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A DYNAMIC THEORY OF CONJECTURAL VARIATIONS*

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I. INTRODUCTION

In his seminal work Bowley [1924] introduced the concept of *conjectural variations.*[1] In short, the idea is that a firm in an oligopolistic market believes that the quantity (price) it chooses will affect the quantities (prices) chosen by its rivals. This belief is taken into account by the firm when selecting the profit maximizing output (price) level. The reactions of the rivals to the quantity (price) decision of firm i, as subjectively perceived by firm i, is called *conjectural variation.* When introducing this concept, Bowley [1924] and later Stackelberg [1934] clearly had in mind a dynamic phenomenon, although their analysis is a static one. Friedman [1977, 1983] criticizes the conjectural variations analysis in static models and lists several arguments against it (Friedman [1983], page 110): (i) The models are not actually dynamic, thus a dynamic interpretation is not possible. (ii) The firms are assumed to maximize one-period profits rather than the discounted stream of profits over a given planning horizon. (iii) Firms have expectations about how their rivals will behave that need not be correct. This latter criticism was first put forward by Fellner [1949] who argued that *ad hoc* conjectural variations are generally inconsistent with rational firm behavior out of equilibrium.

Recently, many authors elaborated on the third point and introduced the concept of consistent conjectural variations, i.e., conjectures that are consistent with the actual responses taken by the rivals after a quantity (price) decision of firm i (Laitner [1980], Breshnahan [1981], Perry [1982], Kamien and Schwartz [1983], Boyer and Moreaux [1983]). But again the analysis is carried out within a static framework.

Despite the criticism put forward by Friedman [1977, 1983] static conjectural variations analyses are very popular. Many writers in industrial organization and/or international trade theory use this approach when analyzing oligopolistic competition (see Dixit [1988a], [1988b], Eaton and Grossman [1986], and Hwang and Mai [1988], etc.). Common to these studies is the notion of modelling *dynamic* interactions even though the analyses are static.

[1]When introducing the new concept, Bowley did not name it conjectural variation. This term stems from Frisch [1933].

In the present paper we explore the relationship between dynamic oligopolistic competition and static conjectural variations equilibria. In particular, we are interested in the long-run (steady state) equilibrium of the dynamic game and compare it to a conjectural variations equilibrium of the corresponding static game. We use the adjustment cost model to study dynamic oligopolistic quantity competition over an infinite horizon and identify the model with single period profit functions as the corresponding static game. It turns out, that the steady state closed-loop (subgame-perfect) equilibrium of the dynamic game coincides with a static conjectural variations equilibrium with nonzero conjectures. Hence, in the limit, if the steady state closed-loop equilibrium is stable, it is *possible* and *justified* to interpret a conjectural variations equilibrium as the outcome of dynamic strategic interactions. With the simplifying assumptions of linear demand and quadratic costs we are able to sharpen our predictions. In this case, the steady state closed-loop (subgame-perfect) equilibrium of the asymmetric dynamic game corresponds to a conjectural variations equilibrium with *constant* and *symmetric* conjectures. This result suggest that it is not only possible to relate conjectural variations to dynamic competition, but also to justify the choice of constant and symmetric conjectures across firms as they correspond to firms' equilibrium behavior. This is particularly important, because many studies that employ a static conjectural variations approach make use of constant and symmetric conjectures (see for example Hwang and Mai [1988]).

The analysis in this paper is related to recent research on dynamic oligopoly theory. Driskill and McCafferty [1989] study dynamic quantity competition in a differential game model with adjustment costs. They derive the closed-loop (subgame-perfect) Nash equilibrium for the symmetric linear quadratic game and find that it results in more competitive behavior than Cournot. This even holds true for the limit game, i.e., the game with adjustement costs approaching zero. In this paper we employ an asymmetric version of the Driskill and McCafferty [1989] model and generalize their result to this case. Moreover, we fully explore the relationship between the closed-loop equilibrium of the dynamic game and the conjectural variations equilibrium of the corresponding static game.

Driskill and McCafferty [1989] are not the only ones who find that dynamic Cournot competition played with closed-loop (or Markovian) strategies results in more competitive behavior than Cournot. The differential game model with sticky prices of Fershtman and Kamien [1987], the strategic investment game of Reynolds [1987], and the discrete time Cournot model with alternating moves of Maskin and Tirole [1987] are other examples that exhibit the same qualitative property. Our analysis clarifies the economic mechanism that generates these results. It is the use of closed-loop (Markovian) strategies that is responsible for this prediction. In the case with closed-loop strategies each firm employs a decision rule for maximizing profits which does take into account the notion that changes in each seller's output level may stimulate reactions by its rivals. This corresponds to a conjectural variations equilibrium behavior. If, moreover, an increase in output of one firm stimulates a reduction of output of the rival (i.e., the closed-loop strategies induce downward sloping reaction functions), the dynamic behavior must correspond to a conjectural variations equilibrium with negative conjectures. All the above examples satisfy this property, therefore resulting in an equilibrium outcome that is more competitive than Cournot.

Similar in spirit to the present discussion is the analysis by Riordan [1985]. He introduces the concept of *dynamic conjectural variations* and relates it to equilibrium behavior in a two period Cournot model in which firms have imperfect information about the market demand, do not observe outputs of rivals but are able to draw inferences about the position of the demand curve from past observations on prices. Thus, changes in one firm's output in the current period cause the market price to change and therefore influence the rivals' estimates about future demand. In this setting, a firm perceives that an increase in its output decreases current market price which will cause rival firms to estimate that demand has gone down and therefore cause them to decrease ouput in the following period. This intertemporal link of perceived actions resulting in more competitive behavior than Cournot is called dynamic conjectural variations by Riordan [1985]. Finally, a different notion of dynamic conjectural variations was recently used in an empirical analysis by Roberts and Samuelson [1988]. They formulate an infinite horizon time dependent supergame to study intertemporal advertising decisions of oligopolistic sellers. They derive what they call a *sophisticated* equilibrium and distinguish it from a *naive* equilibrium.[2] In a sophisticated equilibrium each firm recognizes that its current action may alter its rivals actions in the future. These future reactions of the rivals to the current actions of firm i as subjectively perceived by firm i are called dynamic conjectural variations by Roberts and Samuelson [1988].

Our paper is organized as follows. In the next section we present the dynamic Cournot game. We restrict our attention to two firms and discuss the concepts of an open-loop and perfect equilibrium. Section III presents the main results and is followed by a discussion of a special case with linear demand and quadratic costs. Section V summarizes our findings and concludes the paper.

II. THE MODEL

Consider a market consisting of two firms each producing a homogeneous product. The product price is related to industry output by means of an inverse demand function, $p(Q(t))$, where $p(\cdot)$ is the price at time t and $Q(t) = q_1(t) + q_2(t)$ is industry output at time t, i.e., the sum of outputs produced by each firm. Both firms operate with technologies that are summarized through the cost functions $C_i(q_i(t))$, $i = 1,2$. In addition to the variable production costs we assume that each firm faces adjustment costs when scaling up (or down) output. These adjustment costs can be thought of as investment expenditures, for example, that occur if firms increase their plant sizes. Let $x_i(t) \equiv \dot{q}_i(t)$ denote the rate of change of output of firm i at time t. Adjustment costs are described by the cost functions $A_i(x_i(t))$, $i = 1,2$. Finally, we assume that firms choose their production plans over an infinite planning period so as to maximize the discounted stream of profits, i.e.,

$$\max \Pi_i^d = \int_0^\infty e^{-rt} \left[p(Q(t))\, q_i(t) - C_i(q_i(t)) - A_i(x_i(t))\right] dt, \quad i = 1,2 \quad (1)$$

subject to the given initial condition $q_i(0) = q_{i0}$ and $r > 0$, the common discount rate.

[2]In the terminology of differential game theory a *sophisticated* equilibrium corresponds to a perfect equilibrium played with closed-loop (subgame-perfect) strategies and a *naive* equilibrium corresponds to an open-loop equilibrium.

Through the introduction of adjustment costs the Cournot game (1) becomes what Friedman [1977] calls a *time dependent* or *structurally linked* dynamic game. More precisely, (1) constitutes a two person non-zerosum differential game with the levels of output as the state variables and the rates of change of output as the control variables.

Throughout the paper we make use of the following assumptions:

Assumption 1 $q_i(t) \in [0, M_i]$, $x_i(t) \in [-M_i, M_i]$ for all $t \geq 0$.

Assumption 2 $C_i(q_i(t))$ is C^2 in the interior of its domain and $C_i(0) = 0$.

Assumption 3 $p(Q(t))$ is C^2 in the interior of its domain and there exists a positive level of output $\overline{Q}$ such that $p(Q(t)) > 0$ for $Q(t) < \overline{Q}$ and $p(Q(t)) = 0$ otherwise. Furthermore $p'(Q(t)) < 0$.

Assumption 4 $q_i(t)p''(Q(t)) + p'(Q(t)) < 0$.

Assumption 5 $p'(Q(t)) < C_i''(q_i(t))$.

Assumption 6 $A_i(x_i(t))$ is C^2 in the interior of its domain. $A_i(0) = A_i'(0) = 0$, and $A_i(x_i(t))$ strictly convex.

Assumptions 1 to 6 are standard and frequently employed in oligopoly theory. Assumptions 3 to 6 together imply strict concavity of the instantaneous profit function $p(Q)q_i - C_i(q_i) - A_i(x_i)$ with respect to both q_i and x_i. Assumptions 4 and 5 are the stability conditions introduced by Hahn [1962].

For the dynamic Cournot game (1) to be well defined we need to specify the strategy spaces available to the firms and the equilibrium concept employed. As far as the latter is concerned we choose the Nash/Cournot equilibrium. The choice of the strategy spaces deserves a separate treatment.

There are two concepts of strategy spaces that are frequently employed in economic applications of differential games (see Mehlmann [1988]): the open- loop and the closed-loop (subgame-perfect) strategy space, respectively.[3] If firms use open-loop strategies they design their optimal policies as

[3] The case of a closed-loop (subgame-perfect) strategy space is also referred to as feedback strategy space (see for example Başar and Olsder [1982]).

simple time functions independent of the current state of the system. Since these time paths are set at the beginning of the game and specify actions of the players for the entire game their use requires the ability of commitment to preannounced plans. In that sense an open-loop strategy resembles many features of a one-shot action corresponding to a static game. Open-loop strategy spaces are defined as follows.

Definition 1 *The open-loop strategy space, S_i^{OL}, of player i is defined as*

$$S_i^{OL} = \left\{ \begin{array}{l} x_i(q(0),t) \mid x_i(q(0),t) \textit{ is a piecewise continuous function} \\ \textit{of time } t \textit{ for all } t \geq 0 \end{array} \right\}.$$

If firms choose closed-loop (subgame-perfect) strategies they design their optimal policies as decision rules dependent on the state variables of the game (in our case the levels of production). Since the state variables at time t summarize the latest available information about the system at time t closed-loop strategies can be referred to as Markov strategies.[4] The choice of closed-loop strategies implies that firms take into account the rivals reactions to their own actions as expressed by the state variables of the game. This is exactly the characteristic present in the case of a conjectural variations equilibrium. Furthermore in using closed-loop (subgame-perfect) strategies firms' period of commitment is equal to zero. A formal definition of closed-loop (subgame-perfect) strategy spaces is as follows.

Definition 2 *The closed-loop (subgame-perfect) strategy space, S_i^{CL}, of player i is defined as*

$$S_i^{CL} = \left\{ \begin{array}{l} x_i(q(t),t) \mid x_i(q(t),t) \textit{ is a piecewise continuous function of} \\ \textit{time } t \geq 0, \textit{ and Lipschitz-continuous with respect to } q(t) = \\ (q_1(t), q_2(t)) \end{array} \right\}.$$

Closed-loop equilibria of differential games derived through dynamic programming techniques are subgame-perfect in the sense of Selten [1975].

In the light of the preceeding discussion we define the corresponding Cournot/Nash equilibria.

[4] For a discussion of Markov strategies, see Maskin and Tirole (1988, especially p. 553).

Definition 3 *A pair of strategies* (x_1^*, x_2^*) *constitutes a Cournot/Nash equilibrium of the differential game (1), iff*

$$\Pi_1^d(x_1^*, x_2^*) \geq \Pi_1^d(x_1, x_2^*) \quad \text{and} \quad \Pi_2^d(x_1^*, x_2^*) \geq \Pi_2^d(x_1^*, x_2)$$

for all admissible strategies of the corresponding strategy spaces.

If firms play an open-loop game, i.e., choose open-loop strategies, we call the equilibrium an *open-loop equilibrium*, if they choose closed-loop strategies we call it a *perfect equilibrium*.

As stated in the introductory section of this paper our aim is not to deal with the problem of existence of equilibria. General existence results for differential games can be found in Fershtman and Muller [1984] or Mehlmann [1988]. Instead, in this paper we assume that there exists an open-loop as well as a closed-loop (subgame-perfect) equilibrium.[5]

III. CONJECTURAL VARIATIONS AND DYNAMIC COURNOT COMPETITION

As motivated in the introduction of this paper we are primarily interested in the relationship between the steady state equilibrium (long-run equilibrium) of the dynamic game (1) played with open or closed-loop strategies and the equilibrium of its correpsonding static game. As the corresponding static game we identify the game with the single period profit functions $\Pi_i^s = p(Q)q_i - C_i(q_i)$.

Recent research on differential games applications to oligopoly theory suggests that in the case of linear quadratic models the steady state open-loop equilibrium coincides with the solution of the corresponding static game, whereas the steady state perfect equilibrium does not. This is true even in the limit if we remove the structural links (time dependency) of the original dynamic game (see Fershtman and Kamien [1987], Reynolds [1987], Maskin and Tirole [1987], Dockner [1988], and Driskill and McCafferty

[5]In fact, it is easy to show existence and global asymptotic stability of an open-loop equilibrium in our game as long as Assumptions 1-6 hold. To establish existence and stability of a perfect equilibrium is not so easy. But in Section 4 of this paper we will prove both existence and stability of a perfect equilibrium for the special case with linear demand and quadratic costs.

[1989]). In this section we extend these results to the general nonlinear case and demonstrate how the steady state perfect equilibrium corresponds to a conjectural variations equilibrium of the corresponding static game.

Theorem 1 *The steady state open-loop equilibrium of the dynamic game (1) coincides with the Cournot equilibrium of the corresponding static game.*

Proof:[6] As mentioned above it can be shown that with Assumptions 1 - 6 the open-loop game admits a unique globally and asymptotically stable equilibrium. This equilibrium has to satisfy Pontryagin's maximum principle (see Mehlmann [1988] for details). We formulate the current-value Hamiltonian of player i.

$$H^i = p(Q)q_i - C_i(q_i) - A_i(x_i) + \lambda_i^i x_i + \lambda_j^i x_j, \tag{2}$$

where λ_j^i are the current value adjoint variables. They satisfy the adjoint equations

$$\dot{\lambda}_i^i = r\lambda_i^i - p'q_i - p + C_i', \tag{3}$$

and

$$\dot{\lambda}_j^i = r\lambda_j^i - p'q_i. \tag{4}$$

The maximum condition is given by

$$A_i' = \lambda_i^i. \tag{5}$$

A steady state open-loop equilibrium is defined as the solution to the equation system

$$\dot{q}_i = x_i = \dot{\lambda}_i^i = \dot{\lambda}_j^i = 0.$$

Assumption 6 together with (5) implies $\lambda_i^i = 0$ at the steady state. Thus, (3) reduces to

$$\dot{\lambda}_i^i = -p'q_i - p + C_i' = 0 \tag{6}$$

or equivalently

$$p\left(1 + \frac{S_i}{\eta}\right) = C_i' \tag{7}$$

[6] From now on, unless otherwise stated, we suppress the time argument t.

where $S_i = \frac{q_i}{Q}$ and $\eta = \frac{dQ}{dp} \cdot \frac{p}{Q}$ is the price elasticity of demand. (7), however, is identical to the first order conditions of the corresponding static game. **Q.E.D.**

Theorem 1 provides us with the result that the outcome of a static Cournot game can be considered as the limit of a dynamic game, played with open-loop strategies, as time goes to infinity. This holds because our open-loop model possesses the global asymptotic stability property. The economic reasoning behind this result is straight forward. If firms choose open-loop strategies they announce their optimal plans at the beginning of the game and commit themselves to stick to these time paths for the entire duration of the game. Thus, they base their strategies only on the information available at the beginning of the game and do not update their actions as new information becomes available: they are myopic. As a consequence the steady state open-loop equilibrium is identical to the static Cournot outcome. Put differently, a static Cournot game captures many of the essential characteristics of dynamic oligopolistic competition played with open-loop strategies.

Let us now turn to the case where firms employ closed-loop (subgame perfect) strategies.

Theorem 2 *Any steady state closed-loop (subgame-perfect) equilibrium of the dynamic game (1) can be viewed as a conjectural variations equilibrium of the corresponding static game.*

Proof: A closed-loop equilibrium has to satisfy Pontryagin's maximum principle. But now we have to keep in mind that the strategy of player i, x_i, is a function of $q = (q_1, q_2)$. Thus, the costate equations for a closed-loop equilibrium become

$$\dot{\lambda}_i^i = r\lambda_i^i - p'q_i - p + C_i' - \lambda_j^i \frac{\partial x_j}{\partial q_i} \tag{8}$$

and

$$\dot{\lambda}_j^i = r\lambda_j^i - p'q_i - \lambda_j^i \frac{\partial x_j}{\partial q_j} \tag{9}$$

where the derivative $\frac{\partial x_j}{\partial q_i}$ captures the reaction of firm j to an output change of firm i and $\frac{\partial x_j}{\partial q_j}$ expresses the change by how much firm j scales up (down) its *current* output given a change in its *last period* output. The maximum conditions are given by (5). A steady state closed-loop solution is characterized by

$$\dot{\lambda}^i_j = 0 \Leftrightarrow \lambda^i_j = \frac{p'q_i}{r - \frac{\partial x_j}{\partial q_j}},$$

and

$$\dot{\lambda}^i_i = 0 \Leftrightarrow p'q_i\left(1 + \frac{\frac{\partial x_j}{\partial q_i}}{r - \frac{\partial x_j}{\partial q_j}}\right) + p = C'_i, \tag{10}$$

which in turn can be rewritten to yield

$$p\left(1 + \frac{S_i}{\eta}\left(1 + \frac{\frac{\partial x_j}{\partial q_i}}{r - \frac{\partial x_j}{\partial q_j}}\right)\right) = C'_i. \tag{11}$$

Identifying $\frac{\frac{\partial x_j}{\partial q_i}}{r - \frac{\partial x_j}{\partial q_j}}$ as the conjectural variation that firm i has about the reaction of its rival the result is obvious. **Q.E.D.**

Theorem 2 provides us with an important result that has two main implications. Firstly, it demonstrates that in general the long-run steady state perfect equilibrium price of a dynamic oligopoly model is different from the corresponding static Cournot price. Secondly, it shows that the steady state perfect equilibrium can be viewed as a conjectural variations equilibrium of a static game. This is particularly important if the closed-loop game is stable, i.e., the equilibrium path over time approaches the steady state as time goes to infinity. In such a case a static conjectural variations approach can be viewed as the limit of dynamic strategic interactions. Hence, the interpretation that a conjectural variations equilibrium captures dynamic interactions is justified. From the proof of Theorem 2 it is clear that the conjectures corresponding to the steady state closed-loop equilibrium are not *ad hoc* but rather endogeneously determined. We call them *dynamic conjectures*. The dynamic conjectures are consistent with the model as well as the rival's reactions and are not subject to the criticism put forward by Friedman [1983]. They are entirely determined by the equilibrium decision rules chosen by the firms that relate the rate of change of output of firm i to the current levels of output of all firms in the industry.

The sign of the *dynamic conjecture*, $\frac{\partial x_j/\partial q_i}{r-(\partial x_j/\partial q_j)}$, determines whether the long-run perfect equilibrium price will be below or above the static Cournot equilibrium price. If it is negative the industry equilibrium is more competitive than Cournot if it is positive it is more collusive. In the following section we will prove that for the special example with linear demand and quadratic costs dynamic conjectures are negative.

Additionally, our result suggests that dynamic Cournot competition played with closed-loop (subgame-perfect) strategies allows for a reaction curve analysis (see Samuelson [1987]). The reaction curves are given by the system of differential equations $\dot{q}_i = x_i(q_i, q_j)$. These equations relate the current levels of output of *both* firms in the industry to the rates of change of output. Thus, current levels of output of *both* firms determine furture production in the industry. This reflects the inherent dynamics present in an oligopolistic market. Over time these dynamics result in an outcome consistent with a conjectural variations equilibrium.

IV. AN IMPORTANT EXAMPLE: THE LINEAR-QUADRATIC CASE

In the preceeding section we dicussed the general relationship between steady state closed-loop (subgame-perfect) equilibria of dynamic games and conjectural variations equilibria of the corresponding static games. In this section we introduce specific functional forms for the demand and the cost functions. These simplifying assumptions – linear demand and quadratic costs – allow us to sharpen our predictions. In particular, we will derive explicit formula for the dynamic conjectures and use these results for interesting economic propositions.

We specify the linear market demand curve as

$$p(Q) = a - (q_1 + q_2), \tag{12}$$

assume quadratic asymmetric production costs

$$C_i(q_i) = c_i q_i + \frac{b}{2} q_i^2 \tag{13}$$

with a, b, c_i $(i = 1, 2)$ constant, $a - c_i > 0$, and quadratic adjustment costs

$$A_i(x_i) = \frac{k}{2} x_i^2. \tag{14}$$

With these specifications the dynamic Cournot game (1) becomes an *asymmetric* linear-quadratic differential game. Hence, it generalizes the game discussed by Driskill and McCafferty [1989] that allow only for symmetric technologies across firms. Restricting attention to the linear-quadratic game structure opens up the possibility of characterizing closed-loop (subgame-perfect) equilibria.

Theorem 3 *For $r \leq 1$ and $k \leq 1$ there exists a unique, globally and asymptotically stable perfect equilibrium within the class of linear closed-loop (subgame-perfect) strategies.*

Proof: See Appendix.

Three points of Theorem 3 need to be discussed. Firstly, it generalizes Theorem 1 in Driskill and McCafferty [1989] to the case with differences in production technologies across firms. Secondly, the perfect equilibrium is globally and asymptotically stable. Thus, the long-run behavior of the firms can be characterized by the steady state equilibrium. This is an observation that we will use later on. Thirdly, firm behavior in the perfect equilibrium is governed by the following linear decision rules (reaction functions)

$$\dot{q}_i(t) \equiv x_i(t) = \frac{1}{k}[\beta_i + \delta q_i(t) + \sigma q_j(t)] \quad i \neq j \tag{15}$$

This linear decision rules capture all the strategic dynamics present in the model and are responsible for the key results on dynamic conjectural variations.

Theorem 4 *The steady state closed-loop (subgame-perfect) equilibrium of the dynamic game (1) coincides with a conjectural variations equilibrium of the corresponding static game with constant conjectures equal to $\chi = \frac{\sigma}{rk-\delta}$. The conjectures are symmetric and satisfy*

$$-1 - \frac{b}{2} + \sqrt{b + \frac{b^2}{4}} < \frac{\sigma}{rk - \delta} < 0. \tag{16}$$

Proof: Follows from (15), Theorem 2, and the proof of Theorem 3. **Q.E.D.**

Theorem 4 is a striking result: It demonstrates that dynamic Cournot competition played with linear closed-loop (subgame-perfect) strategies can be approximated in the limit by a static conjectural variations equilibrium with *constant* and *negative* conjectures. In this sense, static conjectural variations analyses really capture what is a truly dynamic process. Hence, the claim put forward by many researchers in industrial organization and/or international trade is justified. But what is the economic mechanism that generates this result? The key to it can be found in the Markovian decision rules (reaction functions) (15) and the negativity of both δ and σ.[7] According to the linear rules (15), each firm takes into account its own current output *and* the *reaction of the rival* when deciding upon future levels of output. With the Cournot assumption (zero conjectural variations) firms ignore the reactions of the rivals and make their output decision only on the basis of the residual demand curve.

If a firm takes its rival's reaction into account, i.e., uses decision rule (15), it knows that if it finds it profitable to decrease current production in order to increase price, firm 2 will react by increasing its output and therefore partially offsetting firm 1's action. Thus, a movement along the residual demand curve is offset by a shift as caused by the rival's reaction. Clearly, in equilibrium, this results in a more competitive behavior than Cournot or put differently, corresponds to a conjectural variations equilibrium with negative conjectures. This explains the relationship between a perfect equilibrium of a dynamic game and a conjectural variations equilibrium of a static game. That the dynamic conjectures χ are constant follows from the linearity of the decision rules (15). Symmetry is a consequence of the structure of the model that is symmetric with respect to the quadratic terms but asymmetric with respect to the linear terms.

Finally, Theorem 4 establishes that the constant conjectures corresponding to the steady state perfect equilibrium are different from consistent conjectures in the sense of Bresnahan [1981]. This, however, does not imply that they are ad hoc. They are endogeneously determined but their numerical value is above that of the static consistent conjectures, i.e., $\chi > -1 - \frac{b}{2} + \sqrt{b + \frac{b^2}{4}}$.

The decision rules (15) corresponding to firms' equilibrium behavior

[7]That the constants σ and δ are both negative is an immediate consequence of the global asymptotic stability of the Nash equilibrium. This implies that the reaction functions corresponding to (15) are downward sloping, or put differently, that the homogeneous product is a strategic substitute.

can also be interpreted as dynamic reaction functions. If we make use of a phase portrait analysis we get downward sloping reaction functions that are depicted in Figure 1. The intersection of the two reaction functions corresponds to the steady state equilibrium that is globally and asymptotically stable. A path leading to the steady state describes an output path for both firms for some initial conditions.

So far we have established that dynamic competition played with closed-loop strategies relates to a conjectural variations equilibrium with nonzero conjectures. What remains to be shown is, if it is also consistent with static Cournot outcomes.

Corollary 1 *In the limit as the discount rate or the adjustment costs become very large, i.e., $r \to \infty$ or $k \to \infty$, the dynamic conjectures λ converge to the Cournot conjectures, i.e., $\chi \to 0$.*

Proof: See Appendix.

The results in Corollary 1 prove that for large enough discount rates (high degree of myopia) or as the adjustment cost parameter, c, becomes large the long-run perfect equilibrium is near the Cournot outcome. This result can also be found in the paper by Maskin and Tirole [1987]. As noted, the model they use differs substantially from ours. Firms in the Maskin and Tirole [1987] framework are assumed not to change their ouput plans for a given finite time (they are able to commit themselves for finite periods) and move alternatingly.

To see why high adjustment costs lead to Cournot outcomes let us look at the decision rules (15). As a firm increases its output the rival firm decreases its own output. But with increasing c (higher adjustment costs) the reaction of the rival becomes smaller (aggressive behavior becomes more costly) and converges to zero in the limit. This, however, is identical with static Cournot behavior.

The last two results are of importance because they show that dynamic duopolistic competition played with closed-loop (subgame perfect) strategies does not only produce more competitive outcomes than Cournot but is also consistent with static Cournot behavior.

V. CONCLUSION

In this paper we studied dynamic duopolistic competition when firms face adjustment costs. A differential game model was used to capture time dependency and structural links present in dynamic markets. We derived open-loop and perfect equilibria for the game and compared the steady state values to the outcome of the corresponding one-shot game. We found that the steady state open-loop equilibrium coincides with the static Cournot solution, whereas the steady state perfect equilibrium does not. The steady state perfect equilibrium of the dynamic game can be viewed as a conjectural variations equilibrium of the corresponding static game. Thus, in the limit a static conjectural variations analysis approximates long-run dynamic interactions. Hence, our findings can be used to justify a static conjectural variations analysis for modelling *dynamic* interactions.

The use of specific functional forms made additional results possible. We demonstrated that the use of *constant* and *symmetric* conjectures in static Cournot models with asymmetric technologies can be justified on the basis that they relate to long-run dynamic competition. Finally, we were able to relate the numerical value of the dynamic conjecture to the corresponding consistent conjecture a la Bresnahan [1981]. We found that the dynamic conjecture is greater than the consistent one.

Our discussion is explicitly based on a dynamic Cournot model with adjustment costs. This does not imply, however, that our results are limited to this class of dynamic games. Alternatively, we could have used the sticky pricing model of Fershtman and Kamien [1987] or the game with alternating moves of Maskin and Tirole [1987]. Although, each of these models differs in terms of its structural dynamics[8] the conclusions as to the characteristics of perfect equilibria are the same. In all cases dynamic competition played with Markov-perfect strategies implies more competitive behavior than Cournot and thus is consistent with a conjectural variations equilibrium with negative conjectures.

[8]The structural dynamics in our model and in that of Driskill and McCafferty [1989] is given by adjustment costs. In the model of Fershtman and Kamien [1987] it is the sticky price that generates the structural links and in Maskin and Tirole [1987] it is the alternating order of moves together with adjustment costs.

APPENDIX

Proof of Theorem 3: We make use of the value function approach to derive closed-loop (subgame-perfect) equilibrium strategies. The value functions $V^i(q_i, q_j)$, $i \neq j$, have to satisfy the Bellman equations

$$rV^i = \max_{x_i} \{(a - q_i - q_j)q_i - c_i q_i - \frac{b}{2}q_i^2 - \frac{k}{2}x_i^2 + V^i_{q_i}x_i + V^i_{q_j}x_j\}. \quad (17)$$

Maximization of the right hand side of (17) yields

$$x_i = \frac{1}{k}V^i_{q_i}. \quad (18)$$

Substitution of (18) into (17) provides us with a system of partial differential equations in $V^i(q_i, q_j)$. Since our problem is of the linear-quadratic type we guess quadratic value functions of the form

$$V^i(q_i, q_j) = \alpha + \beta_i q_i + \gamma q_j + \delta\frac{q_i^2}{2} + \epsilon\frac{q_j^2}{2} + \sigma q_i q_j. \quad (19)$$

Equation (19) shows that the value functions are assumed to be symmetric except for β_i that accounts for the asymmetrie in the linear terms. The value functions (19) solve the partial differential equation system (17) if the parameters satisfy the following system of equations

$$0 = -r\alpha + \frac{1}{2k}\beta_i^2 + \frac{1}{k}\gamma\beta_j, \quad (20)$$

$$0 = -r\beta_i + a - c_i + \frac{\beta_i}{k}\delta + \frac{1}{k}\sigma\beta_j + \frac{1}{k}\sigma\gamma, \quad (21)$$

$$0 = -r\gamma + \frac{\beta_i\sigma}{k} + \frac{\epsilon}{k}\beta_j + \frac{\delta}{k}\gamma, \quad (22)$$

$$0 = -\frac{r}{2}\delta - 1 - \frac{b}{2} + \frac{\delta^2}{2k} + \frac{\sigma^2}{k}, \quad (23)$$

$$0 = -\frac{r}{2}\epsilon + \frac{\sigma^2}{2k} + \frac{\epsilon\delta}{k}, \quad (24)$$

$$0 = -\sigma r - 1 + \frac{2}{k}\delta\sigma + \frac{\epsilon\sigma}{k}. \quad (25)$$

Multiplying the last three equations through by k and introducing the variable $s = \frac{r}{2}$ results in

$$\begin{aligned} -ks\delta - k - \frac{bk}{2} + \frac{\delta^2}{2} + \sigma^2 &= 0, & (26) \\ -2ks\sigma - k + 2\delta\sigma + \epsilon\sigma &= 0, & (27) \\ -ks\epsilon + \frac{\sigma^2}{2} + \epsilon\delta &= 0. & (28) \end{aligned}$$

From (28) we get $\epsilon = \frac{\sigma^2}{2ks-2\delta}$. Equation (26) is quadratic in δ and can be solved as

$$\delta = sk \pm \sqrt{s^2k^2 + 2k + bk - 2\sigma^2}. \qquad (29)$$

Substituting for ϵ in (27) and multiplying through by $(2ks - 2\delta)$, we get

$$4\delta^2\sigma - (8s\sigma k + 2k)\delta = \sigma^3 - 4s^2k^2\sigma - 2sk^2. \qquad (30)$$

Substituting (29) into (30) and rearranging terms yields a polynomial in σ:

$$\begin{aligned} 81\sigma^6 - 72k(s^2k + 2(1+\frac{b}{2}))\sigma^4 + 8k^2[8(1+\frac{b}{2})^2 + 8ks^2(1+\frac{b}{2}) \\ +2s^4k^2 + 1]\sigma^2 - 4k^3(s^2k + 2(1+\frac{b}{2})) &= 0 (31) \end{aligned}$$

After introducing the following changes of variables $\psi = \sigma^2$ and $f = s^2k + 2(1+\frac{b}{2})$, equation (31) becomes

$$81\psi^3 - 72kf\psi^2 + 8k^2(2f^2+1)\psi - 4k^3f = 0. \qquad (32)$$

(32) is a cubic equation in ψ and identical to equation (A.11) in Reynolds [1987]. Thus, we can make use of his result that provides us with a complete solution to (32). Therefore, we get six value functions that satisfy the dynamic programming equations (17). Out of these six candidates we are only interested in those that generate a globally and asymptotically stable closed-loop Nash equilibrium.

Each candidate for a value function generates a pair of linear closed-loop strategies. Substituting these pairs into the state equations gives

$$\dot{q}_1 = \frac{1}{k}[\beta_1 + \delta q_1 + \sigma q_2] \qquad (33)$$

$$\dot{q}_2 = \frac{1}{k}[\beta_2 + \delta q_2 + \sigma q_1] \qquad (34)$$

System (33) - (34) is globally stable iff $\delta + \sigma < 0$ and $\delta - \sigma < 0$. Following Reynolds [1987] this can only be the case if we pick the following solutions for δ and σ:

$$\delta = sk - \sqrt{s^2k^2 + 2k + bk - 2\sigma^2}. \quad (35)$$

$$\sigma = -\sqrt{4k[2f - (4f^2 - 6)^{1/2}\cos(\theta/3)]/27} \quad (36)$$

where $\theta = \arctan(\frac{\sqrt{-D}}{-m})$ with

$$D = 432f^6(-64f^4 + 107f^2 + 128)/9^9 < 0$$
$$m = 4k^3(32f^3 - 99f)/3^9 \quad .$$

It is now easy to show that for a large set of parameter values both stability conditions are satisfied simulaneously. The stability conditions imply that δ and σ are negative. To sum up, we proved existence, stability, as well as uniqueness of the closed-loop (subgame-perfect) Nash equilibrium. **Q.E.D.**

Proof of Theorem 4: The dynamic conjectures are given as $\chi = \frac{\sigma}{rk-\delta}$. Because of stability we get $\delta - rc < \delta < \sigma < 0$. Hence, the dynamic conjectures are negative. With the explicit solutions of δ and σ given by (35) and (36) it can be shown that $-1 - \frac{b}{2} + \sqrt{b + \frac{b^2}{4}} < \chi$ holds. **Q.E.D.**

Proof of Corollary 1: The dynamic conjectures are given as $\chi = \frac{\sigma}{rk-\delta}$ with δ and σ as in (35) and (36). Straight forward calculations show that

$$\lim_{r\to\infty} \frac{\sigma}{rk-\delta} = \lim_{k\to\infty} \frac{\sigma}{rk-\delta} = 0$$

Hence, the dynamic conjectures approach zero as $r \to \infty$ or $k \to \infty$. **Q.E.D.**

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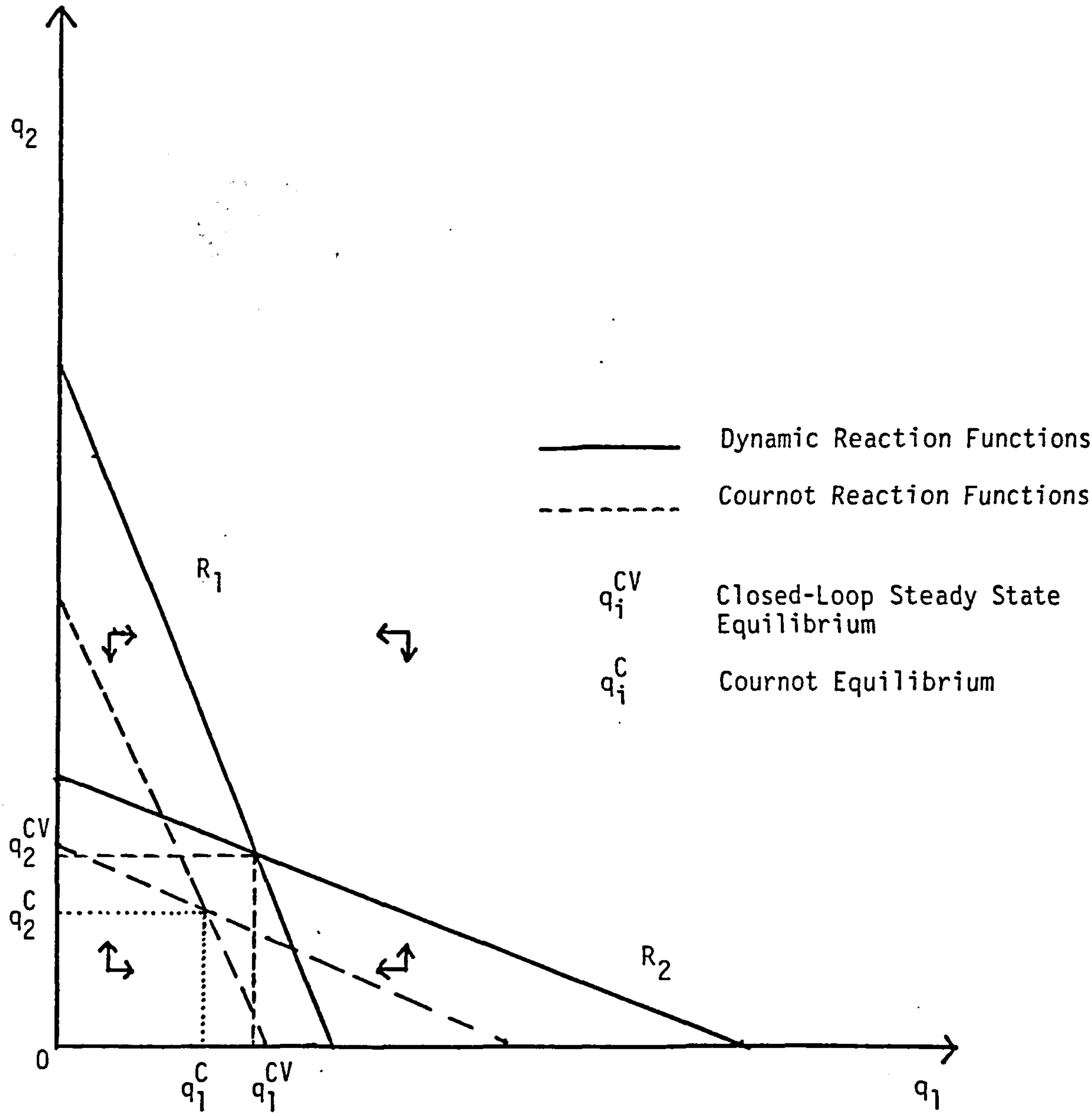

q2
Dynamic Reaction Functions
Cournot Reaction Functions
R1
q_i^CV
Closed-Loop Steady State Equilibrium
q_i^C
Cournot Equilibrium
q2^CV
q2^C
R2
0
q1^C
q1^CV
q1

FIGURE 1

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