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# International Portfolios: A Comparison of Solution Methods\*

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## Abstract

We compare the performance of the perturbation-based (local) portfolio solution method of Devereux and Sutherland (2010a, 2011) with a global solution method. We find that the local method performs very well when the model is designed to capture stylized macroeconomic facts and countries/agents are symmetric, i.e. when the latter have similar size, face similar risks and trade assets with similar risk properties. It performs less satisfactory when the agents engaged in financial trade are asymmetric. The global solution method performs substantially better when the model is parameterized to match the observed equity premium, a key stylized finance fact.

*Keywords:* Country Portfolios, Solution Methods

*JEL-Codes:* E44, F41, G11, G15

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# 1 Introduction

This paper presents and evaluates two solution methods for computing optimal portfolios in incomplete markets general equilibrium settings: the local, i.e. perturbation-based, approach of Devereux and Sutherland (2010a, 2011) (hereafter ‘DS’) and a global solution approach. The DS method is a user-friendly way of solving for optimal portfolios and is easy to incorporate into standard dynamic stochastic general equilibrium (DSGE) models. It provides readily-applicable formulas for optimal constant portfolios (‘steady-state portfolios’) and first-order portfolio dynamics derived using standard local approximation around a non-stochastic steady state.

The DS method, based on the earlier work by Samuelson (1970) and Judd and Guu (2001), together with the noteworthy contributions by Tille and van Wincoop (2007) and Evans and Hnatkowska (2005, 2012) have been path-breaking in many aspects: for most of their existence, general equilibrium international macro models have ignored portfolios altogether.<sup>1</sup> Typically, models featured either incomplete financial markets with only one asset or complete financial markets. In the former case, only net capital flows could be analyzed. In the case with complete financial markets the behavior of macro variables and portfolios are independent. So, the optimal portfolios are typically ‘backed out’ after the optimal allocation has been computed. This prompted interest in models with incomplete financial markets and multiple assets.

The solution approach developed in Devereux and Sutherland (2011, 2010a) applies to models with both complete and incomplete markets and it is based on perturbation techniques, that are commonly used in macroeconomics. This methodological progress allows answering a number of important questions in international macroeconomics: the existence of substantial gross external positions and their rapid growth in recent decades, the increasing empirical importance of two-way asset trade, the role of portfolio re-balancing in determining net capital flows, and the potential influences of size and composition of gross portfolios on macroeconomic outcomes themselves through exchange rate and asset price driven ‘valuation effects’.

Because of its simplicity the DS method has been widely used in the recent (international) macroeconomics literature.<sup>23</sup> Despite the wide-spread adoption, little is known about its accuracy and, therefore, the ‘domain’ of applicability. Our paper tries to fill this gap. To this end, we perform a comprehensive evaluation of the DS solution and the global solution methods. We compare policy functions, simulated short time paths, moments from simulated data, stationary distributions, and Euler equation errors of the DS and the global solutions.

The global solution method is more accurate but it may also be slower and/or limited

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<sup>1</sup>An earlier literature, of the late 1970s and early 1980s, looks at portfolio balance models (e.g. see the review in Branson and Henderson (1985)). Those were, however, typically cast in a partial equilibrium setup. There is also a more recent literature on (also partial equilibrium) country portfolios in continuous time (see Kraay and Ventura (2000, 2003)), and (Kraay et al. (2005)).

<sup>2</sup>For example, see Devereux and Sutherland (2008), Devereux and Sutherland (2009), Coeurdacier et al. (2010), Devereux and Yetman (2010), Devereux and Sutherland (2010c), Devereux and Sutherland (2010b), Amdur (2010), Coeurdacier and Gourinchas (2011), Viani (2011), Nguyen (2011), Gertler et al. (2012), Benigno and Nistico (2012), Berriel and Bhattarai (2013), Berriel (2013), and Karadi et al. (2013).

<sup>3</sup>For this reason we choose the DS method instead of the method in Evans and Hnatkowska (2012). Evans and Hnatkowska (2012) report that their method is more accurate but they only test using symmetric settings where we find that perturbation solutions do perform well. For a comparison of different perturbation-based solution methods we refer the reader to Kazimov (2010).

to simple settings.<sup>4</sup> The perturbation-based method has its advantages. It can handle high-dimensional problems with ease. So, it can be applied to the relatively complex medium- to large-scale models used for macroeconomic policy analysis. It can also be helpful in establishing intuition for the mechanisms at work.

Our test suite consists of two models. The first model contains features that are typical for the ‘macroeconomics’ literature (‘model 1’). The second model contains features that capture key ‘financial’ aspects such as a sizeable equity premium (‘model 2’). Model 1 follows closely Devereux and Sutherland (2011). It is a two-country model with exogenous capital and labor income endowments, and financial markets trading claims to each country’s equity. We look both at symmetric and asymmetric country settings. In the asymmetric case we subject countries to shocks of different size. We find that the DS method performs extremely well in symmetric setups. In asymmetric setups, the DS method performs poorly when long-term (ergodic) properties of the model are of interest. This comes from a difficulty that is inherent in incomplete markets economies: model characteristics such as cross-country differences in shock volatilities fail to pin down aggregate wealth (the net foreign asset position) at the approximation point of the deterministic steady state. Instead, it is custom to choose as an approximation point the steady state wealth position that is pinned down by a purely technical device, for example, the endogenous discount factor. Arguably, asymmetric setups are relevant not only if one wants to study realistically calibrated open economy models (e.g. settings of advanced versus emerging economies), but even more so in any other non-open-economy-two-country settings, where heterogeneous agents face a non-trivial portfolio choice problem.<sup>5</sup> Nevertheless, we find that the DS method continues to perform well if only short simulated paths are considered.

The second model differs from the first in the following two respects. The two countries can now trade a risk-free bond (‘safe’) and a claim to an aggregate capital income endowment (‘risky’ financial asset). We also allow risk attitudes to differ across countries and we parameterize the model such as to obtain a sizeable equity premium, comparable in magnitude to the premia that we see in the data. Asset prices and returns contain information that is necessary for optimal portfolio allocation. Thus matching the observed equity premium brings the model portfolio problem closer to reality. Diverse risk attitudes translate into different willingness to hold the risky and the safe asset. The less-risk averse country is more willing to hold the high risk/high return equity: so, it buys a larger share of the risky asset and sells the safe asset. The excess return that the less-risk averse country earns on average, allows him to accumulate wealth. The global solution method captures this effect well. The DS method fares much worse. The equity premium generated by the DS method is smaller. More importantly, because the DS method takes as approximation point the non-stochastic steady state where excess returns are zero, it fails to capture the effect of the return differential on wealth accumulation. As a result, the dynamics of the net foreign asset position, hence the dynamics of all macroeconomic variables, obtained by the DS method differ substantially from those obtained using the global solution method.

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<sup>4</sup>Nevertheless, the continuing progress in computing power, as well as methodological advances (for example, see Judd et al. (2012). Judd et al. (2011b)) make the use of global solution methods increasingly feasible.

<sup>5</sup>As, for example, in a recent example from the literature on macroeconomic models with financial frictions, Gertler et al. (2012). In their paper banks’ liability side consists of either debt or risky outside equity (‘preferred stocks’), and a portfolio problem needs to be solved that is naturally asymmetric (consumers are creditors, banks debtors)

The paper is organized as follows. Section 2 describes our first test model 1. It closely resembles the model in Devereux and Sutherland (2009) and uses a typical ‘macroeconomic’ parametrization. Section 3 discusses the local (DS) and global solution methods. Section 4 describes the results of the first model, for setting with both symmetric and asymmetric countries. Section 5 introduces our second test model. It captures the most important ‘financial’ aspect of the data – a substantial ‘equity premium’. Section 6 describes the results of the second model. Section 7 concludes.

## 2 Model 1 with two equity claims

In this section we consider the model described in Devereux and Sutherland (2011). It has only essential features that let us demonstrate crucial differences between alternative solution techniques.

We start with a description of the economic environment. Uncertainty in the model is represented by four exogenous stochastic processes:  $\{Y_{ht}^k, Y_{ht}^l, Y_{ft}^k, Y_{ft}^l\} \equiv \mathbf{Y}_t$ . They model home capital income, home labor income, foreign capital income and foreign labor income. All of the above are first-order autoregressive processes:

$$\log \left( Y_{ht}^k / \bar{Y}_h^k \right) = \rho_h^k \log \left( Y_{ht-1}^k / \bar{Y}_h^k \right) + \varepsilon_{ht}^k, \quad (1a)$$

$$\log \left( Y_{ht}^l / \bar{Y}_h^l \right) = \rho_h^l \log \left( Y_{ht-1}^l / \bar{Y}_h^l \right) + \varepsilon_{ht}^l, \quad (1b)$$

$$\log \left( Y_{ft}^k / \bar{Y}_f^k \right) = \rho_f^k \log \left( Y_{ft-1}^k / \bar{Y}_f^k \right) + \varepsilon_{ft}^k, \quad (1c)$$

$$\log \left( Y_{ft}^l / \bar{Y}_f^l \right) = \rho_f^l \log \left( Y_{ft-1}^l / \bar{Y}_f^l \right) + \varepsilon_{ft}^l, \quad (1d)$$

where  $\{\varepsilon_{ht}^k, \varepsilon_{ht}^l, \varepsilon_{ft}^k, \varepsilon_{ft}^l\}$  is a vector of i.i.d. innovations with zero mean and a finite support. We assume that  $\text{cor}(\varepsilon_{ht}^k, \varepsilon_{ft}^k) = \text{cor}(\varepsilon_{ht}^l, \varepsilon_{ft}^l) = 0$  but do not exclude (contemporaneous) dependence between other innovations.

The aggregate output in country  $a \in \{h, f\}$  is the sum of capital and labor income endowments:  $Y_{at} \equiv Y_{at}^k + Y_{at}^l$ .

Financial markets trade claims to home and foreign capital income streams. Let  $q_{at}$  be the price of a claim to a stream of capital income  $\{Y_{a\tau}^k\}_{\tau=t}^{\infty}$  produced in country  $a \in \{h, f\}$ . These prices will be sometimes referred to as countries’ stock market indexes.

The representative agent in country  $a \in \{h, f\}$  ranks different consumption plans  $\{c_{a\tau}\}_{\tau=t}^{\infty}$  according to:<sup>6</sup>

$$U_{at} \equiv E_t \sum_{\tau=t}^{\infty} \delta_{\tau} u(c_{a\tau}), \quad (2)$$

where  $c_{at}$  is consumption. Evolution of the endogenous discount factor  $\delta_{\tau}$  is as follows:

$$\delta_{\tau+1} = \delta_{\tau} \beta(\bar{c}_{a\tau}), \quad \delta_0 = 1,$$

where  $\bar{c}_{at}$  is the average consumption in country  $a$  in period  $t$ . The discount factor function  $\beta : R_+ \rightarrow [0, 1)$  is non-increasing. If  $\beta(\cdot)$  were a constant function and financial markets were

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<sup>6</sup>Leisure is not valued.

- (A1):  $q_{ht}u_{cht} = \beta(c_{ht})E_t u_{cht+1}(q_{ht+1} + Y_{ht+1}^k)$ ,  
(A2):  $q_{ft}u_{cht} = \beta(c_{ht})E_t u_{cht+1}(q_{ft+1} + Y_{ft+1}^k)$ ,  
(A3):  $q_{ht}u_{cft} = \beta(c_{ft})E_t u_{cft+1}(q_{ht+1} + Y_{ht+1}^k)$ ,  
(A4):  $q_{ft}u_{cft} = \beta(c_{ft})E_t u_{cft+1}(q_{ft+1} + Y_{ft+1}^k)$ ,  
(A5):  $c_{ht} + c_{ft} = Y_{ht} + Y_{ft}$ ,  
(A6):  $c_{ht} + \theta_{ht}^h q_{ht} + \theta_{ft}^h q_{ft} = \theta_{ht-1}^h (q_{ht} + Y_{ht}^k) + \theta_{ft-1}^h (q_{ft} + Y_{ft}^k) + Y_{ht}^l$ ,  
(A7):  $\theta_{ht}^h + \theta_{ht}^f = 1$ ,  
(A8):  $\theta_{ft}^h + \theta_{ft}^f = 1$ .

Table 1: System of equilibrium conditions, model 1.

incomplete then in a local solution, that is based on a first-order Taylor series approximation, countries' net financial positions would be non-stationary.<sup>7</sup>

Unless noted otherwise, we assume that the utility function is of the constant relative risk aversion class:  $u(c_{at}) = c_{at}^{1-\gamma}/(1-\gamma)$ .

The representative agent in country  $a$  maximizes his life-time utility (2) subject to the budget constraint:

$$c_{at} + \theta_{ht}^a q_{ht} + \theta_{ft}^a q_{ft} = \theta_{ht-1}^a (q_{ht} + Y_{ht}^k) + \theta_{ft-1}^a (q_{ft} + Y_{ft}^k) + Y_{at}^l, \quad (3)$$

where  $\theta_{ht}^a$  and  $\theta_{ft}^a$  denote country  $a$ 's purchases of domestic and foreign equity claims.

The goods market clearing condition is:

$$c_{ht} + c_{ft} = Y_{ht} + Y_{ft}. \quad (4)$$

Market clearing conditions for the two traded assets are:

$$\theta_{ht}^h + \theta_{ht}^f = 1, \quad (5a)$$

$$\theta_{ft}^h + \theta_{ft}^f = 1. \quad (5b)$$

Table 1 summarizes the set of the model's equilibrium conditions.

### 3 Global and local solution methods

In the following we provide a description of global and local numerical solution methods.

#### 3.1 Global solution method

Following Judd et al. (2011a), Kubler and Schmedders (2003), and Stepanchuk and Tsyrennikov (2011) we recast the above equilibrium conditions in a form that is consistent with a wealth-recursive equilibrium. So, the dimensionality of the problem is reduced as the model's only endogenous state variable is the wealth share,  $\omega_t$ . More precisely, this transformed state variable expresses the domestic country's financial wealth share in total (world) financial wealth, which can be written as:

<sup>7</sup>That is, the solution allows reaching financial positions that are known to be infeasible.

$$\omega_t = \frac{\theta_{ht-1}^h(q_{ht} + Y_{ht}^k) + \theta_{ft-1}^h(q_{ft} + Y_{ft}^k) + Y_{ht}^l}{q_{ht} + Y_{ht} + q_{ft} + Y_{ft}}, \quad (6)$$

Using the definition in (6), we can rewrite the budget constraint of the home economy:

$$c_{ht} + \theta_{ht}^h q_{ht} + \theta_{ft}^h q_{ft} = (q_{ht} + Y_{ht} + q_{ft} + Y_{ft})\omega_t. \quad (7)$$

The above and equation (6) replace (A6) in the original system of equilibrium conditions.

Let  $\mathbf{Y}$  and  $\omega$  denote respectively current, date  $t$ , values of the exogenous income states and the wealth share. We start the iterative algorithm by guessing  $q_h(\omega; \mathbf{Y})$ ,  $q_f(\omega; \mathbf{Y})$ ,  $c_h(\omega; \mathbf{Y})$ , and  $c_f(\omega; \mathbf{Y})$ . We then use these guesses to solve for next period values of  $\omega'$  for each possible next-period realization of  $\mathbf{Y}'$ :

$$\omega' = \frac{(q_h(\omega', \mathbf{Y}') + Y_h^{k'})\theta_h^h + (q_f(\omega', \mathbf{Y}') + Y_f^{k'})\theta_f^h + Y_h^l}{q_h(\omega', \mathbf{Y}') + Y_h^l + q_h(\omega', \mathbf{Y}') + Y_f^l}, \quad (8)$$

Using (8) and the guessed policy functions we can compute the expectations in the Euler equations (A1-A4). This leaves us with a system of 8 non-linear conditions that can be solved for 8 current equilibrium variables:  $c_{ht}, c_{ft}, \theta_{ht}^h, \theta_{ft}^h, \theta_{ht}^f, \theta_{ft}^f, q_{ht}, q_{ft}$ . We do so for a range of  $(\omega, \mathbf{Y})$  and use the solution to compute an update for the guessed policy functions. We continue this procedure until convergence is achieved. Our stopping criterion is fulfilled when the relative distance between consecutive solution updates is less than a given threshold.

We choose 81 discretization points for  $\mathbf{Y}$ , three values for each element of the vector. We discretize the VAR process given in (1) as in Lkhagvasuren and Gospodinov (2011).<sup>8</sup> Finally, we choose 41 discretization points for our endogenous state variable,  $\omega$ .

### 3.2 Local solution method

To obtain a local (perturbation) solution we follow the method of Devereux and Sutherland (2011), henceforth DS. The DS method provides readily applicable solution formulas for the zero-order and first-order parts of an approximation to portfolio holdings, and has, because of its user-friendliness become widely used in recent contributions in macroeconomics. Other noteworthy contributions to solving portfolios with local approximation methods are Samuelson (1970), Judd and Guu (2001), Tille and van Wincoop (2007), and Evans and Hnatkovska (2005).<sup>9</sup> The DS perturbation solution method is straightforward to implement and in simple settings it is possible to obtain an analytic characterization of the approximate portfolio solution, which can be helpful for building intuition for the mechanisms at play. Its main

<sup>8</sup>A number of recent papers have shown that the widely used discretization approach described in Tauchen and Hussey (1991) can perform rather poorly when the number of discretization nodes is low or when underlying processes are very persistent: (Flodén (2006), Kopecky and Suen (2010)). For this reason we avoid using the Hussey-Tauchen procedure.

<sup>9</sup>The DS method relates to these other contributions in the following ways. In particular, it builds up on and extends the principles developed by Samuelson (1970) to a dynamic *general* equilibrium setting. The zero-order (steady state) solution of portfolio holdings obtained by the DS method is equivalent to the zero-order portfolio solution obtained by Judd and Guu's (2001) bifurcation approach to solving portfolios – yet, the Judd and Guu approach is not directly applicable to a dynamic setting. The DS solution method delivers an equivalent solution (for zero- and first-order portfolio holdings) as the iterative method by Tille and van Wincoop (2007). Finally, the DS method is quite different from Evans and Hnatkovska (2005), who combine perturbation methods with continuous-time approximations.



advantage is that it can be used in rich models, in the presence of several (endogenous) state variables.

We begin by re-stating the budget constraint of the home country as follows:

$$(\theta_{ht}^h - 1)q_{ht} + \theta_{ft}^h q_{ft} = (\theta_{ht-1}^h - 1)(q_{ht} + Y_{ht}^k) + \theta_{ft-1}^h (q_{ft} + Y_{ft}^k) + Y_t - c_{ht}. \quad (9)$$

Let  $(\alpha_{ht}^h, \alpha_{ft}^h) = ((\theta_{ht}^h - 1)q_{ht}, \theta_{ft}^h q_{ft})$  be *net* funds invested in home and foreign equity claims by the home country.<sup>10</sup> Net funds invested by the foreign country are:  $(\alpha_{ht}^f, \alpha_{ft}^f) = (\theta_{ht}^f q_{ht}, (\theta_{ft}^f - 1)q_{ft})$ . The asset market clearing conditions (5) are then replaced by:

$$\begin{aligned} \alpha_{ht}^h + \alpha_{ht}^f &= 0, \\ \alpha_{ft}^h + \alpha_{ft}^f &= 0. \end{aligned}$$

We can write the budget constraint of the home country in terms of  $\alpha$ 's:

$$\alpha_{ht}^h + \alpha_{ft}^h = \alpha_{ht-1}^h r_{ht} + \alpha_{ft-1}^h r_{ft} + Y_{ht} - c_{ht},$$

and asset returns:

$$r_{ht} = \frac{q_{ht} + Y_{ht}^k}{q_{ht-1}}, \quad r_{ft} = \frac{q_{ft} + Y_{ft}^k}{q_{ft-1}}.$$

The net foreign asset (NFA) position of country  $h$  then evolves according to the following law of motion:

$$\begin{aligned} W_{ht} &\equiv \alpha_{ht}^h + \alpha_{ft}^h \\ &= r_{ht} W_{ht-1} + \alpha_{ft-1}^h (r_{ft} - r_{ht}) + Y_{ht} - c_{ht}. \end{aligned} \quad (10)$$

The NFA position of the foreign country is  $W_{ft} = -W_{ht}$ . The solution – the country's policy functions and the price system – are functions of  $W_{ht}$  and exogenous shocks  $\mathbf{Y}$ . This is equivalent to using the home country's wealth share,  $\omega_t$ .<sup>11</sup> Generally, applying a perturbation method to a system of nonlinear difference equations consists of two steps. The first step is to construct a Taylor series approximation to the system of equilibrium conditions. The second step is to solve this system – this step is relatively standard across different models and order of approximation. It is the first step that poses problems when agents face a non-trivial portfolio choice. The reason for this complication is that the approximation point is typically chosen to be the solution to a deterministic version of a model. But in a deterministic setting all assets must yield the same return and thus are perfect substitutes. As a consequence, there is a continuum of solutions to a deterministic version as first emphasized in Judd and Guu (2001). DS show how to overcome this problem: they solve for the zero-order component of the portfolio solution by combining a first-order approximation to the 'macroeconomic part' of the model with a second-order approximation to the 'portfolio part', Euler equations A1-A4. A second-order approximation to Euler equations and a first-order approximation to the macroeconomic part are in general interdependent. But DS show that this simultaneous

<sup>10</sup>Net is relative to a portfolio of one unit of domestic equity and zero units of other claims. This is the convention used by DS.

<sup>11</sup>Because  $W_{ht}$  can be expressed as a function of  $\omega_t$ .

system can be used to obtain an analytical solution for the steady-state portfolios. Similarly, to solve for the first-order portfolio dynamics:

$$\alpha_t \simeq \alpha(\bar{x}) + \alpha'(\bar{x})\hat{x}_t \quad (11)$$

one should combine a second-order approximation to the 'macroeconomic part' with a third-order approximation to Euler equations. In the above expression,  $\hat{x}_t$  denotes the vector of state variables, in terms of percentage deviations from steady state (apart from NFA which is in terms of absolute deviations).

DS also state that their solution principle, which builds up on earlier work by Samuelson (1970), could be successively applied to higher orders: to obtain an  $n$ -th order accurate portfolio solution, one needs to approximate the portfolio optimality conditions up to order  $n + 2$ , in conjunction with an approximation to the model's other optimality and equilibrium conditions of order  $n + 1$ . E.g., going one order higher, one would obtain the approximate portfolio solution as  $\alpha_t = \bar{\alpha} + \alpha'\hat{x}_t + \frac{1}{2}\hat{x}_t'\alpha''\hat{x}_t$ .

It is important to realize that the expression in equation (11) is, however, not the same as what would result from a Taylor series expansion of the true policy function  $\alpha_t$ , around the deterministic steady state. Following Schmitt-Grohé and Uribe (2004) and Jin and Judd (2002) we can think of the true policy function in a recursive economy as a function that depends on the model's state variables,  $x_t$ , and on a parameter that scales the variance-covariance matrix of the model's exogenous shock processes,  $\varepsilon$ ; that is,  $\alpha_t = \alpha(x_t, \varepsilon)$ . A Taylor series to policy function  $\alpha_t$ , evaluated at approximation points  $x_t = \bar{x}$  and  $\varepsilon = 0$ , would then result in:

$$\alpha_t = \alpha(\bar{x}, 0) + \alpha_x(\bar{x}, 0)\hat{x}_t + \alpha_\varepsilon(\bar{x}, 0)\varepsilon + \frac{1}{2}\hat{x}_t'\alpha_{xx}(\bar{x}, 0)\hat{x}_t + \alpha_{x\varepsilon}(\bar{x}, 0)\hat{x}_t\varepsilon + \frac{1}{2}\alpha_{\varepsilon\varepsilon}(\bar{x}, 0)\varepsilon^2 + \dots \quad (12)$$

That is, in contrast to the Taylor series expansion in equation (12) the DS approximate portfolio solution does only consider how variations in the model's state variables affect the optimal portfolio solution, but ignores the effect of variations in the size of uncertainty.<sup>12</sup> In the general case of a dynamic model as in the present setup, this still does not imply that the size of uncertainty cannot have an effect on optimal portfolios. In principle there could be an effect of the size of uncertainty,  $\varepsilon$ , on the portfolio through the effect of  $\varepsilon$  on the states themselves. This, however, would only be happening at higher orders, as the (state) variables are not affected by  $\varepsilon$  at first-order (certainty equivalence) and only through a constant at second-order (see Schmitt-Grohé and Uribe (2004)).

We now draw attention to another problem that is not explicitly addressed in the description of the DS solution method. The problem arises because in general, without a stationarity inducing-device such as an endogenous discount factor, the steady-state NFA positions cannot be determined uniquely. Instead there exists a continuum of steady states, one for each assumed value of  $\bar{W}$ . It should be noted that this problem arises even when only one asset is

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<sup>12</sup>The comparison of the DS solution with equation (12) is simply for reasons of exposition. We are of course not suggesting that an approximate solution to the true unknown portfolio function actually can be obtained by taking a simple Taylor series expansion around the non-stochastic steady state. This is not feasible using standard local approximation methods (using the standard implicit function theorem) – the portfolio is indeterminate both at the non-stochastic steady state and in a first-order approximation of the stochastic setting. This is exactly the problem that the DS method (and Judd and Guu (2001) in their bifurcation approach) have addressed and proposed (different) ways of solving for.

traded and an explicit portfolio choice problem is absent.<sup>13</sup> But it is much more consequential in the setting with a non-trivial portfolio choice.

Since the DS method relies on a Taylor series approximation to the budget constraint around a steady-state value  $\bar{W}$ , the solution for steady-state portfolios depends on the assumed value of  $\bar{W}$ . A unique  $\bar{W}$  can be obtained, among other possibilities described in Schmitt-Grohé and Uribe (2003), by endogenizing the discount factor. However, it can be argued that this approach is not more satisfactory than simply postulating the desired level of  $\bar{W}$ . This issue is inconsequential for models with symmetric countries where  $\bar{W} = 0$  is a natural steady-state. But it may pose problems in models with asymmetric countries when the ergodic distribution of  $W_{ht}$  is not centered around zero. In the latter case it may seem appealing to use the mean value<sup>14</sup> of the ergodic distribution of  $W_{ht}$ , determined jointly with the steady-state portfolios  $\bar{\alpha}$ 's. This can be done in an iterative procedure that continuously refines  $\bar{W}$  as proposed by Devereux and Sutherland (2009). We evaluate this approach in section 4.4 and demonstrate that it can fare relatively poorly compared to the global solution.

Finally, we would like to emphasize another technical difficulty with the perturbation method. It arises when simulations are generated using second or higher order approximation to the model equilibrium system. In this case the dynamics of control variables are affected by higher than second order terms. These in turn feed into dynamics of the state. This can lead to explosive system dynamics because, as emphasized by Kim et al. (2003), these extra high-order terms in general do not correspond to high-order coefficients in a Taylor series approximation. A ‘stable simulation’ can be obtained by ‘pruning’ out extraneous high-order terms in each iteration by computing projections of second-order terms based on a first-order approximation. Our simulations obtained using the perturbation solution use ‘pruning’. Yet, the latter lacks theoretical justification and so it is merely a trick. See Kim et al. (2003), Den Haan and de Wind (2009), Lombardo (2010) and Lan and Meyer-Gohde (2013) for a discussion of advantages and disadvantages of ‘pruning’.

## 4 Results for model 1

We start our comparison of the two solution methods by analyzing the setting with symmetric countries. This facilitates the comparison because in this case it is ‘natural’ to assume that the NFA position in a steady state is zero:  $\bar{W} = 0$ . This is equivalent to assuming that both countries hold equal wealth shares:  $\bar{\omega} = 0.5$ . Section 4.2 repeats the analysis with asymmetric setups.

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<sup>13</sup>Consider a one-good two-country economy. Financial markets trade only a risk-free bond. Each period countries receive a deterministic endowment and decide how much to consume and save. The equilibrium conditions of this model:

$$\begin{aligned} q_{bt} &= \beta u_{cht+1}/u_{cht}, \\ q_{bt} &= \beta u_{cft+1}/u_{cft}, \\ q_{bt}b_{t+1} + c_{ht} &= b_{ht} + y_{ht}, \\ c_{ht} + c_{ft} &= y_h + y_f, \end{aligned}$$

determine the time paths of  $c_{ht}, c_{ft}, b_{ht}, q_{bt}$ , for a given  $b_{h0}$ . In a deterministic steady state, all equilibrium variables are constant:  $c_{ht} = \bar{c}_h, c_{ft} = \bar{c}_f, b_{ht} = \bar{b}_h, q_{bt} = \bar{q}_b, \forall t$ . But at the constant values the two Euler equations reduce to the same  $q_{bt} = \beta$ . As a result, one is left with two independent equations for three variables ( $\bar{c}_h, \bar{c}_f, \bar{b}_h$ ) leaving us with a continuum of solutions.

<sup>14</sup>We refer here to the ergodic distribution and its mean that correspond to the perturbation solution.

## 4.1 Symmetric setting

Parameter		Value
Discount factor	$\beta$	0.95
Endogenous discount factor	$\eta$	0.001
Risk aversion	$\gamma$	2.00
Capital income share	$Y_k^h/Y^h, Y_k^f/Y^f$	0.30
Persistence	$\rho_{Y^h}, \rho_{Y^f}, \rho_{Y_k^h}, \rho_{Y_k^f}, \rho_{Y_l^h}, \rho_{Y_l^f}$	0.80
Volatility	$\sigma_{Y^h}, \sigma_{Y^f}, \sigma_{Y_k^h}, \sigma_{Y_k^f}, \sigma_{Y_l^h}, \sigma_{Y_l^f}$	0.02
Correlation	$cor(Y_k^h, Y_l^h) = cor(Y_k^f, Y_l^f)$	0.20

Table 2: Parameters for the symmetric setup, model 1.

The parameter values for the setup with symmetric countries are reported in table 2, they fall into the range of values that are commonly used in macroeconomics.<sup>15</sup> The left column (panels A,C,E) in figure 1 presents policy functions for the home country’s consumption share, portfolio shares and asset prices for the global solution method. We plot policies as a function of the home country’s NFA and conditional on  $\mathbf{Y} = E[\mathbf{Y}]$ . The solution is highly accurate as evidenced by the Euler equation errors presented in figure 10 in appendix A. Because of its high accuracy we refer to the global solution as to the true solution of the model.

The differences between the perturbation and global methods’ policy functions are plotted in the right column. First, the two solutions predict slightly different consumption shares for country  $h$  when country  $h$ ’s NFA is low. The relative difference can be as large as 0.019. But the levels of NFA where the difference is large are unlikely. Interestingly, the consumption policy function corresponding to the perturbation solution is more non-linear than its global solution counterpart. We see the reverse with asset price functions. Asset prices corresponding to the global solution increase when one of the countries becomes significantly richer than the other. While these increases are very small they happen to be sufficient to prevent wealth from diverging. But the discrepancy in asset prices is negligible.

Figure 2 compares time paths generated using the perturbation and global solution methods. Simulations for the perturbation solution are based on a second-order approximation and were ‘pruned’. Except for portfolio holdings, the two solution methods generate similar simulation paths: the maximum difference for the NFA, consumption share, and the asset prices are respectively 0.002, 0.008, and 0.017%. The maximum difference between the simulated series of portfolio holdings is 3.08%. The ‘portfolio errors’ are strongly negatively correlated:  $\rho(\Delta\theta_h^h, \Delta\theta_f^h) = -0.934$ . So, despite a large discrepancy in simulated portfolios the two NFA paths are close.

Next we compare first- and second-order moments obtained using the two solution methods. Table 3 reports moments from ‘panel simulations’: 10000 series of 100 periods, starting at  $W_0 = 0$ , each.<sup>16</sup> The two solution methods generate identical means and standard deviations. Yet, the solutions differ in their predictions for correlations of portfolios with H’s and F’s output. While the perturbation solution predicts the correlation signs correctly it under-

<sup>15</sup>Moments of the output processes,  $\mathbf{Y}_t$ , are the ‘targets’ that we use to create discrete approximations to continuous VAR processes. Because discrete approximations are not exact, we use numerically computed moments as inputs to the DS method.

<sup>16</sup>Because the NFA position is highly persistent, ergodic moments and moments obtained from a panel of short simulations could be different.

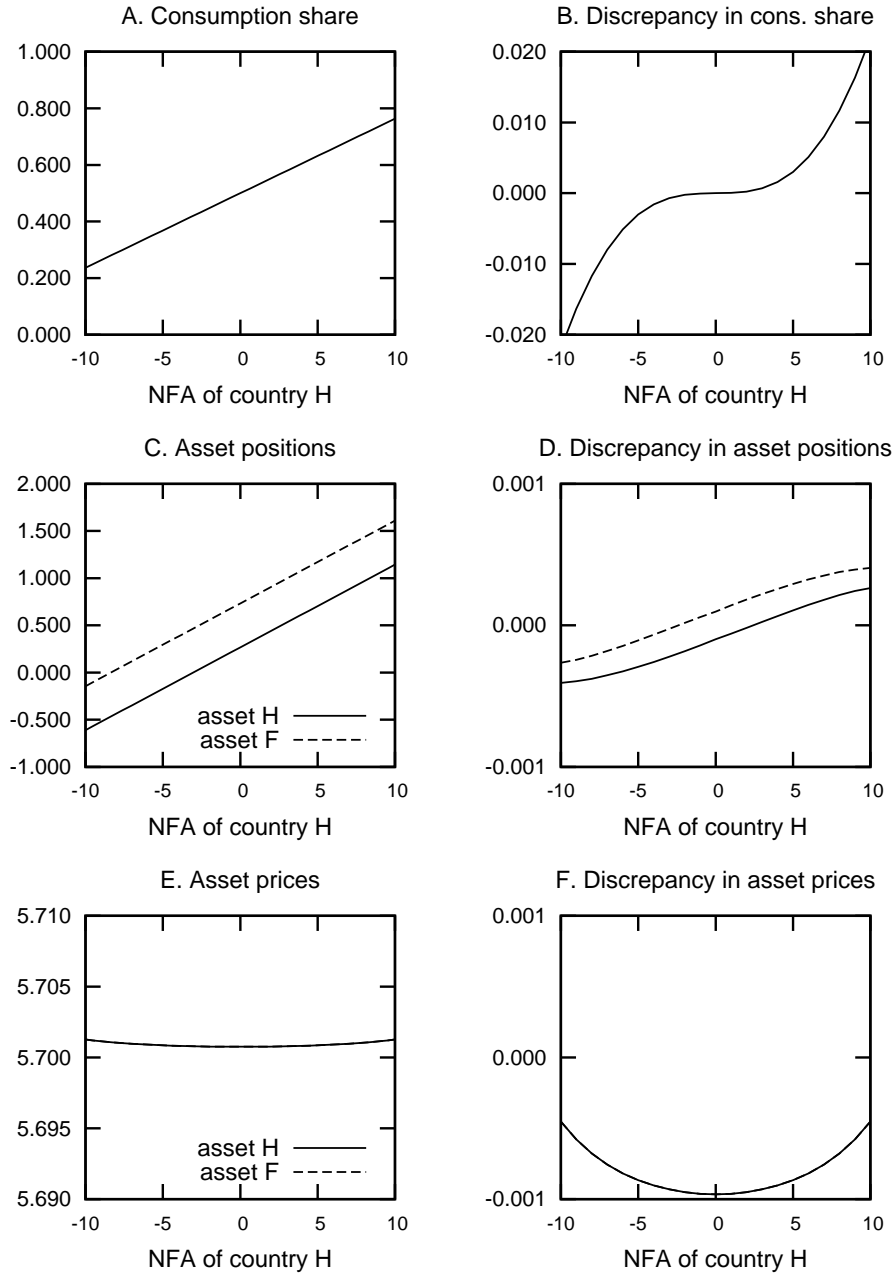


Figure 1: Country  $h$ 's policy functions in the symmetric setting, model 1. Panels A,C,E present the policy functions for the global solution method. Panels B,D,F plot the discrepancy between the global and perturbation policy functions.

estimates the strength of the relation. For example, it predicts that country  $h$ 's ownership of asset H is nearly uncorrelated with output in the two countries. The global solution method implies a relation of mild strength:  $\rho(\theta_h^h, Y^h) = -0.196$ ,  $\rho(\theta_f^h, Y^h) = -0.260$ . The perturbation solution method under-performs because it imposes that “to a first-order approximation,

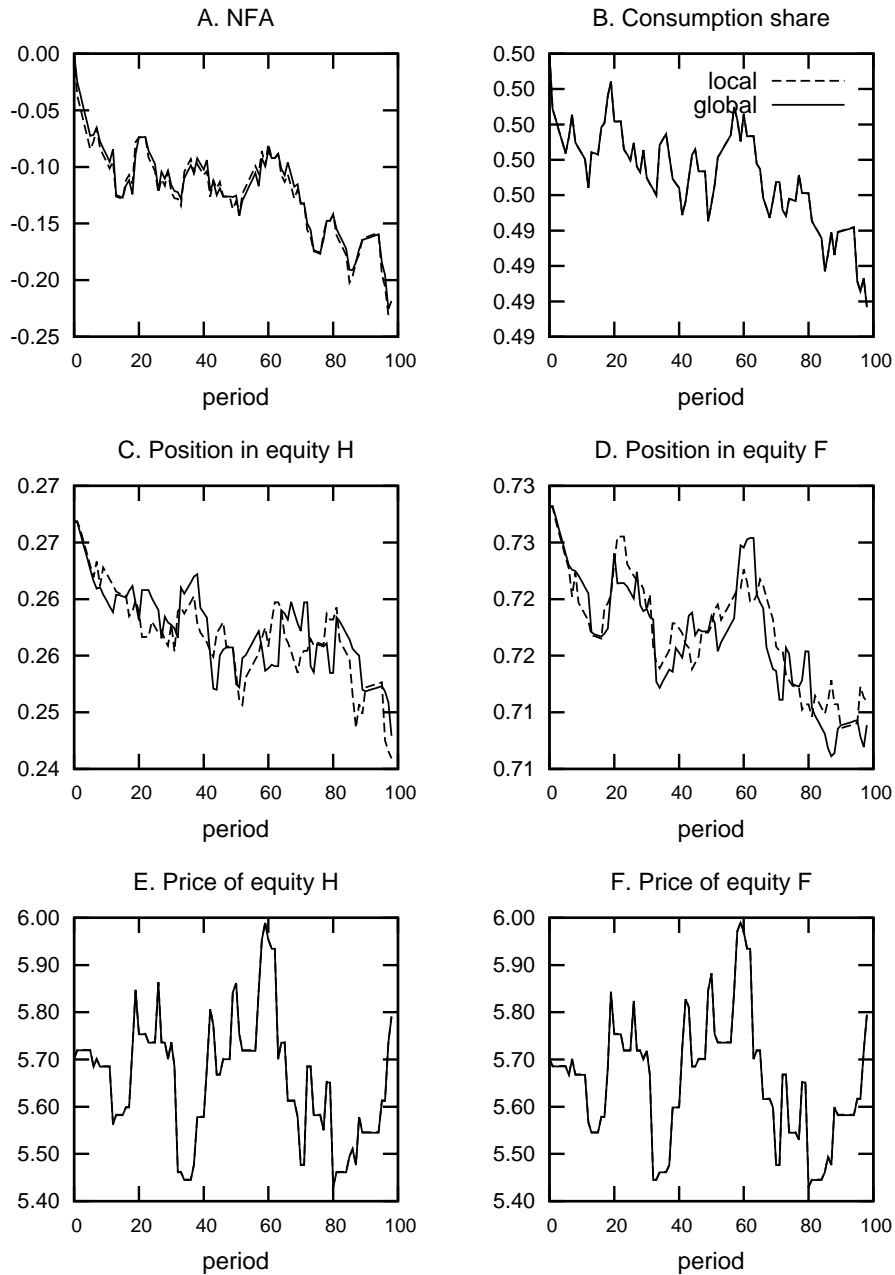


Figure 2: Simulated time paths for country  $h$  in the symmetric setting, model 1.

the portfolio excess returns are zero-mean i.i.d. random variables.”<sup>17</sup> The excess returns drive the portfolio choice and render it largely unrelated to the fundamentals.

To summarize, in a symmetric setting parameterized to match output processes of developed economies the perturbation method performs well. In particular, it matches closely the evolution of the macroeconomic variables and the NFA position. But it produces inaccurate

<sup>17</sup>For details see page 1329 in Devereux and Sutherland (2010a).

	Global				Perturbation			
	$\mu(\cdot)$	$\sigma(\cdot)$	$\rho(\cdot, Y^h)$	$\rho(\cdot, Y^f)$	$\mu(\cdot)$	$\sigma(\cdot)$	$\rho(\cdot, Y^h)$	$\rho(\cdot, Y^f)$
$NFA^h$	0.000	0.083	-0.062	0.061	0.000	0.083	-0.058	0.056
$c_h$	0.500	0.008	0.245	-0.247	0.500	0.002	0.243	-0.244
$\theta_h^h$	0.267	0.007	-0.196	-0.262	0.267	0.007	-0.068	-0.030
$\theta_f^h$	0.733	0.007	0.260	0.195	0.733	0.007	0.029	0.066
$q_h$	5.703	0.111	0.737	0.649	5.704	0.111	0.737	0.649
$q_f$	5.703	0.111	0.648	0.738	5.704	0.111	0.648	0.738
$r_h$	1.053	0.014	0.243	0.180	1.053	0.014	0.243	0.180
$r_f$	1.053	0.014	0.180	0.243	1.053	0.014	0.180	0.243

Table 3: Comparison of model moments from panel simulations, model 1.

predictions about cyclical properties of countries' portfolios. These findings are also robust with respect to increasing shock volatility, increasing shock persistence or higher risk aversion.

## 4.2 Asymmetric setting

In this section we study a setting in which country  $f$  faces income shocks with higher volatility. In particular, we assume  $\sigma^f = 2\sigma^h$ . Because markets are incomplete precautionary motives are active. Since shocks, that country  $f$  faces, are more volatile, its precautionary demand is higher. So, we expect country  $f$  to accumulate more wealth, or, equivalently, we expect country  $h$  to reduce its aggregate asset holdings. We study this setting because the perturbation solution method instructs us to choose  $\bar{W} = 0$  because in the deterministic version of the model the two countries are symmetric. This case presents us with a realistic setting<sup>18</sup> where we expect the perturbation solution quality to deteriorate. At the same time the solution accuracy of the global solution method should not be compromised. This is indeed true as measured by the errors in the equilibrium conditions plotted in figure 11, in appendix A.

Figure 3 plots simulated series for the setting with diverse output volatility. Results are qualitatively similar to those for the symmetric setting (see figure 2). Consumption and asset prices are approximated well. But the NFA and portfolio dynamics differ across the two solution methods. The maximal error for the NFA is 3.3% of country  $h$ 's output. The maximal error for the portfolios is 9.0%. These differences are economically significant.

Table 4 shows moments computed from 10000 randomly generated samples of length 100, each starting at  $W_0 = 0$ . The results are qualitatively similar to the symmetric setting (see table 3). So, we only highlight differences. The perturbation solution method predicts that the portfolio shares are up to 33% more volatile than they are in the global solution.<sup>19</sup> Cyclical properties of the portfolio are also predicted incorrectly. For example,  $\rho(\theta_f^h, Y^f) = -0.339$  in the perturbation solution while it is  $-0.155$  in the global solution. As before, we attribute these underestimated correlations to the way excess returns are approximated in the perturbation solution method.

Figure 4 presents average simulated paths, averaged over 10000 runs, starting each simulation at the means of exogenous variables,  $\mathbf{Y} = E[\mathbf{Y}]$ , and at  $W_0 = 0$ . This allows us to

<sup>18</sup>When studying portfolio holdings of advanced versus emerging market economies, such asymmetries in the volatility of shocks may be important to incorporate (see, for example, Devereux and Sutherland (2009), Coeurdacier et al. (2013)); e.g., here we can think of country  $h$  as an advanced economy and of country  $f$  as an emerging economy.

<sup>19</sup>Compare  $\sigma(\theta_h^h) = 0.009$  for the global solution vs 0.012 for the perturbation solution.

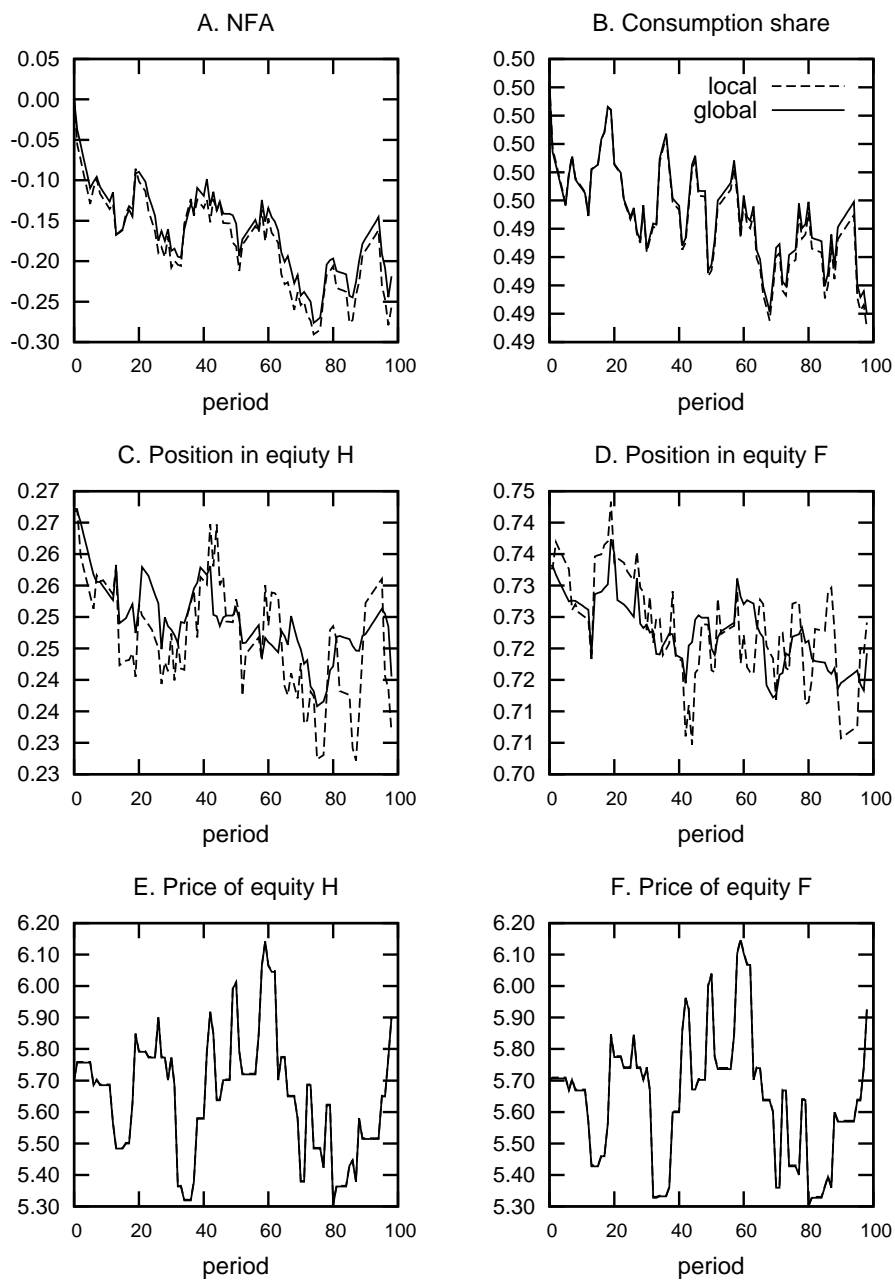


Figure 3: Simulated time paths for country  $h$  in the asymmetric setting with  $\sigma^f = 2\sigma^h$ , model 1.



	Global				Perturbation			
	$\mu(\cdot)$	$\sigma(\cdot)$	$\rho(\cdot, Y^h)$	$\rho(\cdot, Y^f)$	$\mu(\cdot)$	$\sigma(\cdot)$	$\rho(\cdot, Y^h)$	$\rho(\cdot, Y^f)$
$NFA^h$	-0.003	0.130	-0.076	0.155	-0.016	0.131	-0.060	0.125
$c_h$	0.500	0.004	0.312	-0.155	0.500	0.004	0.309	-0.152
$\theta_h^h$	0.267	0.015	-0.235	0.141	0.266	0.018	-0.175	0.284
$\theta_f^h$	0.733	0.009	0.350	-0.155	0.731	0.012	0.223	-0.339
$q_h$	5.708	0.169	0.482	0.863	5.710	0.169	0.482	0.863
$q_f$	5.709	0.184	0.389	0.900	5.712	0.184	0.389	0.900
$r_h$	1.053	0.020	0.162	0.239	1.053	0.020	0.162	0.239
$r_f$	1.053	0.023	0.109	0.293	1.053	0.023	0.109	0.293

Table 4: Comparison of model moments from panel simulations in the asymmetric setting with  $\sigma^f = 2\sigma^h$ , model 1.

identify any systematic differences in the two solution methods. Panel A of figure 4 shows that the NFA position of country  $h$  drifts away, on average, from the initial wealth position assumed in the simulation. The direction of this trend is not surprising: country  $f$  is subject to more volatile endowment shocks than county  $h$ , as a consequence of which it has stronger precautionary motives and should be expected to accumulate positive precautionary assets in a stochastic equilibrium. As a result, we observe country  $f$ 's NFA position to, on average, increase, or equivalently, country  $h$ 's NFA position to decrease. The strength of this relative precautionary motives is, however, not perfectly captured in the local approximation method, leading the NFA paths of DS method and global method to diverge. The NFA position at the end of the 100-period simulation horizon implied by the DS method lies more than three times below the one obtained from the global method. While, in principle, there are also discrepancies in the average time paths of other model variables, they are quantitatively not significant.

To summarize, the DS perturbation solution in an asymmetric country setting of model 1 is very accurate except for portfolios, especially their cyclical properties.<sup>20</sup> Despite these inaccuracies the perturbation solution for the NFA position remains largely accurate over a single simulated path.

### 4.3 Ergodic moments

The perturbation solution, being a Taylor series approximation, is a sum of polynomial components of different order. It is well known, see Schmitt-Grohé and Uribe (2003), that with a constant discount factor,  $\beta(c) = \beta$ , the first-order component of the NFA equation is non-stationary. That is  $NFA_{ht} = NFA_{ht-1} +$  linear function of  $\mathbf{Y}_{t-1}$ . Because the first-order component is dominant, the ergodic distribution of  $NFA_t^h$  and, hence, country  $h$ 's wealth cannot be computed using the perturbation solution method. This issue can be resolved by introducing an endogenous discount factor as described in section 2.  $\beta(c) = \beta c^{-\eta}$  is a commonly assumed functional form. Among the work on international portfolio choice it is used in Devereux and Sutherland (2011, 2010a, 2009). The endogenous discount is a technical device that induces stationarity and we have throughout the paper set  $\eta = 10^{-3}$ , a ‘small’

<sup>20</sup>We also explored other asymmetric settings, such as differences in country sizes, but none of the results change fundamentally.

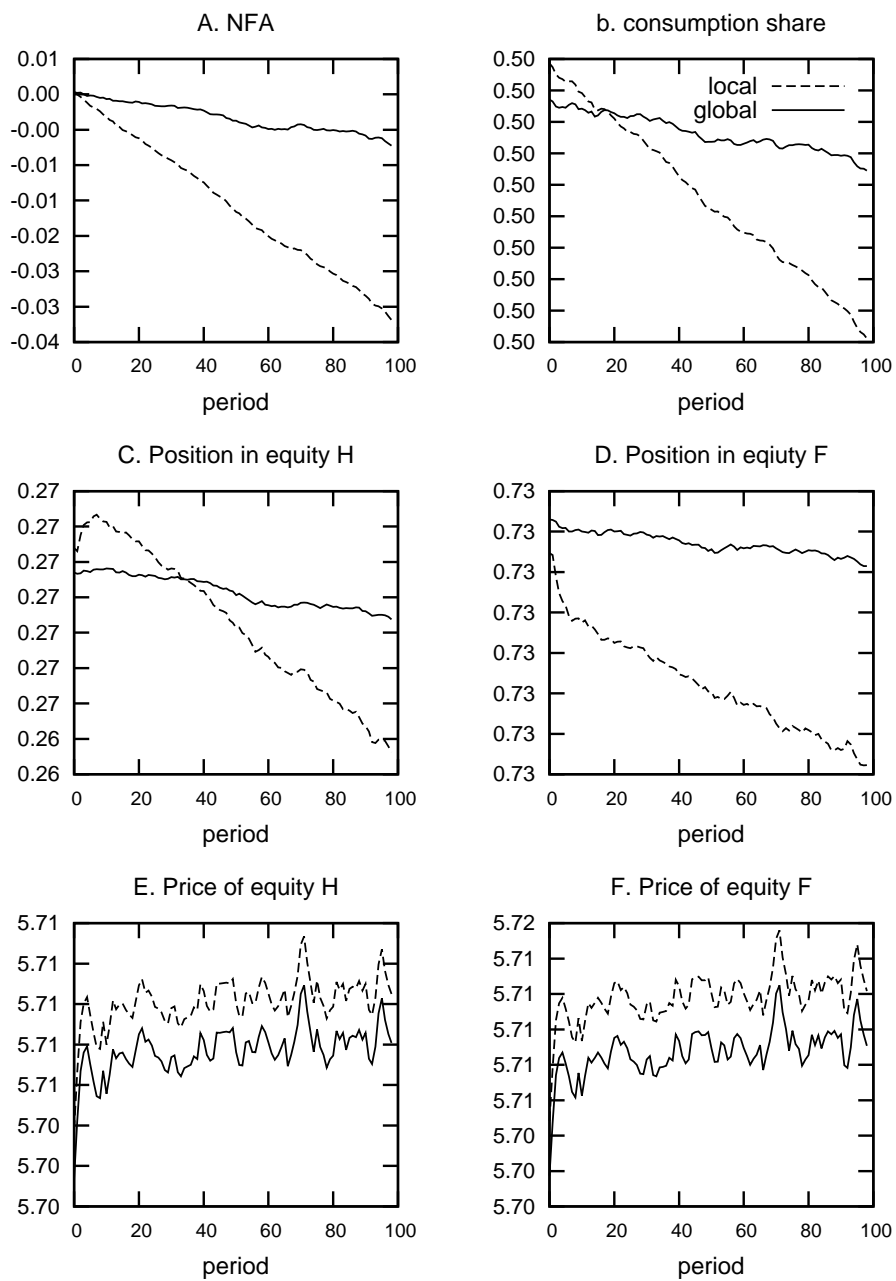


Figure 4: Average simulated paths for country  $h$  in the asymmetric setting with  $\sigma^f = 2\sigma^h$ , model 1.

value.<sup>21</sup> Keeping  $\eta$  relatively small lets us compare the methods' ability to capture "true" economic forces. In appendix B.4 we demonstrate how the ergodic distribution obtained using the global solution method depends on the choice of  $\eta$ . For the time paths studied so far, changing  $\eta$  (within reasonable bounds) had little impact on the results.

#### 4.4 Iterative computation of approximating point

The perturbation solution instructs us to use  $NFA^h = 0$  as the approximation point. In asymmetric settings it is possible to 'refine' the approximation point using a heuristic procedure, that is inspired by the routine described in Devereux and Sutherland (2009).<sup>22</sup> It seems reasonable to approximate the model solution around the level of NFA that the economies tend to 'on average.' We define the so-called 'stochastic' steady state of NFA as a rest point of the economy when it is 'hit' in every period by the mean values of the shock vector. It can be used as a new approximation point and the whole procedure repeated. This procedure is not guaranteed to converge but it does in most cases. We apply this algorithm to the asymmetric setting with  $\sigma^f = 2\sigma^h$ . The iterative procedure converges to  $NFA^h = \bar{W} = -6.19$ . The implied portfolio is  $(\theta_h^h, \theta_f^h) = (-0.463, 0.378)$ .<sup>23</sup>

	Global		Perturbation			
	$\mu(\cdot)$	$\sigma(\cdot)$	no updating		with updating	
	$\mu(\cdot)$	$\sigma(\cdot)$	$\mu(\cdot)$	$\sigma(\cdot)$	$\mu(\cdot)$	$\sigma(\cdot)$
$NFA^h$	-0.254	1.096	-0.6976	1.1483	-6.1676	0.9130
$c_h$	0.493	0.029	0.4820	0.0295	0.3461	0.0236
$\theta_h^h$	0.237	0.128	0.1862	0.1347	-0.4582	0.1042
$\theta_f^h$	0.718	0.064	0.6916	0.0672	0.3779	0.0517
$q_h$	5.708	0.178	5.7099	0.1783	5.7094	0.1785
$q_f$	5.709	0.194	5.7115	0.1940	5.7110	0.1942

Table 5: Comparison of ergodic model moments in the asymmetric setting with  $\sigma^f = 2\sigma^h$  and  $\eta = 0.001$ , model 1.

In table 5 we compare the ergodic moments (mean and standard deviation) generated using the global solution and the perturbation solution without ( $\bar{W} = 0$ ) and with approximation point updating ( $\bar{W} = -6.19$ ). We first compare the global solution and the perturbation solution without updating. Country  $h$ 's NFA is  $-69.8\%$  of steady state output according to the perturbation solution and it is only  $-25.4\%$  of output under the global solution. Because the mean NFA is estimated too low, so is the mean consumption share: 0.482 versus 0.493. These differences stem from incorrectly estimated portfolios: the perturbation solution is  $(\theta_h^h, \theta_f^h) = (0.186, 0.691)$  while the global solution is  $(0.237, 0.718)$ . The differences in the ergodic means are economically paramount. Yet volatilities of all variables are estimated correctly.

<sup>21</sup>This is a small number when compared to  $\eta = 0.01$  used in Devereux and Sutherland (2009). But it is very significant if one realizes that it depresses  $\sigma(NFA^h)$  4.35 times. If we chose to match the observed volatility of the US NFA we would have to set  $\eta \geq 0.03$ . At this level the mean level of NFA is only marginally different from 0. That is the endogenous discount factor dominates all other economic forces and the model solution is symmetric for all practical purposes.

<sup>22</sup>We thank Alan Sutherland for laying out the details of their routine to find the approximate stochastic steady state NFA position.

<sup>23</sup>When  $NFA^h = \bar{W} = 0$  the portfolio  $(\theta_h^h, \theta_f^h)$  is  $(0.266, 0.733)$ .

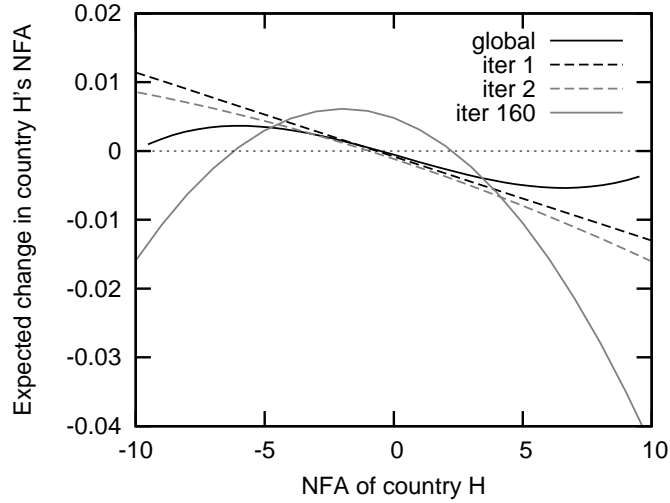


Figure 5: NFA transition map for country  $h$  in the asymmetric setting with  $\sigma^f = 2\sigma^h$ , model 1.

The results from the perturbation solution when the updated approximation point is used deteriorate hugely. In this case, country  $h$ 's NFA is  $-617\%$  of steady state output. As a result, the mean consumption share is found to be orders of magnitude too low (0.346), and the portfolio solutions obtained, do not even resemble the portfolios obtained from the global method. Such a poor performance can be explained as follows. As iterations proceed the approximation point diverges from zero to the region where the policy functions are more non-linear. So, as iterations progress the neighborhood where the perturbation solution is accurate shrinks.<sup>24</sup> We illustrate our logic with a plot of NFA transition functions: we plot the expected change in NFA as a function of current NFA conditional on  $Y = E[\mathbf{Y}]$ . Stationary points of NFA can be found at the intersection with the horizontal zero line. Figure 5 shows that as the iteration count increases the perturbation solution maintains the right slope at NFA=0 but at the cost of shifting the solution point towards large negative values.<sup>25</sup> As the approximation point shifts away from zero the solution becomes less accurate around the ergodic mean of NFA which is  $-0.254$ . At the final iteration 160 the solution at the approximation point NFA= $-6.19$  is more accurate than at the true ergodic mean. This approximation point is unstable and the obtained portfolio is exactly what is needed to support such dynamics.<sup>26</sup>

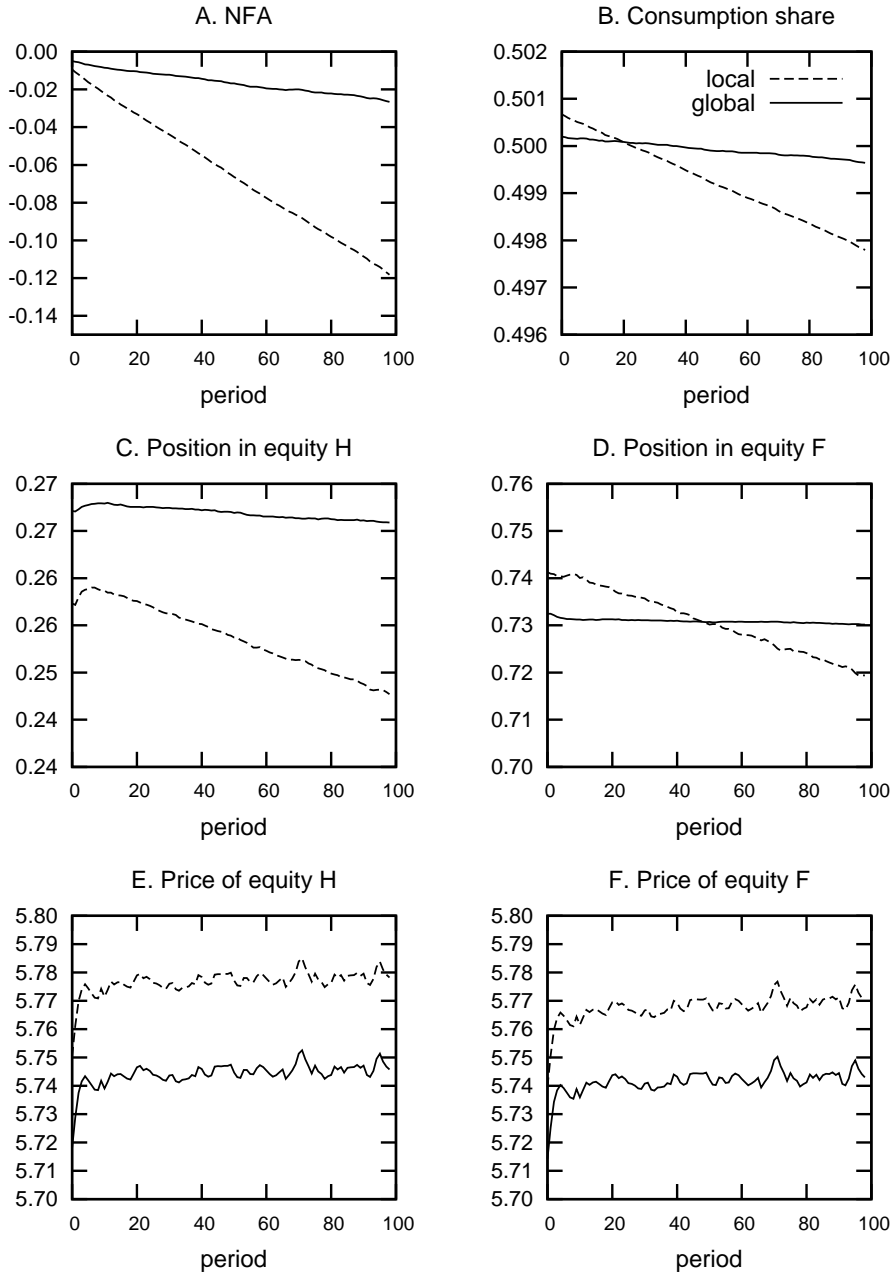


Figure 6: Average simulated paths for country  $h$  in the asymmetric setting with  $\sigma^f = 2\sigma^h$ ,  $\gamma = 5$ , model 1.

#### 4.5 Sensitivity analysis in the asymmetric setting

In this section we study how our results change when we increase risk-aversion from the benchmark value of 2 to 5. The effect is very small in the symmetric setting and we do

<sup>24</sup>This can be related to Judd and Guu (2001): we should look for the approximation point where the relevant policies have slope sufficiently different from zero. But at the  $\bar{W} = -6.19$  found by the iterative procedure the ‘true’ transition function is nearly flat. The perturbation method being based on the implicit function theorem may be very inaccurate. So, the iterative procedure may well take the solution *away* from

not report the results. In the asymmetric setting the difference between the two solutions increases. As risk-aversion increases, precautionary demands of both countries increase. But country  $f$ , facing more volatile shocks, increases its demand more. Hence, country  $h$ 's NFA is decreases even more strongly (starting at an initial value of  $W_0 = 0$ ) than under the baseline parameterization with  $\gamma = 2$ . While both solution methods capture this effect the difference between the two increases. Figure 6 presents average simulated paths for the case with high risk-aversion. At the end of period  $t = 100$  the NFA is predicted to be -0.119 on average under the DS method; under the global solution it is only -0.027. This difference of 0.093 in the NFA position is sizeable, when compared to steady state annual output that equals 1. The effect on differences in the optimal portfolios and the consumption share are of similar significance.

With the CRRA preferences that we analyze here the coefficient of relative risk-aversion and intertemporal elasticity of substitution (IES) are inversely related. To determine which of the two is the crucial element we explore recursive preferences following Epstein and Zin (1989) that allow separating the two. To this end, we set the IES at 0.5 but increased risk aversion to  $\gamma = 5$ . The results are very similar to those with CRRA preferences and high risk-aversion as can be seen in figure 14 of appendix B. This suggests our results are driven primarily by the degree of risk-aversion.

## 5 Model 2 with a bond and an equity claim

In model 2 we evaluate our portfolio solution methods under the model setup in which key 'finance' stylized facts are at center stage. Standard macroeconomic models with CRRA preferences perform poorly in matching the asset-pricing facts such as the observed equity premium. Explaining asset-pricing facts not only makes models more realistic but also increases the cost-of-business-cycles estimates and justifies policy intervention.<sup>27</sup> It is even more important to be consistent with these facts in international macroeconomic models with portfolio choice. In the latter asset prices determine relative wealth positions and, therefore, real allocations. Gourinchas and Rey (2013) argue that asset-pricing facts are also important ingredients to understanding the composition of international capital flows.

To match the observed equity premium we modify model 1 as follows. First, instead of two equity claims countries can now invest in a risk-free bond or a claim to 'world' equity (to a world capital income endowment). We parameterize the model such that the risky asset earns a substantial excess return comparable in magnitude to those that we see in the data ('equity premium'). Second, we allow countries to differ in their tastes towards risk, i.e. degrees of risk aversion. Because the investors of the two countries are heterogeneous in their tastes towards the 'higher risk' – 'higher returns' tradeoff, this naturally separates countries into equity and bond investors, as observed in the data. In this setup the DS method does not perform well. The risk premium generated under the DS method is smaller than under

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the best approximation point.

<sup>25</sup>At  $NFA=0$ , regardless of the approximation point consumption levels of the two countries must be the same on average according to the perturbation solution. Changes in volatility have only high-order effects on consumption. So, the slope of the transition function at  $NFA=0$  is determined only by preference parameters.

<sup>26</sup>Notice that only by starting simulations from  $NFA > 0$  the iterative procedure could have converged to the stable stationary point.

<sup>27</sup>Tallarini (2000) and Ellison and Sargent (2012) show that in the model that matches observed risk premium business cycle fluctuations are much costlier than in Lucas (1987).

the global solution. Because the less risk-averse country holds more (risky) equity claims and less (safe) bonds, the return to its portfolio is underestimated. This compromises dynamics of the net foreign asset position, and consequently, dynamics of all macroeconomic variables.

## 5.1 Model setup

Both countries receive their income from the two sources. The first source is the world 'capital income' endowment  $Y_t^k$  units of good each period. The world endowment is a first-order autoregressive process:

$$\log \left( Y_t^k / \bar{Y}^k \right) = \rho^k \log \left( Y_{t-1}^k / \bar{Y}^k \right) + \varepsilon_t^k. \quad (13a)$$

The second source is the country-specific 'labor income' endowment  $Y_{at}^l$ ,  $a \in \{h, f\}$ . Country-specific endowments are i.i.d. processes:

$$\log \left( Y_{ht}^l / \bar{Y}_h^l \right) = \varepsilon_{ht}^l, \quad (13b)$$

$$\log \left( Y_{ft}^l / \bar{Y}_f^l \right) = \varepsilon_{ft}^l, \quad (13c)$$

We assume that the world and country-specific endowments are independent processes. We think of world endowment as 'capital income' and of country-specific endowments as 'labor income'. We denote the vector of endowment processes by  $\mathbf{Y}_t \equiv \{Y_t^k, Y_{ht}^l, Y_{ft}^l\}$ .

The representative agent in country  $a \in \{h, f\}$  values different consumption plans  $\{c_{a\tau}\}_{\tau=t}^{\infty}$  according to:

$$V_{at} \equiv \max_{c_{at}} \left[ (1 - \beta(c_{at})) c_{at}^{\frac{1-\gamma_a}{\psi}} + \beta(c_{at}) \left( E_t V_{at+1}^{(1-\gamma_a)} \right)^{\frac{1}{\psi}} \right]^{\frac{\psi}{1-\gamma_a}}, \quad (14)$$

where  $c_{at}$  is consumption,  $\beta(c_{at})$  is the endogenous discount factor,  $\psi$  is the intertemporal elasticity of substitution and  $\gamma_a$ ,  $a \in \{h, f\}$ , is the country-specific coefficient of risk-aversion.

The set of tradable assets consists of the claims to the world 'capital income' endowment and a risk-free bond that is in zero net supply. This means that consumers cannot directly hedge against the fluctuations in their country-specific shocks. Let  $\theta_t^a$  be investor  $a$ 's shares of the global tree, and  $b_t^a$  be his holdings of the bond. Also, denote the price of equity with  $q_t^e$ , and the price of the bond with  $q_t^b$ . Then we can write investor  $a$ 's budget constraint as:

$$\theta_t^a q_t^e + b_t^a q_t^b = \theta_{t-1}^a (q_t^e + Y_t^k) + b_{t-1}^a + Y_{at}^l - c_{at}. \quad (15)$$

Table 6 summarizes the set of the model's equilibrium conditions. Equations A6-A8 are the good and asset market clearing conditions.

## 5.2 Model solution and parameterization

To solve the model with the global solution method, we define country  $h$ 's share in world financial wealth,  $\omega_t$ , and use it to rewrite the budget constraint as:

$$\omega_t = \frac{\theta_{t-1}^h (Y_t^k + q_t^e) + b_{t-1}^h}{Y_t^k + q_t^e} \quad (16)$$

$$\begin{aligned}
\text{(A1-A2): } & q_t^e = \beta(c_{at})E_t \{m_{at+1}\} (q_{t+1}^e + Y_{t+1}^k), \quad a = h, f, \\
\text{(A3-A4): } & q_t^b = \beta(c_{at})E_t \{m_{at+1}\}, \quad a = h, f, \\
& m_{at+1} = \left(\frac{c_{at+1}}{c_{at}}\right)^{\frac{1-\gamma_a}{\psi}-1} \left[\frac{V_{at+1}^{1-\gamma_a}}{E_t V_{at+1}^{1-\gamma_a}}\right]^{1-\frac{1}{\psi}}, \quad a = h, f, \\
\text{(A5): } & \theta_t^h q_t^e + b_t^h q_t^b = \theta_{t-1}^h (q_t^e + Y_t^k) + b_{t-1}^h + Y_{ht}^l - c_{ht}, \\
\text{(A6): } & c_{ht} + c_{ft} = Y_t^k + Y_{ht}^l + Y_{ft}^l, \\
\text{(A7): } & \theta_t^h + \theta_t^f = 1, \\
\text{(A8): } & b_t^h + b_t^f = 0.
\end{aligned}$$

Table 6: System of equilibrium conditions for model 2

Parameter		Value
Discount factor	$\beta$	0.95
Endog. discount factor	$\eta$	0.001
Coef. of risk aversion, H	$\gamma_h$	8
Coef. of risk aversion, F	$\gamma_f$	16
EIS parameter	$\psi$	0.33
Capital income share	$Y^k / (Y^k + Y_h^l + Y_f^l)$	1/3
Capital persistence	$\rho_k$	0.8
Capital volatility	$\sigma_k$	0.09
Labor volatility	$\sigma_h^l = \sigma_f^l$	0.06

Table 7: Parameter values, model 2.

$$\theta_t^h q_t^e + b_t^h q_t^b = \omega_t (Y_t^k + q_t^e) + Y_{ht}^l - c_{ht}. \quad (17)$$

Equations (16) and (17) replace (A6) in the original system defined in table 6.

To apply the DS method, we introduce a ‘default’ division of the world endowment,  $\theta_0$ , and rewrite county  $h$ ’s budget constraint as follows:

$$(\theta_t^h - \theta_0)q_t^e + b_t^h q_t^b = (\theta_{t-1}^h - \theta_0)q_{t-1}^e \left(\frac{Y_t^k + q_t^e}{q_{t-1}^e}\right) + b_{t-1}^h q_{t-1}^b \left(\frac{1}{q_{t-1}^b}\right) + \theta_0 Y_t^k + Y_{ht}^l - c_{ht}.$$

Upon defining  $\alpha_{et}^h = (\theta_t^h - \theta_0)q_t^e$ ,  $\alpha_{bt}^h = b_t^h q_t^b$ ,  $r_t^e = (Y_t^k + q_t^e) / q_{t-1}^e$ ,  $r_t^b = 1 / q_{t-1}^b$ , and  $W_t = \alpha_{et}^h + \alpha_{bt}^h$ , we can write the above as:

$$W_t = \alpha_{et-1}^h (r_t^e - r_t^b) + W_{t-1} r_t^b + \theta_0 Y_t^k + Y_{ht}^l - c_{ht}. \quad (18)$$

Equation (18) replaces (A6), and (A7) and (A8) are replaced by the asset market clearing conditions in terms of net assets,  $\alpha_{et}^h + \alpha_{et}^f = 0$ , and  $\alpha_{bt}^h + \alpha_{bt}^f = 0$ .

Table 7 reports parameter values. We set country  $f$ ’s coefficient of risk aversion to be twice that of country  $h$ ,  $\gamma_f = 2\gamma_h = 16$ . The intertemporal elasticity of substitution,  $\psi$ , in both countries is set to one third. We set the means of the exogenous endowment processes such as to have a world capital share of income equal to 0.3:  $\bar{Y}^k = 0.6, \bar{Y}_h^l = \bar{Y}_f^l = 0.7$ . The volatility of the ‘labor’ income shocks is set to  $\sigma_{Y_a^l} = 0.06, a \in \{h, f\}$ . The volatility of ‘capital income’ is taken to be 1.5 times higher,  $\sigma_{Y^k} = 0.09$ . The persistence of capital income shocks,  $\rho_{Y^k}$ , is set to 0.8.



## 6 Results for model 2

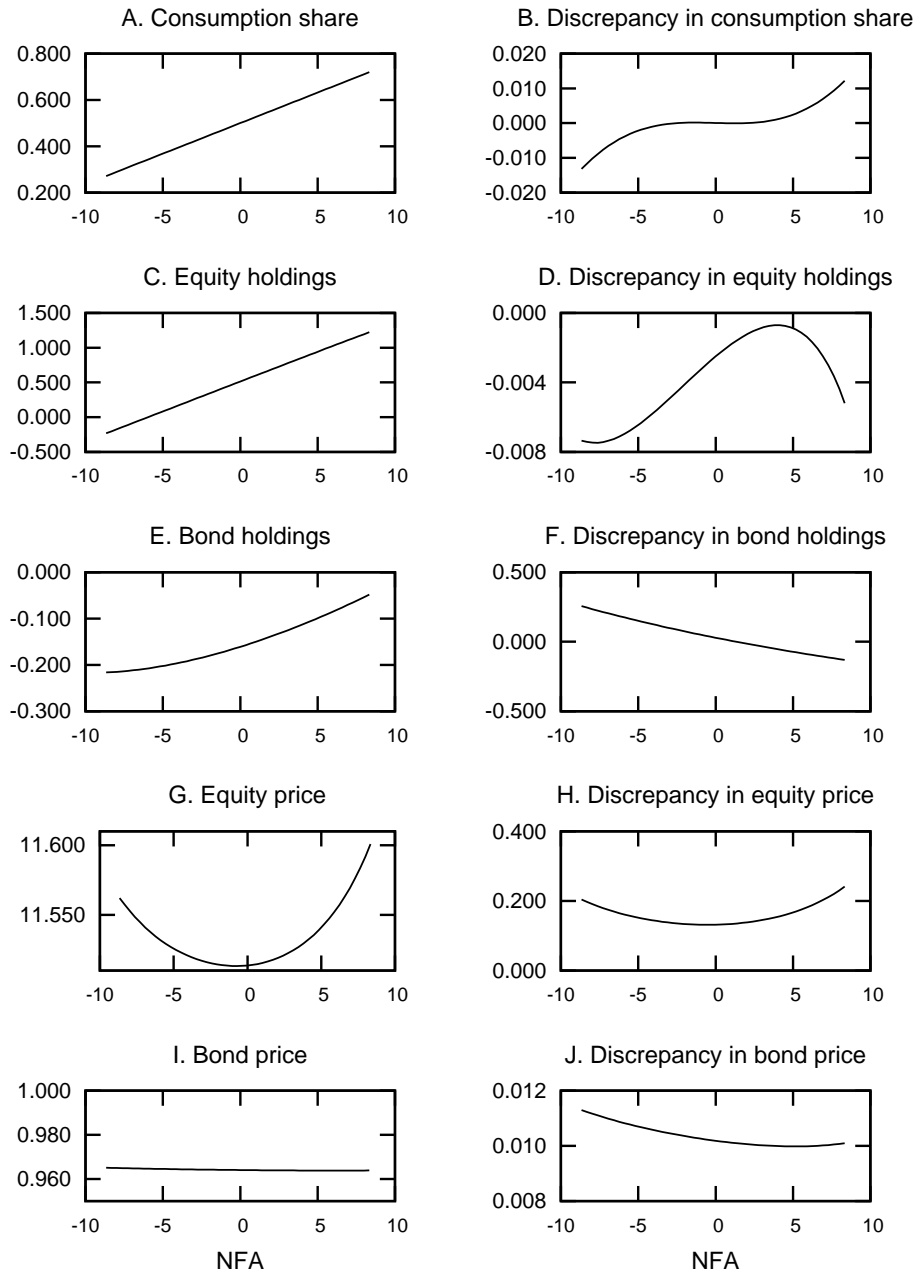


Figure 7: Country  $h$ 's policy functions, model 2. Panels A,C,E,G,I present the policy functions for the global solution method. Panels B,D,F,H,J plot the discrepancy between the global and perturbation policy functions.

Figure 7 presents policy functions for country  $h$ 's consumption share, equity holdings, bond holdings, equity price and bond price under the global solution, shown in panels A, C, E, G, I. Panels B, D, F, H and J of figure 7 show discrepancies between the global and the DS

solution. The discrepancy for the consumption share of country  $h$  is similar to that obtained in model 1. The discrepancies in the global and local policy functions of all other variables are substantially larger than in model 1. Those discrepancies can be as large as 0.136, 0.253, and 0.011 for bond holdings, equity prices, and bond prices respectively.

Figure 8 compares time paths from a single series of realizations of the exogenous shock process of length 100. The differences between the time paths generated by the two solution methods show, unlike in model 1, visible differences. In particular, the maximum difference for the NFA, consumption share, equity holdings, bond holdings, equity price, and bond price are, respectively, 28.997, 0.258, 0.811, 24.843, 16.442, and 1.266%. Those differences are economically paramount.

	Global				Perturbation			
	$\mu(\cdot)$	$\sigma(\cdot)$	$\rho(\cdot, Y^h)$	$\rho(\cdot, Y^f)$	$\mu(\cdot)$	$\sigma(\cdot)$	$\rho(\cdot, Y^h)$	$\rho(\cdot, Y^f)$
$NFA^h$	0.028	0.179	-0.087	0.081	-0.050	0.189	-0.117	0.045
$c_h$	0.501	0.005	0.157	-0.020	0.499	0.005	0.160	-0.004
$\theta^h$	0.516	0.015	-0.116	0.059	0.512	0.016	-0.117	0.050
$b^h$	-0.161	0.008	-0.307	-0.277	-0.201	0.078	0.481	-0.272
$q_e$	11.640	1.741	0.756	0.755	11.507	1.723	0.756	0.755
$q_b$	0.970	0.125	0.690	0.689	0.960	0.124	0.691	0.690
$r_e$	1.071	0.199	0.465	0.464	1.071	0.199	0.465	0.464
$r_b$	1.048	0.136	-0.028	-0.028	1.059	0.137	-0.029	-0.029

Table 8: Comparison of model moments from panel simulations, model 2.

Table 8 presents model moments based on ‘panel simulations’: 10000 series of 100 periods. As expected, the less-risk averse country  $h$  buys more of the risky asset ( $h$ ’s equity position is above 0.5) and finances these purchases by selling bonds ( $h$ ’s bond position is negative). Both solution methods are able to capture this. However, unlike under model 1, the model moments implied by the two solution methods are no longer close, and show economically significant differences. Means of almost all model variables are different. They are particularly pronounced for the NFA (even the sign is wrongly predicted by the DS method), bond holdings, the asset prices, and, as a consequence, asset returns. The equity premium, defined as  $r^e - r^b$ , obtained under the global approximation method is as large as 2.3%. The equity premium generated by the DS solution is much smaller, equal to 1.2%.

To understand these differences it is instructive to study the average simulated paths implied by the two methods. Figure 9 shows average paths, period-by-period averages over 10000 simulations, starting each simulation run in the ‘average’ state with  $\mathbf{Y}_t = E_t(\mathbf{Y}_t)$ , and starting with  $W_0 = 0$ . We distinguish two dimensions along which the average paths obtained using the DS and the global solution methods differ. First, the initial period ( $t = 0$ ) levels of the equity and bond holdings differ across the two portfolio solution methods. Second, as time progresses the average paths implied by the global method and by the DS method trend into different directions. We discuss each of these dimensions of differences in turn.

The differences in the initial period of the average path, when  $t = 0$  and  $W_0 = 0$  (and therefore all of the model’s state variables are at their steady state values), can be understood as follows: as panel C of figure 9 demonstrates, both DS and global solution method predict that the less risk averse agent, country  $h$ , holds a larger share of the risky asset, i.e.  $\theta^h > 0.5$ . We also find, that *relative* to the DS method the equity holdings of the less risk averse agent are smaller under the global solution: equity holdings at  $t = 0$  are 0.5135 under the global

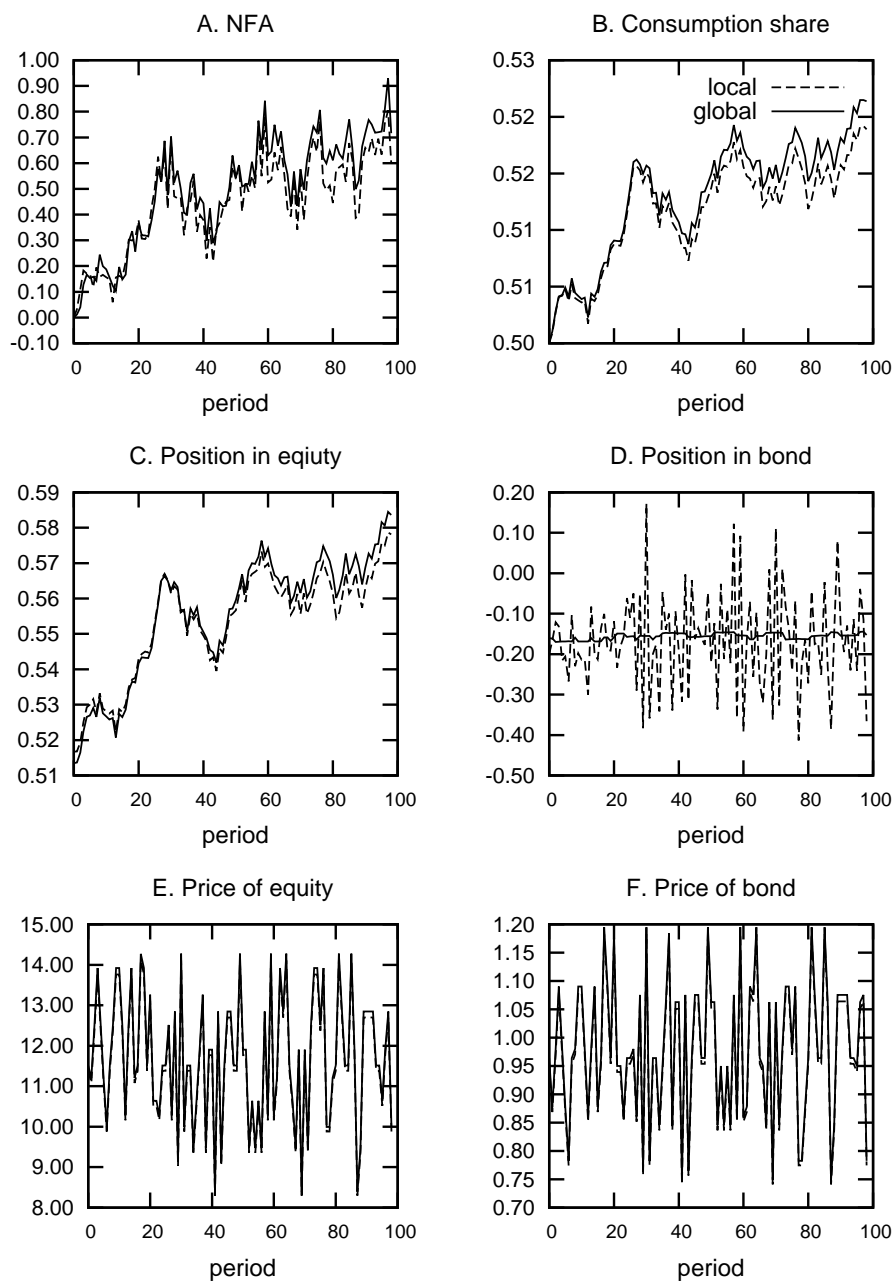


Figure 8: Single simulated time paths, model 2.

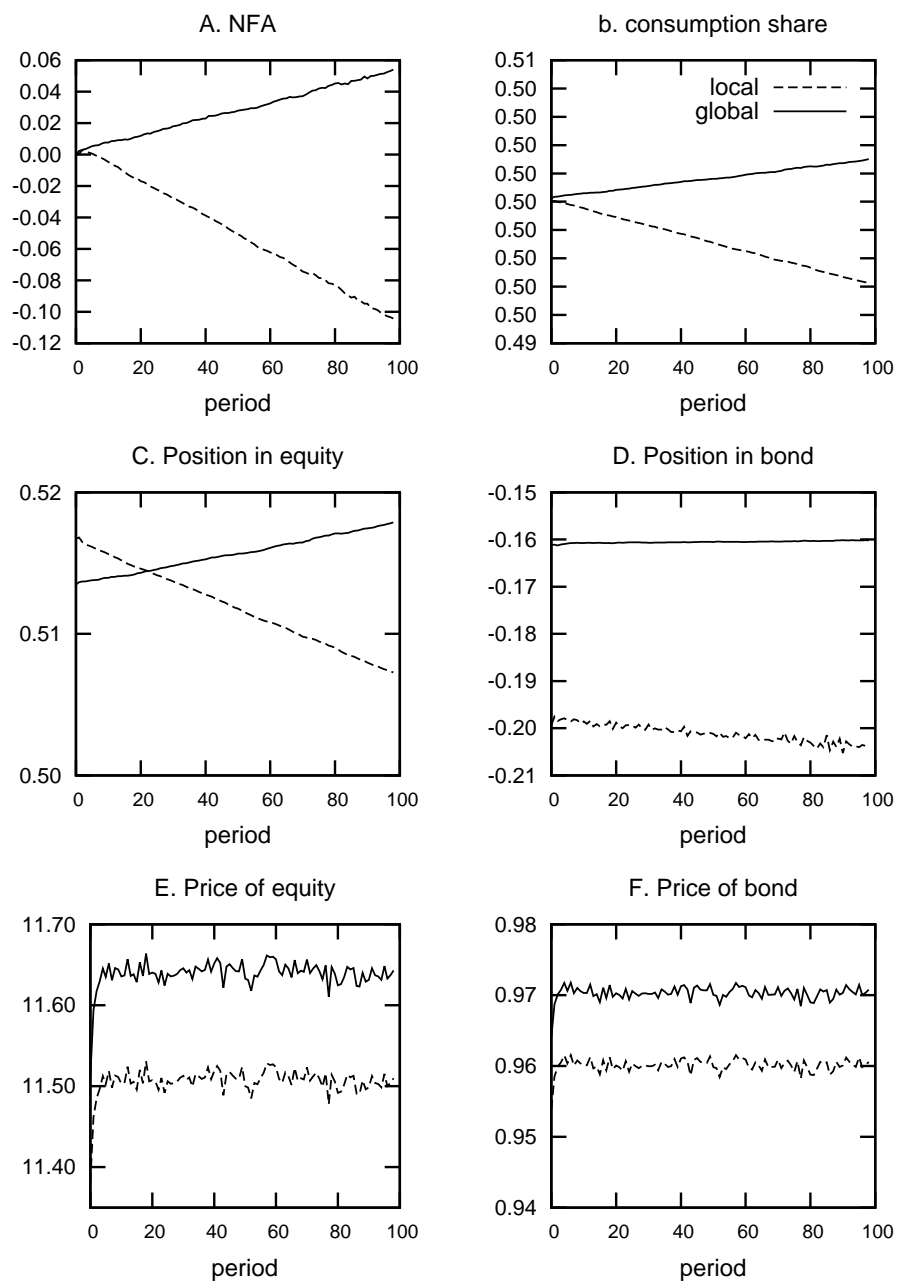


Figure 9: Average simulated paths, model 2.

method, but 0.5167 under the DS method. This finding is reminiscent of our findings in a two period version of this model, that we have explored in a companion paper (see Rabitsch and Stepanchuk (2013)). In the two period version –in which, by definition, the net asset position across periods is equal to  $W = 0$ , and therefore all state variables are at their steady state values– we also obtained that the nonlinear portfolio solution results in a smaller equity position for the less risk averse agent compared to the DS method. In the two period case, we could attribute this finding to the fact that the DS method does not consider effects of the size of uncertainty on the portfolio positions. Technically, the Taylor approximation to the portfolio policy function under the DS method only considers an expansion in the direction of state variables, but not an expansion in the direction of the size of shocks, as mentioned in section 3.2. Under the DS method, therefore, as long as all state variables are at their steady states, around which the economy is approximated, the portfolio solution obtained is the same for any size of uncertainty. On the other hand, in the nonlinear solution method of the two period model, we could show that for increasing size of shocks, the equity holdings of the less risk averse agent decrease somewhat –the less risk averse agent continues to hold a higher share of equity, but to a lesser degree. The period 0 findings of the infinite horizon version mirrors those results. We can therefore attribute the period 0 differences in the average paths to the fact that the DS method lacks a role of the size of uncertainty on the approximate portfolio solutions.

We now turn to the differences in the trends of the average paths. Similar to the asymmetric setting in model 1, the two countries have different precautionary demands. Here, in model 2, the different strength of precautionary motives comes from country  $f$  being more risk averse than country  $h$ . We should expect this channel to lead to an increase in country  $f$ 's NFA position over time, or, equivalently, to a decrease in country  $h$ 's NFA position. However, there is an additional channel that we should expect to impact the evolution of the NFA position over time. Our model is parameterized to display large excess returns of the risky asset over the safe asset ('equity premium'), and features a setting with heterogeneous investors, where one of the investors is more willing to trade higher risk for higher future returns. Since the less risk-averse investor invests more in the higher risk, and higher return asset, he earns the equity premium and we can expect this to positively affect his wealth position, that is, his net foreign asset position. The expected path of the NFA position obtained by the global method captures this effect. It appears that, when the model is solved by the global method, the positive effect of higher average return on country  $h$  outweighs the effect of lower relative precautionary demand on its wealth: over time, the NFA position of the less risk-averse agent country  $h$  improves. The 'equity premium effect' is much weaker in the DS solution for two reasons. First, the equity premium that is generated by the perturbation method is much smaller (1.2% in comparison to 2.3%). Second, because the DS method approximates the model solution around the non-stochastic steady state in which all assets deliver the same return and in which excess returns are zero. So, it drastically understates the role that the higher average returns of the less risk-averse agent have on the dynamics of his NFA position. Technically, the terms  $\bar{\alpha}_e^h (\bar{r}^e - \bar{r}^b)$  are set to zero in country  $h$ 's budget constraint because at the non-stochastic steady state excess returns,  $r^e - r^b$ , are zero by definition. Also, the terms  $E_{t-1} \bar{\alpha}_e^h (\hat{r}_t^e - \hat{r}_t^b)$  and  $E_{t-1} \hat{\alpha}_{et-1}^h (\hat{r}_t^e - \hat{r}_t^b)$  are assumed to behave as mean-zero i.i.d. shocks when solving for the zero- and first-order portfolio solution, which may be ill-described in the current setting of a model with strong excess returns.

Figure 16 in appendix B.4 plots the stationary distribution obtained by the global solution

method. The long simulation of the model solution obtained by the DS method diverged, so no stationary distribution could be presented for the local method. Notice that this is the case, despite having an endogenous discount factor ( $\eta = 0.001$ ). Similarly, the iterative computation of the approximation point failed in the present parameterization of model 2.<sup>28</sup>

## 7 Conclusions

This paper compared the performance of the local portfolio solution method of Devereux and Sutherland (2010a, 2011) relative to a global portfolio solution method. We find that the DS method performs very well in symmetric country settings and under parameterizations common in the macroeconomic literature. In asymmetric country setups the performance of the DS solution method may be compromised when the local approximation is taken around a 'wrong' point of the net foreign asset position. Unfortunately, an algorithm in which one aims to solve for the approximate mean stochastic steady state of the NFA (based on the second-order approximation to the policy functions) in search of a better approximation point, does not improve results. Finally, we show that in a model setup that generates large and positive excess returns and where investors have different attitudes towards risk-return tradeoffs, the DS method can perform much worse compared to global portfolio solution methods. This is because the equity premium generated by the DS method is smaller than the premium generated under the global method. More importantly, because the DS method takes as approximation point the non-stochastic steady state where excess returns are zero, it fails to capture the effect of the return differential on wealth accumulation.

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<sup>28</sup>For more modest parameter values of the standard deviations of the shock processes, iterative computation of the approximation point converges. However, using thus obtained approximation point does not improve, and rather worsens, quality of the DS solution.

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## A Solution accuracy

We evaluate solution accuracy by computing errors in the system of equilibrium conditions on a grid of wealth with 1001 nodes. (Recall that we used only 41 node to solve the system.)

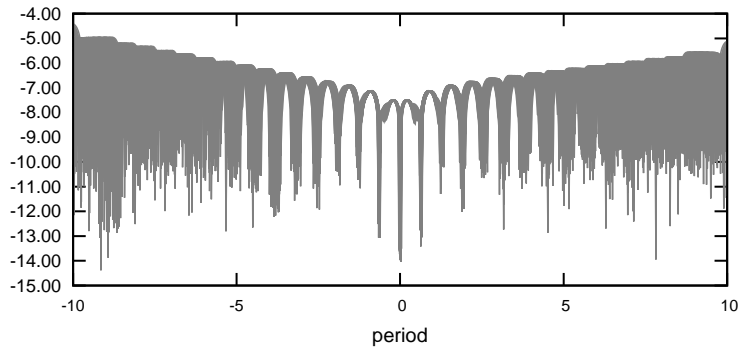


Figure 10: Equilibrium errors in the symmetric setting, model 1.

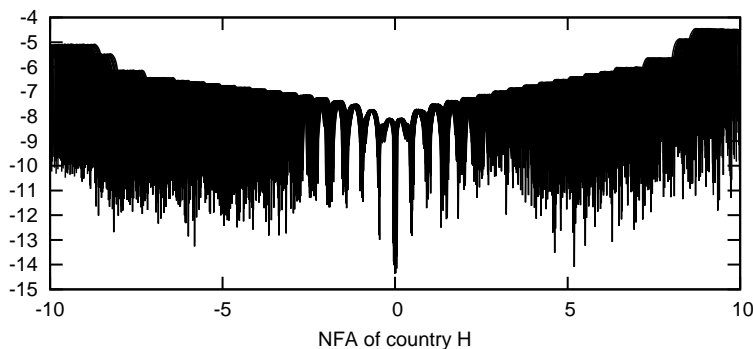


Figure 11: Equilibrium errors in the asymmetric setting with  $\sigma^h = 2\sigma^f$ , model 1.

## B Additional figures and tables

- B.1** Average of simulated paths in symmetric setting of model 1
- B.2** Iterative updating
- B.3** Asymmetric setting of model 1 with Epstein-Zin preferences
- B.4** Ergodic distributions of NFA

Figure 15 plots ergodic distributions of NFA in the asymmetric case of model 1, with  $\sigma(Y^h) = 2\sigma(Y^f)$  and endogenous discount factor,  $\eta > 0$ . Ergodic distributions were computed from samples of 1 million simulated observations. We used the more accurate perturbation solution that corresponds to  $\bar{W} = 0$ . With a higher value of  $\eta$  NFA does not wander far away from zero where the quality of the perturbation solution is degraded. So, the higher value of  $\eta$  the closer are the two solution methods.

Figure 16 plots ergodic distribution of NFA in model 2, obtained by the global solution method. The ergodic distribution for the DS method could not be obtained, as the simulations diverged.

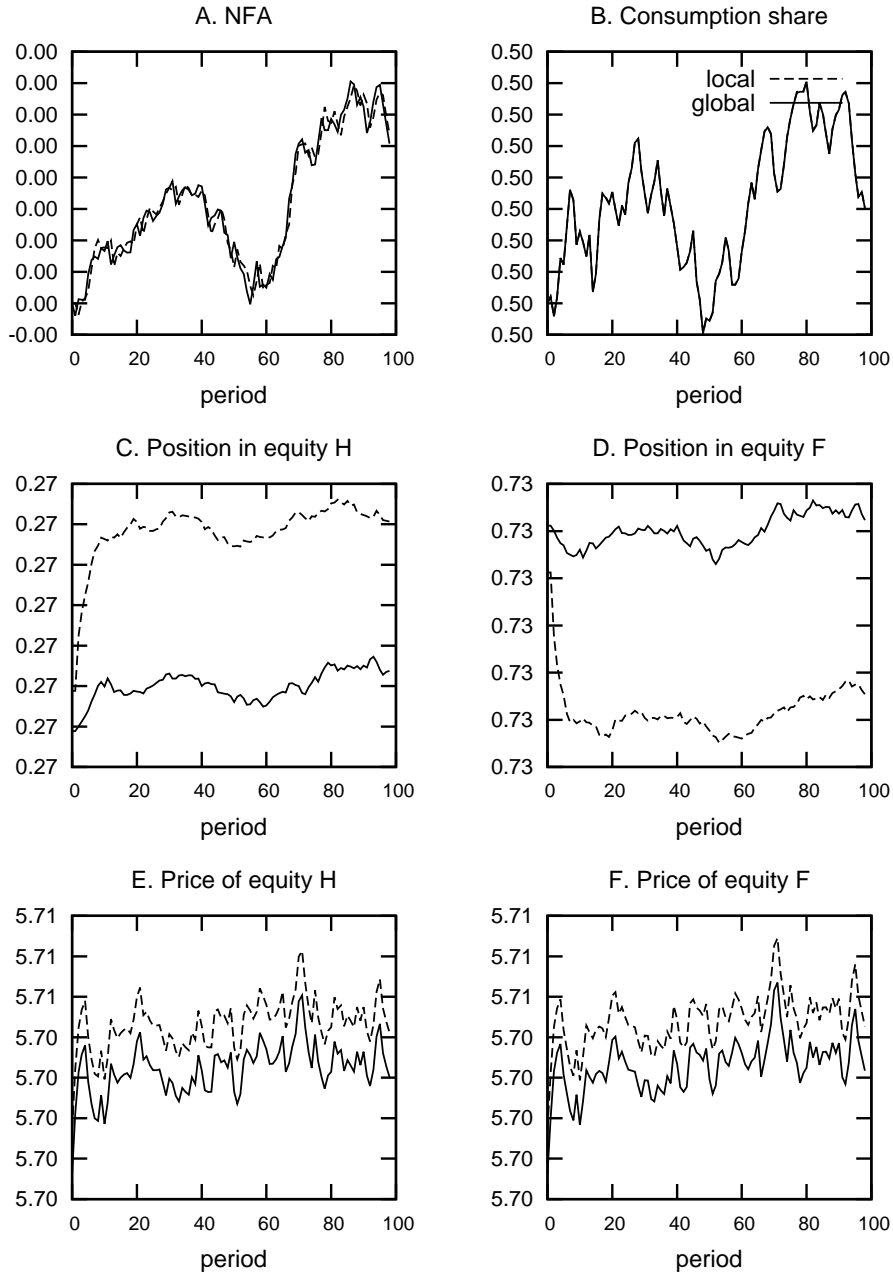


Figure 12: Average simulated paths for country  $h$  in the symmetric setting, model 1.

### C Construction of policy functions

We now explain how to construct policies for the perturbation solution method that are functions of the wealth as in the global solution. To construct optimal policy for any variable  $X$  we need to know both current and past state realizations. To this end, fix current  $\mathbf{Y}$  and wealth  $W$ . For each past realization  $\mathbf{Y}_{-1}$  compute the implied past realization of wealth  $W_{-1}$ . We can now construct optimal policy for variable  $X$  for each  $\mathbf{Y}_{-1}, \mathbf{Y}$ .

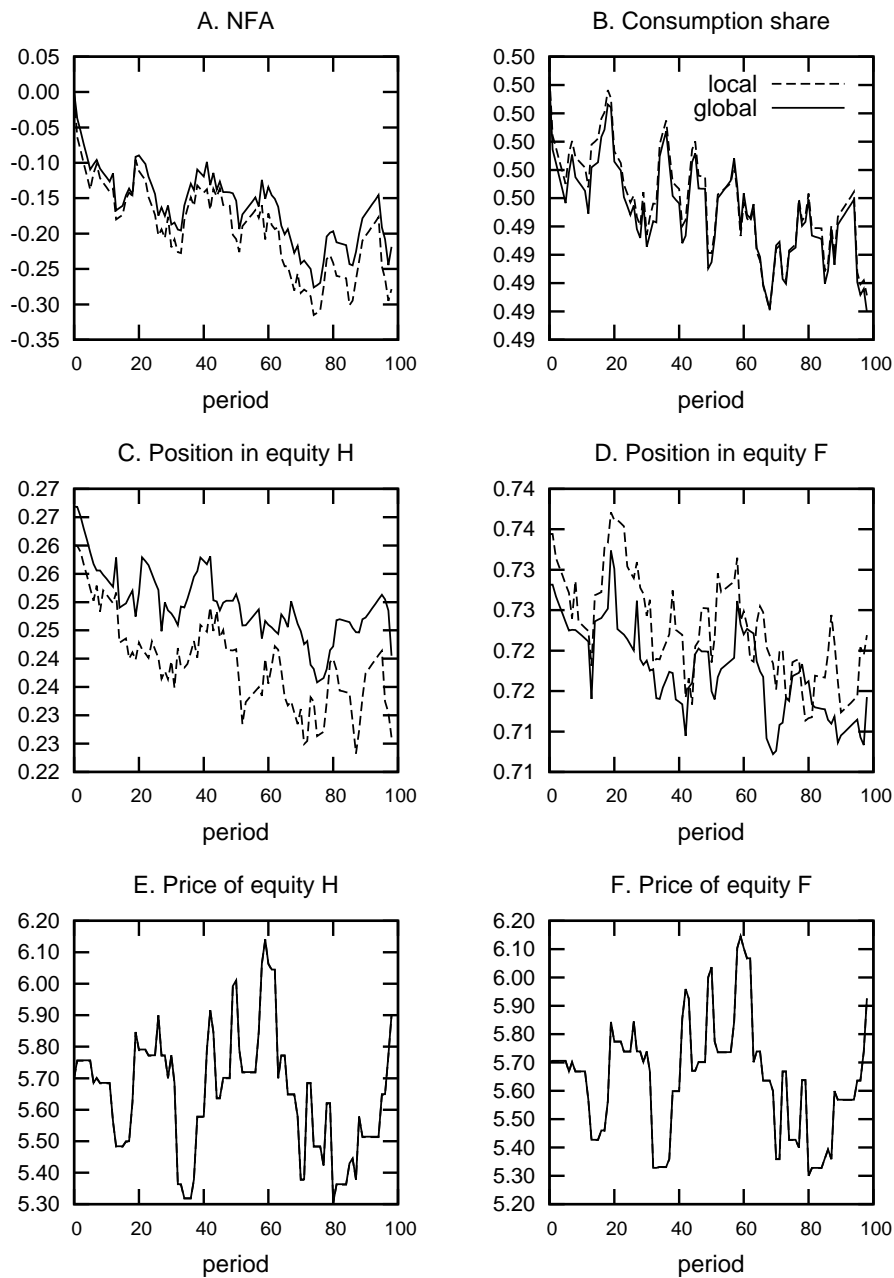


Figure 13: Simulated time paths for country  $h$  in the asymmetric setting with  $\sigma^f = 2\sigma^h$ . Approximation point:  $NFA^h = -6.19$ , model 1.

We then integrate out  $\mathbf{Y}_{-1}$  using its ergodic distribution. Formally, we compute the following object:

$$\begin{aligned} \rho_x(w, \mathbf{Y}) &= E_{w_{-1}, \mathbf{Y}_{-1}} \rho_{x, ds}(w, w_{-1}, \mathbf{Y}, \mathbf{Y}_{-1}) \\ &= E_{\mathbf{Y}_{-1}} \rho_{x, ds}(w, h(w, \mathbf{Y}, \mathbf{Y}_{-1}), \mathbf{Y}, \mathbf{Y}_{-1}), \end{aligned}$$

where  $h$  is the inverse of the wealth transition function  $w = \Omega_{ds}(w_{-1}, \mathbf{Y}_{-1}, \mathbf{Y})$ .

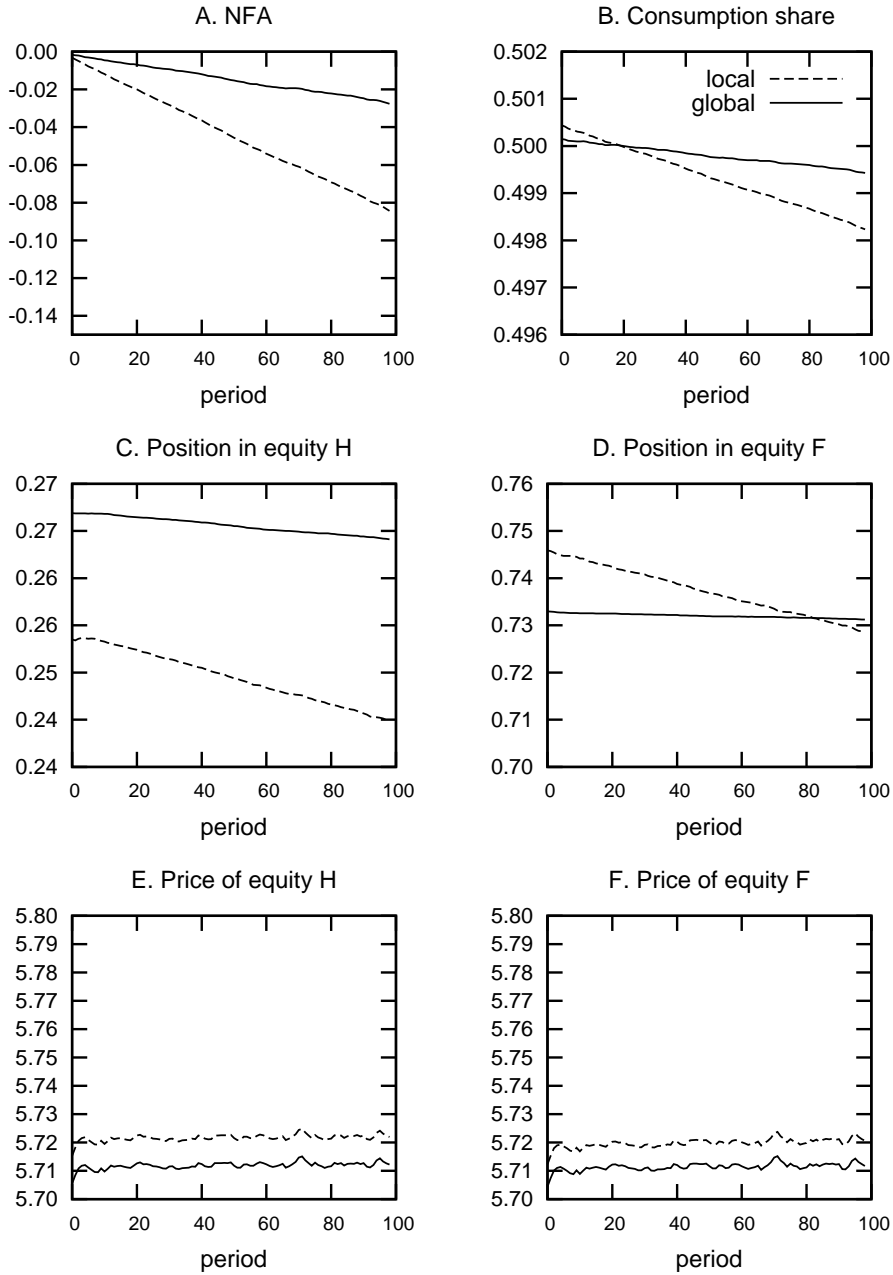


Figure 14: Average simulated paths for country  $h$  in the asymmetric setting with  $\sigma^f = 2\sigma^h$ , Epstein-Zin preferences with  $\gamma = 5$  and EIS 0.5, model 1.

Consider a symmetric setting. There is a continuum of steady state allocations, one for each of exogenously chosen  $NFA_{h0}$  position (first type of multiplicity). When we introduce an endogenous discount factor this multiplicity disappears. Only  $NFA_{ht} = 0$ , and a symmetric allocation that corresponds to it, is now acceptable.<sup>29</sup> The endogenous adjustment in the discount factor induce stationarity of a stochastic solution:

<sup>29</sup> Allocation that correspond to  $NFA_{h0} \neq 0$  converge to the steady state allocation. For example, if we set  $NFA_{h0} < 0$  consumption of country  $h$  will be lower initially and country  $h$  will discount future flows of utility

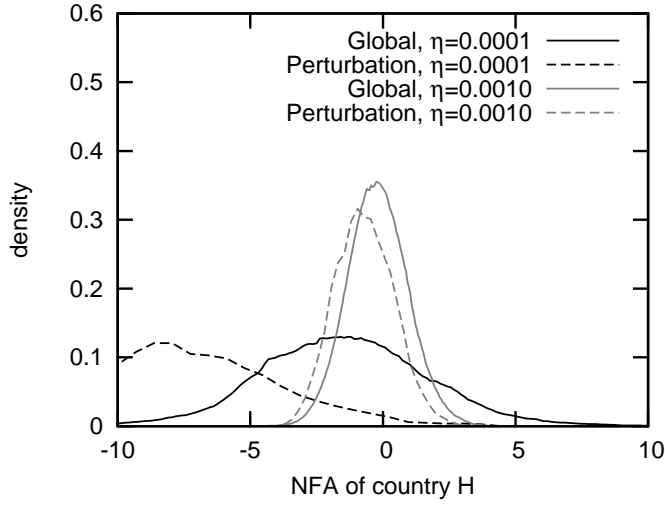


Figure 15: Ergodic distributions of country  $h$ 's NFA in the asymmetric setting with  $\sigma^f = 2\sigma^h$  and endogenous discount factor,  $\eta \in \{0.0010, 0.0001\}$ , model 1.

when a country's NFA decreases this country becomes more patient and saves more until its NFA recovers.

Now suppose that the two economies are not symmetric. Then with an endogenous discount factor  $\beta(c) = \beta c^{-\eta}$  steady state may not exist!<sup>30</sup> It is tempting then to approximate the system of equilibrium conditions around the mean of equilibrium variables. However, this point is generally not a solution to the deterministic version of the system of equilibrium conditions. Thus, it is a judicial choice.

$$NFA_{ht} = q_{ft}\theta_{ft}^h - q_{ht}\theta_{ht}^f = q_{ft}\theta_{ft}^h + q_{ht}\theta_{ht}^h - q_{ht}.$$

less. That is country  $h$  will save until consumption of the two countries equalize.

<sup>30</sup>The argument is simple. For the allocation to be stationary countries must discount at the same rate. Therefore countries' consumption must be the same. But this is impossible, say, when countries receive different but constant income.

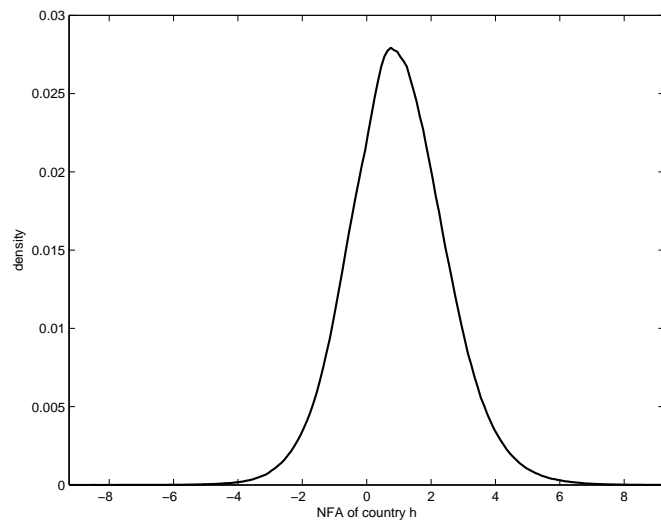


Figure 16: Stationary distribution, model 2.