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INTERNATIONAL POLLUTION CONTROL: COOPERATIVE VERSUS NON-COOPERATIVE STRATEGIES

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1 Introduction

Recent concerns over environmental issues like the greenhouse effect, the depletion of the ozone layer, or acidification of soil make clear that pollution control strategies of countries can best be studied by using dynamic game models (see Clemhout and Wan [1991], Hoel [1990] Kaitala et al. [1991] and Long [1991].)[1] Game theoretic models of international pollution control are capable of explaining why and when negotiations about bilateral emission reductions fail, how bilateral agreements come about, and why once an agreement over emission controls is reached countries have a unilateral incentive to deviate from this agreement.

In this paper we formulate a simple dynamic game that is interpreted as a two country pollution model and address some of these coordination issues. Each country is assumed to produce a single good that is consumed at home but its production emits pollutants that are added to the existing stock of pollution which is common to both countries (stock externality). Consumers derive positive utility from consuming the good and costs from the current stock of pollution.

The level of pollution can be controlled through the amount of pollutants emitted with the production process. It is the government in each country that decides upon the pollution control strategy. Assuming benevolent governments we study two fundamentally different scenarios: one in which they are able to reach a pollution control agreement (cooperative scenario) and one in which they fail to do so (non-cooperative scenario). It turns out that when the governments are restricted to use global linear strategies non-cooperative behaviour results in overall losses for both countries. This is reflected in higher steady state pollution stocks as well as lower net benefits for the consumers. Desirable cooperative outcomes on the other hand are shown not to possess an equilibrium property. Hence, without binding agreements or threats there always exists an incentive for one country to unilaterally deviate from the agreement, i.e., the cooperative outcome is Pareto-efficient but not an equilibrium. If, on the contrary, governments use nonlinear Markov-perfect strategies and the discount rate

[1] For an excellent introduction to game theoretic modelling in environmental economics see Weimann [1990].

is small enough a Pareto-efficient steady state pollution stock can be supported as a differentiable subgame perfect equilibrium. This is an interesting result and has the following implications. Neither trigger strategies nor any kind of retaliation mechanism is necessary to bring about cooperative outcomes. The emergence of cooperative outcomes requires a *sophisticated* choice of nonlinear Markov strategies as well as a sufficiently low rate of discount.

The paper is organized as follows. In the next section we present the pollution control model. In Section 3 the coordinated pollution control strategy is derived. This coordinated strategy constitutes a benchmark for non-coordinated strategies that are derived in Section 4. Finally, Section 5 concludes the paper.

2 The Model

There are two countries, indexed by $i = 1,2$. Each country produces a single consumption good, Q_i, with a given fixed endowment of factors of production and a given technology. The production of a unit of the consumption good results in an amount of pollutants, E_i, given by the following emission consumption trade-off function (see Forster [1973], [1975]):

$$Q_i = F_i(E_i). \tag{1}$$

The "technology" $F_i(E_i)$ indicates how much pollutants are produced when the current output of country i is Q_i. $F_i(.)$ is assumed to be strictly concave and to satisfy $F_i(0) = 0$.

The amount of pollutants emitted today by both countries adds to the current stock of pollution, $P(t)$, according to the following kinematic equation

$$\dot{P}(t) = E_1(t) + E_2(t) - kP(t), \tag{2}$$

with the initial stock $P(0) = P_0$. In equation (2) it is assumed that the natural purification is proportional to the existing stock $P(t)$, i.e., $kP(t)$.

Consumers in each country derive utility from consuming Q_i given by $U_i(Q_i)$ and face costs of the polluted environment given by $C_i(P)$. Consumer preferences, U_i, are assumed to be strictly concave and the cost functions C_i are strictly convex. Net benefits of a representative consumer in country i in period t are given by

$$W_i(Q_i(t), P(t)) = U_i(Q_i(t)) - C_i(P(t)). \tag{3}$$

Using the emission consumption trade-off function F_i net benefits in period t can be represented as

$$W_i(E_i(t), P(t)) = U_i(F_i(E_i(t))) - C_i(P(t)). \tag{4}$$

The objective of each government is to choose a pollution control strategy $E_i(t)$ (or equivalently an ouput strategy) that maximizes the discounted stream of net benefits from consumption of a representative consumer, i.e.,

$$\max_{E_i} \int_0^\infty e^{-rt}[U_i(F_i(E_i(t))) - C_i(P(t))]dt, \tag{5}$$

subject to the accumulation equation (2) and the initial condition. r is the discount rate that is assumed constant and identical for both countries.

(5) and (2) constitutes a dynamic two player game for which we will derive cooperative (Pareto-optimal) as well as non-cooperative strategies. The outcome of the cooperative game is interpreted as the scenario where the countries are able to reach a pollution control agreement (they coordinate their control efforts), non- cooperative equilibrium outcomes are interpreted as the scenario where they fail to do so.

3 Coordinated Pollution Control

As a reference scenario we look at the cooperative outcome of the game. Any cooperative outcome constitutes a first best solution but requires preplay communication that is not explicitly modeled in this paper.

To characterize first best solutions for our model we impose some additional assumptions on the functional forms of preferences and technologies.[2] In particular, we assume that the cost functions, C_i, are quadratic,

$$C_i(P(t)) = \frac{s}{2}P^2(t)$$

[2]Our specification of explicit functional forms is motivated by the choice of the strategy spaces for the non-cooperative game (Markov-perfect strategy spaces). Only for specific classes of differential games existence and other qualitative properties of Markov-perfect equilibria can be derived analytically.

where $s > 0$, and that the preferences of consumers and the emission consumption trade-off functions are such that $U_i(F_i(E_i(t)))$ becomes quadratic, i.e.,

$$U_i(F_i(E_i(t))) = AE_i(t) - \frac{1}{2}E_i^2(t)$$

where $A > 0$.

With these simplifications our pollution model becomes a linear quadratic game, i.e.,

$$\max_{E_i} \int_0^\infty e^{-rt}[AE_i(t) - \frac{1}{2}E_i^2(t) - \frac{s}{2}P^2(t)]dt \tag{6}$$

subject to

$$\dot{P}(t) = E_1(t) + E_2(t) - kP(t), \quad P_0 \text{ given.} \tag{7}$$

To solve for the cooperative game we make use of Pontryagin's maximum principle (see Leonard and Long [1991]). In particular, we calculate the collusive solution that is obatined by maximizing joint welfare of both countries. This amounts to solving a simple one state variable optimal control problem. We formulate the current value Hamiltonian[3]

$$H(E_1, E_2, P, \lambda) = A(E_1 + E_2) - \frac{1}{2}(E_1^2 + E_2^2) - sP^2 + \lambda[E_1 + E_2 - kP], \tag{8}$$

where λ is the current value adjoint variable.

It is easily shown that the collusive game admits a unique solution that is characterized by the following necessary and sufficient conditions: (i) the maximum conditions

$$H_{E_1} = 0 \;\Rightarrow\; A + \lambda = E_1, \tag{9}$$

$$H_{E_2} = 0 \;\Rightarrow\; A + \lambda = E_2, \tag{10}$$

(ii) the adjoint equation

$$\dot{\lambda} = (r + k)\lambda + 2sP, \tag{11}$$

and (iii) the transversality condition

$$\lim_{t\to\infty} \lambda(t)P(t) = 0. \tag{12}$$

[3]From now on, unless otherwise stated we suppress the time argument t.

The necessary conditions can be summarized in the canonical equations given by

$$\begin{aligned} \dot{P} &= 2A - kP + 2\lambda \qquad (13) \\ \dot{\lambda} &= 2sP + (r+k)\lambda \qquad (14) \end{aligned}$$

with the initial condition P_0 and the boundary condition (12). The unique collusive solution has to satisfy system (13) and (14).

Proposition 1 *There exists a unique globally and asymptotically stable collusive outcome that results in a steady state pollution stock given by*

$$P^C = \frac{2A(r+k)}{k(r+k)+4s}. \qquad (15)$$

Net benefits are given by

$$W_1 = W_2 = -\frac{1}{4}\alpha_c P_0^2 - \frac{1}{2}\beta_c P_0 - \frac{\mu_c}{2}, \qquad (16)$$

where

$$\begin{aligned} \alpha_c &= \frac{1}{2}[-(k+\frac{r}{2}) + \sqrt{(k+\frac{r}{2})^2 + 4s}], \qquad (16\text{ a}) \\ \beta_c &= \frac{2A\alpha_c}{r+k+2\alpha_c}, \qquad (16\text{ b}) \\ \mu_c &= -\frac{(\beta_c - A)^2}{r}. \qquad (16\text{ c}) \end{aligned}$$

Proof: It is easily seen that system (13) and (14) admits a unique steady state that is saddle-point stable. The steady state pollution stock (15) can be calculated as the solution to the equation system $\dot{P} = \dot{\lambda} = 0$. Discounted net welfare for each country can be calculated using dynamic programming techniques. We define the value function as

$$\frac{1}{2}W(P_0) = \max \int_0^\infty e^{-rt}[AE_i(t) - \frac{1}{2}E_i^2(t) - \frac{s}{2}P^2(t)]dt. \qquad (17)$$

The value function has to satisfy the dynamic programming equation

$$rW(P) = \max_{E_1,E_2}\{A(E_1+E_2) - \frac{1}{2}(E_1^2+E_2^2) - sP^2 + W'(P)[E_1+E_2-kP]\}. \qquad (18)$$

Given the linear quadratic structure of the dynamic game we guess a quadratic value function

$$W(P) = -\frac{1}{2}\alpha_c P^2 - \beta_c P - \mu_c$$

Comparing coefficients leads to the desired results (16). In (16a), the positive root is choosen as required by the saddle-point stability. **Q.E.D.**

The first best solution determines the reference levels of the long-run pollution stock and welfare that can be achieved through international coordination of pollution controls. This outcome, however, is not structurally stable, i.e., the collusive outcome does not admit an equilibrium property. Hence, unilateral deviations can make one of the players better off. Therefore the problem arises as to what mechanism to use to ensure that the collusive outcome is sustained by a pair of equilibrium strategies. If it is a Nash equilibrium then there are no incentives for individual players to unilaterally deviate from the agreement.

The literature on supergames (see for example Friedman [1986]) provides some guidelines. The enforcement for both countries to stick to their initially agreed upon cooperative strategies can be achieved through credible threats and retaliation summarized in so called trigger strategy equilibria. These equilibria are subgame perfect and are implemented in the following way: As long as a player's opponent plays the cooperative strategy the other player honors this behavior by playing the cooperative strategy himself. If the opponent deviates the other player punishes this behavior by switching to any non-cooperative strategy that constitutes a credible threat, i.e., really makes the opponent worse off and hence punishes him.

In this paper we do not follow the route of constructing trigger strategy equilibria. Instead, we allow the countries to play nonlinear Markov-perfect strategies and demonstrate that such choices can support first best solutions as equilibrium outcomes. The nonlinear strategies that we are going to derive are, however, not global strategies, i.e., are not defined globally for any initial value of the state space. But we will characterize the subsets of the state space for which they can be derived. Before discussing the nonlinear strategies we calculate a unique linear Markov-perfect equilibrium for our game that is supported by a pair of global strategies.

4 Non-Cooperative Pollution Control

In the preceeding section we have characterized the first best solution that can be achieved if both countries are able to reach a pollution control agreement. In this section we look at the non-cooperative game and characterize its equilibrium outcomes. This requires a detailed description of the strategy spaces available for the two players in the game. Here we choose that both goverments use Markov strategies. This choice implies that the corresponding outcomes are subgame-perfect and do not require any commitment on the part of either government. Formally, Markov-perfect strategies are decision rules that result in actions of the respective players conditional on the current state variable that summarizes the latest available information of the dynamical system. Markov strategy spaces for differential games are defined as [4]

$$\begin{aligned} S_i^{MP} &= \{E_i(P(t),t) \mid E_i(P(t),t) \text{ is Lipschitz-continuous w.r.t. } P(t) \\ &\quad \text{and continuous w.r.t. } t\} \end{aligned}$$

Markov-perfect equilibria are subgame-perfect equilibria in which players use Markov strategies. They can be derived through the application of dynamic programming techniques. In what follows we allow the governments to play linear as well as non-linear Markov-perfect equilibrium strategies.

4.1 Linear Markov Strategies

When players are restricted to linear Markov strategies they design their equilibrium strategies as linear state dependent decision rules, i.e.,

$$E_i(P(t)) = \alpha_i P(t) + \beta_i,$$

where α_i and β_i are constants that are independent of the initial conditions of the game.

[4]In the traditional differential games literature (see Başar and Olsder [1982]) Markov strategies are referred to as feedback or closed-loop (subgame-perfect) strategies. For a discussion of Markov strategies, see Maskin and Tirole (1988, especially p. 553).

Proposition 2 *There exists a unique pair of linear Markov strategies that constitute an asymptotically stable Markov-perfect equilibrium that results in a steady state pollution stock given by*

$$P^{MP} = \frac{2A(r+k+\alpha_m)}{(k+2\alpha_m)(r+k+3\alpha_m)}. \tag{19}$$

The steady state pollution level corresponding to the non-cooperative Markov-perfect equilibrium exceeds that of the cooperative game.

Discounted net welfare in the non-cooperative game is given by

$$W_1 = W_2 = -\frac{1}{2}\alpha_m P_0^2 - \beta_m P_0 - \mu_m, \tag{20}$$

where

$$\begin{aligned} \alpha_m &= \frac{1}{3}[-(k+\frac{r}{2}) + \sqrt{(k+\frac{r}{2})^2 + 3s}], \\ \beta_m &= \frac{2A\alpha_m}{r+k+3\alpha_m}, \\ \mu_m &= -\frac{(\beta_m - A)^2}{2r} + \frac{(\beta_m - A)(A - 2\beta_m)}{r}. \end{aligned}$$

Proof: As noted, Markov-perfect equilibria can be derived through dynamic programming techniques. They have to satisfy the dynamic programming equations (see Basar and Olsder [1982]):

$$rW_i(P) = \max_{E_i}\{AE_i - \frac{1}{2}E_i^2 - \frac{s}{2}P^2 + W_i'(P)[E_i + E_j - kP]\}. \tag{21}$$

Given the linear-quadratic structure of the game we restrict attention to the class of quadratic value-functions $W_i(P)$:

$$W_i(P) = -\frac{1}{2}\alpha_m P^2 - \beta_m P - \mu_m, \tag{22}$$

where α_m, β_m and μ_m are to be determined. Maximization of the right hand side of (21) yields

$$E_i = A - \alpha_m P - \beta_m. \tag{23}$$

Substituting the value function and the maximization condition into the Bellman equations results in

$$-\frac{1}{2}\alpha_m r P^2 - r\beta_m P - r\mu_m = A(A - \alpha_m P - \beta_m) - \frac{1}{2}(A - \alpha_m P - \beta_m)^2 - \frac{s}{2}P^2 - (\alpha_m P + \beta_m)[2(A - \alpha_m P - \beta_m) - kP] \quad (24)$$

Since the last equation has to hold for any level of P we get

$$\alpha_m = \frac{1}{3}[-(k+\frac{r}{2}) \pm \sqrt{(k+\frac{r}{2})^2 + 3s}], \quad (24\text{ a})$$

$$\beta_m = \frac{2A\alpha_m}{r+k+3\alpha_m}, \quad (24\text{ b})$$

$$\mu_m = -\frac{(\beta_m - A)^2}{2r} + \frac{(\beta_m - A)(A - 2\beta_m)}{r}. \quad (24\text{ c})$$

To guarantee stability of the steady state pollution stock we restrict our analysis to the positive root for α_m. The steady state stock level is given by

$$P^{MP} = \frac{2A(r+k+\alpha_m)}{(k+2\alpha_m)(r+k+3\alpha_m)}. \quad (25)$$

It is now easily seen that the non-cooperative stock level exceeds the cooperative one.[5] **Q.E.D.**

The last result has the following implications. Firstly, it demonstrates that non-cooperative behavior of the two governments results in higher long-run pollution levels than coordination of pollution control. Secondly, non-cooperative behavior modeled as linear Markov-perfect equilibrium is inefficient. This inefficiency is an immediate consequence of the stock as well as the strategic externality that are present in the model (equation (2)). When the two countries play linear Markov-perfect equilibrium strategies

[5] From (24), $s = 3\alpha_m^2 + (2k+r)\alpha_m$. Therefore

$$P^{MP} = \frac{2A(r+k+\alpha_m)}{k(r+k) + 4s - 6\alpha_m^2 - 2\alpha_m r - 3\alpha_m k} > P^C.$$

their behavior is respresented by the decision rules (23). These rules imply that with an increase (decrease) in the stock $P(t)$ optimal emissions and hence production should be decreased (increased). Suppose that country 1 finds it optimal to drastically reduce its emissions. This causes the level of the pollution stock to decline. Since a clean environment in this model is a public good the neighbor country benefits from this decline in pollution and reacts according to the decision rule (23) by producing more and hence increasing the level of pollutants. In long-run equilibrium this behavior results in a higher steady state pollution stock.

4.2 Non-Linear Markov Strategies

The linear Markov-perfect strategy characterized in the preceeding section is not the only Markov-perfect equilibrium in differentiable strategies. In this section we demonstrate (following Tsutsui and Mino [1990]) that our pollution game admits non-linear Markov equilibria. These set of non-linear equilibria turns out to support a wide range of long-run pollution stock levels including the Pareto-efficient one.

As noted above a Markov-perfect equilibrium has to satisfy the Bellman equations

$$rW_i(P) = \max_{E_i}\{AE_i - \frac{1}{2}E_i^2 - \frac{s}{2}P^2 + W_i'(P)[E_i + E_j - kP]\}. \qquad (26)$$

Using the maximization condition $E_i - A = W_i'(P)$ and looking at symmetric solutions we get

$$rW_i(P) = \{AE_i - \frac{1}{2}E_i^2 - \frac{s}{2}P^2 + (E_i - A)[2E_i - kP]\}. \qquad (27)$$

Temporarily assuming that the discount rate $r = 0$, the Bellman equation (27) becomes the following quadratic equation in E_i (with the subscript dropped for simplicity):

$$0 = AE - \frac{1}{2}E^2 - \frac{s}{2}P^2 + (E - A)[2E - kP] \qquad (28)$$

with two non-linear solutions

$$E_{1,2} = \frac{1}{3}(A + kP) \pm \sqrt{(A + kP)^2/9 + sP^2/3 - 2AkP/3}. \qquad (29)$$

The last result, in principle, demonstrates the characteristics of the Markov-perfect equilibria that we are going to derive in this section. Firstly, the solutions (29) are highly non-linear and continuously differentiable in the state variable P and secondly, they are not defined globally. For (29) to be well defined we have to assume that $(A+kP)^2/9+sP^2/3-2AkP/3 \geq 0$ holds. This results in a restriction of the state space, i.e., feasible pollution levels.

Guided by the results for $r=0$ we guess a non-linear decision rule for the general case ($r>0$) as

$$E(P)=\frac{1}{3}(A+kP)+h(P), \tag{30}$$

with $h(P)$ a nonlinear function in P. Substituting (30) into the Bellman equation (27) yields

$$rW(P)=-\frac{1}{9}A^2-\frac{1}{18}A^2-\frac{3}{18}k^2P^2-\frac{s}{2}P^2+\frac{12}{18}AkP+\frac{3}{2}h^2(P). \tag{31}$$

Assuming that the value-function $W(P)$ is continuously differentiable and differentiating (31) with respect to P yields

$$r[-\frac{2}{3}A+\frac{k}{3}P+h(P)]=-\frac{3}{9}k^2P-sP+\frac{2}{3}Ak+3h(P)h'(P), \tag{32}$$

which upon rewriting gives

$$3h(P)h'(P)=rh(P)+P[\frac{rk}{3}+\frac{1}{3}k^2+s]-\frac{2A}{3}(k+r). \tag{33}$$

Introducing the definition of variables $F=\frac{rk+k^2+3s}{3}$ and $C=\frac{2A(k+r)}{3}$ equation (33) becomes

$$h'(P)=\frac{rh(P)+PF-C}{3h(P)}. \tag{34}$$

(34) is a simple first order nonlinear differential equation in P that characterizes non-linear Markov-perfect equilibria. In what follows we are interested in solutions to equation (34). Note that if $r=0$ than $h(P)=-\sqrt{(A+kP)^2/9+sP^2/3-2AkP/3}$ is a solution to (34), hence (30) is identical to (29) in this case.

Equation (34) can be rewritten to yield

$$\frac{dh}{dP} = \frac{rh + PF - C}{3h}. \tag{35}$$

To solve the last equation we introduce a change of variables

$$X = P - \frac{C}{F} \tag{36}$$

so that (35) becomes

$$\frac{dh}{dX} = \frac{FX + rh}{3h}. \tag{37}$$

The last differential equation is a familiar type of equation that can be solved explicitly when introducing $Z = \frac{h}{X}$. This implies

$$\frac{dZ}{dX}X = \frac{F + rZ - 3Z^2}{3Z}, \tag{38}$$

or equivalently

$$\frac{1}{X}dX = \frac{3Z}{F + rZ - 3Z^2}dZ. \tag{39}$$

A solution of (39) is given by

$$K = (h - XZ_a)^{\xi_1}(h - XZ_b)^{\xi_2} \tag{40}$$

where K is a constant of integration,

$$Z_a = \frac{r}{6} + \sqrt{\frac{r^2}{36} + \frac{F}{3}}, \quad Z_b = \frac{r}{6} - \sqrt{\frac{r^2}{36} + \frac{F}{3}},$$

and

$$\xi_1 = -1 - \xi_2, \quad \xi_2 = \frac{-Z_b}{Z_b - Z_a}.$$

Solution (40) can be rewritten to yield

$$K = [E - \frac{A}{3} - \frac{k}{3}P - PZ_a + Z_a\frac{C}{F}]^{\xi_1}[E - \frac{A}{3} - \frac{k}{3}P - PZ_b + Z_b\frac{C}{F}]^{\xi_2}. \tag{41}$$

The set of integral curves given by (41) includes two singular solutions given by

$$E_a = (Z_a + \frac{k}{3})P + \frac{A}{3} - Z_a\frac{C}{F} \quad (42)$$

$$E_b = (Z_b + \frac{k}{3})P + \frac{A}{3} - Z_b\frac{C}{F} \quad (43)$$

as well as an uncountable number of hyperbolic curves. Graphically the integral curves are depicted in Figure 1. The two straight lines E_a and E_b correspond to the two singular solutions. The integral curves of Figure 1 are drawn under the assumption of s being sufficiently large. In the limiting case of $s = 0$ (no costs of environmental pollution) our game admits a unique stable and efficient equilibrium given by $E_1 = E_2 = A$ and the maximum sustainable stock of pollution $P = \frac{2A}{k}$. (E_b is the horizontal line $E = A$, if $s = 0$.)

Every integral curve in Figure 1 corresponds to a nonlinear Markov equilibrium. Integral curves crossing through the steady state curve SS (defined by $2E = kP$) support different levels of steady state pollution stocks. From those different levels of steady state stocks we are interested in those that can be supported as asymptotically stable ones.

Proposition 3 *Any pollution level in the interval*

$$\frac{12rA + 36C - 6Ak}{6kr + 36F - 3k^2} < P^{\infty} < \frac{2A}{k}$$

can be supported as an asymptotically stable steady state.

Proof: As derived, non-linear Markov strategies are given by (30) where $h(P)$ corresponds to a solution curve of (40). With these nonlinear strategies the state equation (2) becomes

$$\dot{P} = 2E(P) - kP.$$

Linearizing the last equation around the steady state $P^{\infty} = \frac{2E(P^{\infty})}{k}$ gives the stability condition

$$E'(P^{\infty}) < \frac{k}{2} \quad (44)$$

Using (30) this condition can be rewritten to yield

$$\frac{-2rA + krP^{\infty} + 6P^{\infty}F - 6C}{-6A + 3kP^{\infty}} < \frac{k}{6}. \tag{45}$$

Keeping in mind that the maximum sustainable pollution stock is $P^{\infty} = \frac{2A}{k}$ the last inequality implies the result. **Q.E.D.**

This last proposition has an important economic implication. It turns out that in the limiting case when r tends to zero the collusive long-run pollution stock can be supported as a steady state of nonlinear differentiable Markov strategies, i.e., $P^{\infty} = \frac{2Ak}{k^2+4s}$. These nonlinear strategies can be reached from the following domains of initial pollution stock. Nonlinear strategies from the region I in Figure 1 can be reached from initial conditions lying in the interval $[0, P_b)$. Strategies from region II can be reached by initial conditions lying in the interval $(P_b, P_o]$. This characterizes the local nature of the nonlinear Markov strategies.

What remains to be shown, however, is that the above constructed nonlinear differentiable Markov-perfect strategies constitute equilibrium strategies.

Proposition 4 *For each $E(P) = \frac{1}{3}(A + kP) + h(P)$ given by (41) the function $W(P)$ defined by*

$$W(P) = \frac{1}{r}\{AE(P) - \frac{1}{2}E^2(P) - \frac{s}{2}P^2 + (E(P) - A)[2E(P) - kP]\}$$

is a twice differentiable value-function that generates nonlinear Markov-perfect equilibria that support steady state pollution stock levels from the interval

$$\frac{12rA + 36C - 6Ak}{6kr + 36F - 3k^2} < P^{\infty} < \frac{2A}{k}$$

Proof: A proof of this proposition follows along the arguments outlined in Tsutsui and Mino [1990] and will not be repeated here. **Q.E.D.**

The last proposition establishes multiplicity as well as existence of nonlinear Markov-perfect strategies. The driving force behind this non-uniqueness result is the incomplete transversality condition as well as the infinite horizon of the dynamic game. It is a well known result (Folk-Theorem) that in infinite horizon dynamic games any individually rational payoff vector can be supported as a subgame perfect equilibrium.

The economic interpretation of our results on nonlinear Markov-perfect equilibria in the model of international pollution control is as follows: Strictly speaking they imply that if the discount rate is sufficiently low, the use of non-linear Markov strategies can be a "substitute" for coordinating environmental policies. Theoretically this is an appealing result but what is its policy relevance (normative conclusion)? Can we really infer that the best environmental policy is laissez-faire? Certainly not! Firstly, our approach makes use of a lot of idealizing assumptions (deterministic model, complete information, symmetry, etc.) that are hardly met. Secondly, the non-uniqueness result imposes a non-trivial equilibrium selection problem for the two governments. In our case there is an additional difficulty due to the local nature of the nonlinear Markov equilibria. It is only if both players choose mutually consistent strategies from the uncountable set of equilibrium strategies that they maximize welfare. Finally, for agents with a high enough discount rate policy coordination is still Pareto-superior to a laissez-faire policy.

5 Conclusion

In this paper we presented a dynamic game that was interpreted as a model of international pollution control. We characterized cooperative as well as non-cooperative strategies that were identified as coordinated and uncoordinated environmental policies. It turned out that for the class of Markov-perfect equilibria in linear strategies non-cooperation results in high long-run pollution stocks as well as in overall inefficiences. If, on the contrary, agents use nonlinear Markov strategies and have a low discount rate Pareto-efficient pollution stock levels can be supported as a long-run equilibrium. Theoretically this is a remarkable result and demonstrates that cooperation can be brought about through an appropriate choice of Markov-strategies without using institutional arrangements like threats, etc. Application of this result to actual environmental policies requires some degree of caution. As pointed out above, it would be too simplistic to argue that based on our results laissez-faire is the best possible environmental policy.

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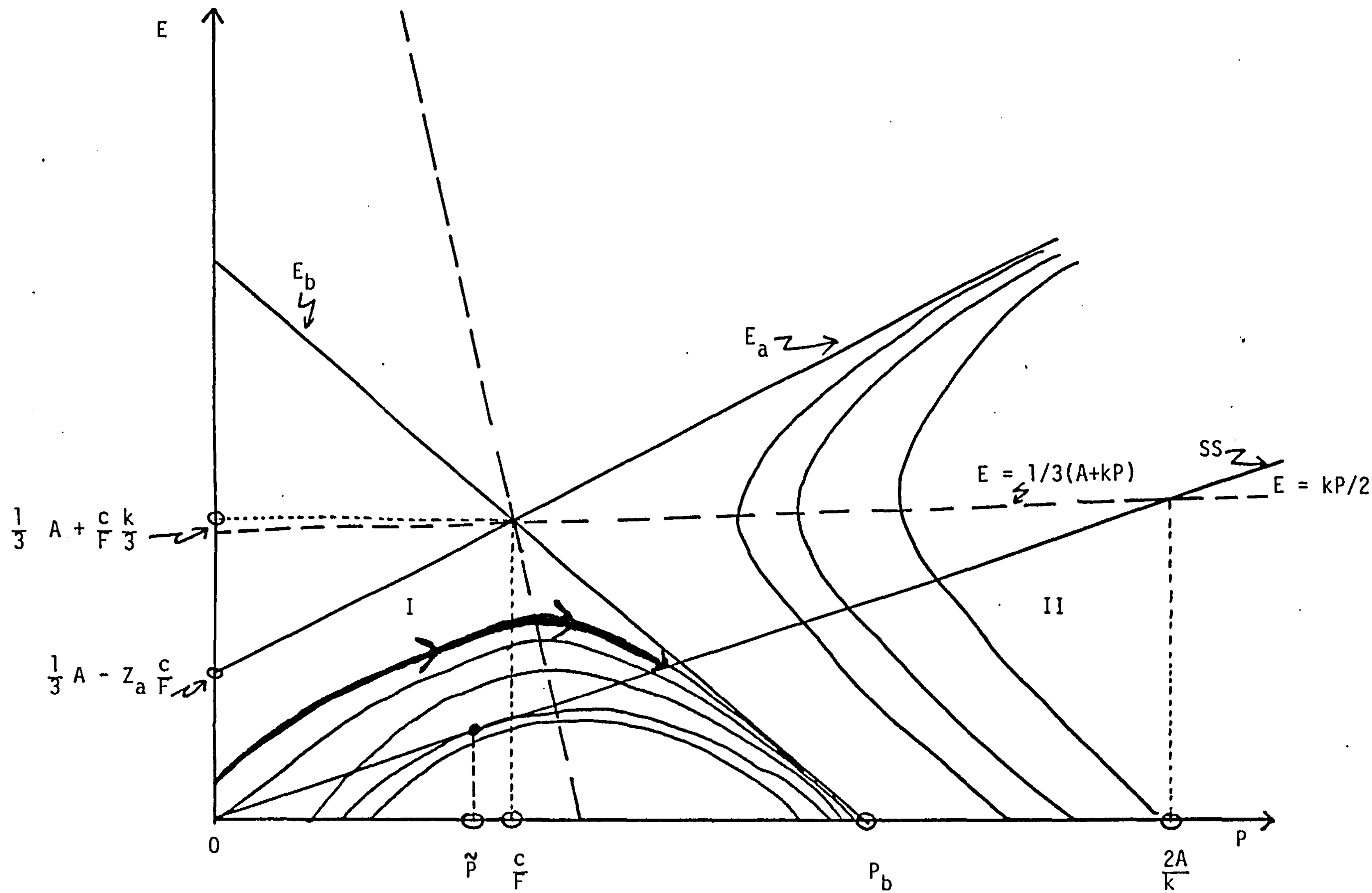

FIGURE 1: $\left[\tilde{P}, \frac{2A}{k}\right]$ Range of Stable Steady States

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