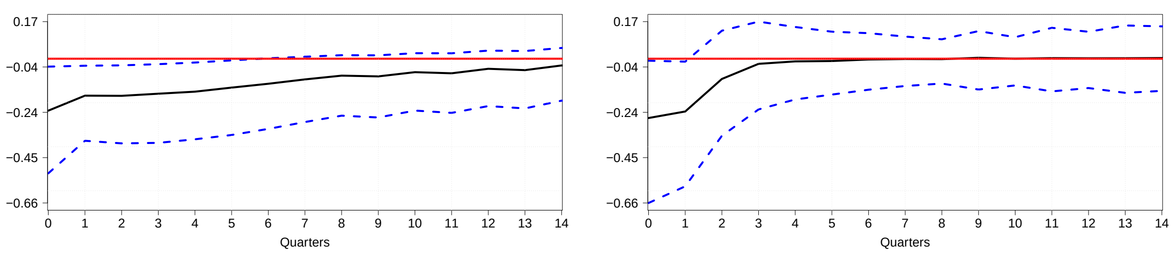
(a) Real gross domestic product (GDP)



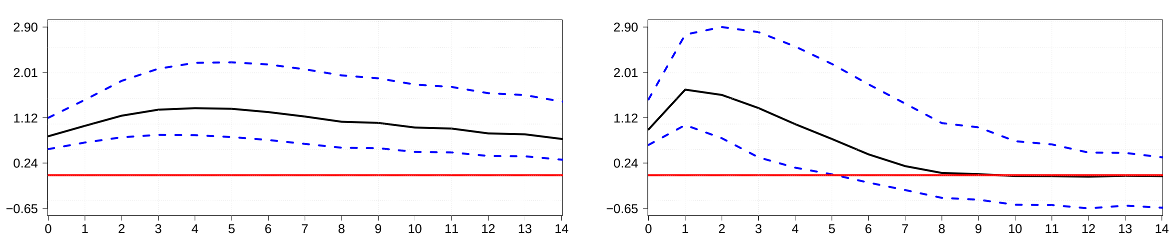
(b) Personal income (PI)



(c) Personal consumption expenditures (PCEPI)



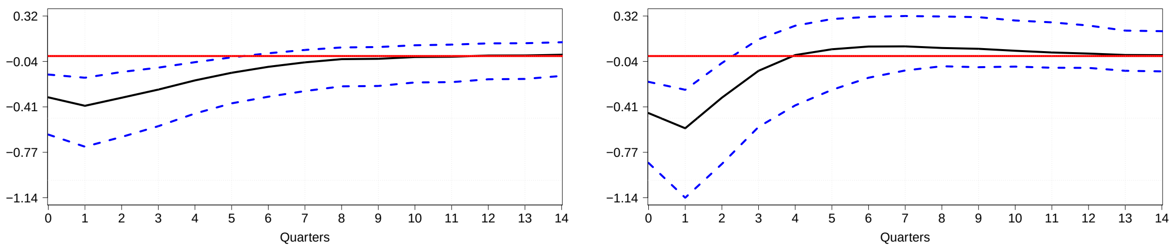
(d) Unemployment rate (UNRATE)



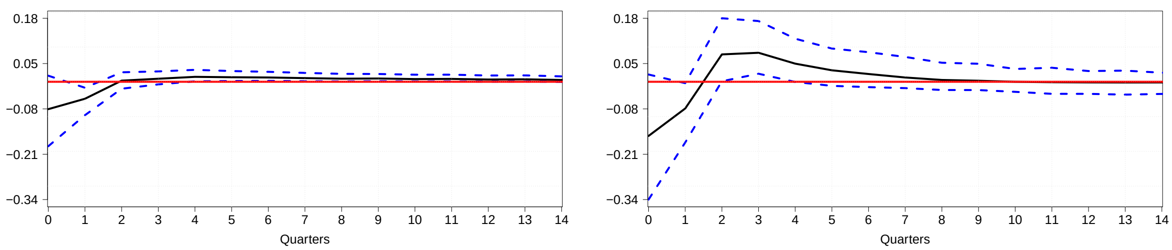
Notes: Posterior distribution of impulse responses in percentage points. Median in black. Dashed blue lines correspond to the 16th and 84th percentiles. Results are based on 35,000 posterior draws. The red line indicates the zero line.

Response in 'expansionary phase' **Response in 'recessionary phase'**

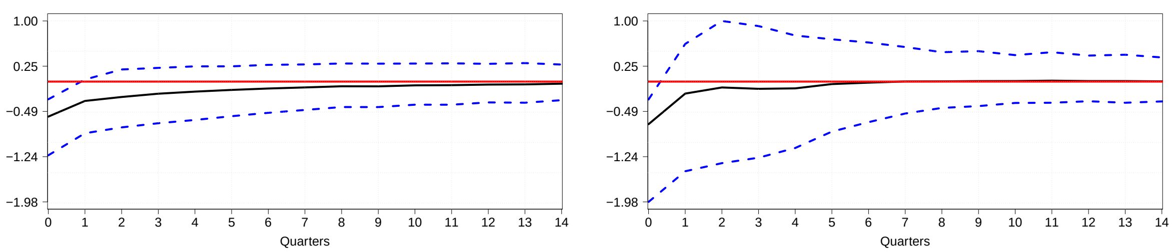
(e) Average weekly hours of production and nonsupervisory employees (AWHMAN)



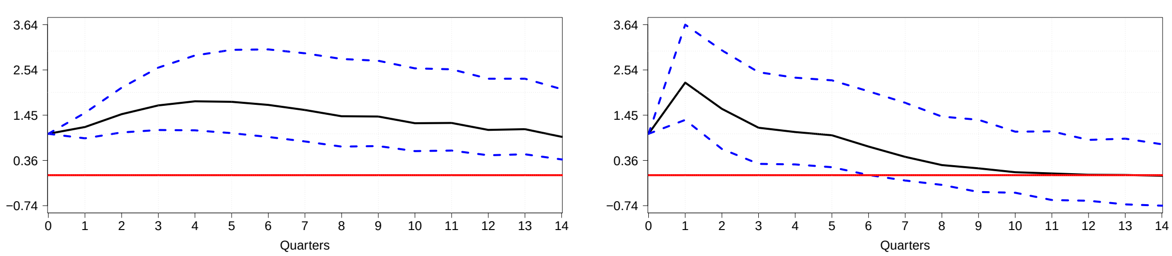
(f) ISM manufacturing: employment index (NAPMEI)



(g) Consumer price inflation (CPIAUCSL)



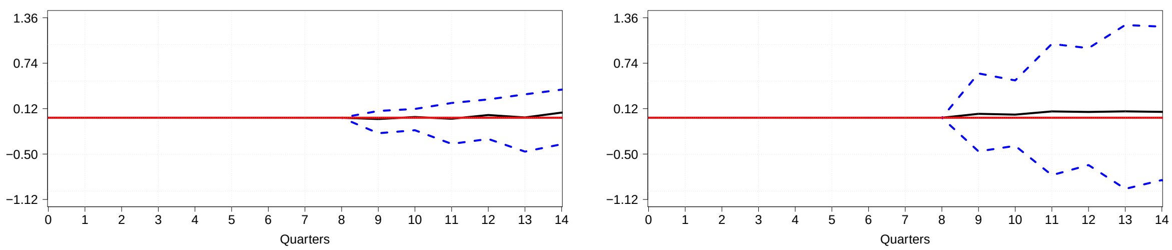
(h) 10-year treasury constant maturities minus the federal funds rate (T10YFFM)



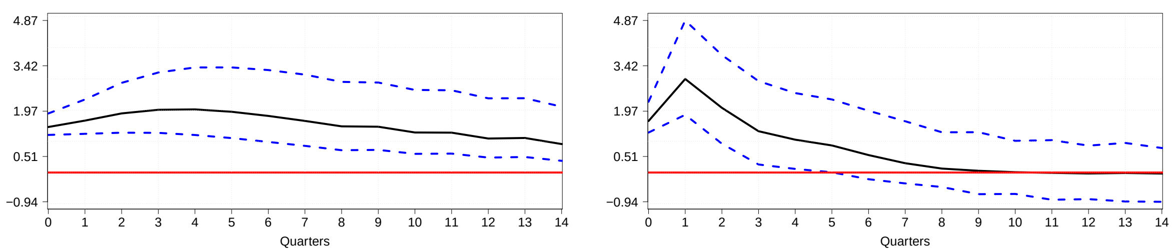
Notes: Posterior distribution of impulse responses in percentage points. Median in black. Dashed blue lines correspond to the 16th and 84th percentiles. Results are based on 35,000 posterior draws. The red line indicates the zero line.

Response in ’expansionary phase’ **Response in ’recessionary phase’**

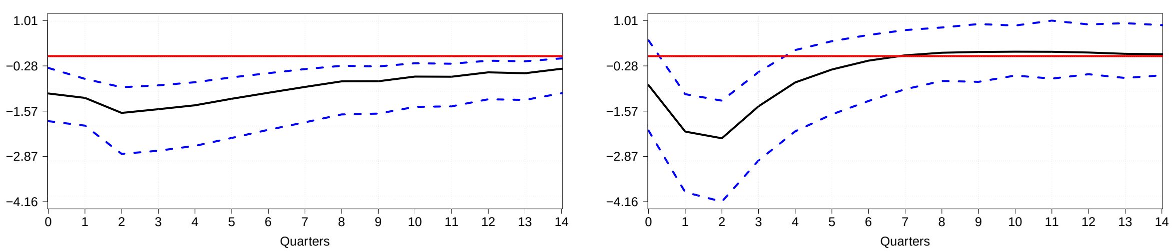
(i) Fed’s funds rate (FEDFUNDS)



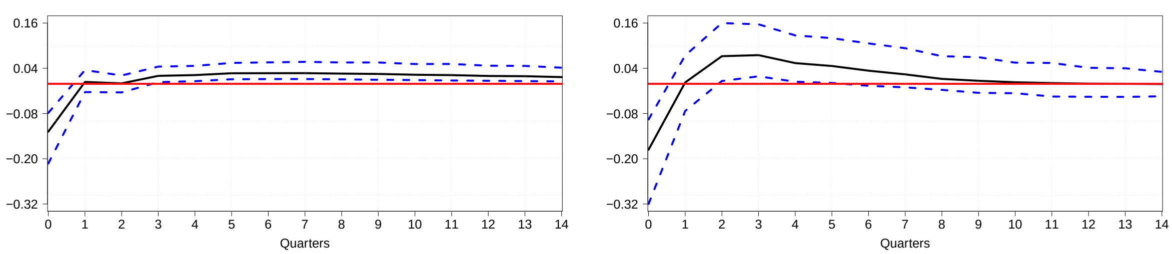
(j) Moody’s seasoned BAA-FFR spread (BAAFFM)



(k) Commercial and industrial loans: all commercial banks (BUSLOANS)



(l) Housing starts (HOUST)

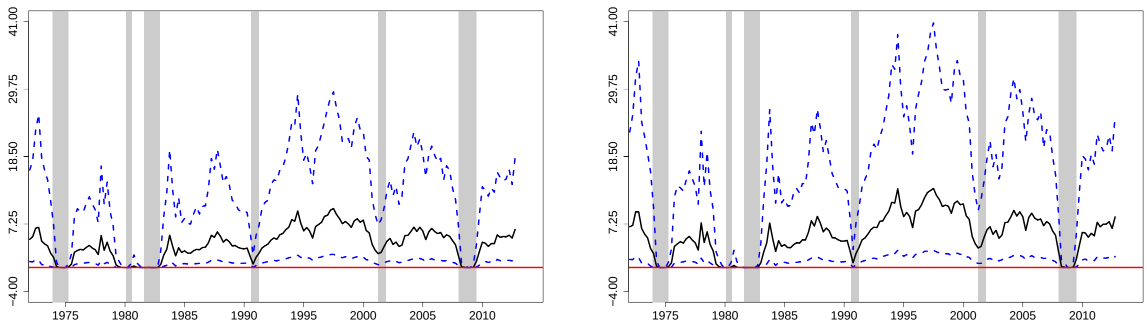


Notes: Posterior distribution of impulse responses in percentage points. Median in black. Dashed blue lines correspond to the 16th and 84th percentiles. Results are based on 35,000 posterior draws. The red line indicates the zero line.

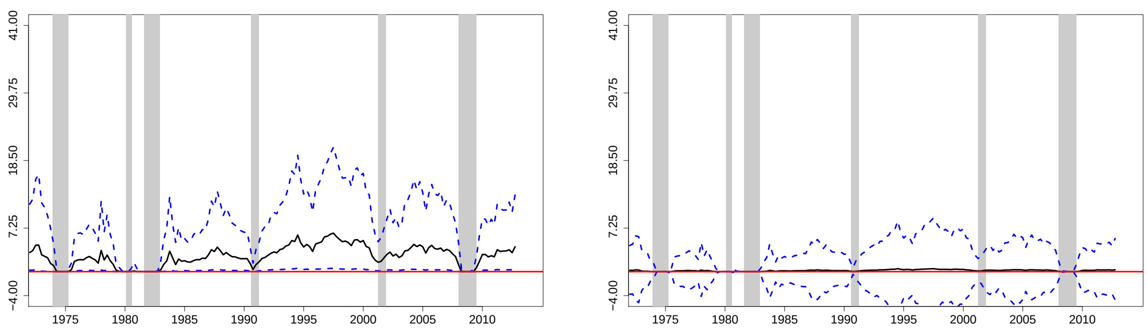
Fig. 2: Dynamic responses of selected macroeconomic quantities to a 100 bps unconventional monetary policy shock

Response in 'expansionary phase' **Response in 'recessionary phase'**

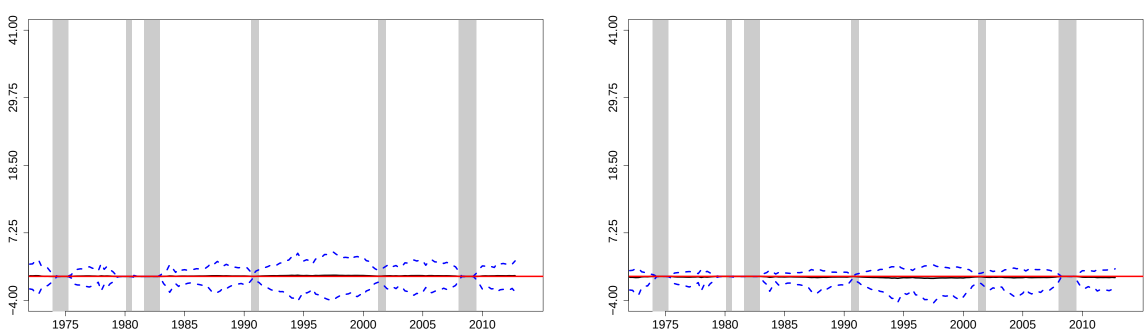
(a) Response after one quarter



(b) Response after one year



(c) Response after three years



Notes: Posterior distribution of impulse responses in percentage points. Median in black. Dashed blue lines correspond to the 16th and 84th percentiles. Grey shaded areas refer to recessions dated by the Business Cycle Dating Committee of the National Bureau of Economic Research (www.nber.org). Results are based on 35,000 posterior draws. The red line indicates the zero line.

Fig. 3: Dynamic responses of recession probabilities to a 100 basis point (bp) unconventional monetary policy shock

Appendix A Posterior distributions

This appendix provides details on the corresponding posterior distributions and how to simulate them. In what follows,

$$\boldsymbol{\pi}^t = (\boldsymbol{\pi}_1', \ldots, \boldsymbol{\pi}_t')' \tag{A.1}$$

denotes the entire history of a generic vector $\boldsymbol{\pi}$ up to time t, and

$$\boldsymbol{\Pi}^t = (vec(\boldsymbol{\Pi}_1)', \ldots, vec(\boldsymbol{\Pi}_T)')' \tag{A.2}$$

the history of a generic matrix $\boldsymbol{\Pi}$ up to time T. Moreover, let us use the following notation to indicate estimates of some random quantity χ based on information available at time t,

$$\chi_{t|t} = E(\chi_t|\mathcal{I}_t) \tag{A.3}$$

with $\mathcal{I}_t$ denoting a generic information set. Accordingly, we denote a forecast of χ by

$$\chi_{t+1|t} = E(\chi_{t+1}|\mathcal{I}_t). \tag{A.4}$$

Conditional posterior distributions for the state equation

The (conditional) posterior distributions of the parameters in Eq. (2.3) take a particularly simple form,

$$vec(\boldsymbol{A}_{S_t})|\boldsymbol{\Xi}^T, \mathcal{D}^T \sim \mathcal{N}(vec(\overline{\boldsymbol{A}}_{S_t}), \boldsymbol{\Sigma}_{S_t} \otimes \overline{\boldsymbol{V}}_{AS_t}) \tag{A.5}$$

$$\boldsymbol{\Sigma}_{\varepsilon S_t}|\boldsymbol{\Xi}^T, \mathcal{D}^T \sim \mathcal{IW}(\overline{\boldsymbol{C}}_{S_t}, \overline{v}_{S_t}) \tag{A.6}$$

where $\boldsymbol{\Xi}^T$ stores the remaining parameters, regime indicators and latent factors and $\mathcal{D}^T$ denotes the available data up to time T.

The posterior moments for $\boldsymbol{A}_{S_t}$ are given by

$$\overline{\boldsymbol{A}}_{S_t} = (\overline{\boldsymbol{D}}_{S_t}'\overline{\boldsymbol{D}}_{S_t})^{-1}\overline{\boldsymbol{D}}_{S_t}'\overline{\boldsymbol{Z}}_{S_t} \tag{A.7}$$

$$\overline{\boldsymbol{V}}_{S_t} = (\overline{\boldsymbol{D}}_{S_t}'\overline{\boldsymbol{D}}_{S_t})^{-1} \tag{A.8}$$

with $\overline{\boldsymbol{D}}_{S_t} = (\boldsymbol{D}_{S_t}', \underline{\boldsymbol{D}}_{S_t}')'$ and $\overline{\boldsymbol{Z}}_{S_t} = (\boldsymbol{Z}_{S_t}', \underline{\boldsymbol{Z}}_{S_t}')'$. The posterior scale matrix of $\boldsymbol{\Sigma}_{\varepsilon S_t}$, $\overline{\boldsymbol{C}}_{S_t}$ is given by

$$\overline{\boldsymbol{C}}_{S_t} = (\overline{\boldsymbol{Z}}_{S_t} - \boldsymbol{D}_{S_t}'\overline{\boldsymbol{A}}_{S_t})'(\overline{\boldsymbol{Z}}_{S_t} - \boldsymbol{D}_{S_t}'\overline{\boldsymbol{A}}_{S_t}). \tag{A.9}$$

Conditional posterior distributions for the probit model

The parameters of the latent regression model obey posterior distributions which are of a well-known form (George and McCulloch, 1993), namely a normal distribution for

$\boldsymbol{\gamma}$ and a Bernoulli distribution for each δ_k.

$$\boldsymbol{\gamma}|\boldsymbol{\Xi}^T, \mathcal{D}^T \sim \mathcal{N}(\overline{\gamma}, \overline{\boldsymbol{V}}_\gamma) \tag{A.10}$$

where

$$\overline{\boldsymbol{V}_\gamma} = (\boldsymbol{w}'\boldsymbol{w} + \boldsymbol{H}'\boldsymbol{H})^{-1} \tag{A.11}$$

$$\overline{\gamma} = \overline{\boldsymbol{V}_\gamma}(\boldsymbol{w}'\boldsymbol{r}). \tag{A.12}$$

Consistent with the notation used above $\boldsymbol{w}$ and $\boldsymbol{r}$ are the corresponding full-data counterparts of $\boldsymbol{w}_t$ and $\boldsymbol{r}_t$.

The posterior of δ_k follows a Bernoulli distribution,

$$\delta_k \sim \textit{Bernoulli}\ (\overline{p}_k) \tag{A.13}$$

with the corresponding posterior probability given by

$$\overline{p}_k = \frac{\frac{1}{\tau_0} \exp\left(-\frac{1}{2}(\frac{\gamma_k}{\tau_0})^2\right) \underline{p}_k}{\frac{1}{\tau_0} \exp\left(-\frac{1}{2}(\frac{\gamma_k}{\tau_0})^2\right) \underline{p}_k + \frac{1}{\tau_1} \exp\left(-\frac{1}{2}(\frac{\gamma_k}{\tau_1})^2\right) (1 - \underline{p}_k)}. \tag{A.14}$$

The posterior of r_t takes a particularly simple distributional form, namely a truncated standard normal distribution as described in Albert and Chib (1993).

Conditional posterior distributions for the observation equation

Since we assume that the variance-covariance matrix associated with the innovations in Eq. (2.2) is diagonal and in light of the restrictions described in subsection 2.4, the conditional posterior for $\boldsymbol{\Lambda}$ is described exclusively in terms of the remaining $M - K$ rows of $\boldsymbol{\Lambda}$,

$$\boldsymbol{\Lambda}_{j\bullet}|\boldsymbol{\Xi}^T, \mathcal{D}^T \sim \mathcal{N}(\overline{\boldsymbol{\Lambda}}_{j\bullet}, \overline{\boldsymbol{V}}_{\boldsymbol{\Lambda}_{j\bullet}}) \tag{A.15}$$

where $\boldsymbol{\Lambda}_{j\bullet}$ selects the jth row of $\boldsymbol{\Lambda}$ for $K < j \leq M$. The corresponding posterior moments are given by

$$\overline{\boldsymbol{V}}_{\boldsymbol{\Lambda}_{j\bullet}} = (\varsigma_j^{-1}\boldsymbol{f}'\boldsymbol{f} + \boldsymbol{L}_j'\boldsymbol{L}_j)^{-1} \tag{A.16}$$

$$\overline{\boldsymbol{\Lambda}}_{j\bullet} = \overline{\boldsymbol{V}}_{\boldsymbol{\Lambda}_{j\bullet}}(\varsigma_j^{-1}\boldsymbol{f}'\boldsymbol{Y}_{\bullet j}). \tag{A.17}$$

Here, $\boldsymbol{f} = (\boldsymbol{f}_1, \ldots, \boldsymbol{f}_T)'$, $\boldsymbol{L}_j$ denotes the block of $\boldsymbol{L}$ associated with the coefficients of the jth row in Eq. (2.2), and $\boldsymbol{Y}_{\bullet j}$ selects the jth column of a $T \times M$ matrix $\boldsymbol{Y}_t = (\boldsymbol{y}_1, \ldots, \boldsymbol{y}_T)'$.

The posterior of ι_k is Bernoulli distributed with the corresponding posterior probability $\overline{\rho}_k$ given by

$$\overline{\rho}_k = \frac{\frac{1}{\varrho_0}\exp\left(-\frac{1}{2}(\frac{\iota_k}{\varrho_0})^2\right)\overline{\rho}_k}{\frac{1}{\varrho_0}\exp\left(-\frac{1}{2}(\frac{\iota_k}{\varrho_0})^2\right)\overline{\rho}_k + \frac{1}{\varrho_1}\exp\left(-\frac{1}{2}(\frac{\iota_k}{\varrho_1})^2\right)(1-\overline{\rho}_k)}. \tag{A.18}$$

For all other quantities like the discrete states $\boldsymbol{s}$ the conditional posteriors require more complex forward filtering-backward sampling algorithms. Fortunately, several convenient and efficient algorithms are available to obtain posterior estimates.

Sampling the latent factors $\boldsymbol{f}_t$

The latent factors are obtained by using the well-known algorithm put forth in Carter and Kohn (1994), and Frühwirth-Schnatter (1994). The density of $\boldsymbol{f}_t$ can be factored as

$$p(\boldsymbol{f}^T|\boldsymbol{\Xi}^T,\mathcal{D}^T) = p(\boldsymbol{f}_T|\boldsymbol{\Xi}^T,\mathcal{D}^T)\prod_{t=1}^{T-1} p(\boldsymbol{f}_t|\boldsymbol{f}_{t+1},\boldsymbol{\Xi}^T,\mathcal{D}^T)$$

where the moments are given by

$$\begin{aligned} \boldsymbol{f}_t|\boldsymbol{f}_{t+1},\boldsymbol{\Xi}^T,\mathcal{D}^T &\sim \mathcal{N}(\boldsymbol{f}_{t|t+1},\boldsymbol{\Omega}_{t|t+1}) \\ \boldsymbol{f}_{t|t+1} &= E(\boldsymbol{f}_t|\boldsymbol{f}_{t+1},\boldsymbol{\Xi}^T,\mathcal{D}^T) \\ \boldsymbol{\Omega}_{t|t+1} &= var(\boldsymbol{f}_t|\boldsymbol{f}_{t+1},\boldsymbol{\Xi}^T,\mathcal{D}^T). \end{aligned}$$

If $\boldsymbol{f}_{t|t+1}$ and $\boldsymbol{\Omega}_{t|t+1}$ is available, the full history of the latent factors can be sampled in a straightforward fashion from $\mathcal{N}(\boldsymbol{f}_{t|t+1},\boldsymbol{\Omega}_{t|t+1})$. $\boldsymbol{f}_{t|t+1}$ and $\boldsymbol{\Omega}_{t|t+1}$ are obtained using Kalman filtering and the corresponding backward recursions. More specifically, let us assume without loss of generality that Q equals one and no observable quantities are included. Then Eq. (2.3) can be rewritten as

$$\boldsymbol{f}_t = \boldsymbol{A}_{1S_t}\boldsymbol{f}_{t-1} + \boldsymbol{\varepsilon}_t. \tag{A.19}$$

In addition, the observation equation (2.2) can be written more compactly as

$$\boldsymbol{y}_t = \boldsymbol{\Lambda}^f\boldsymbol{f}_t + \boldsymbol{e}_t. \tag{A.20}$$

Conditional on $\boldsymbol{f}_{0|0}$ and $\boldsymbol{\Omega}_{0|0}$, the Kalman filter produces

$$\begin{aligned}
\boldsymbol{f}_{t|t-1} &= \boldsymbol{A}_{1S_t}\boldsymbol{f}_{t-1|t-1} \\
\boldsymbol{\Omega}_{t|t-1} &= \boldsymbol{A}_{1S_t}\boldsymbol{\Omega}_{t-1|t-1}\boldsymbol{A}'_{1S_t} + \boldsymbol{\Sigma}_{\varepsilon t} \\
\boldsymbol{K}_t &= \boldsymbol{\Omega}_{t|t-1}\boldsymbol{\Lambda}^{f'}(\boldsymbol{\Lambda}^f\boldsymbol{\Omega}_{t|t-1}\boldsymbol{\Lambda}^{f'} + \boldsymbol{\Sigma}_{\boldsymbol{e}})^{-1} \\
\boldsymbol{f}_{t|t} &= \boldsymbol{f}_{t|t-1} + \boldsymbol{K}_t(\boldsymbol{y}_t - \boldsymbol{\Lambda}^f\boldsymbol{f}_{t|t-1}) \\
\boldsymbol{\Omega}_{t|t} &= \boldsymbol{\Omega}_{t|t-1} - \boldsymbol{K}_t\boldsymbol{\Lambda}^f\boldsymbol{K}'_t\boldsymbol{\Omega}_{t|t-1}.
\end{aligned}$$

Note that at time $t = T$ we obtain $\boldsymbol{f}_{T|T}$ and $\boldsymbol{\Omega}_{T|T}$, which permits us to sample $\boldsymbol{f}_T$. This draw of $\boldsymbol{f}_T$, in conjunction with $\boldsymbol{f}_{T|T}$ and $\boldsymbol{\Omega}_{T|T}$ is then used to obtain $\boldsymbol{f}_{t|t+1}$ and $\boldsymbol{\Omega}_{t|t+1}$ until time $t = 0$ is reached. The corresponding recursions are given by

$$\begin{aligned}
\boldsymbol{f}_{t|t+1} &= \boldsymbol{f}_{t|t} + \boldsymbol{\Omega}_{t|t}\boldsymbol{A}'_{1S_t}\boldsymbol{\Omega}^{-1}_{t+1|t}(\boldsymbol{f}_{t+1} - \boldsymbol{A}_{1S_t}\boldsymbol{f}_{t|t}) \\
\boldsymbol{\Omega}_{t|t+1} &= \boldsymbol{\Omega}_{t|t} - \boldsymbol{\Omega}_{t|t}\boldsymbol{A}'_{1S_t}\boldsymbol{\Omega}^{-1}_{t+1|t}\boldsymbol{A}'_{1S_t}\boldsymbol{\Omega}^{-1}_{t|t}.
\end{aligned}$$

Sampling the regime indicators s_t

Following Kim and Nelson (1999), and Amisano and Fagan (2013) we obtain the filtered and predicted probabilities, $\hat{p}_{jt|t} = Prob(S_t = j|\Xi^t, \mathcal{D}^t)$ and $\hat{p}_{it+1|t} = Prob(S_t = i|\Xi^t, \mathcal{D}^t)$ through a standard filter (Kim and Nelson, 1999). The prediction and updating probabilities are given by

$$\begin{aligned}
\hat{p}_{jt+1|t} &= \sum_{i=1}^{2} p_{ij,t|t}\hat{p}_{jt|t} \\
\hat{p}_{jt+1|t+1} &= \frac{\hat{p}_{jt+1|t}p(\boldsymbol{z}_{t+1}|\boldsymbol{A}_{1S_{t+1}=j}, \boldsymbol{\Sigma}_{\varepsilon S_{t+1}=j})}{\sum_{h=1}^{2}\hat{p}_{ht+1|t}p(\boldsymbol{z}_{t+1}|\boldsymbol{A}_{1S_{t+1}=h}, \boldsymbol{\Sigma}_{\varepsilon S_{t+1}=h})}.
\end{aligned}$$

The filtered probabilities are then used in the next step to sample the full history of regime indicators $\boldsymbol{s}^T$. Similar to the decomposition of the joint conditional density of the latent factors, it is possible to use the following factorization,

$$p(\boldsymbol{s}^T|\Xi^T, \mathcal{D}^T) = p(S_T|\Xi^T, \mathcal{D}^T)\prod_{t=1}^{T-1} p(S_t|S_{t+1}, \Xi^T, \mathcal{D}^T)$$

where $p(S_T|\boldsymbol{f}^T, \Xi^T, \mathcal{D}^T)$ is obtained from the final iteration of the Hamilton (1989) filter. S_t conditional on S_{t+1} and the remaining parameters can be obtained in a straightforward fashion by noting that

$$p(S_t|S_{t+1}, \Xi^T, \mathcal{D}^T) \propto p(S_{t+1}|S_t)p(S_t|\Xi^T, \mathcal{D}^T).$$

The first term on the right hand side refers to the transition probability and the second term is obtained from the Hamilton filter. Thus $p(S_t|S_{t+1}, \Xi^T, \mathcal{D}^T)$ can be obtained by iterating backwards until time $t = 0$ is reached. To be more precise, let

$$Prob(S_t = i | S_{t+1} = j, \mathbf{\Xi}^T, \mathcal{D}^T) = \frac{\hat{p}_{jt|t} p_{ij,t+1}}{\sum_{h=1}^{2} \hat{p}_{ht|t} p_{hj,t+1}}.$$

Finally, the corresponding transition probabilities $p_{ij,t}$ are obtained straightforwardly through Eq. (2.5).

Appendix B Forecasting US macroeconomic quantities

In this appendix we briefly assess whether allowing for time-varying transition probabilities in the MS-FAVAR improves point and density forecasts for four macroeconomic quantities: the growth rate of real gross domestic product (GDP), the unemployment rate (UNRATE), consumer price inflation for all urban consumers (CPIAUCSL), and Moody's seasoned Baa corporate bond minus the federal funds rate (BAAFFM). Table B.1 presents the results of a simple recursive forecasting exercise. More specifically we pick the period ranging from $t_0 =$1974:Q1 to $t_1 =$1998:Q1 as an initial estimation sample and compute the one-quarter and one-year ahead predictive densities. After obtaining the predictions we expand the estimation sample, keeping t_0 fixed. This procedure is repeated until the $t_1 = T$ is reached.

As measures of forecast accuracy we use the well-known log predictive score, motivated in Geweke and Amisano (2010), and the root mean square forecast error. The predictive density of the Markov switching models is in general a two-components mixture of normal distributions which is not available in closed form. We thus follow Amisano and Fagan (2013) and simulate the predictive density using Monte Carlo integration. The corresponding log predictive score is computed by means of a Gaussian approximation.

As competing models we exclusively include models that are nested variants of our MS-FAVAR-TVP model. All models in the forecasting exercise are estimated within a Bayesian framework with the same type of prior calibration as for the MS-FAVAR-TVP. We include a vector autoregressive model alongside with a factor-augmented VAR model. These models provide some intuition on the predictive capabilities of linear models with small and large information sets. Furthermore we consider a Markov switching VAR and a Markov switching FAVAR, and finally, a Markov-switching VAR model with time-varying transition probabilities.

The upper panel of Table B.1 presents the one-quarter and four-quarter ahead log predictive scores for the models across the different variables. While the MS-FAVAR-TVP outperforms all competing models when it comes to predicting GDP growth and the BAAFFM spread, it tends to perform worse when used to predict the unemployment rate and the consumer price index. For these variables the linear FAVAR model beats all competing models at the one-quarter ahead horizon. Increasing the forecasting horizon yields a different picture. At the one-year ahead horizon the MS-FAVAR-TVP model improves upon all competing models for unemployment, inflation and the BAAFFM. For GDP growth, the only model that performs slightly better is the MS-VAR-TVP.

Inspection of the root mean square forecast errors in the bottom panel of Table B.1 reveals that the MS-FAVAR-TVP outperforms all competitors at the one-quarter ahead horizon for unemployment, inflation and the BAAFFM spread. Looking at the one-year ahead horizon reveals that a simple linear VAR coupled with a Minnesota prior exhibits the strongest performance for GDP growth while the MS-FAVAR-TVP model performs best when used to predict the rate of unemployment and the BAAFFM spread.

In summary, there is no clear picture emerging. While the MS-FAVAR-TVP model performs rather well for the majority of variables under consideration, simpler model variants also tend to produce precise predictions. However, it is worth noting that the modeling approach presented in this paper is not tailored to produce accurate predictions, but to provide detailed insights on the relationship between unconventional monetary policy and business cycles. To further improve the model it might thus be tempting to expand the information set and exploit more information to improve the predictions obtained from the MS-FAVAR-TVP model, but this is outside the scope of this forecasting exercise.

Table B.1: Out-of-sample performance relative to the VAR model in terms of the sum of log predictive scores (LPS) and the root mean square error (RMSE): 1998:Q2 to 2013:Q2

Sum of log predictive scores (LPS) relative to the VAR model								
	One-step ahead				Four-steps ahead			
	GDP	UNRATE	CPIAUCSL	BAAFFM	GDP	UNRATE	CPIAUCSL	BAAFFM
VAR (absolute LPS)	214.886	167.272	201.660	166.754	193.233	164.933	225.670	160.356
FAVAR	-0.574	**66.357**	**-1.209**	7.005	0.028	66.412	-0.848	8.047
MS-VAR	-3.008	0.146	-3.229	4.234	0.279	3.401	9.128	11.662
MS-FAVAR	-7.418	1.380	-23.200	5.422	-6.403	98.625	5.840	23.609
MS-VAR-TVP	-0.041	-0.348	-6.508	0.973	**5.653**	4.047	5.544	7.327
MS-FAVAR-TVP	**2.611**	25.737	-5.408	**22.076**	2.128	**99.786**	**10.968**	**27.685**

Average root mean square forecast errors (RMSE) relative to the VAR model								
	One-step ahead				Four-steps ahead			
	GDP	UNRATE	CPIAUCSL	BAAFFM	GDP	UNRATE	CPIAUCSL	BAAFFM
VAR (absolute RMSE)	0.006	0.014	0.007	0.015	0.008	0.015	0.005	0.014
FAVAR	1.025	0.342	0.992	0.875	1.015	0.285	1.109	0.973
MS-VAR	1.015	0.986	0.997	0.903	1.024	0.987	0.944	0.894
MS-FAVAR	1.140	0.302	1.003	0.753	1.183	0.201	1.104	0.852
MS-VAR-TVP	**0.996**	0.985	0.997	0.932	**1.001**	0.981	**0.939**	0.932
MS-FAVAR-TVP	1.051	**0.286**	**0.890**	**0.605**	1.067	**0.189**	1.137	**0.794**

Notes: VAR stands for a vector autoregressive model, FAVAR for a factor-augmented VAR model, MS-VAR for a Markov switching VAR model, MS-FAVAR for a Markov switching FAVAR model, MS-VAR-TVP for a Markov switching VAR model with time-varying transition probabilities and MS-FAVAR-TVP for the Markov switching FAVAR model with time-varying transition probabilities. GDP stands for the real gross domestic product, UNRATE for the civilian unemployment rate, CPIAUCSL for the comsumer price index for all urban consumers (all items), and BAAFFM for Moody's seasoned Baa corporate bonds minus the federal funds rate. The bold figures indicate the best performing model for a given variable and time horizon.

Appendix C Data description

The time series used to construct the vectors $\boldsymbol{x_t}$ and $\boldsymbol{y_t}$ are presented in Tables B.1 and B.2, respectively. The format is as follows: series number, series mnemonic, transformation code, and brief series description. The transformation codes are 1=no transformation, and 2=first difference of logarithms. The series were taken from the US Federal Reserve database.

Table C.1: Data series used for $\boldsymbol{x_t}$ in Eq. (2.2)

1	FEDFUNDS	1	interest rate: federal funds (effective)
2	T10YFFM	1	10-year treasury constant maturities minus federal funds rate

Table C.2: Data series used for $\boldsymbol{y_t}$ in Eq. (2.2)

Real output and income			
3	GDP	2	real gross domestic product
4	IPMANSICS	2	industrial production: manufacturing (SIC)
5	IPFPNSS	2	industrial production: final products and nonind. supplies
6	IPFINAL	2	industrial production: final products (market group)
7	IPCONGD	2	industrial production: consumer goods
8	IPDCONGD	2	industrial production: durable consumer goods
9	IPNCONGD	2	industrial production: nondurable consumer goods
10	IPMAT	2	industrial production: materials
11	IPDMAT	2	industrial production: durable materials
12	IPNMAT	2	industrial production: nondurable materials
13	IPBUSEQ	2	industrial production: business equipment
14	IPFUELS	2	industrial production: fuels
15	NAPMPI	2	ISM (Institute for Supply Management) manufacturing: production index
16	PI	2	personal income
17	RPI	2	real personal income
18	W875RX1	2	real personal income less transfer payments
Employment and unemployment			
19	NAPMEI	2	ISM (Institute for Supply Management) maunfacturing: employment index
20	UNRATE	1	civilian unemployment rate
21	AWHMAN	2	avg. weekly hours of prod. and nonsupervisory employees
22	AWOTMAN	2	avg. weekly overtime hours of prod. and nonsupervisory employees
Consumption expenditures (chain-type price indices)			
23	PCEPI	2	personal consuption expenditures
24	DPCERA3M086SBEA	2	real personal consuption expenditures
25	DDURRG3M086SBEA	2	personal consumption expenditures: durable goods
26	DNDGRG3M086SBEA	2	personal consumption expenditures: nondurable goods
27	DSERRG3M086SBEA	2	personal consumption expenditures: services
Interest rates, spreads and credit quantities			
28	T1YFFM	1	1-year treasury constant maturities minus federal funds rate
29	T5YFFM	1	5-year treasury constant maturities minus federal funds rate
30	BUSLOANS	2	commercial and industrial loans: all commercial banks
31	BAAFFM	2	bond yields: Moody's seasoned Baa corporate minus federal funds rate
Price indices			
32	CPIAUCSL	2	consumer price index for all urban consumers: all items
33	NAPMPRI	2	ISM (Institute for Supply Management) manufacturing: price index
Orders, inventories and sales			
34	NAPMNOI	2	ISM (Institute for Supply Management) manufacturing: new orders index
35	NAPMII	2	ISM (Institute for Supply Management) manufacturing: inventories index
36	CMRMTSPL	2	real manufacturing and trade industries sales
Housing			
37	HOUST	2	housing stats: total
38	HOUSTNE	2	housing starts: northeast census region
39	HOUSTMW	2	housing starts: midwest census region
40	HOUSTS	2	housing starts: south census region
41	HOUSTW	2	housing starts: west census region
42	PERMIT	2	new private housing units authorized by building permits: total
43	PERMITNE	2	new private housing units authorized by building permits: northeast census region
44	PERMITMW	2	new private housing units authorized by building permits: midwest census region
45	PERMITS	2	new private housing units authorized by building permits: south census region
46	PERMITW	2	new private housing units authorized by building permits: west census region
Miscellaneous			
47	NAPM	2	ISM (Institute for Supply Management) manufacturing: PMI composite index
48	NAPMSDI	2	ISM (Institute for Supply Management) manufacturing: supplier deliverer index