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*DOI:*
10.57938/7852613a-1eb6-4ef0-8587-7b5380f02b1b

Published: 01/09/2023

*Document Version*
Publisher's PDF, also known as Version of record

Department of Economics
Working Paper No. 350

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September 2023

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# From Linear to Nonlinear: Rethinking Inflation Dynamics in the Calvo Pricing Mechanism *

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September 26, 2023

**Abstract**

Modern macroeconomics is increasingly leaning towards nonlinear solution methods. Our paper addresses the importance of nonlinearities in price setting. We demonstrate how nonlinearity in endogenous price adjustments, due to misalignments in relative prices, can trigger a price dispersion inflation spiral. This phenomenon yields globally unstable dynamics, even in instances where the model is locally stable around the non-stochastic steady state. We introduce the concept of the stability region as a nonlinear counterpart to the determinacy region. Our findings indicate that in a nonlinear world, the Taylor principle alone does not guarantee inflation stability and stable macroeconomic model moments. This new understanding not only challenges the conventional wisdom on inflation stabilization but also underscores the urgency for recalibrating monetary policy strategies in response to these dynamics.

*Keywords:* Determinacy, stability; price dispersion; monetary policy; nonlinear solution methods; macro-finance
*JEL-Codes:* E13, E31, E43, E44

*We wish to thank our working paper referees Martin Andreasen and Guido Ascari for excellent comments. We have also benefited from discussions with Fernando Alvarez, Larry Christiano, Roberto Billi, Alex Grimaud, Michal Marencak and seminar participants at DNB, CNB, NBU, NBS, UC Louvain, 60th SUERF Anniversary Conference. The authors gratefully acknowledge financial support from the Austrian National Bank, Jubilaeumsfond Grant No. 17791.
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## 1 Introduction

Investigating inflation dynamics and monetary transmission fundamentally relies on macroeconomic models that incorporate nominal rigidities, with Calvo's (1983) staggered price-setting mechanism being the prevalent approach. Traditionally, macro dynamics within the Calvo framework have predominantly been analyzed using (log-)linear approximations. However, the limitations of these conventional methods have been starkly revealed by volatile episodes such as the Great Financial Crisis, the pandemic, or global inflation spikes. These episodes necessitate a deeper exploration into the nonlinearities of business cycles, a shift emphasized by Fernandez-Villaverde et al. (2016), Mendoza (2016), and Mendoza and Villalvazo (2020), and lead to a discernible move towards nonlinear solution methods. These methods also play a pivotal role in asset pricing, especially in understanding risk premia and precautionary savings (see, e.g., Rudebusch and Swanson (2012), Andreasen et al. (2018), Horvath et al. (2022)). An increasing body of recent work has also highlighted the significance of accounting for nonlinearities in price setting behavior in explaining a variety of macroeconomic puzzles, e.g., Harding et al. (2022), Linde and Trabandt (2018), Ascari et al. (2023).

While the aforementioned literature has made substantial progress in understanding the implications of nonlinearities, a notable oversight persists in ignoring the full spectrum of model-implied nonlinearities in price-setting behavior: dispersion of prices and firms' inherent nonlinear markups over marginal costs, that come from consideration of risk and precautionary behavior, imply that relative prices of good varieties can diverge. Previous studies utilizing the nonlinear Calvo framework, such as Rudebusch and Swanson (2012), Andreasen and Kronborg (2020), Anderson et al. (2010), Marsal et al. (2019a), and Gasteiger and Grimaud (2023) have selectively adopted specific model calibrations or integrated features that mitigate the pronounced nonlinearities in price setting. This approach aims to secure model stability and anchored inflation expectations while bypassing the complexities arising from divergent relative prices. However, employing models that preemptively rule out the possibility of such divergence may result in model mispecification, biased identification, and flawed counterfactual policy analyses, leaving a comprehensive exploration of monetary policy in such contexts largely uncharted.

This paper fills this gap by studying the region of monetary policy reactions capable of maintaining inflation stability even in the presence of possible divergence in relative prices. The monetary policy ensuring model stability needs to robustly counteract the consequences of a price dispersion driven inflation spiral, a phenomenon exclusive to nonlinear settings. In elucidating these dynamics, our paper offers three major contributions.

First, a key insight is that inflation dynamics (and macroeconomic dynamics in general) can quickly turn unstable in a nonlinear setting even if the linear approximation is stable. We develop the novel concept of the '*stability region*' as a nonlinear counterpart to the determinacy region which provides prescriptions on how endogenous monetary policy can preserve stable inflation. The determinacy region spans all parameter combinations of the monetary policy rule that ensure stable inflation based on first-order, linearized dynamics. Intuitively, the central bank must follow the Taylor principle, according to which the nominal interest rate should rise more than proportionally with inflation (Blanchard and Kahn (1980) and Bullard and Mitra (2002)). The stability region similarly spans all parameter combinations of the monetary policy rule that ensure stable inflation *in the nonlinear setting*. Intuitively, this is achieved when monetary policy reactions contain a divergence in relative prices and avoid the price dispersion inflation spiral. Our research underscores that, in a nonlinear setting, the threat of the price dispersion driven inflation spiral significantly curbs the monetary policy space ensuring price stability. In particular, monetary policy is constrained in its ability to respond to the real side of the economy when stabilizing prices. Many of the combinations of policy rule parameters commonly used in the literature imply stable inflation dynamics in linearized models, but lead, in nonlinear settings, to explosive inflation and de-anchored inflation expectations. This happens especially under persistently high inflation, i.e. in the presence of positive trend inflation. Already in a linearized setting, Ascari and Sbordone (2014) and Coibion and Gorodnichenko (2011) have documented that, with trend inflation, the Taylor principle is not sufficient and that a more aggressive policy response to inflation is required. We find that, once we move to a nonlinear setting with trend inflation, the shrinkage of policy room to maneuver is even more pronounced than previously documented. Trend inflation prohibits virtually any aim to stabilize the real side of the economy.

Second, building on Fernandez-Villaverde et al. (2016), who attest that perturbation methods provide a high level of accuracy, we underscore that outside of the stability region, the accuracy of these methods wanes, and they become not just imprecise but fundamentally flawed. Moreover, outside of this region solutions from global approximations do not exist. Consequently, the results of work that neglects this stability caveat become biased and suffer from large approximation errors, a leading example being the seminal work of Rudebusch and Swanson (2012).

Third, our analysis reveals the underlying reason for the necessity of a more aggressive monetary policy in a nonlinear setting reflected by the contraction from the determinacy region to a more confined stability region. It is the looming threat of a price dispersion-induced inflation spiral not present in linear models. Linearly approximated models ignore the dispersion of prices in the economy (see Schmitt-Grohe and Uribe (2006)) and therefore the possibility of starting the price inflation spiral. In nonlinear models, this spiral arises from the divergence in relative prices, induced by the nonlinearity in price markup. Using nonlinear solution methods, we identify a novel threshold level of inflation beyond which inflation dynamics become unstable; inflation triggers further inflation by widening the dispersion of prices across products and starts a self-reinforcing price-inflation spiral. In more detail, the price inflation spiral results from Calvo firms' setting of (relative) prices. In each period, only a random fraction of firms can reset their prices, with the remaining firms maintaining their existing prices. These firms are otherwise the same in every aspect, except for the price they charge. In this situation, a shock that drives up inflation creates misalignment in relative prices and widens the distribution of prices in the economy. This is primarily because firms unable to adjust their prices in response to an inflationary shock fall behind, locating themselves in the left tail of the price distribution. Subsequently, even in the absence of any further inflationary shocks, these firms attempt to align with the increased aggregate price level. Being forward looking, they set a profit maximizing price above the aggregate price level, creating large endogenous price changes, that, when inflation is above the threshold, further widen the distribution of prices. In short, inflation generates inflation. Whether a spiral is triggered is contingent upon the magnitude of the shock and the degree of model nonlinearity with factors like trend inflation and decreasing returns intensifying this nonlinearity. This model's intrinsic nonlinearity impacts the risk adjustments made by firms in their sticky price markups due to precautionary motives. Aware of a potential price-dispersion spiral, firms increase their average markup. Such unstable inflation dynamics pose a new challenge for monetary policy in anchoring inflation expectations.

Our analysis proceeds in three distinct steps. In the first step, we show that the price-inflation spiral is a global property of the model (no approximation involved) and is independent of the solution method. We document this by studying price dispersion dynamics from the exact, non-approximated nonlinear law of motion. We identify threshold values of inflation across state dependence of past price dispersion. Inflation above the threshold value triggers the price- inflation spiral. In the second step, we use a simple New Keynesian model with Calvo prices similar to Galí (2015), Anderson et al. (2010), Miao and Ngo (2019) to demonstrate that our findings in step one can be generalized in a full-scale model. Following the literature studying the determinacy of linearized solutions (Ascari and Ropele (2009), Coibion and Gorodnichenko (2011), Lubik and Schorfheide (2004)), we simulate the determinacy region and extend it by displaying, in addition, the stability region relevant for nonlinear models. Whenever the price-inflation spiral drives economic fluctuations in the nonlinear model, a third-order perturbation approximation to the solution produces substantially different (amplified) moments compared to the linear approximation to the solution. This is equivalent to saying that the linear approximation falls into the determinacy region and produces stable inflation, but the third-order approximation falls outside the stability region, producing unstable inflation (and macroeconomic) dynamics. To be precise, even if the third-order approximate solution can be obtained with standard perturbation solution techniques, the amplified and counterfactually elevated simulated moments derived from it are a reflection of the fact that it is not a valid solution. An important implication of this is also that there is no (stable) global solution outside the stability region, even if the corresponding linear solution is determined.

In the third step, we present stochastic simulation results for the full macro-finance model of Rudebusch and Swanson (2012). This model serves as an influential example of a case for which the solution lies outside of the stability region[1]. As a result, part of the

[1]To answer the question why many of the applications in macro using global methods did not report

generated volatility of macro and financial variables emphasized as the main result stems from inflation instability triggered by the price dispersion-inflation spiral.

### 1.1 Literature review

Our paper relates to the literature on recent advances in solution methods for rational expectations dynamic general equilibrium models. The price-inflation spiral was first noticed by Andreasen and Kronborg (2020) in the context of their novel extended perturbation method. They find that: "standard perturbation generates a price-inflation spiral, which eventually may lead to explosive dynamics" ... "because standard perturbation is unable to account for a previously unnoticed upper bound on inflation with Calvo pricing,". Our work extends and further develops Andreasen and Kronborg's findings. We differ in several dimensions; in the depth of the analysis and in showing that the price-inflation spiral is a global property of Calvo pricing independent of perturbation solutions. We show that in the Calvo (1983) pricing model there exists a previously unidentified threshold value on inflation which gives rise to a price-inflation spiral. We also show that the upper bound on inflation is reached because of the price inflation spiral and not the other way around as found by Andreasen and Kronborg (2020). Further, we go beyond Andreasen and Kronborg (2020) by discussing the nonlinear model's stability properties. We provide a detailed analysis of the price-inflation spiral both in the nonlinearly solved model with Calvo pricing and as a phenomenon independent of the model solution method by focusing on the exact nonlinear equation for price dispersion.

Our paper also strongly relates to previous research on Calvo pricing and trend inflation. Ascari (2004), Ascari and Ropele (2009) and Ascari and Sbordone (2014) study effects of trend inflation on the deterministic steady state in the models of time dependent contracts. Amano et al. (2007) study the impact of trend inflation on stochastic means of macroeconomic variables and find that the nonlinearity introduced by price dispersion implied by the second order approximate solution increases the welfare costs of inflation. We extend this literature by studying Calvo pricing in a stochastic nonlinear setting more generally, by *i*) considering higher order perturbation solutions and global solutions, *ii*) considering other forms of nonlinearities which, similarly to the introduction of trend inflation, increase the dispersion of prices across product varieties, and by *iii*) adding a formal exposition of the channels through which the dispersion of prices transmits to the real economy. Ascari (2004) and Ascari and Sbordone (2014) show that trend inflation significantly affects the cost of inflation. We add to their results, showing our finding that the price-inflation spiral, present in nonlinear solutions, significantly increases the cost of inflation in the model further. Ascari (2004) concludes that Taylor pricing should be the preferable implementation of price stickiness because of the sensitivity of Calvo pricing to

the problems with instability we replicate the globally solved models by Anderson et al. (2010) and Miao and Ngo (2019) to show that their calibration falls well into the stability region and therefore in these applications the issue was not reported before.

trend inflation. We build on his findings and show, in the application to Rudebusch and Swanson (2012), that Taylor and Rotemberg pricing avoid the problems related to the price dispersion spiral.

Harding et al. (2022) shows that nonlinearities in price setting together with kinked demand curve can rationalize the elusive deflation puzzle. Ascari et al. (2023) further underscores the pivotal role of nonlinearities in delineating the long-run relationship between inflation and economic activity, referred to as the long-run Phillips curve. Empirical validations for the existence of nonlinearities in price setting are presented in works such as Alvarez et al. (2019),Sheremirov (2020), and Adam et al. (2023). In early results of our research agenda, put forth in Marsal et al. (2019b) we extensively study the explosive dynamics of the Calvo pricing mechanism in the nonlinear macro-finance model of Rudebusch and Swanson (2012), which recently sparked the interest also of other authors such as Iania et al. (2023), who published results with significant overlap to our early work.

# 2 The Baseline Model

Our baseline model is a standard textbook style New Keynesian DSGE model with Calvo pricing (e.g., Galí (2015), Anderson et al. (2010), Miao and Ngo (2019)) which can be viewed as a simplified version of the Rudebusch and Swanson (2012) (RS) model we adopt later. The model consists of a continuum of firms which operate under monopolistic competition and are subject to nominal rigidities à la Calvo. Firms produce with a technology that uses labor and fixed capital as production inputs. Households have CRRA preferences over consumption and labor, and make optimal choices over consumption, labor supply and bond holdings. The monetary authority follows a Taylor rule. Model dynamics are driven by shocks to total factor productivity.[2] Since the model is quite standard, experienced readers may want to only skim over most of the model description, and jump directly to the section where we discuss price dispersion and its properties, section 3.

## 2.1 Households

The household maximizes lifetime utility of the form

$$U(C_t, N_t) = E_t \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\varphi}}{1-\varphi} + \chi_0 \frac{(1-N_t)^{1-\chi}}{1-\chi} \right], \quad \varphi,\, \chi > 0, \tag{1}$$

where $\varphi,\ \chi,\ \chi_0 > 0$. The intertemporal elasticity of substitution (IES) is $1/\varphi$, and the Frisch labor supply elasticity is given by $(1-\overline{N})/\chi\overline{N}$, where $\overline{N}$ is the steady state level of hours worked. The households' problem is subject to the flow budget constraint:

$$B_t + P_t C_t = W_t N_t + D_t + R_{t-1} B_{t-1} - \tau_t. \tag{2}$$

[2] As we solve the model globally and the global solution is subject to the curse of dimensionality, we consider a parsimonious setup with a low number of state variables.

In equation (1), $\beta$ is the discount factor. Utility ($U$) at period $t$ is derived from consumption ($C_t$) and leisure ($1 - N_t$). $E_t$ denotes expectations conditional on information available at time $t$ . As the time endowment is normalized to one, leisure time ($1 - N_t$) is what remains after spending some time working ($N_t$). $W_t N_t$ is labor income, $R_t$ is the return on the one-period nominal bond, $B_t$. $D_t$ is dividend income, $\tau_t$ are lum-sum taxes by the government.[3]

## 2.2 Firms

Final good firms operate under perfect competition with the objective to minimize expenditure subject to the aggregate price level $P_t = \left[\int_0^1 P_t^{1-\epsilon}(i)(di)\right]^{\frac{1}{1-\epsilon}}$, where $P_t(i)$ is the price of the intermediate good produced by firm $i$, using production technology $Y_t = \left[\int_0^1 Y_t^{\frac{\epsilon-1}{\epsilon}}(i)di\right]^{\frac{\epsilon}{\epsilon-1}}$. Final good firms aggregate the continuum of intermediate goods $i$ on the interval $i \in [0,1]$ into a single final good. Parameter $\epsilon$ determines the elasticity of substitution between varieties. The cost-minimization problem of final good firms delivers demand schedules for intermediary goods of the form:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t. \tag{3}$$

A continuum of intermediate firms operates in the economy. Intermediate firm $i$ produces according to a Cobb-Douglas production function, where $\theta$ denotes the capital share, and where $\bar{K}$ refers to the fact that firms have fixed capital:[4]

$$Y_t(i) = A_t \bar{K}^\theta (N_t(i))^{1-\theta}, \tag{4}$$

Technology follows the autoregressive process:

$$\log A_t = \rho_A \log A_{t-1} + \sigma_A \varepsilon_t^A, \varepsilon_t^A \sim N(0,1). \tag{5}$$

Intermediate firms facing Calvo contracts maximize the present value of future profits by choosing price, $P_t(i)$,

$$E_t \left\{ \sum_{k=0}^{\infty} \zeta^k Q_{t,t+k} \frac{P_t}{P_{t+k}} \left[ P_t(i) Y_{t+k}(i) - P_{t+k} W_{t+k} N_{t+k}(i) \right] \right\}, \tag{6}$$

[3] The government is assumed to follow a balanced budget each period financing constant government expenditure, $G$, each period, i.e. $\tau_t = G$.

[4] Fixed capital can be interpreted as a model with endogenous investment that features high adjustment costs in investment. Capital which is treated as fixed implies decreasing return to scale (DRS) and is supported by the arguments in McCallum and Nelson (1999), for instance a very small correlation between capital and output at business cycle frequency, and the fact that capital is in general changing little in the short run.

where $Q_{t,t+k} = \beta \left(\frac{C_{t+k}}{C_t}\right)^{-\varphi}$ is the real stochastic discount factor from period $t$ to $t+k$. The Calvo parameter $\zeta$ represents the probability that a firm is not able to reset its price in a given period, which controls the average frequency of price changes. $W_t$ denotes the real wage, the term $P_{t+k}W_{t+j}N_{t+j}(i)$ represents the cost of labor in nominal terms. The optimal price chosen by the price resetting firm, $P_t^*$, relative to aggregate price index, $P_t$, is $p_t^* = \frac{P_t^*}{P_t}$ and equals a weighted average of current and future expected real marginal costs. The ratio, $p_t^* = \frac{P_t^*}{P_t}$, is often referred to as the price-adjustment gap in the literature (c.f. Ascari and Sbordone (2014)). We follow this terminology throughout the paper, and write:

$$(p_t^*)^{1+\frac{\theta\epsilon}{1-\theta}} = \frac{\epsilon}{\epsilon-1}\sum_{k=0}^{\infty}\Upsilon_{t+k}MC_{t+k}, \tag{7}$$

where $\Upsilon_{t+k} = \frac{\zeta^k E_t Q_{t,t+k}\Pi_{t+k}^{\frac{\epsilon}{1-\theta}}Y_{t+k}}{\sum_{k=0}^{\infty}\zeta^k E_t Q_{t,t+k}\Pi_{t+k}^{\epsilon-1}Y_{t+k}}$, and where $\Pi_t$ is gross inflation. Throughout the paper, $\Pi_t$ denotes the gross inflation rate, defined as $\Pi_t = P_t/P_{t-1}$; lower case variable $\pi_t$ instead denotes the (annualized) net inflation rate in percent, $\pi_t = 100\log(\Pi_t^4)$. $\Upsilon_{t+k}$ is the time varying mark-up implied by price rigidity and $\frac{\epsilon}{\epsilon-1}$ is the mark-up implied by monopolistic competition. $MC_t$ is average real marginal cost, defined as:

$$MC_t = \frac{1}{1-\theta}\left(\frac{W_t}{A_t}\right)\left(\frac{Y_t}{\bar{K}A_t}\right)^{\frac{\theta}{1-\theta}}. \tag{8}$$

### 2.3 Monetary Policy

The model is closed by a monetary policy rule:

$$\log(i_t) = \log(\bar{\imath}) + \phi_\pi[\log(\Pi_t) - \log(\overline{\Pi})] + \phi_Y \log\left(\frac{Y_t}{\bar{Y}}\right), \tag{9}$$

where $i_t$ is the quarterly policy rate and $\bar{\imath} = \overline{\Pi}/\beta$. $\overline{\Pi}$ is the inflation target of central bank defining the value of trend inflation. $\bar{Y}$ denotes the steady-state level of output, $Y_t$ .

## 3 Nonlinear Implications of Price Dispersion

The properties of price dispersion in nonlinear settings differ substantially from those of the linearized version. In the following, we discuss how price dispersion, as implied by the exact nonlinear law of motion, given by equation (14) below, affects the global properties of the model. Importantly, the analysis of this section is independent of any choice of (approximate) solution method. Section 3.1 derives the recursive formulation of price dispersion and shows that it can be interpreted as a resource cost, reducing output produced per unit of inputs. Section 3.2 discusses the presence and effect of upper and lower bounds on price dispersion. These bounds define a region over which price dispersion is well-defined, but beyond which a (real) solution for price dispersion ceases to exist and where the model economy's production stops. Section 3.3 provides a novel analysis of when inflation dynamics turn unstable, defining a previously unnoticed threshold level of inflation which triggers an inflation spiral and de-anchors inflation expectations in the model.[5].

### 3.1 Resource Cost of Price Dispersion

Price dispersion is defined by aggregating output across firms (firm $i$'s production functions), where we equalize the supply of intermediate good $i$, equation (4), with the final good producers's demand curve for intermediate good $i$, equation (3), to obtain $A_t\bar{K}^{\theta}N_t^{1-\theta}(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon}Y_t$ . Assuming that workers are homogeneous across $i$, the aggregation of labor input is $N_t = \int_0^1 N_t(i)di$ and together with the definition of price distortion[6], $S_t = \left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} di\right]^{1-\theta}$, it delivers the aggregate production function,

$$Y_t S_t = A_t\bar{K}^{\theta}N_t^{1-\theta}. \tag{10}$$

We define $S_t$ so that the cost of prices dispersed across product varieties can be directly expressed in terms of output loss. Variable $S_t$ is, therefore, a model-consistent index of price distortion and summarizes the resource costs induced by the inefficient price dispersion featured in the Calvo model. As a consequence of price dispersion the equilibrium allocation is not optimal. The subset of firms which cannot reset their price, charge either a lower or higher price for their product varieties than price-resetting firms. Since the demand for intermediate output comes from a perfectly competitive final good sector, where intermediate goods enter symmetrically and with equal weight, the firm charging a lower (higher) price than the price-resetting firm is not efficiently using its production capacities and produces too much (little). The efficient resource allocation dictates that each firm produces the same amount of goods. Formally, the loss of GDP due to price dispersion, $\Delta_t$, in percent can be defined as

$$\Delta_t = 100\left(1 - S^{-1}\right). \tag{11}$$

We can use the Calvo (1983) result and rewrite $S_t^{\frac{1}{1-\theta}}$ recursively as[7]

[5] In addition, in appendix A.2 we study the numerical properties of the true, fully nonlinear price dispersion (as described by the actual nonlinear law of motion) and its Taylor series expansions. This analysis helps us to build intuition for how the behavior of price dispersion affects the solution and stability of the simulation in our nonlinearly solved New Keynesian model of section 4

[6] Note that the literature typically differentiates between price distortion, $S_t$, and price dispersion, $S_t^{\frac{1}{1-\theta}}$.

[7] $S_t^{\frac{1}{1-\theta}} = (1-\zeta)\left(\frac{P_t^*}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} + (1-\zeta)\zeta\left(\frac{P_{t-1}}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} + (1-\zeta)\zeta^2\left(\frac{P_{t-2}}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} \dots$
where the expansion from time $t-1$ can be summarized by $S_{t-1}$.

$$S_t^{\frac{1}{1-\theta}} \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\varepsilon}{1-\theta}} di, \qquad (12)$$
$$= (1-\zeta)(p_t^*)^{\frac{-\varepsilon}{1-\theta}} + \zeta(\Pi_t)^{\frac{\varepsilon}{1-\theta}} S_{t-1}^{\frac{1}{1-\theta}}.$$

### 3.2 Lower and Upper Bound on Price Dispersion

Several points regarding the properties of equation (12) are worth highlighting for future discussion. First, the resource cost of price dispersion, $S_t$, is a concave function of the ratio of two price indexes, $\left[\int_0^1 P_t(i)^{\frac{-\epsilon}{1-\theta}}\right]$ and aggregate price index $P_t^{\frac{-\epsilon}{1-\theta}} = \left[\int_0^1 P_t^{1-\epsilon}(i)di\right]^{\frac{-\epsilon}{(1-\epsilon)(1-\theta)}}$. The bigger the difference between these indexes the wider is the distribution of prices across product varieties. Yun (1996) and Schmitt-Grohe and Uribe (2006) show that this ratio implies that price distortion $S_t$ is bounded from below by 1, $S_t \geq 1$.[8] When $S_t = 1$ all firms have the same prices in the economy.

Second, the Calvo pricing mechanism also implies an upper bound on inflation, $\Pi_t < \Pi^{upper}$. The upper bound consequently determines a maximum admissible level of price dispersion, $S_t^{upper}$ for given $S_{t-1}$. To see this, let us make use of the definition of the aggregate price index, and express the price-adjustment gap, $p_t^* = P_t^*/P_t$, as a function of inflation only,

$$p_t^* = \left[\frac{1-\zeta(\Pi_t)^{\epsilon-1}}{1-\zeta}\right]^{\frac{1}{(1-\epsilon)}}. \qquad (13)$$

Substituting this into equation (12) we obtain:

$$S_t^{\frac{1}{1-\theta}} = (1-\zeta)\left[\frac{1-\zeta(\Pi_t)^{\epsilon-1}}{1-\zeta}\right]^{\frac{\epsilon}{(\epsilon-1)(1-\theta)}} + \zeta(\Pi_t)^{\frac{\epsilon}{1-\theta}} S_{t-1}^{\frac{1}{1-\theta}}. \qquad (14)$$

The power term in (14), $(\frac{\epsilon}{(\epsilon-1)(1-\theta)})$, implies that price dispersion turns complex unless $\left[\frac{1-\zeta(\Pi_t)^{\epsilon-1}}{1-\zeta}\right] > 0$, which defines the inflation upper bound as $\Pi_t < \Pi_{max} \equiv \zeta^{\frac{1}{1-\epsilon}}$[9]. The dispersion of prices is therefore bounded both from below and above, creating two bounds on the underlying true policy function of $S_t$.

[8] We show in the appendix that the same result holds for price dispersion under decreasing returns to scale, $S_t^{\frac{1}{1-\theta}}$.

[9] Note that Ascari (2004) derives an inflation upper bound for the deterministic steady state. The bound we present here holds also for the cyclical component of inflation and is often tighter. In our model the inflation upper bound is 11.72% for quarterly steady-state inflation versus 5.92% for for the cyclical deviation of inflation from its steady-state.

### 3.3 Price (Dispersion) Inflation Spiral

Inflation (together with structural parameters $\zeta$, $\epsilon$, $\theta$) determines the dynamic stability of the recursive formulation of price dispersion given by the nonlinear first-order difference equation (14). From this equation, we can identify, for each value of $S_{t-1}$, the specific threshold value of inflation, $\Pi_t^{threshold}$, beyond which $S_t > S_{t-1}$, i.e., inflation for which price dispersion becomes increasing in its own past, so that, the process for $S_t$ is not stable. Figure 1 plots this threshold value of inflation, $\Pi_t^{threshold}$ as a function of $S_{t-1}$: the red-shaded region defines the region of inflation values $\Pi_t$ below the inflation threshold $\Pi_t^{threshold}$, which imply stable price dispersion dynamics, i.e. for which $S_t < S_{t-1}$. Instead, levels of $\Pi_t$ above $\Pi_t^{threshold}$ trigger explosive price dispersion dynamics, $S_t > S_{t-1}$.

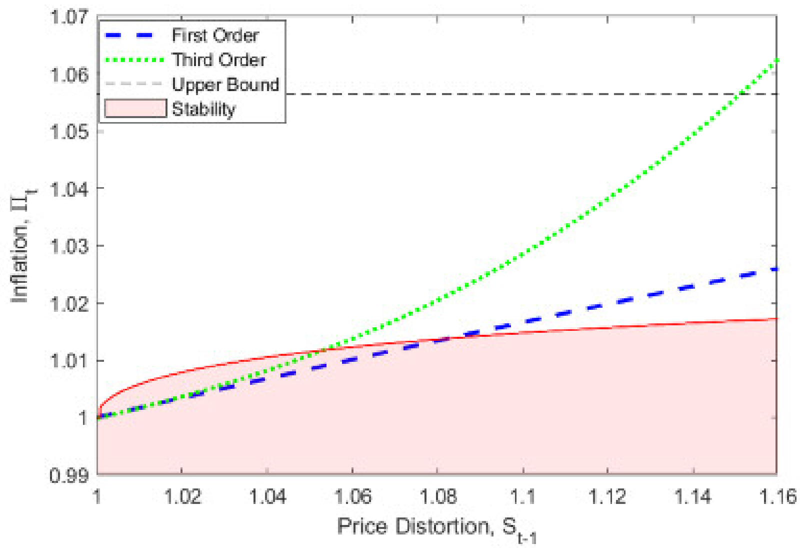
Inflation above the threshold value, $\Pi_t > \Pi_t^{threshold}$, triggers the price (dispersion) inflation spiral. In this case, inflation and price dispersion are self-reinforcing. High inflation, $\Pi_t$, at time $t$, leads to high dispersion of prices across product varieties, $S_t$, as only some firms can adjust their prices. In this region, the high dispersion of prices, $S_t$, in turn calls for such strong an adjustment in prices, and thus high inflation in the next period, $\Pi_{t+1}$, that price dispersion is rising even further. These reinforcing dynamics continue, and hence starts the price-inflation spiral.

We show in the next section that standard model parameterizations in a wide variety of (including the textbook) New Keynesian models quickly lead to values of inflation outside of the region of stable dynamics (the red-shaded region). In practice, in these applications, dynamics are locally stable around the non-stochastic steady state and for small shocks that do not push inflation above the threshold $\Pi_t^{threshold}$. For larger shocks –or more precisely, over some part of the state space, for high past price dispersion $S_{t-1}$ and low productivity realizations– inflation exceeds the inflation threshold $\Pi_t^{threshold}$ and dynamics become unstable. In this region dynamics of price dispersion and inflation may spiral off, without the chance to return to the locally stable region. For illustrative reasons, to better understand the relevance of the inflation threshold for the approximate perturbation solutions we discuss in section 4, we include policy functions, $\Pi_t = g(S_{t-1}, \overline{A}, \sigma)$ from the first and third perturbation solution in Figure 1 as an example[10] Plotted policy functions represent the relationship between $\Pi_t$ and state variable $S_{t-1}$, when exogenous state $A_t = \overline{A}$. For the example in Figure 1, the inflation spiral is triggered already for price dispersion implying mild inflation (above 1.3% quarterly). Quarterly inflation of 1.3% lies far away from the level of inflation implied by the inflation upper bound, of 5.92%. Andreasen and Kronborg (2020) have previously documented the importance of respecting the inflation upper bound for the precision of the approximate model solution. In contrast to Andreasen and Kronborg (2020), we stress here that the unstable dynamics of higher-order approximate solutions of a model with Calvo pricing are not caused by ignoring a upper bound on inflation, but, instead, arise from exceeding the threshold on inflation, $\Pi_t^{threshold}$. The threshold we identify

[10]Policy functions plotted here depend on a precise model and model parameterization: here they come from the benchmark model parameterization (see Table 1).

is obtained directly from the actual nonlinear equation that characterizes price dispersion dynamics, so that the explosive dynamics beyond that threshold are a true feature of the Calvo model and not an artefact of an approximation or the choice of solution method.

Figure 1: Threshold and Stability Region, Implying Stable Price Dynamics




*Note: The (red) line defines the inflation threshold,* $\Pi^{threshold}$. *Inflation values in the red-shaded region, below the inflation threshold,* $\Pi_t < \Pi_t^{threshold}$, *imply stable dynamics, where* $S_t < S_{t-1}$. *Inflation values above the inflation threshold,* $\Pi_t > \Pi_t^{threshold}$, *trigger a self-enforcing inflation spiral and lead to unstable dynamics,* $S_t > S_{t-1}$. *For illustrative purposes, we include policy functions from first (blue) and third (green) order perturbation solution, showing that unstable dynamics may arise already at mild inflation levels far below the inflation upper bound.*

To understand the economic mechanism leading to inflation values surpassing $\Pi_t^{threshold}$ and initiating a price inflation spiral, we must endogenize inflation and take a general equilibrium view. Consider an economy in a non-stochastic steady state, hit by a shock at $t = 1$ causing persistent inflation. Within the Calvo setting, firms react differently: some immediately set new profit-maximizing prices, $P_1^*$, while others maintain old prices, $P_1^{old}$. The aggregate price level at $t = 1$, $P_1$ is a (Calvo parameter-weighted) average of $P_1^{old}$ and $P_1^*$, so that $P_1^{old} < P_1 < P_1^*$. In the next period, prices will change (even in absence of any other shock) due to firms repricing their products. A fraction of firms that were stuck with the old price in period 1, $P_1^{old}$, can now set their profit optimizing price at $t = 2$, $P_2^*$, which contributes to further increasing the aggregate price level $P_2$. However, these second-round effects of price changes necessitate additional adjustments to the new relative prices by the firms that have already adjusted their prices in the first round.

This dynamic of firms catching up to the aggregate price level is standard for Calvo pricing in general. Whether it leads to divergence in relative prices depends on inflation's magnitude and degree of nonlinearity in price setting. When inflation is mild, firms adjust their prices by small increments and thus contribute little to further increases in the aggregate price level; inflation dynamics are stable. The (relative) price adjustments will continue and eventually will converge to the only optimal case when all firms have the same prices. Instead, when the level of inflation is high and exceeds the inflation threshold, $\Pi_t > \Pi_t^{threshold}$, these price re-setters increase their price by such an amount that the resulting dispersion of prices is even higher than it used to be. This starts a spiral from inflation to an increase in price dispersion, back to an increase in inflation and so on – producing unstable inflation dynamics. The convergence in relative prices depends on the size of the initial shock but also on the degree of model nonlinearity.

Firms set their prices in a forward-looking manner, knowing the probability that they might not be able to adjust their prices. This means that, in case of an inflation-inducing shock, firms face an immediate increase in the nominal wage bill, i.e., labor costs go up. For firms unable to re-optimize, their output is sold at the old price they are stuck with, and thus profits go down. This logic is symmetric, so that for the case of a shock that decreases inflation, costs go down and profits stay high (because of their constant price).[11] In a world with zero trend inflation and symmetric shocks the chances are equal that cost will be higher or lower than revenues. Up to a first order approximation (i.e., under certainty equivalence) inflation dynamics are contained.

However, when moving to higher-order approximations, where the certainty equivalence property no longer holds, there is an additional risk correction and firms' average sticky price markup reflects also the risk of a price-inflation spiral. Quantitatively, for values of inflation above $\Pi_t^{threshold}$, this risk correction contributes to on average more dispersed prices. The risk correction is the driving mechanism leading firms to set a high nonlinear sticky price markup, similar to (but distinct from) the markup discussed already in the linear case of models with trend inflation (see Ascari and Sbordone (2014)). With a high sticky price markup forward-looking firms set, for precautionary reasons, the price $P^*$ above the certainty equivalent optimal price. The higher nonlinear sticky price markup means that inflation generated by the price change from $P_1^{old}$ to $P_1^*$ will be larger and that prices will be even more dispersed. Which will lead to even larger price adjustments and even more dispersed prices. When inflation levels exceed the inflation threshold, the process of setting relative prices will therefore lead to explosive inflation dynamics.

In case of positive steady state inflation, i.e., trend inflation, the probability that exogenous inflationary shocks will trigger this price-inflation spiral further increases. Trend inflation spreads out the distribution of prices as those firms which cannot change their low prices are left behind further and further from the optimal price as the price level grows. When these firms can finally change their price they create large inflation and, in fact, instead of decreasing the dispersion of prices they further contribute to its widening.

[11] Appendix C and equation (C.7) presents formally these economic channels.

# 4 Stability of the Baseline NK Model

Section 3 established the nonlinear properties of price dispersion by studying the equation that governs its true nonlinear law of motion in isolation. This section instead shows that the consequences of Calvo pricing we outlined dominate the dynamic behavior of the nonlinearly solved dynamic general equilibrium New Keynesian model with important implications for monetary policy. For this purpose, we solve and simulate the example model from section 2 with both higher-order perturbation (local) and with global approximation methods.[12] Subsection 4.1 discusses the model parameterization, where we follow standard parameter choices in the literature. Subsection 4.2 introduces the concept of the *stability region* as a nonlinear analogue to the determinacy region, defining the combinations of policy parameters of the Taylor rule that imply stable inflation (and macroeconomic) dynamics *in a nonlinear context*. We show that solutions outside of the stability region are characterized by the price dispersion inflation spiral and deliver explosive dynamics. Importantly, this is the case for many Taylor rule parameter combinations that imply (determinate and) stable inflation dynamics in a linear (and otherwise equivalent) model setting. Outside of the stability region, explosive dynamics precludes one from obtaining a global solution and the third-order perturbation solution becomes invalid. In subsection 4.4 we consider the role of trend inflation and decreasing in, quantitatively, shaping the stability reagion. We build on the findings of Ascari and Ropele (2009) and Coibion and Gorodnichenko (2011) that the Taylor principle breaks under trend inflation and show that the stability region quickly shrinks with positive trend inflation. Similarly, following the literature on real rigidities as in Levin et al. (2008) we discuss the effect of decreasing returns to scale (as the most widespread form of real rigidity) on the stability region.

## 4.1 Parameterization

We summarize the choice of parameters in Table 1. Parameter values fall well into the region typically chosen in the literature (e.g. Galí (2015), Rudebusch and Swanson (2012)). The bottom part of Table 1 presents three model scenarios we consider to demonstrate the role of monetary policy in anchoring inflation expectations. The benchmark scenarios (cases A) rely on a standard assumption of zero trend inflation and decreasing returns to scale. Scenarios B relax the assumption of zero trend inflation and scenarios C consider the economy in the absence of real rigidity (adopting instead a linear-in-labor production function). We analyze several cases for the Taylor rule calibrations. We choose popular

[12]The perturbation solutions are obtained as implemented in Dynare routines. Simulated time series from higher-order solutions are pruned. As a global approximation method we adopt a policy function/ time iteration method developed by Coleman (1990, 1991) and adopted, e.g. in Prades and Rabitsch (2012); Rabitsch (2016). Appendix A provides a description of the algorithm for our example model. Alternatively, we also implement a projection method (Chebyshev collocation) as in Anderson et al. (2010), which yields virtually identical results.

Table 1: Parameterization and Scenarios

| | | | | | |
|---|---|---|---|---|---|
| $\beta$ | Discount factor | 0.99 | $\zeta$ | Calvo parameter | 0.76 |
| $\varphi$ | Coeff. of RRA | 2 | $\rho_A$ | Autocorrelation, TFP shock | 0.95 |
| $\epsilon$ | Elasticity of subst. | 6 | $\sigma_A$ | Volatility, TFP shock | 0.007 |
| $\frac{1-\bar{N}}{\bar{N}}\frac{1}{\chi}$ | Frisch elast. | 0.28 | $G/Y$ | government spending to GDP | 0.17 |

| | | | |
|---|---|---|---|
| **Benchmark** (A): zero trend inflation, decreasing returns-to-scale: $\bar{\pi}=0$, $\theta=1/3$: | | | |
| case A1: | $\phi_\pi = 2.5$ | $\phi_Y = 0$ | *Simple TR, Woodford(2003)* |
| case A2: | $\phi_\pi = 2.2$ | $\phi_Y = 0.43$ | *Coibion and Gorodnichenko (2011)* |
| case A3: | $\phi_\pi = 1.53$ | $\phi_Y = 0.77$ | *Taylor (1999)* |
| **Trend inflation (B):** 2% trend inflation, decreasing returns-to-scale: $\bar{\pi}=2$, $\theta=1/3$: | | | |
| case B1: | $\phi_\pi = 2.5$ | $\phi_Y = 0$ | *Simple TR, Woodford(2003)* |
| case B3: | $\phi_\pi = 2.5$ | $\phi_Y = 0.5$ | *Guerrieri and Iacoviello (2015)* |
| **Real rigidity (C):** zero trend inflation, linear-in-labor production: $\bar{\pi}=0$, $\theta=0$ | | | |
| case C1: | $\phi_\pi = 2.5$ | $\phi_Y = 0$ | *Simple TR, Woodford(2003)* |
| case C2: | $\phi_\pi = 1.53$ | $\phi_Y = 0.77$ | *Taylor (1999)* |

*Note: The first part presents baseline parameters common across all model parameterization scenarios. The second part specifies the parameters for cases A1-C2, where parameters $\phi_\pi$, $\phi_Y$, $\bar{\pi}, \theta$ differ across scenarios.*
*Scenarios A consider zero trend inflation and DRS in production.*
*Scenarios B consider positive trend inflation and DRS in production.*
*Scenarios C consider zero trend inflation and no DRS in production.*

estimates of the Taylor rule so that they lie *i*) in, *ii*) on the border and *iii*) outside of the stability region. Consequently, we demonstrate the resulting model dynamics.

## 4.2 Equilibrium Stability and the Anchoring of Expectations

The standard way of closing a macroeconomic NK model is the assumption that monetary policymakers set interest rates based on a simple Taylor rule. It is well known that to anchor inflation expectations in the linearized version of the model the weight on inflation must follow the Taylor (1993) principle, according to which the nominal interest rate should rise more than proportionally with inflation. Blanchard and Kahn (1980) and Bullard and Mitra (2002), among others, generalize the Taylor (1993) principle and define necessary and sufficient conditions for the rational expectations equilibrium to be unique, which is known as the *determinacy region*. The determinacy region provides a model-consistent definition of what it means to anchor inflation expectations by the central bank. Weber et al. (2022) define anchored expectations as: "changes in short-run inflation expectations that are largely uncorrelated with changes in long-run expectations". In other words, the central bank achieves its targeted inflation in the medium to long run. In NK macro models shocks which might, e.g., temporally lead to increased inflation are responded to by a monetary tightening, so that inflation returns to its steady state. The determinacy region ensures that the monetary policy response is strong enough.

We add to these well-known results by showing that model determinacy is not a sufficient condition for model stability in nonlinearly solved models with Calvo prices. We show numerically that ensuring that $S_t \leq S_{t-1}$ over the largest part of the state-space is the necessary condition for model stability. This result holds both for globally solved models and higher order (local) perturbation methods. Using the insights and results from section 3 we define the *stability region* as a nonlinear analogue to the determinacy region. The stability region is the space spanned by $\phi_\pi$ and $\phi_y$ for which $S_t \leq S_{t-1}$, i.e., for which inflation falls below the inflation threshold. Alternatively, the stability region can be thought of as the region for which the global solution exists and for which higher-order perturbation methods generate stable model dynamics. We define the stability region as the region where in a model simulation 'almost all' realizations of inflation (as obtained from third order approximated policy functions) fall below the inflation threshold. Formally, the region defined by

$$\max\Big(\Pi^{3rd.o.}(P_1(S_{t-1}, A_t), \gamma, \sigma), \ldots, \Pi^{3rd.o.}(P_{99}(S_{t-1}, A_t), \gamma, \sigma)\Big) \leq \Pi^{threshold}|S_{t-1} \quad (15)$$

provides a sufficient condition for the stability of simulated model dynamics. The function $\Pi^{3rd.o.}$ is the third order approximation to the policy function of inflation, which depends on the two state variables of our example model, $S_{t-1}, A_t$, and on $\gamma$ and $\sigma$. $\gamma$ is the vector of model parameters, which includes the policy parameters $\phi_\pi$ and $\phi_y$ and all other model parameters. $\sigma$ scales the amount of uncertainty in the model. The inflation policy function is evaluated over the relevant state space traveled in a (long) model simulation. As the relevant state space we take values that lie between the 1st and 99th percentile of the simulated bivariate distribution of $S_{t-1}$ and $A_t$, denoted by $P_1(S_{t-1}, A_t)$ and $P_{99}(S_{t-1}, A_t)$.[13] $\Pi^{threshold}|S_{t-1}$ calculates the inflation threshold based on the exact nonlinear form of price dispersion in equation (14) as the level of inflation at which $S_t = S_{t-1}$ conditional on $S_{t-1}$.

We use the following algorithm to calculate the stability region. We construct a grid over policy parameters $\phi_\pi$ and $\phi_y$, and, for each combination of the policy parameters follow four steps. First, we compute the threshold inflation from the non-approximated recursive form of price dispersion using a numerical equations solver. Second, we use the third order perturbation solution[14] to simulate the model and calculate the relevant state

[13] This choice is driven by the experience that model moments are well contained at this percentile. In contrast, we experienced first signs of instability (amplification of moments) when using the $P_5(S_{t-1}, A_t)$ and $P_{95}(S_{t-1}, A_t)$.

[14] Aruoba et al. (2006) show that higher order perturbation and projection methods deliver very similar results. We use the third order perturbation solution, because, as we show, it delivers virtually the same policy functions as a global solution within the stability region, yet global solutions cannot be obtained when model dynamics imply instability. Appendix A explains this is the case because the relevant state space in dimension $S_{t-1}$ over which the solution would need to be computed becomes unbounded for regions where $S_t > S_{t-1}$. Appendix B shows policy function solutions for the various scenarios of this section to develop these points further.

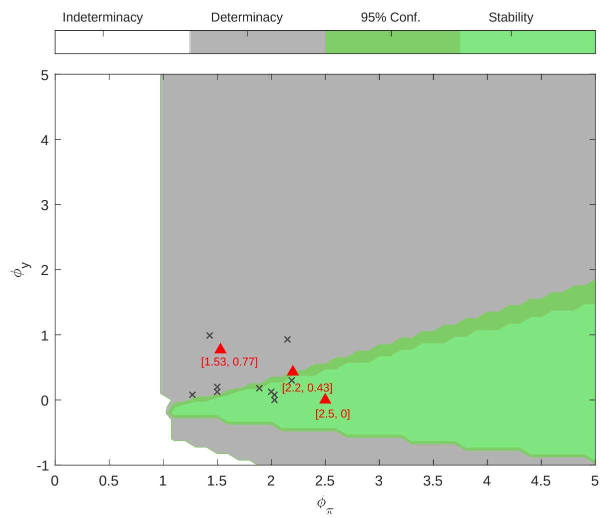
space traveled in this simulation as the one implied by the 1st to 99th percentile of the simulated bivariate distribution of $S$ and $A$. Third, we use the inflation policy function from the third-order approximate solution to calculate the implied levels of inflation over this state space, and find the maximum implied inflation. Fourth, we evaluate whether the implied inflation lies below or above the inflation threshold (i.e., within or outside the stability region (see Figure 1). Inflation outside of the stability region implies unstable dynamics.

### 4.3 Stability under Benchmark Scenario

To assess stability of the nonlinear model we build on the example of Ascari and Sbordone (2014) and Coibion and Gorodnichenko (2011). We simulate the *determinacy region* for the calibration of our benchmark example model and extend it by displaying the *stability region* relevant for nonlinear models. The gray area in Figure 2 depicts the determinacy region which demonstrates the well-known generalized Taylor principle that the central bank should respond more than one-to-one to inflation. It equals the combinations of $\phi_y$ and $\phi_\pi$ for which the model satisfies the Blanchard and Kahn (1980) conditions and the linear rational expectations system can be solved. The green area in Figure 2 instead is specific to the nonlinearly solved model and shows the stability region for which $S_t \leq S_{t-1}$. In other words, the determinacy region represents the first order condition for inflation stability whereas the condition $S_t \leq S_{t-1}$ defines the stability region in nonlinear models. Once out of the green region, approximate macroeconomic dynamics computed from the third order perturbation to the policy functions can (still) be obtained, but feature large local instabilities in the simulation and, as a consequence, yield wild dynamics of second moments as we present in Table 2. In practice, even a small part of the state space in a model simulation plagued by unstable price dispersion dynamics leads to unreasonable amplification in simulated model moments. For these reasons, obtaining a global solution outside of the stability region is not feasible. The red triangles in Figure 2 represent the calibration of the Taylor rule coefficients employed in benchmark scenarios A1-A3 (c.f., Woodford (2003), Coibion and Gorodnichenko (2011), Taylor (1999)). The gray marks depicts other seminal estimates from a literature survey (summarized in Table E.1). While these examples of Taylor rule coefficients fall well into the determinacy region, once the nonlinear aspects of the solution (e.g. dispersion of prices and inflation uncertainty) are taken into account many of the empirically relevant estimates quickly leave the stability area. Namely, a higher weight on the output gap[15], plays a potentially destabilizing role and calls for a more aggressive policy response to inflation even if an economy stays in the determinacy region.

[15] $\phi_y > 0.2$ is an common estimate, see Taylor (1999), Judd and Rudebusch (1998) and Clarida et al. (2000) and others.

Figure 2: Determinacy vs. Stability Region for the Simple Model, Benchmark Scenario A




*Note: Simulated determinacy (gray) and stability (green) region. Red triangles represent scenarios A1-A3. The stability region denotes the region where $S_t < S_{t-1}$ over the 1st to 99th percentile of the simulated bivariate distribution state variables, i.e. from $P_1(S_{t-1}, A_t)$ to $P_{99}(S_{t-1}, A_t)$. The region 95 % Conf. does the same from $P_5(S_{t-1}, A_t)$ to $P_{95}(S_{t-1}, A_t)$. Gray 'x' marks correspond to other empirically found Taylor rule coefficients in the literature (Table E.1 ).*

Table 2 accompanies Figure 2 and shows simulated moments for the benchmark scenarios represented by the red triangles. The simple feedback to inflation-only Taylor rule in case A1 falls well into the stability region. The model solutions of third-order and global solution method yield almost identical and stable moments (and policy functions). This attests higher-order perturbations a high solution accuracy. This is in line with previous findings by Fernandez-Villaverde et al. (2016), who show that higher order perturbation methods feature small approximation errors. Our results confirm these findings but only within the stability region. The lower part of Table 2 shows how much output in percent is lost due to dispersion of prices across products. Well within the stability region price dispersion is contained and in the range of empirical estimates (c.f. Nakamura et al. (2018)) and the lower and upper bounds on $\pi$ and $S$ are respected.

Table 2: Model-Based Unconditional Moments: Benchmark scenarios (A1-A3)

| | case A1: $\phi_\pi = 2.5$, $\phi_Y = 0$ | | | case A2: $\phi_\pi = 2.2$, $\phi_Y = 0.43$ | | case A3: $\phi_\pi = 1.53$, $\phi_Y = 0.77$ | |
|---|---|---|---|---|---|---|---|
| | 1st order | 3rd order | global | 1st order | 3rd order | 1st order | 3rd order |
| Moments of macroeconomic variables | | | | | | | |
| std($C$) | 2.73 | 2.74 | 2.74 | 2.43 | 4.47 | 2.16 | 16.37 |
| std($N$) | 0.93 | 0.94 | 0.94 | 1.21 | 2.20 | 1.47 | 11.89 |
| std($i$) | 1.76 | 1.76 | 1.76 | 3.72 | 6.23 | 5.55 | 39.54 |
| std($\pi$) | 0.70 | 0.70 | 0.70 | 2.89 | 5.01 | 4.94 | 36.12 |
| Behavior of price dispersion and implied output loss, $\Delta_t = 100\left(1 - S^{-1}\right)$ | | | | | | | |
| mean($\Delta$) | -0.00 | 0.05 | 0.05 | -0.00 | 0.87 | -0.00 | 1.85 |
| std($\Delta$) | 0.00 | 0.06 | 0.06 | 0.00 | 2.11 | 0.00 | 14.02 |
| max($\Delta$) | 0.00 | 0.42 | 0.43 | 0.00 | 16.85 | 0.00 | 67.47 |
| min($\Delta$) | -0.00 | 0.00 | 0.00 | -0.00 | -7.76 | -0.00 | -143.34 |
| $\pi_t > \pi^{upper}$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.67 | 0.00 | 20.35 |
| $S < 1$ | 0.00 | 0.00 | 0.00 | 0.00 | 12.63 | 0.00 | 44.98 |

*Note: Implied model moments for calibration from the stable (A1), borderline (un)stable (A2) and unstable (A3) but determined region. All variables are quarterly values expressed in percent, apart from inflation and interest rates, which are expressed at an annual rate. The mean, standard deviation, minimum and maximum value of price dispersion are expressed in terms of implied output loss. We report the percentage of simulation periods, in which price dispersion travels to regions of the state space that are in violation of the feasible regions implied by upper and lower bound on $\pi$ and $S$. Macro moments are for consumption, hours worked, the nominal interest rate and inflation.*

Under Taylor rules of scenarios A2 (borderline (un)stable) and A3 (unstable) no global approximation can be obtained due to explosive dynamics, as (part of) the relevant state space is plagued by unstable dynamics of the price dispersion inflation spiral. Appendix A discusses the reasons behind this finding in detail. While a third-order approximate solution can be derived, it is fundamentally flawed. Table 2 reflects this by documenting that the simulated moments of macro variables under the third-order approximation show a largely elevated volatility compared to the linear approximation; The third-order approximation in scenarios A2 and A3 implies significant output losses each quarter (0.87% and 1.85% respectively). Even if mean output losses are already far from what is suggested by empirical evidence (c.f. Nakamura et al. (2018)), the volatility of $\Delta_t$ rises to even more clearly counterfactual levels. Higher-order approximations to the solution produce states of the world when the lost output due to price dispersion reaches 16.85% (A2) or respectively 67.47% (A3). The 'negative losses' of output -7.76% or -143.34% in the third-order approximative solution reflect the perturbation solution's inability to respect the bound of $S_t \geq 1$, which is violated in 12.63% or 44.98% of cases. The percentage of simulation periods with violations of bounds on $\pi_t \leq \pi^{upper}$, lies at 0.67% or 20.35%.[16,17]

To better visualize the part of the model's state space that suffers from local non-stability, we also inspect policy functions under the various settings. To save space this analysis is relegated to appendix B for interested readers. In the rest of this section, we investigate how two – for monetary policy major and empirically important – modifications of the model affect the stability region.

### 4.4 Stability Under Positive Trend Inflation and Absence of Real Rigidity

Recent theoretical and empirical work of Ascari and Sbordone (2014), Coibion and Gorodnichenko (2011), Ascari and Ropele (2009), Bauer and Rudebusch (2017) among others shows that the conduct of monetary policy should be analyzed by appropriately accounting for positive trend inflation targeted by policymakers. Using different theoretical and empirical models the literature on trend inflation finds that positive trend inflation tends to destabilize inflation expectations and requires a much stronger response of monetary policy to inflation.

In a similar fashion, the monetary policy literature (see e.g. chapter 3 of Woodford (2003), Levin et al. (2008) and Marencak (2022)) has emphasized that the degree of real rigidity in the economy is another key element forming the determinacy space and anchoring inflation expectations. Woodford (2003) argues that nominal rigidities in quantitative models for monetary policy analysis need to be accompanied by some form of real rigidity to be able to amplify the real effects of monetary disturbances. Real rigidities are the mechanism which lowers firms' incentives to increase prices in the face of a rise in nominal demand or productivity. To analyze the way monetary policy anchors inflation expectations in the nonlinear model we choose, for illustration, the most wide spread way of introducing real rigidity in the form of decreasing returns to scale.

In Figure 3 we show that *i*) positive trend inflation significantly shrinks the stability region and requires monetary policy to renounce responding to the real side of the economy; *ii*) the absence of real rigidity widens the stability region and significantly improves the ability of monetary policy to anchor inflation expectations.

Figure 3a shows, in line with Ascari and Sbordone (2014), that already with mild positive annualized trend inflation of 2% the determinacy region shrinks significantly. After taking the nonlinear aspects of the perturbation approximation into account the model prescribes monetary policy to abstain from virtually any aim to stabilize the real side of the

---

[16] We should note at that the amplification in moments in the unstable cases does not arise only because of violation of lower and upper bounds on price dispersion and inflation. The counterfactual amplification remains when excluding simulation periods where bound were violated from the sample.

[17] Figure 1 presented the inflation threshold (as function of $S_{t-1}$) where dynamics turn unstable, $S_t \leq S_{t-1}$. Figure A.1 in Appendix A.2 adds to this our analysis of the analytical approximation of the underlying law of motion governing price dispersion, and a discussion on how it respects the lower and upper bounds on price dispersion and inflation.

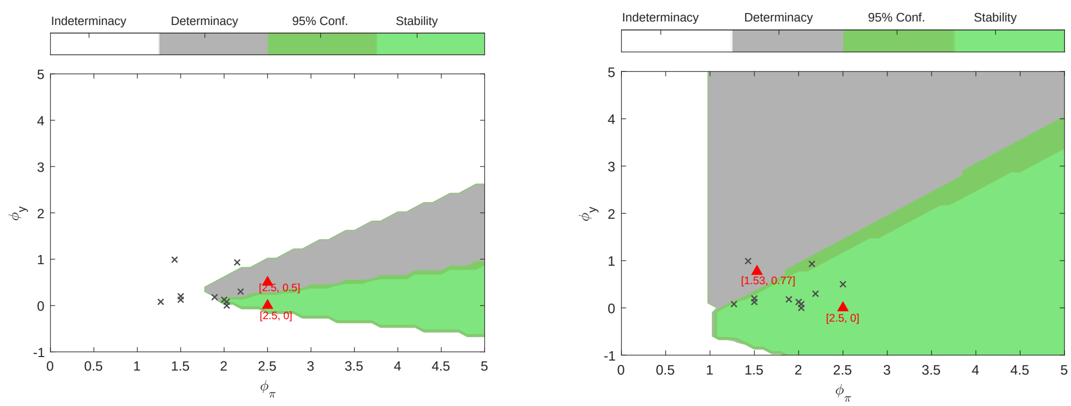
economy and respond solely to inflation. In Appendix C we analyze in detail the channels through which trend inflation contributes to the instability of the nonlinear solution. Ascari and Sbordone (2014) show that the determinacy of the linear approximation shrinks with trend inflation because price-setting firms are more forward-looking and the inflation rate becomes less sensitive to current economic conditions. In nonlinear settings, with an active price-inflation spiral, the current output levels do not reflect future inflation induced by price dispersion and the stability region shrinks even more substantially.

Figure 3b shows that the presence of real rigidity has no impact on the determinacy region and, up to dynamics based on a first-order approximation, as soon as the Taylor principle is satisfied inflation expectations are anchored. However, the absence of real rigidity has a significant impact on the stability region. Without the real rigidity, the ability of monetary policy to respond to the real economy increases. The stability region increases over the $\phi_y$ dimension with the decreasing level of real rigidity. In Appendix C we show in detail that the reason is the wedge between optimal marginal costs at the firm level and aggregate marginal costs (see equation (C.4)). Firms with higher market share produce with higher marginal cost as they move along on the concave (as opposed to linear) production function to the right which makes the inability to adjust prices more costly in terms of aggregate output. Decreasing returns to scale magnify the dispersion of prices and the power on $S_t$). Signals from the real side of the economy do not fully reflect the price inflation spiral. Monetary policy responding to deviations of output does not anchor inflation expectations in the model with real rigidities.

Table 3 shows the implied (cases B1 and B2) model moments from the stable and unstable calibration for positive trend inflation and moments for the benchmark model without decreasing returns to scale (case C2)[18]. The unstable scenario B2 demonstrates the strong impact of trend inflation on model dynamics. For instance, the standard deviation of consumption grows from 3.13% quarterly standard deviation to 104.67 % when moving from the stability to the instability region by increasing the response to the real side of the economy in the Taylor rule. Moments for case C2 demonstrate that even when the NK model is characterized only by mild nonlinearities (no trend inflation, no decreasing returns to scale), standard Taylor rule specifications can easily fall outside the stability region.

[18]Case C1 is stable and moments of third order and global solution coincide.

Figure 3: Stability region under Positive Trend Inflation and Absence of Real Rigidity




(a) Trend Inflation $\pi = 2\%$.

(b) No DRS, $\theta = 0$.

*Note: Simulated determinacy (gray) and stability (green) region for scenarios B (left) and C (right). Red triangles represent model parameterization scenarios B1-B2 and C1-C2 respectively. The stability region denotes the region where $S_t < S_{t-1}$ over the 1st to 99th percentile of the simulated bivariate distribution state variables, i.e. from $P_1(S_{t-1}, A_t)$ to $P_{99}(S_{t-1}, A_t)$. The region 95 % Conf. does the same from $P_5(S_{t-1}, A_t)$ to $P_{95}(S_{t-1}, A_t)$. Gray 'x' marks correspond to other empirically found Taylor rule coefficients in the literature (Table E.1 )*

Table 3: Model-Based Unconditional Moments: Scenarios B and C (B1-B2, C2)

| | case B1: $\phi_\pi = 2.5$, $\phi_Y = 0.$ | | | case B2: $\phi_\pi = 2.5$, $\phi_Y = 0.5$ | | case C2: $\phi_\pi = 1.53$, $\phi_Y = 0.77$ | |
|---|---|---|---|---|---|---|---|
| | 1st order | 3rd order | global | 1st order | 3rd order | 1st order | 3rd order |
| Moments of macroeconomic variables | | | | | | | |
| std($C$) | 3.11 | 3.13 | 3.13 | 5.12 | 104.67 | 2.13 | 10.52 |
| std($N$) | 1.05 | 1.06 | 1.06 | 2.21 | 47.91 | 1.00 | 5.54 |
| std($i$) | 1.80 | 1.79 | 1.79 | 6.11 | 108.81 | 9.02 | 42.60 |
| std($\pi$) | 0.72 | 0.72 | 0.72 | 5.01 | 93.33 | 8.60 | 41.12 |
| Behavior of price dispersion and implied output loss, $\Delta_t = 100\left(1 - S^{-1}\right)$ | | | | | | | |
| mean($\Delta$) | 0.94 | 1.01 | 0.54 | 0.97 | -27.33 | -0.00 | 1.72 |
| std($\Delta$) | 0.35 | 0.37 | 0.37 | 2.44 | 223.17 | 0.00 | 9.01 |
| max($\Delta$) | 1.94 | 2.72 | 2.23 | 7.86 | 99.95 | 0.00 | 52.27 |
| min($\Delta$) | 0.01 | 0.48 | 0.01 | -5.61 | -3999.1 | -0.00 | -71.58 |
| $\pi_t > \pi^{upper}$ | 0.00 | 0.00 | 0.00 | 0.00 | 26.25 | 0.50 | 23.38 |
| $S < 1$ | 0.00 | 0.00 | 0.00 | 35.57 | 38.42 | 0.00 | 42.56 |

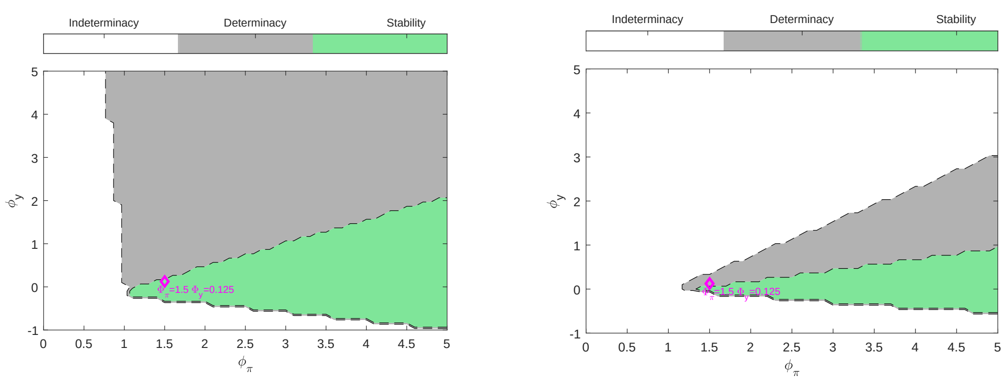
*Note: Implied model moments for calibration from the stable and unstable but determined region. All variables are quarterly values expressed in percent, apart from inflation and interest rates, which are expressed at an annual rate. The mean, standard deviation, minimum and maximum value of price dispersion are expressed in terms of implied output loss. We report the percentage of simulation periods, in which price dispersion travels to regions of the state space that are in violation of the feasible regions implied by upper and lower bound on $\pi$ and $S$. Macro moments are for consumption, hours worked, the nominal interest rate and inflation.*

# 5 An Application to Macro-Finance

## 5.1 Globally solved models

The inflation threshold determining the stability region is a global property of the model with Calvo pricing independent of the solution method applied to the model. There are several papers in the (optimal) monetary policy literature using the framework we describe solved by projection methods (i.e. Anderson et al. (2010), Leith and Liu (2014) and Miao and Ngo (2019)). None of these three papers report issues with stability. The model structure of these papers is very similar. Whereas Anderson et al. (2010) and Leith and Liu (2014) solve for optimal monetary policy Miao and Ngo (2019) use a calibrated Taylor rule.

Figure 4: Determinacy vs. Stability region in Miao and Ngo (2019)




(a) annual $\pi_t = 0\%$ (b) annual $\pi_t = 4\%$

*Note: The left panel plots the stability region based on zero trend inflation. The stability region denotes the region where $S_t < S_{t-1}$ over the 1st to 99th percentile of the simulated bivariate distribution state variables, denoted by $P_1(S_{t-1}, A_t)$ to $P_{99}(S_{t-1}, A_t)$. The region 95 % Conf. does the same for $P_5(S_{t-1}, A_t)$ to $P_{95}(S_{t-1}, A_t)$. The diamond indicates the model calibration by Miao and Ngo (2021). The right panel plots the stability region with 4% annual trend inflation.*

We replicate these papers and demonstrate, based on the example of Miao and Ngo (2019), in the left panel of Figure 4 that the reason for the stability of the global solution is that these models are calibrated to be in the stability region. Nevertheless, setting the coefficient on output gap only slightly higher would already set the model on an explosive path. By assuming non-zero trend inflation the right panel of Figure 4 shows that the stability region narrows[19] and the calibration of Miao and Ngo (2019) does fall outside the stability region despite the linear solution being still determined.

[19] Ascari and Ropele (2009) shows that trend inflation narrows the determinacy region.

### 5.2 Macro-finance

We now present an application that illustrates that the mechanism described in the previous sections is relevant and quantitatively pronounced in a widely used modeling framework of the term structure of interest rates. As our example application we adopt the model of Rudebusch and Swanson (2012), hereafter RS, which has become the state-of-the-art model in the macro asset pricing literature.[20] We choose this model because it represents a central model in the literature which uses Calvo pricing and is bound to rely on nonlinear solution methods. In particular, the RS model explains the economics of risk premia and precautionary saving effects which are inherently nonlinear phenomena. In addition, price dispersion implied by Calvo pricing has been stressed in the macro asset pricing literature as an important driver explaining risk premia. In a New Keynesian model with Calvo prices, Swanson (2015) uses the price dispersion index as a source of conditional volatility in consumption to generate sizable term premia in bond prices. Andreasen et al. (2018) argue, in a mid-scale model with Calvo contracts, that their model can match the volatility of term premia by using trend inflation to amplify the nonlinearities in the price dispersion index.

Price dispersion therefore plays an important role in the macro-asset pricing literature to explain the behavior of risk. A negative TFP shock received at the time with high dispersion in prices might lead to a price-inflation spiral which in turn leads to high conditional volatility of the stochastic discount factor and, thus, a high risk premium.

The literature on asset pricing in New Keynesian models with Calvo contracts resorts dominantly to third-order approximate solutions to the policy functions. The third-order approximation is justified by the fact that only third and higher-order approximations can capture positive and volatile risk premia (i.e. van Binsbergen et al. (2012) and Andreasen et al. (2018).

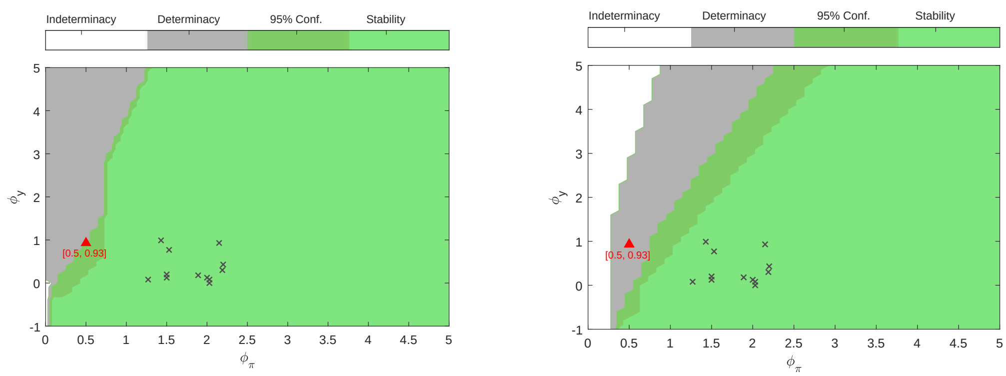
Figure 5 shows the stability and determinacy region for the RS model. The left panel of Figure 5 shows the original calibration in Table 3 of RS. It shows that the model is calibrated on the boarder of the stability region; in a model simulation about five percent of observations feature $S_t > S_{t-1}$. The right panel of Figure 5 demonstrates that already for small inflation of 1.6% annually the RS benchmark calibration features unanchored inflation expectations explosive moments despite the fact that it lies well in the determinacy region.

Table 4 follows RS and reports moments of macro and finance variables, where $NTP^{(40)}$ denotes the 10-year nominal term premium, and $i^{(40)}$ denotes the 10-year bond yield. In addition to RS we also report, consistent with Table 2, the statistics describing the loss due to price dispersion, $\Delta_t$, and the statistics on the violations of bounds.

The first column of Table 4 provides targeted empirical moments. The subsequent columns are model-based unconditional moments, calculated from third-order approxi-

[20]In the interest of space we do not review here the model of Rudebusch and Swanson (2012) and refer the reader to Appendix D for the summary of equilibrium conditions and to the original article. The online appendix on authors web-page provides details on the model derivation.

Figure 5: Determinacy vs. Stability region in Rudebusch and Swanson (2012)




(a) annual $\pi_t = 0\%$

(b) annual $\pi_t = 1.6\%$

*Note: The left panel plots the stability region based on zero trend inflation. The stability region denotes the region where $S_t < S_{t-1}$ over the 1st to 99th percentile of the simulated bivariate distribution state variables., denoted by $P_1(S_{t-1}, A_t)$ to $P_{99}(S_{t-1}, A_t)$. The region 95 % Conf. does the same for $P_5(S_{t-1}, A_t)$ to $P_{95}(S_{t-1}, A_t)$. The diamond indicates the calibration by Rudebusch and Swanson (2012). The right panel plots the stability region with 1.6% annual trend inflation.*

mated and pruned model simulations of various RS model versions, where we highlight the modeling features and channels discussed. Column 'RS' presents simulated moments from the original baseline RS model (with zero trend inflation), using the RS best fit calibration from Table 3 of their paper (which we summarize in Table D.2). All remaining columns assume positive annual trend inflation, $\bar{\pi} = 1.6\%$.

Column '$RS$ with trend inflation' shows that as soon we add even moderate trend inflation into the RS model the simulation is becomes unstable; both lower (51.5% of periods) and upper bound (7.46% of periods) on $S_t$ are violated and for instance, the standard deviation of inflation reaches an enormous 32.92%. Column '$RS$ with linear-in-labor' production function highlights the quantitative impact of the marginal cost channel driven by the degree of real rigidity in the model. Both macro and finance moments show that the absence of the real rigidity (by using a linear-in-labor production function) significantly tames model dynamics. For instance, the volatility of consumption drops from 10.59 to 0.54, and the model also rarely violates the bounds on $S_t$.

Column '$RS$ with $2 \times \phi_\pi$' shows that doubling the weight on inflation in the Taylor rule helps to anchor inflation expectations which lowers the trend inflation markup. The increased reaction of the central bank to inflation leads to lower price changes of optimizing firms. As price-resetting firms adjust their prices by smaller magnitudes the trend inflation markup drops. We see from this column that, although the dynamics of macro variables are much improved and the model is able to generate surprisingly large nominal term premia

($NTP^{(40)} = 1.66$) without compromising the macro moments[21], the numerical properties related to price dispersion remain compromised (45.68% of periods below the bound on $S_t$) and the match of the dynamics of output loss implied by price dispersion remains poor (in some periods up to 26.14% loss of quarterly output).

Column '$RS$ with indexation' substantially decreases the occurrence of the price-inflation spiral, by introducing full inflation indexation into the model with trend inflation. In this case the upper bound on inflation is violated only in 0.11% of periods as opposed to 7.46% and the model dynamics is comparable to original RS model. We identify indexation, therefore, as one of the modeling remedies attenuating the amplification effects of trend inflation in nonlinear Calvo settings. In the last two columns we report the most common alternative pricing mechanisms found in the literature: Rotemberg quadratic price adjustment costs and the staggered Taylor pricing mechanism. Results in these columns show that these alternative methods of modeling nominal rigidities match equally well both the macro and finance stylized facts, but do not generate the counterfactual size of the cost of inflation.

We therefore confirm the results from the analysis conducted in the standard New Keynesian DSGE model. Price dispersion drives both finance and macroeconomic fluctuations. Anchoring inflation expectations, destabilized by the price dispersion inflation spiral, is a necessary condition for accuracy of perturbation solution and existence of global solution.

## 6 Concluding Remarks

Applications in modern macroeconomics increasingly require nonlinear solution methods, among them models with asset pricing implications. Our paper studies the consequences of the widely used Calvo pricing mechanism in a nonlinear world. We show, in a standard New Keynesian and later in a workhorse macro-finance model, that accounting for the model's nonlinearities in price setting – dispersion of prices and the impact of risk and precautionary behavior on firms' markups – can give rise to divergence in (relative) prices. This can initiate a price dispersion driven inflation spiral, constituting a new challenge for the central bank to anchor inflation expectations.

We introduce the concept of the stability region as a nonlinear counterpart to the determinacy region. Outside of the stability region, the explosive character of price dispersion over some part of the traveled state space implies the impossibility of computing a stable global solution. Perturbation methods with pruning give the appearance that an approximate solution to the model can be obtained even outside of the stability region. Yet, it is not a valid (stable) solution: the explosive dynamics of the price dispersion inflation spiral strongly compromise model dynamics of all other variables and the perturbation approximation to the model solution outside of the stability region and implies counterfactual second moments.

[21] This has been shown to be challenging in the literature, see, e.g., Rudenbusch and Swanson (2008).

We stress the implications of our findings, particularly for macro-finance literature, revealing that the notable achievements of the pioneering work of Rudebusch and Swanson (2012), explaining the upward sloping yield curve, are predominantly attributed to model instability. The potential emergence of unstable dynamics due to the price dispersion inflation spiral is heavily contingent on the model's nonlinearities and is especially prominent under conditions such as trend inflation or diminishing returns to scale.

Previous contributions in the literature that employed Calvo price setting in a nonlinear context has predominantly circumvented model instability, opting for model features that shut down a large part of nonlinearities. We show that, when these nonlinearities are genuinely considered, it becomes indispensable for monetary policy to respond to inflation much more aggressively. From a modeling perspective the main implication of our findings is that moving to and from linearized and nonlinear model settings is not simple or straightforward. The presence of a price-inflation spiral in nonlinear settings means that one cannot simply adopt parameter values commonly used in linear settings. Instead a nonlinear model setting requires careful re-calibration and tuning of Taylor rule parameters to anchor inflation expectations.

Table 4: Empirical and Model-Based Unconditional Moments

| Unconditional Moment | US data 1961-2007 | RS replication $\bar{\pi} = 0$ | RS with trend inflation $\bar{\pi} > 0$ | RS with linear $N_t(i)$ in PF $\bar{\pi} > 0$ | RS with $2 \times \phi_\pi$ on $\pi_t$ $\bar{\pi} > 0$ | RS indexation $\bar{\pi} > 0$ | RS Rotemberg pricing $\bar{\pi} > 0$ | RS Taylor pricing $\bar{\pi} > 0$ |
|---|---|---|---|---|---|---|---|---|
| Moments of macroeconomic variables | | | | | | | | |
| std($C$) | 0.83 | 0.88 | 10.59 | 0.54 | 1.20 | 0.89 | 0.50 | 0.70 |
| std($N$) | 1.71 | 2.51 | 30.01 | 1.42 | 2.91 | 2.50 | 1.50 | 1.82 |
| std($i$) | 2.71 | 3.41 | 38.69 | 2.49 | 2.97 | 3.43 | 2.13 | 3.07 |
| std($\pi$) | 2.52 | 3.01 | 32.92 | 2.35 | 1.98 | 2.98 | 2.14 | 2.78 |
| Moments of finance variables | | | | | | | | |
| std($i^{(40)}$) | 2.41 | 3.94 | 4.79 | 5.42 | 4.16 | 5.24 | 4.03 | 4.34 |
| mean($NTP^{(40)}$) ) | 1.06 | 0.91 | 3.68 | 0.65 | 1.66 | 1.08 | 0.84 | 1.47 |
| std($NTP^{(40)}$) | 0.54 | 0.42 | 7.04 | 0.11 | 0.98 | 0.55 | 0.36 | 0.13 |
| Behavior of price dispersion and implied output loss, $\Delta_t = 100\left(1 - S^{-1}\right)$ | | | | | | | | |
| violation of $\pi_t \leq \pi^{upper}$ | - | 0.05 | 7.46 | 0.00 | 0.06 | 0.11 | 0.00 | 0.00 |
| violation of $S \geq 1$ | - | 8.77 | 51.50 | 0.08 | 45.68 | 12.95 | 0.00 | 19.10 |
| mean($\Delta$) | 0.4 | 0.55 | -1.24 | 0.29 | 0.45 | 0.52 | 0.76 | 0.10 |
| std($\Delta$) | - | 0.99 | 61.87 | 0.35 | 1.73 | 1.03 | 0.88 | 0.15 |
| max($\Delta$) | - | 17.45 | 98.55 | 4.94 | 26.14 | 41.00 | 0.00 | 4.25 |
| min($\Delta$) | - | -7.48 | -6227.45 | -0.42 | -14.80 | -7.17 | 0.00 | -0.12 |

*Note: All variables are quarterly values expressed in percent. Inflation, interest rates and the term premium are expressed at an annual rate. All models feature trend inflation, $\bar{\pi} = 1.6\%$, but the RS replication. The column with firm i's linear-in-labor production function shows the quantitative effect of the marginal cost channel. The column with $2 \times \phi_\pi$ shows the effect of the reduced trend markup channel. The column with indexation shows the effects of decreased occurrences of the price-inflation spiral. The last two columns show that alternative pricing mechanism do a better job at matching stylized facts.*

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# Appendix A Global Approximation Method

The obtain a global solution of our benchmark model we adopt a time iteration method as developed by Coleman (1990, 1991), where we solve for the approximate nonlinear policy functions in an iterative algorithm. The nonlinear model equations are summarized in Table A.1. Below we briefly outline the steps of the algorithm used:

- In the following, denote $t+1$ variables and non-predetermined state variables at time $t$ with a prime, e.g. $C = C_t$, $C' = C_{t+1}$, $S' = S_t$, $S = S_{t-1}$, etc. we construct a two-dimensional grid over the model's state variables at time $t$, that is, over $S = S_{t-1}$, $A = A_t$ consisting of $n_s n_A$ grid points. The grid in dimension of $S$ ranges from 1 to $S_{\max}$, where we obtain $S_{\max}$ from the relevant region of price dispersion that is traveled in a simulation of the third-order perturbation solution. The gridpoints in dimension of $A$ are obtained by discretizing the continuous AR process with the Rouwenhorst (1995) method, which, for persistent processes, has been shown to yield more accurate approximations than, e.g., the conventional discretization by Tauchen and Hussey (1991) (c.f. Kopecky and Suen (2010)). The number of gridpoints is chosen to be $n_s = 61$ and $n_A = 11$.

- Set counter equal to 1. Using the perturbation solution make initial guesses on the consumption policy, inflation policy, and the policy functions of the two auxiliary variables in the Calvo pricing block.

- Having guesses $C(S,A)$, $\Pi(S,A)$, $K(S,A)$, $F(S,A)$ in hand, we can write the model's conditional expectations, $CE_{EE} = \left\{ \frac{C_{t+1}^{-\varphi}}{\Pi_{t+1}} \right\}$, $CE_K = \left\{ C_{t+1}^{-\varphi} \Pi_{t+1}^{\frac{\epsilon}{1-\theta}} K_{t+1} \right\}$ and $CE_F = \left\{ C_{t+1}^{-\varphi} \Pi_{t+1}^{\epsilon-1} F_{t+1} \right\}$, as

$$\begin{aligned} CE_{EE}(S', A) &= \sum_{A'} \pi(A'|A) \frac{[C'(S', A')]^{-\varphi}}{\Pi'(S', A')}, && \text{(A.1)} \\ CE_K(S', A) &= \sum_{A'} \pi(A'|A) \left[C'(S', A')\right]^{-\varphi} \left[\Pi'(S', A')\right]^{\frac{\epsilon}{1-\theta}} K'(S', A'), && \text{(A.2)} \\ CE_F(S', A) &= \sum_{A'} \pi(A'|A) \left[C'(S', A')\right]^{-\varphi} \left[\Pi'(S', A')\right]^{\epsilon-1} F'(S', A'), && \text{(A.3)} \end{aligned}$$

  where we integrate out the expectations over future $A'$ using the Markov transition matrix $\pi(A'|A)$.

Table A.1: System of model equations, baseline NK model

| | |
|---|---|
| (NK1): | $1 = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\varphi} \frac{(1+i_t)}{\Pi_{t+1}} \right\}$ |
| (NK2): | $K_t = \frac{\epsilon}{\epsilon-1} MC_t Y_t + \beta\zeta \left( \frac{C_{t+1}}{C_t} \right)^{-\varphi} \Pi_{t+1}^{\frac{\epsilon}{1-\theta}} K_{t+1}$ |
| (NK3): | $F_t = Y_t + \beta\zeta \left( \frac{C_{t+1}}{C_t} \right)^{-\varphi} \Pi_{t+1}^{\epsilon-1} F_{t+1}$ |
| (NK4): | $\chi_0 (1 - N_t)^{-\chi} C_t^{\varphi} = W_t$ |
| (NK5): | $(p_t^*)^{1+\frac{\theta\epsilon}{1-\theta}} = \frac{K_t}{F_t}$ |
| (NK6): | $MC_t = \frac{1}{1-\theta} \overline{K}^{\frac{\theta}{1-\theta}} \frac{W_t}{A_t} \left( \frac{Y_t}{A_t} \right)^{\frac{\theta}{1-\theta}}$ |
| (NK7): | $S_t Y_t = A_t \bar{K}^{\theta} (N_t)^{1-\theta}$ |
| (NK8): | $S_t^{\frac{1}{1-\theta}} = (1-\zeta)\,(p_t^*)^{\frac{-\epsilon}{1-\theta}} + \zeta (\Pi_t)^{\frac{\epsilon}{1-\theta}} S_{t-1}^{\frac{1}{1-\theta}}$ |
| (NK9): | $\Pi_t^{1-\epsilon} = (1-\zeta)\,(p_t^* \Pi_t)^{1-\epsilon} + \zeta$ |
| (NK10): | $Y_t = C_t + \overline{I} + G$ |
| (NK11): | $\left( \frac{1+i_t}{1+\bar{i}} \right) = \left( \frac{\Pi_t}{\overline{\Pi}} \right)^{\phi_\pi} \left( \frac{Y_t}{\bar{Y}} \right)^{\phi_Y}$ |
| (NK12): | $\log A_t = \rho_A \log A_{t-1} + \sigma_A \varepsilon_{A,t}$ |

- Using the guesses for the conditional expectations and given the time $t$ values of grid variables, $(S, A)$, the model's endogenous variables $S'$, $C$, $N$, $(1 + i_t)$, $\Pi$, $p_t^*$, $K$, $F$, $W$, $MC$, $Y$ can be computed by solving the system given in Table A.1 with a nonlinear equations solver, at each gridpoint $(S, A)$. Price dispersion next period, $S'$, is part of the variables that need to be solved for, and is also the argument in evaluating expressions A.1 at value $S'$. We use linear interpolation (extrapolation) when evaluating expressions $CE_{EE}(S', A)$, $CE_K(S', A)$, and $CE_F(S', A)$ at value $S'$ off of (outside of) the exogenously specified $S$-grid.

- Having in hand solutions for $C(S, A)$, $\Pi(S, A)$, $K(S, A)$, $F(S, A)$ provides us with new policy function guesses to compute an updated guess of the conditional expectation terms, A.1. We set counter = counter+1 and return to step 3. The above steps are repeated until convergence is achieved. The convergence criterium is specified as the maximum absolute value of the difference from policy function guesses in two consecutive iterations to be smaller than 1e-6.

## A.1 No global solution outside the stability region

The reason why no global solution can be obtained whenever the model falls outside of the stability region is that for some values of the exogenous state $A_t$, the relevant state space in the $S_{t-1}$ dimension becomes unbounded, so that no specified grid over $S_{t-1}$ is large enough. In particular, this is the case for low values of the exogenous state $A_t$ which imply (high) inflation rates that exceed the level of threshold inflation, i.e., $\Pi_t > \Pi_t^{threshold}$, so that (as explained in section 3.3) $S_t > S_{t-1}$. This means that whatever maximum value of the $S_{t-1}$-grid one takes, the value of $S_t$ falls outside of the specified grid. Section 3.2 discussed the existence of an upper bound, so that one may be tempted to think that it could be of guidance in specifying a maximum grid in the $S_{t-1}$ dimension. However, unfortunately, the upper bound is solely an upper bound on inflation, $\Pi^{upper}$, and the fact that $\Pi^{upper} > \Pi_t^{threshold}$ continues to imply that because $S_t > S_{t-1}$ no 'upper bound' on price dispersion can be specified. In the practise of the time iteration algorithm described above, this problem implies that the algorithm is unable to converge. This is because policy function guesses $CE_{EE}(S', A)$, $CE_K(S', A)$ and $CE_F(S', A)$ need to be evaluated at $S'$, when solving the exact nonlinear system of model equations; and $S'$ falls beyond the maximum value of $S$ and thus outside the prespecified ($S$-$A$) grid. As a result, the very nature that price dispersion is explosive in some regions ($S_t > S_{t-1}$), implies that a current guess for the policy function needs to be evaluated outside of the grid over which it is computed, i.e. one needs to extrapolate. Because extrapolation is likely to be inexact (particularly for this region of high nonlinearity), convergence cannot be achieved. As explained, extending the grid is not an option to resolve these issues.

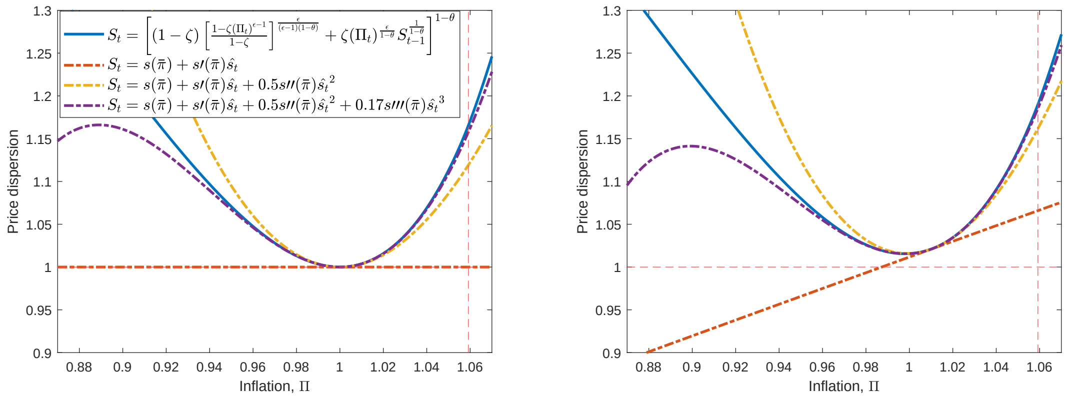
We have experimented with a number of alternatives to obtain a globally approximated model solution. This includes turning to a version of our time iteration/ policy function iteration algorithm that adopts an endogenous grid, specifying the grid over $S_t$ and $A_t$ and solving for $S_{t-1}$ as part of the variables in the nonlinear equations solver step. Alternatively we have also explored other, alternative global solution methods. As a robustness check we have explored using a projection method (Chebyshev collocation) as in, e.g., Anderson, Kim and Yun (2010). Similarly, we explored the 'extended perturbation method' of Andreasen and Kronborg (2017). Whenever the solution falls into the stability region, all these methods imply almost identical policy functions and simulated model moments are virtually identical; a global solution for parameterization cases falling outside of the stability region can, however, not be obtained.

## A.2 Nonlinear Law of Motion of Price Dispersion and Approximation Accuracy

Figure A.1 provides intuition for our main quantitative results (Table 2 in the section 4). It visually shows how different orders of a polynomial approximation of the underlying price dispersion function respect the lower and upper bound on price dispersion. It also highlights when high and low realization of inflation yields approximations outside of the radius of convergence of Taylor series. Figure A.1 plots the true functional form of price dispersion, given by equation (14), together with the analytical approximations given by first, second and third-order Taylor series expansions to that equation. Price dispersion, $S_t$, is plotted as a function of $\Pi_t$ at the point $S_{t-1} = \bar{S}$. Dashed-dotted lines are first, second and third-order approximation of $S_t$ at the point $\pi = 0\%$ in panel A.1a and $\pi = 4\%$ in panel A.1b. Price dispersion, $S_t$, is a highly nonlinear function in inflation which has been historically approximated simply by a line (red dashed-dotted line). Up to a first-order approximation around zero steady state inflation price dispersion is always constant and equal to 1 (panel A.1a), and price dispersion does not have an effect on the model solution. Instead, linear approximation around a non-zero inflation steady state or higher-order approximation at any point of steady state inflation introduces the process of price dispersion into the model equilibrium (panels A.1a andA.1b).

Figure A.1 illustrates several important properties of the model approximation discussed in the next section. First, a linear approximation is likely to violate the lower bound on $S_t$ as seen in A.1b. Second, at empirically relevant values for steady-state inflation the slope of the line approximating the true $S_t$ implies lower values of approximated $S_t = s(\bar{\pi}) + s'(\bar{\pi})\hat{s}_t$ than the true $S_t$ as $\Pi_t$ increases and therefore the upper bound on inflation is rarely reached in linearly approximated models. Third, the second-order approximation respects the lower bound on $S_t$ and approximates the underlying function $S_t$ fairly well. Fourth, the third-order approximation matches the underlying function $S_t$ remarkably well for inflation increases but delivers large approximation errors in case of inflation decreases. For large price drops it violates the lower bound on $S_t$.

Figure A.1: Analytical approximation of $S_t$




(a) Approximation at $\pi = 0\%$.

(b) Approximation at $\pi = 4\%$.

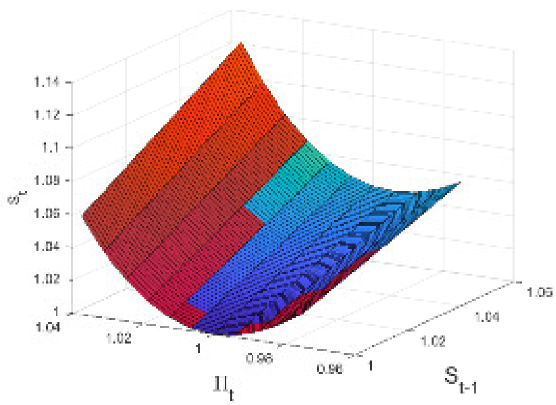
*Note: Blue solid line, $S_t$, represents price dispersion as a function of $\Pi_t$, as given by equation (14), at point $S_{t-1} = \bar{S}$. Dash-dotted lines are first (red), second (yellow) and third (purple) order approximations of $S_t$ at $\pi = 0\%$ and at $\pi = 4\%$. Light red Horizontal and vertical dashed lines represent the lower bound on $S_t$ and upper bound on $\Pi_t$.*

# Appendix B Policy Functions of the Simple NK Model

This section deepens the intuition for unstable price dispersion dynamics by looking at the policy function plots for the parameterization cases A1-C2 from section 4. Policy functions in our simple NK example model depend on two state variables, on endogenous state $S_{t-1}$, and on exogenous state $A_t$. Before doing so, Figure B.1, however, plots price dispersion, $S_t$, as a function of both arguments it depends on according to its true nonlinear law of motion, equation (12), which is past price dispersion, $S_{t-1} \geq 1$, and on inflation, $\Pi_t$. In a model solution $\Pi_t$ is itself a function of state variables $S_{t-1}$,$A_t$, so that there is a correspondence between the plot of price dispersion as a function of $S_{t-1}, \Pi_t$ and as a function of $S_{t-1}, A_t$. The advantage of the former is that it is not dependent on a model solution; Figure B.1 here does not need to rely on any approximation method at all: what is plotted is the true nonlinear dynamics, as given by the nonlinear law of motion of price dispersion.[22] It can clearly be seen that, for a large region of $\Pi_t$ price dispersion is an increasing function of past price dispersion, so that dynamics are explosive, confirming that unstable dynamics are a true feature of the model. We know from section 3 that this is the case whenever $\Pi_t > \Pi^{threshold}$, then $S_t > S_{t-1}$. For easier visibility this region of $S_t$ is highlighted in red.

[22]The true nonlinear law of motion depends on three model parameters, $\theta$, $\epsilon$, and $\zeta$. These are chosen as in the benchmark parameterization (case A).

Figure B.1: Price Dispersion Dynamics According to the True Nonlinear Law of Motion




*Note: Price dispersion $S_t$ plotted as implied by the exact nonlinear law of motion (equation (14)) in dependance of $S_{t-1}$ and $\Pi_t$. The red part of the surface indicates the region where price dispersion dynamics are unstable, i.e. where $\Pi_t > \Pi^{threshold}$ and $S_t > S_{t-1}$.*

Figure B.2 presents policy function plots for price dispersion, $S_t$, for parameterization cases A1-A3 (benchmark scenario), for cases B1-B2 (scenarios with positive trend inflation) and cases C1-C2 (scenarios with zero trend inflation and no decreasing returns to scale), respectively. Policy functions for $S_t$, are now plotted as a function of the model's fundamental state variables, which are past price dispersion, $S_{t-1}$, and total factor technology, $A_t$. Also included in the panels is a transparent surface which plots the S-Grid on itself, i.e. providing a surface with slope equal to one in the $S_{t-1}$-dimension. Similar to Figure B.1 regions with increasing price dispersion dynamics, $S_t > S_{t-1}$, are colored in red. As discussed in the main text, scenarios A1, B1, and C1, are all parameterization examples that fall well into the stability region. The policy function plots deliver the same insight: policy function $S_t$ falls below the transparent surface indicating the instability threshold, thus price dispersion dynamics are stable over the entire relevant state space to which price dispersion travels. Also, we observe that the policy function obtained from the third order solution is virtually identical to the one obtained from the global solution, mirroring the simulation results in the main text (section 4). This drastically changes when moving to parameterization scenarios A2-A3, B2, or C2, for which we know that they lie outside of the stability region. Here, we observe that –highlighted in red– price dispersion dynamics are unstable, $S_t > S_{t-1}$, for the region of the state space where $A_t$ is low (e.g., resulting from a negative supply shock), which is associated with high levels of inflation, leading to high price dispersion, and vice versa (the price-inflation spiral). The dynamic behavior visualized in the policy function plots is Figure reminiscent of the unstable region highlighted in B.1, the difference being that the policy function plots recognize that inflation is itself a function of the model's fundamental state variables, $S_{t-1}$ and $A_t$. For parameterization cases A2-A3, B2, and C2, because of the wildly unstable price dispersion dynamics no global solution can be obtained, as explained in section A.

# Appendix C Economic Interpretation - Transmission Channels

In section 3 we explained that efficient resource allocation dictates that each firm produces the same amount of goods. This section discusses in detail the economic mechanism through which the nonlinear model solution, together with trend inflation and decreasing returns-to-scale, amplifies economic shocks and leads to excessive model dynamics. Let, in the following, variables carrying an asterisk denote prices and quantities of a firm that, in period $t$, is allowed to re-set its price optimally. Let variables without asterisk denote aggregate, economy-wide variables, that include firms that are not allowed to re-set their price in the current period and are stuck with prices from the past.

Figure C.1 plots the distribution of prices of optimizing firms ($P_t^*$) relative to aggregate price ($P_t$) and visualizes that in a nonlinear Calvo pricing model firms with many different prices co-exist.[23] Trend inflation adds a drift to the evolution of prices and, thus, drives the distribution of optimal prices, $P_t^*$, further apart from the average price index $P_t$. Aggregate output in the New Keynesian model is driven by the downward sloping demand for product variety, $Y_t(i)$. Firms always adjust their supply to meet demand for good $i$ given their price, $P_t(i)$. As only a fraction of firms are allowed to re-set their prices optimally, each firm produces a different amount of goods. In particular, a fraction $1 - \zeta$ is allowed to set its price; for the fraction $\zeta$ of non-optimizing firms for which $P_t(i) < P_t^*$ (respectively $P_t(i) > P_t^*$) the produced output is $Y_t(i) > Y_t^*$ (respectively $Y_t(i) < Y_t^*$).

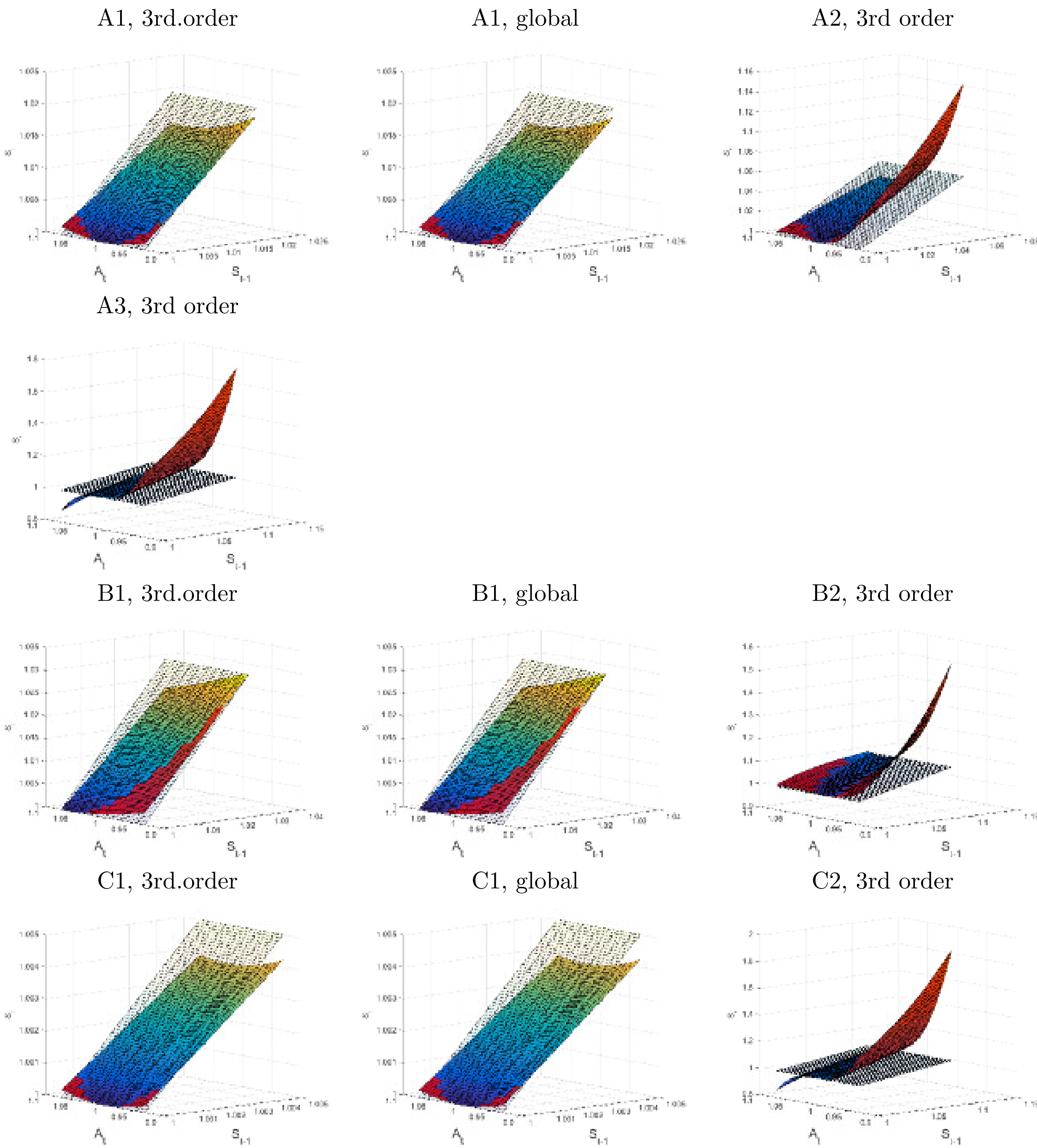
Firms with low prices produce more than firms with high prices. In other words, the dispersion of prices across products leads to $Y_t(i) \lesseqgtr Y_t^*$ which implies that resources are not allocated optimally across firms. As shown in section 3, on the aggregate level this introduces a wedge, $S_t^{-1}$, between aggregate output and the amount of inputs, $Y_t = S_t^{-1} A_t K^\theta N_t^{1-\theta}$.[24]

We disentangle the effects through which price dispersion amplifies the impact of macroeconomic shocks on prices and quantities into two channels: i) a marginal-cost channel, and ii) a trend-inflation markup channel. The marginal cost channel shows how the real rigidity (DRS) amplifies the impact of the fact that firms cannot reoptimize and are stuck with prices from the past. The trend-inflation markup on the other hand highlights the effects of trend inflation on firms that can currently reoptimize and how they act in setting their new price.

[23] The cross-sectional distribution of prices is captured by the measure of price dispersion, $S_t$, equation (12). The distribution of prices across time is linked to cross sectional distribution of prices because prices optimally set in time $t-1$, $P_{t-1}^*$ are fixed at time $t$ with probability $1 - \zeta$.

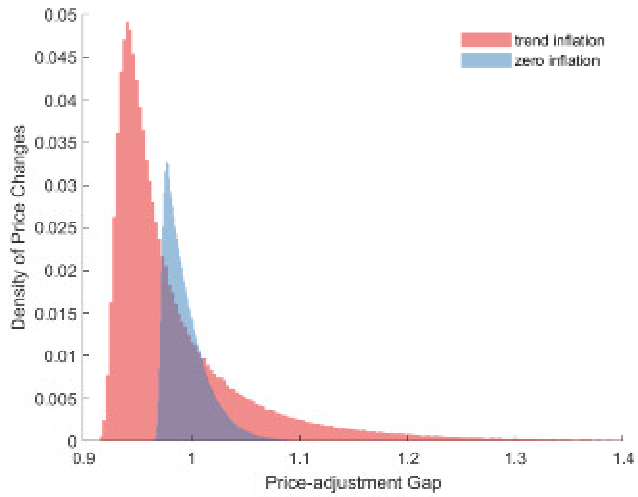
[24] This wedge was emphasized in the literature by Ascari (2004), Ascari and Sbordone (2014)). Ascari and Sbordone (2014) discuss the steady-state implications of trend inflation, whereas our focus is on the dynamics, which is crucial for asset pricing.

Figure B.2: Policy functions of price dispersion for cases A1-A3, B1-B2, and C1-C2




*Note: Policy functions for price dispersion $S_t$ for scenarios A1-A3, B1-B2, and C1-C2. Policy function $S_t$ is plotted in dependance of state variables $S_{t-1}$ and $A_t$. The transparent surface included plots the $S_{t-1}$ grid on itself for each given value of $A_t$, i.e. having slope 1 in $S_{t-1}$ dimension. The red part of the policy function surface indicates the region where price dispersion dynamics are unstable, i.e. where $\Pi_t > \Pi^{threshold}$ and $S_t > S_{t-1}$.*

Figure C.1: Simulated Distribution of Price-adjustment Gap




*Note: The shaded areas plot the simulated distribution of optimal relative prices,* $p_t^* = \frac{P_t^*}{P_t}$*, which captures the changes in the prices of the optimizing firms relative to aggregate price in the economy at every period with (red) and without (blue) trend inflation.*

We should note that these two channels are already to some extent present in the literature. Our contribution is to outline the channels' mechanism in the newly structured exposition tailored to nonlinear settings. The marginal-cost channel was already mentioned in Ascari (2004). The fact that price dispersion creates an inefficiency wedge between production inputs and aggregate output is stressed in King and Wolman (1996), Schmitt-Grohe and Uribe (2006) and Ascari and Sbordone (2014). The upper bound on inflation is discussed also in Ascari and Sbordone (2014) and from a numerical perspective in a nonlinear setting by Andreasen and Kronborg (2020). The trend inflation mark-up channel is related to the discussion of the price adjustment gap in Ascari and Sbordone (2014).

## C.1 The Marginal-Cost Channel

We show here that, in a world with trend inflation, the subset of firms which cannot change their price and are stuck with a price from the past faces higher marginal costs, which aggravates the impact of exogenous shocks on the economy. The wedge between optimal marginal costs of a current price-resetter, $MC_t^*$, and aggregate marginal costs, $MC_t$ increases with trend inflation and the curvature of the production function (i.e., the degree of DRS). This wedge is present only in a nonlinear model solution.

A firm which can change its price in period $t$, will set its price equal to $P_t(i) = P_t^*$, which, from equations (3) and (4), implies that $Y_t^* = \left(\frac{P_t^*}{P_t}\right)^{-\epsilon} Y_t = A_t \bar{K}^\theta N_t^{*(1-\theta)}$. This expression defines labor demand of a price changing firm as

$$N_t^* = \left( \left( \frac{P_t^*}{P_t} \right)^{-\epsilon} \frac{Y_t}{A_t \bar{K}^\theta} \right)^{\frac{1}{1-\theta}}. \tag{C.1}$$

Relating the optimal labor demand $N_t^*$ to aggregate labor demand $N_t$ – which we express from the aggregate production function – delivers

$$N_t^* = \phi_{n,t} N_t \qquad \text{where} \qquad \phi_{n,t} = \frac{\left( \frac{P_t^*}{P_t} \right)^{-\frac{\epsilon}{1-\theta}}}{\int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{\frac{-\epsilon}{1-\theta}} di}, \tag{C.2}$$

where $\phi_{n,t}$ can be interpreted as a measure of labor market inefficiency and is determined by the ratio of two price indexes.[25]

Using the wedge between labor demands, we can similarly describe the wedge between marginal costs for the price re-setting firm, $MC_t^* = \frac{W_t}{(1-\theta) A_t K^\theta N_t^{*-\theta}}$, and the average firm, $MC_t = S_t \frac{W_t}{(1-\theta)\left( A_t K^\theta N_t^{-\theta} \right)}$[26], as

$$MC_t^* = \phi_{mc,t} MC_t \qquad \text{where} \qquad \phi_{mc,t} = \frac{\left( \frac{P_t^*}{P_t} \right)^{-\frac{\theta\epsilon}{1-\theta}}}{\int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{\frac{-\epsilon}{1-\theta}} di}. \tag{C.3}$$

Average marginal costs are proportional to marginal costs of the price re-setting firm and this proportion is determined by a similar ratio of price indices as in the case of the labor market wedge.

In Appendix C.2 we show that

$$\phi_n, \phi_{mc} \begin{cases} < 1 & \text{for } \pi_t > 0, \\ = 1 & \bar{P} = P_t^* = P_t(i) = P_t, \\ > 1 & \text{for } \pi_t < 0, \end{cases} \tag{C.4}$$

where $\bar{P}$ is the deterministic steady state of the price level. In a setting *without trend inflation* ($\bar{\pi} = 0$), the ratios involving the price adjustment gap to price dispersion in equations C.2 and C.3 will be smaller than one, $\phi_{n,t} < 1$, $\phi_{mc,t} < 1$, in states of the

[25]We develop these results in the baseline setting with fixed capital, which is a case of decreasing returns to scale (DRS). The appendix develops a similar set of results for the case of variable capital or constant returns to scale (CRS).

[26]Using aggregate labor demand $N_t = \left[ \frac{S_t Y_t}{A_t K^\theta} \right]^{\frac{1}{1-\theta}}$, we can calculate total costs, $W_t N_t$ , from which, using the definition of marginal costs, $MC_t = \frac{\partial W_t N_t}{\partial Y_t}$, the expression in the text follows.

economy with positive inflation realizations, $\pi_t > 0$; equivalently it is larger than one, $\phi_{n,t} > 1$, $\phi_{mc,t} > 1$ in states in which $\pi_t < 0$. In a nonlinear model solution the average value of the wedges is smaller than 1, $\phi_{n,t} < 1$, $\phi_{mc,t} < 1$, even when inflation is centered around zero at steady state. This comes from the fact that, in a nonlinear world, when resources are used unequally, this results in a welfare loss. Therefore, as on average $S_t > 1$ and with an average price adjustment gap of one, $\frac{P_t^*}{P_t} = 1$, it follows that $\phi_{n,t}$, $\phi_{mc,t} < 1$ on average as well.

In the case of *positive trend inflation*, $(\bar{\pi} > 0)$, the probability of realizations of deflationary states of the world, $\pi_t < 0$, decreases, because to reach an overall negative realization of inflation the decrease in prices would need to exceed the positive steady-state inflation value. Equivalently, the likelihood of observing states of nature where $\pi_t > 0$ increases. For this reason, the values of $\phi_{n,t}$ and $\phi_{mc,t}$ will be less than one, $\phi_{n,t} < 1$ and $\phi_{mc,t} < 1$, for most of the states of the world and accordingly also move the average value of $\phi_{n,t}$ and $\phi_{mc,t}$ (more strongly) below one. The intuition is straightforward: in periods with growing prices, firms that can react and adjust their prices will be able to reflect the price growth in their profit-maximizing-price and consequently employ a lower optimal amount of labor inputs and produce with lower marginal costs. The fact that the average firm will produce at a higher marginal cost than the price optimizing firm at time $t$ adds an additional inefficiency in production and amplifies the real costs of price dispersion in the economy. Positive trend inflation thus amplifies the inefficiency coming from price stickiness. As firms produce under different cost conditions some of them are better equipped to accommodate exogenous (productivity) shocks. The wedge between aggregate and optimal quantities, $1/\phi_{n,t}$ and $1/\phi_{mc,t}$ nonlinearly increases with decreasing-returns-to-scale, pinned down by parameter $\theta$, and the elasticity of substitution between good varieties, $\epsilon$. The intuition is again straightforward: to satisfy the higher demand for $Y_t(i)$, implied by $P_t(i) < P_t^*$, firms move along the concave production function to the right. In the case of a linear production function the increase in production is associated with no change in marginal costs. The presence of $\epsilon$ in the wedge $\phi_{n,t}$ and $\phi_{mc,t}$ represents the amplification that monopolistic competition adds to the inefficiency from price dispersion.

Furthermore, following from $Y_t^* = A_t \bar{K}^\theta N_t^{*1-\theta} = \left(\frac{P_t^*}{P_t}\right)^{-\epsilon} Y_t$, the ratio between output of a price re-setting firm and aggregate output is determined by the price adjustment gap which can be re-written using the aggregate price index as

$$Y_t^* = \phi_{o,t} Y_t, \qquad \text{where} \qquad \phi_{o,t} = \left[\frac{1 - \zeta\,(\Pi_t)^{\epsilon-1}}{1-\zeta}\right]^{\frac{\epsilon}{\epsilon-1}}. \tag{C.5}$$

It follows that,

$$Y_t \begin{cases} = Y_t^* & \text{for } \Pi_t = 1, \phi_{o,t} = 1, \\ > Y_t^* & \text{for } \Pi_t > 1, \phi_{o,t} < 1, \\ < Y_t^* & \text{for } \Pi_t < 1, \phi_{o,t} > 1 \end{cases} \tag{C.6}$$

In the economy with trend inflation where, on average, $\Pi_t > 1$, the average output is higher than optimal output, $Y_t^* < Y_t$ . As prices are on average too low the economy produces on average too much. A negative productivity shock, therefore, has a much stronger impact on macroeconomic quantities in this situation. In the nonlinear world with Calvo prices and trend inflation, firms need to meet the demand for goods and on average overproduce. If in this situation their production technology becomes, as a result of a negative TFP shock, even less efficient without the possibility to adjust prices, the only option for accommodation of these adverse conditions is adjustment in quantities. It is this shock accommodation through quantities which leads to large model volatility. The next sub-section discusses the behavior of the subset of firms given the chance to adjust its price. Overproducing firms, given the chance to change their price, will need to, on average, adjust their low prices more strongly. This jump in adjustment leads to more volatile inflation in the economy.

## C.2 Trend-Inflation Markup Channel

The presence of trend inflation leads firms to set their price at an additional markup over (current and future expected) marginal costs, which we denote the trend-inflation markup: a markup implied by sticky prices and elevated by trend inflation that occurs over and above the traditional markup from monopolistic competition. Trend inflation enters the firm's price decision problem, and therefore the first order condition for the optimal price represents another important channel. The price re-setting firm is forward-looking, it can foresee trend inflation and will therefore, on average, set its price above the aggregate price level (which includes non-resetting firms' prices from the past), $P_t^* > P_t$. It is because the optimal price has to equate the present value of future marginal revenues with marginal costs,

$$\sum_{k=0}^{\infty} \zeta^k E_t Q_{t,t+k} \Pi_{t+k}^{\epsilon-1} Y_{t+k} \left(\frac{P_t^*}{P_t}\right)^{1+\frac{\epsilon\theta}{1-\theta}} = \frac{\epsilon}{\epsilon-1} \sum_{k=0}^{\infty} \zeta^k E_t Q_{t,t+k} \Pi_{t+k}^{\frac{\epsilon}{1-\theta}} Y_{t+k} MC_{t+k}(i). \quad \text{(C.7)}$$

Trend growth in prices increases both firms' costs of production and revenues from output sold. Nonetheless, nominal marginal costs (the expression in the infinite sum on the right hand side of equation (C.7)) grow at a faster rate than nominal revenues (the left hand side of equation (C.7)).[27] So, to keep the equality of marginal revenues with marginal cost in present value terms, the price setting firm must set $P_t^*$ above $P_t$.[28] The difference between the rate of growth in marginal cost and marginal revenue shapes the firm's markup over (present and future) marginal costs. Equation (C.8) defines the price

[27]As $\Pi$ goes up, the right hand side of equation (C.7) grows faster – at rate $\Pi_{t+k}^{\frac{\epsilon}{1-\theta}}$ – than the left hand side – which grows by $\Pi_{t+k}^{\epsilon-1}$.

[28]Note that the trend-inflation markup channel is amplified through the parameter determining the strength of DRS, $\theta$, which further widens the gap between costs and revenues.

adjustment gap, which depends on the weighted average of the firm's current and expected future real marginal costs:

$$\left(\frac{P_t^*}{P_t}\right)^{1+\frac{\epsilon\theta}{1-\theta}} = \frac{\epsilon}{\epsilon-1}\sum_{k=0}^{\infty}\Upsilon_{t+k}MC_{t+k}(i), \quad \text{where} \quad \Upsilon_{t+k} = \frac{\zeta^k E_t Q_{t,t+k}\Pi_{t+k}^{\frac{\epsilon}{1-\theta}}Y_{t+k}}{\sum_{k=0}^{\infty}\zeta^k E_t Q_{t,t+k}\Pi_{t+k}^{\epsilon-1}Y_{t+k}}. \tag{C.8}$$

Ascari and Sbordone (2014) show that the mark-up, $\Upsilon_{t+k}$, increases with inflation and, thus, as trend inflation increases, the firm's trend-inflation markup amplifies the distortion implied by monopolistic competition. The rise in $\Upsilon_{t+k}$ means that firms put more weight on marginal costs far in the future compared to current marginal costs[29].

## C.3 Proofs and Propositions

**Proposition C.1.** *Price dispersion is bounded by one, $S_t \geq 1$.*

*Proof.* The aggregate price index is $P_t = \left[\int_0^1 P_t^{1-\epsilon}(i)\right]^{\frac{1}{1-\epsilon}}$. Dividing by $P_t$ gives $1 = \left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$. Defining $v_{i,t} = \left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon}$ we get that $\left[\int_0^1 v_{i,t}\right]^{\frac{1}{1-\epsilon}} = 1$. Writing price dispersion, $S_t = \left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} di\right]^{1-\theta}$, in terms of $v_{i,t}$ yields $v_{i,}^{\frac{-\epsilon}{1-\epsilon}\frac{1}{1-\theta}} = \left[\left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon}\right]^{\frac{-\epsilon}{1-\epsilon}\frac{1}{1-\theta}}$ Thus, price dispersion can be written in terms of variable $v$ as, $S_t^{\frac{1}{1-\theta}} = \int_0^1 v_{i,t}^{\frac{\epsilon}{\epsilon-1}\frac{1}{1-\theta}}$ And as $\frac{\epsilon}{\epsilon-1}\frac{1}{1-\theta} > 1$, Jensen's inequality implies that

$$1 = \left[\int_0^1 v_{i,t}\right]^{\frac{\epsilon}{\epsilon-1}\frac{1}{1-\theta}} \leq \int_0^1 v_{i,t}^{\frac{\epsilon}{\epsilon-1}\frac{1}{1-\theta}} = S_t^{\frac{1}{1-\theta}}. \tag{C.9}$$

□

**Proposition C.2.** *The ratio of price indexes, $\phi_n = \frac{\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}}}{\left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} di\right]} \gtreqless 1$, for*

$$\phi_n \begin{cases} < 1 & \textit{for } \bar{\pi} = 0 \quad \& \quad \hat{\pi}_t > 0, \\ & \textit{for } \bar{\pi} > 0 \quad \& \quad \hat{\pi}_t > -\bar{\pi}, \\ = 1 & \textit{for } \bar{P} = P_t^* = P_t(i) = P_t, \\ > 1 & \textit{for } \bar{\pi} = 0 \quad \& \quad \hat{\pi}_t < 0, \\ & \textit{for } \bar{\pi} > 0 \quad \& \quad \hat{\pi}_t < -\bar{\pi}, \end{cases} \tag{C.10}$$

[29] Ascari and Sbordone (2014) shows that overly forward looking agents de-anchor inflation expectations and decrease the determinacy region.

*where $\bar{P}$ is the deterministic steady state of price and $\hat{\pi}_t$ is deviation of inflation from its steady state.*

*Proof.* The ratio $\phi_n < 1$ if $\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}} < \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} di$. From the Proposition C.1, $\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} di \geq 1$. Thus, it must be true that if $\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}} \leq 1$ then $\phi_n \leq 1$. This will hold for all cases when $P_t^* \geq P_t$. Because $\frac{P_t^*}{P_t} = \left[\frac{1-\zeta(\Pi_t)^{\epsilon-1}}{1-\zeta}\right]^{\frac{1}{1-\epsilon}}$ (equation (D.2)) for $\Pi_t \geq 1$ it holds that $P_t^* \geq P_t$ In case of positive steady state inflation, $\bar{\pi}_t > 0$ the inflation deviation from its steady state can reach $\hat{\pi}_t > -\bar{\pi}$ for $\phi_n \leq 1$. □

# Appendix D Rudebusch and Swanson (RS) Model

This appendix gives a summary of the equilibrium conditions of Rudebusch and Swanson (2012). Table D.1 summarizes the system of equations of the Rudebusch Swanson model in terms of stationary allocations and real (relative) prices (i.e., in term of detrended and deflated variables, denoted by lowercase variables) defined as $c_t = \frac{C_t}{Z_t}$, $y_t = \frac{Y_t}{Z_t}$, $\Pi_t = \frac{P_t}{P_{t-1}}$, $w_t = \frac{W_t}{P_t Z_t}$, $p_t^* = \frac{P_t^*(i)}{P_t}$, $mc_t(i) = \frac{MC_t(i)}{P_t}$, $y_t = \frac{Y_t}{Z_t}$, $\mu_t = \frac{Z_t}{Z_{t-1}}$. The best fit calibration of the RS model based on their Table 3 is summarized in Table D.2. In this setting, model dynamics are driven by three types of shocks, stationary technology shocks, government spending shocks, and inflation target shocks (in particular, there are no trend productivity shocks, so that $\mu_t = \frac{Z_t}{Z_{t-1}} = \mu$ is constant).

## D.1 Bond Pricing

The price of a default-free $n$-period zero coupon bond that pays \$1 at maturity can be described recursively as:

$$p_t^{(n)} = E_t\{Q_{t,t+1} p_{t+1}^{(n-1)}\}$$

where $Q_{t,t+1}$ is the stochastic discount factor; $p_t^{(n)}$ denotes the price of the bond at time $t$ with maturity $n$, and $p_t^{(0)} \equiv 1$, i.e. the time-$t$ price of \$1 delivered at time $t$ is \$1.

The price of a bond can be decomposed into the risk-neutral price and a term premium. The risk-neutral bond price, $\hat{p}_t^{(n)}$, is defined through the expectations hypothesis of the term structure:

$$\hat{p}_t^{(n)} = e^{-i_t} E_t \hat{p}_{t+1}^{(n-1)} \qquad NTP_{n,t} = i_t^{(n)} - \frac{1}{n}\sum_{j=0}^{n-1} E_t[i_{t+j}] \tag{D.1}$$

where the bond price is discounted by one period rate, $i_t$. The price of bond reflects, in this case, expectations about inflation and economic activity but abstracts from the uncertainty

Table D.1: System of model equations, Rudebusch Swanson model

| | |
|---|---|
| (RS1): | $V_t = \frac{c_t^{1-\varphi}}{1-\varphi} + \chi_0 \frac{(1-N_t)^{1-\chi}}{1-\chi} + \beta(E_t[(V_{t+1}\mu_{t+1}^{1-\gamma})^{1-\alpha}])^{\frac{1}{1-\alpha}}$ |
| (RS2): | $Q_{t-1,t} = \mu_t^{-\gamma} \left( \frac{(V_t \mu_t^{1-\gamma})}{[E_{t-1}(V_t\mu_t^{1-\gamma})^{1-\alpha}]^{\frac{1}{1-\alpha}}} \right)^{-\alpha} \left( \frac{c_t}{c_{t-1}} \right)^{-\varphi}$ |
| (RS3): | $\chi_0(1-N_t)^{-\chi} c_t^{\varphi} = w_t$ |
| (RS4): | $1 = \beta E_t \left\{ Q_{t,t+1} \frac{(1+i_t)}{\Pi_{t+1}} \right\}$ |
| (RS5): | $(p_t^*)^{1+\frac{\theta\epsilon}{1-\theta}} = \frac{aux_{1t}}{aux_{2t}}$ |
| (RS6): | $aux_{1t} = \frac{\epsilon}{\epsilon-1} mc_t y_t + \beta\zeta Q_{t,t+1} \Pi_{t+1}^{\frac{\epsilon}{1-\theta}} aux_{1t+1}$ |
| (RS7): | $aux_{2t} = y_t + \beta\zeta Q_{t,t+1} \Pi_{t+1}^{\epsilon-1} aux_{2t+1}$ |
| (RS8): | $S_t Y_t = A_t \bar{K}^{\theta} (N_t)^{1-\theta}$ |
| (RS9): | $S_t^{\frac{1}{1-\theta}} = (1-\zeta)(p_t^*)^{\frac{-\epsilon}{1-\theta}} + \zeta(\Pi_t)^{\frac{\epsilon}{1-\theta}} S_{t-1}^{\frac{1}{1-\theta}}$ |
| (RS10): | $\Pi_t^{1-\epsilon} = (1-\zeta)(p_t^* \Pi_t)^{1-\epsilon} + \zeta$ |
| (RS11): | $MC_t = \frac{1}{1-\theta} \overline{K}^{\frac{\theta}{1-\theta}} \frac{W_t}{A_t} \left( \frac{y_t}{A_t} \right)^{\frac{\theta}{1-\theta}}$ |
| (RS12): | $y_t = c_t + \bar{I} + g_t$ |
| (RS13): | $4i_t = 4\rho_i i_{t-1} + (1-\rho_i) \left[ 4(\bar{i} - \bar{\pi}) + (\pi_t^{avg}) + \phi_\pi (4(\pi_t^{avg}) - (\pi_t^*)) + \phi_Y \left( \frac{\mu_t Y_t}{\bar{\mu}\bar{Y}} - 1 \right) \right]$ |
| (RS14): | $\pi_t^* = (1-\rho_{\pi^*}) 4\pi_t^{avg} + \rho_{\pi^*} \pi_{t-1}^* + \zeta_{\pi^*} (4\pi_t^{avg} - \pi_t^*) + \sigma_{\pi^*} \varepsilon_{\pi^*,t}$ |
| (RS15): | $\pi_t^{avg} = \theta_{\pi^{avg}} \pi_{t-1}^{avg} + (1-\theta_{\pi^{avg}}) \pi_t$ |
| (RS16): | $\log A_t = \rho_A \log A_{t-1} + \sigma_A \varepsilon_{A,t}$ |
| (RS17): | $\log(g_t/\bar{g}) = \rho_G \log(g_{t-1}/\bar{g}) + \varepsilon_t^G$ |

surrounding the expectations[30]. The continuously compounded yield to maturity of the $n$-period zero-coupon bond can be written as $i_t^{(n)} = -\frac{1}{n}\log p_t^{(n)}$, (see for instance Cochrane (2001)). The term premium, $NTP_{n,t}$ is defined as the difference between the yield expected by the risk-averse investor ($i_t^{(n)}$) minus the yield awaited by the risk-neutral investor ($\hat{\imath}_t^{(n)} = \frac{1}{n}\sum_{j=0}^{n-1} E_t[i_{t+j}]$).

## D.2 Aggregation

Here we describe in detail the aggregation across the i-firms in case of decreasing return to scale (i.e., the model version with fixed capital, as in the original model specification of RS), and constant return to scale production function (i.e., with variable capital).

### D.2.1 Aggregate Price Index

The aggregate price index $P_t = \left[\int_0^1 P_t^{1-\epsilon}(i)di\right]^{\frac{1}{1-\epsilon}}$ can be written using the Calvo result as,

$$\frac{P_t^*}{P_t} = \left[\frac{1-\zeta\left(\Pi_t\right)^{\epsilon-1}}{1-\zeta}\right]^{\frac{1}{1-\epsilon}}, \tag{D.2}$$

### D.2.2 Aggregation for DRS

The production function of intermediate firm i is given by $Y_t(i) = A_t K^\theta N_t^{1-\theta}(i)$. Using this, plug in for $Y_t(i)$ into the demand for variety $i$, equation 3, solve for $N_t(i)$ and integrate over all varieties $i$. Since workers are all the same the aggregation of hours worked is $N_t = \int_0^1 N_t(i)di$. Aggregation thus delivers,

$$N_t = \left(\frac{Y_t}{A_t K^\theta}\right)^{\frac{1}{1-\theta}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\theta}} di, \tag{D.3}$$

which can be re-written as

$$Y_t = S_t^{-1} A_t K_t^\theta N_t^{1-\theta}, \tag{D.4}$$

where variable $S_t^{\frac{1}{1-\theta}} = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} di$ defines price dispersion.

[30] This can be understood as the bond price of a 10-year bond expected by the so-called risk-neutral investor who is rolling over a one-period investment for 10 years.

### D.2.3 Re-setting firm vs. aggregate quantities for DRS

The demand function at time $t+k$ for the firm re-setting its price at time t is given by,

$$Y_{t+k}^* = A_{t+k}\bar{K}^\theta N_{t+k}^{*(1-\theta)} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}, \tag{D.5}$$

where $P_t^*$ is the optimal price of firm resetting its price at time $t$ for the horizon $k$. Factor demand of the price re-setting firm, $N_t^*$ is,

$$N_{t+k}^* = \left(\left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} \frac{Y_{t+k}}{A_{t+k}\bar{K}^\theta}\right)^{\frac{1}{1-\theta}}. \tag{D.6}$$

The ratio of the price re-setting (equation (D.6)) and the aggregate firm's factor demands (D.3)), expressed in terms of time t quantities, is given by

$$\frac{N_t^*}{N_t} = \frac{\left(\left(\frac{P_t^*}{P_t}\right)^{-\epsilon}\frac{Y_t}{A_t K^\theta}\right)^{\frac{1}{1-\theta}}}{\left[\frac{Y_t S_t}{A_t K^\theta}\right]^{\frac{1}{1-\theta}}} = \frac{\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}}}{S_t^{\frac{1}{1-\theta}}} = \frac{\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}}}{\left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} di\right]}. \tag{D.7}$$

An analogous ratio can be derived for aggregate marginal cost and marginal costs of the price resetting firm. Marginal costs for the price resetting firm are,

$$MC_t^* = \frac{W_t}{(1-\theta)A_t K^\theta N_t^{-\theta}} \frac{N_t^{-\theta}}{N_t^{*-\theta}}, \tag{D.8}$$

Aggregate marginal cost come from $\frac{\partial W_t N_t}{\partial Y_t}$ and, using $N_t = \left[\frac{Y_t}{A_t K^\theta}\right]^{\frac{1}{1-\theta}} S_t^{\frac{1}{1-\theta}}$, delivers,

$$\frac{MC_t}{S_t} = \frac{W_t}{(1-\theta)\left(A_t K^\theta N_t^{-\theta}\right)}. \tag{D.9}$$

Plugging equation (D.7) into (D.8) and rearranging delivers,

$$MC_t^* = MC_t \frac{\left(\frac{P_t^*}{P_t}\right)^{-\frac{\theta\epsilon}{1-\theta}}}{S_t^{\frac{1}{1-\theta}}} \tag{D.10}$$

### D.2.4 Aggregation for CRS

For the case of a constant returns to scale production function, where capital is variable, the cost minimization problem is given by

$$\min_{N_t(i)} W_t N_t(i) + R_t^k K_t + MC_t^r(i)\left[Y_t(i) - A_t K_t(i)^{\theta} N_t^{1-\theta}(i)\right], \tag{D.11}$$

subject to production function, $Y_t(i) = A_t K_t(i)^{\theta} N_t^{1-\theta}(i)$, and where $MC_t(i)$ is the multiplier associated with the constraint.

The firm's demands for labor and capital are, respectively,

$$W_t = MC_t^r(i)(1-\theta)A_t K_t(i)^{\theta} N_t^{-\theta}, \tag{D.12}$$

$$R_t^k = MC_t^r(i)A_t\theta K_t(i)^{\theta-1} N_t^{1-\theta}(i), \tag{D.13}$$

Plugging the factor demands into the definition of total costs, $TC_t(i) = W_t N_t(i) + R_t^k K_t(i)$ delivers,

$$TC_t(i) = \left[MC_t^r(i)\right] Y_t(i). \tag{D.14}$$

Marginal costs are defined as a change in total cost when output changes, $\frac{dTC_t(i)}{dY_t(i)} = MC_t^r(i)$, which shows that the Lagrange multiplier equals real marginal costs. From the ratio of equation (D.12) and equation (D.13) we get that,

$$\frac{1-\theta}{\theta} = \frac{W_t N_t(i)}{R_t^k K_t(i)}. \tag{D.15}$$

Since factor prices are common for all the firms, the ratio of $\frac{1-\theta}{\theta}\frac{R_t}{W_t} = \frac{N_t(i)}{K_t(i)}$ is the same for all firms. Plugging factor demands from equation (D.12) and equation (D.13) into production function of firm $i$ we get $Y_t(i) = A_t\left(\frac{MC_t^r(i)\theta Y_t(i)}{R_t^k}\right)^{\theta}\left(\frac{MC_t^r(i)(1-\theta)Y_t(i)}{W_t}\right)^{1-\theta}$ which after expressing for $MC_t^r$ delivers,

$$MC_t^r = \int_0^1 MC_t^r(i)di = \frac{\left(R_t^k\right)^{\theta} W_t^{1-\theta}}{A_t\theta^{\theta}(1-\theta)^{1-\theta}}, \tag{D.16}$$

Marginal costs are therefore the same for all firms, both of price setters and firms with staggered prices.

### D.2.5 Re-setting firm vs. aggregate quantities for CRS

From the relationship $Y_t^* = A_t K_t^{*\theta} N_t^{*1-\theta} = \left(\frac{P_t^*}{P_t}\right)^{-\epsilon} Y_t$ we can express for amount of labor input hired by the price re-setting firm as

$$N_t^* = \frac{\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}}}{\left[\int_0^1\left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}}di\right]}\left(\frac{K_t}{K_t^*}\right)^{\frac{\theta}{1-\theta}} N_t. \tag{D.17}$$

The ratio of capital demand equations for the price resetting firm and the aggregate firm delivers,

$$\frac{K_t}{K_t^*} = \frac{Y_t}{Y_t^*}, \tag{D.18}$$

We have shown that $Y_t^* \leq Y_t$ in the presence of trend inflation. Therefore, as $\frac{K_t}{K_t^*} = \frac{Y}{Y_t^*}$, then $K_t^* \leq K_t$ and $N_t^* \leq N_t$.

## D.3 Calibration

Table D.2: Calibration of the RS table 3 (best fit) model

| Symbol | Variable | Value |
|---|---|---|
| $\beta$ | Discount factor | 0.99 |
| $CRRA$ | Risk aversion | 110 |
| $IES$ | Intertemporal elasticity | 0.09 |
| $\epsilon$ | Elasticity of substitution | 6 |
| $Frisch$ | Frisch elasticity | 0.28 |
| $\phi_\pi$ | Response to inflation | 0.53 |
| $\phi_y$ | Response to output | 0.93 |
| $\rho_i$ | $i_t$ smoothing | 0.73 |
| $\zeta$ | Price adjustment | 0.76 |
| $\bar{G}/\bar{Y}$ | Government spending on output | 0.17 |
| $\rho_G$ | Autocorrelation Government spending shock | 0.95 |
| $\sigma_G$ | Volatility of Government spending shock | 0.004 |
| $\rho_A$ | Autocorrelation of TFP shock | 0.95 |
| $\sigma_A$ | Volatility of TFP shock | 0.005 |
| $\theta_{\rho_{pi^*}}$ | Inflation target shock persistence | 0.995 |
| $\sigma_{\pi^*}$ | Volatility of inflation target shock | 0.0007 |
| $\zeta_{\pi^*}$ | Inflation target adjustment | 0.003 |
| $\theta$ | Capital share of output | 1/3 |
| $\bar{\Pi}$ | Steady state inflation | 1.004 |
| $\delta$ | Capital depreciation | 0.02 |

## D.4 Sensitivity analysis to RS model

In what follows, we focus our analysis further on the dispersion of prices in the economy. As the model parameters of RS were calibrated to match moments for the case of $\bar{\pi} = 0$, it may be argued that the model might be not well calibrated, when simply allowing for $\bar{\pi} > 0$ but leaving other model parameters unchanged. We first confirm that the patterns documented in Table 2 hold across a wide range of parameter values.

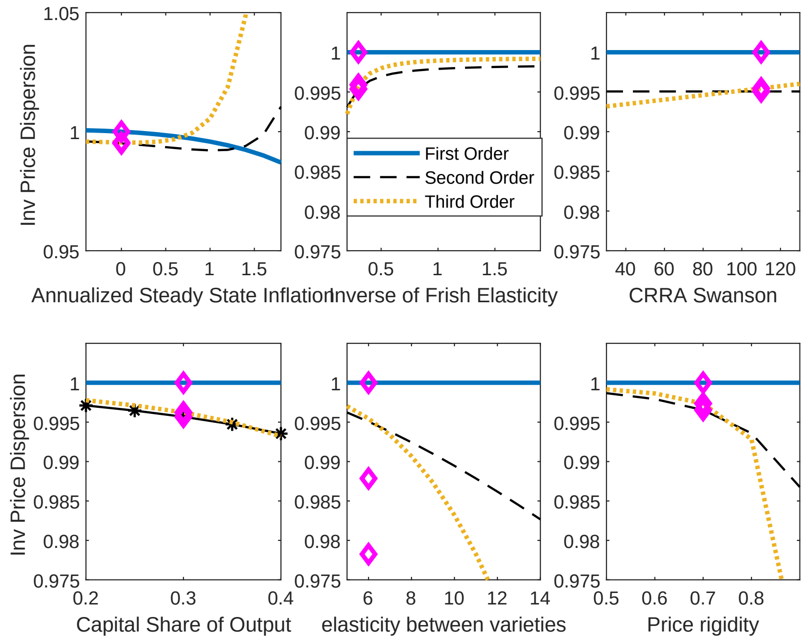
Figure D.1 shows how the mean of the inverse price dispersion changes over different ranges of other model parameter values and orders of approximation. The first set of panels shows the sensitivity of mean simulated price dispersion to changes in key model parameters for the case of zero trend inflation, for different orders of approximation (first, second, third-order approximations). Pink diamonds reflect the case of the 'RS Table 3'-baseline parameterization. Whereas the mean simulated price dispersion is affected strongly by varying trend inflation (panel 1), including pushing $S^{-1}$ to the infeasible region bigger than one[31], varying other model parameters does not affect the simulated mean price dispersion drastically (and never pushes $S^{-1}$ to an infeasible region). Other than variations in trend inflation, only regions of relatively high elasticities of substitution or high price rigidities lead to large costs from price dispersion (of, e.g. more than 1%, reflected in $S^{-1}$ falling below 0.99). The second set of panels presents comparable figures for the case of positive trend inflation. Pink diamonds reflect the 'RS Table 3'-baseline parameterization, apart for steady-state inflation, which now is $\bar{\pi} = 1\%$. Since the accuracy of the mean price dispersion is already somewhat compromised at $\bar{\pi} = 1\%$, regions of relatively high elasticities of substitution or high price rigidities quickly lead to problems (flat lines in the last two reported panels represent cases with indeterminate solutions). Variations in other key parameters continue to leave mean price dispersion mostly unaffected.

# Appendix E Results

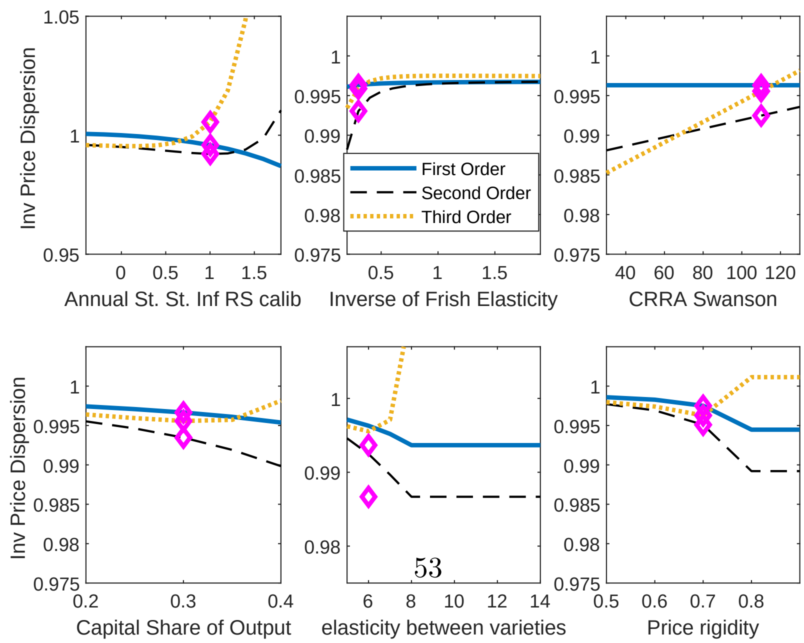
[31] $S^{-1}$ is bounded from above by one. See Proposition C.1

Figure D.1: Parameters sensitivity in the RS (2012) model

Parameter sensitivity of RS (2012) model, case of zero trend inflation, $\bar{\pi} = 0\%$




Parameter sensitivity of RS (2012) model, case of positive trend inflation, $\bar{\pi} = 1\%$






*Note: The first set of panels shows the sensitivity of the mean simulated price dispersion to changes in key model parameters for the case of zero trend inflation, for different orders of approximation (first, second and third order approximations). Pink diamonds reflect the case of the 'RS Table 3'-baseline parameterization. The second set of panels presents analogous figures for the case of positive trend inflation. Flat lines in the last two reported panels represent cases with indeterminate solutions.*

Table E.1: Taylor rule estimates for US

| Study | Period | $\phi_{pi}$ | $\phi_y$ |
|---|---|---|---|
| Taylor (1999) | 1987 - 1997 | 1.53 | 0.77 |
| Judd and Rudebusch (1998) | 1987 - 1997 | 1.54 | 0.99 |
| Clarida et al. (2000) | 1979 - 1996 | 2.15 | 0.93 |
| Orphanides (2003) | 1979 - 1995 | 1.89 | 0.18 |
| Coibion and Gorodnichenko (2011) | 1983 - 2002 | 2.2 | 0.43 |
| Smets and Wouters (2007) | 1966 - 2004 | 2.03 | 0.08 |
| Jermann and Quadrini (2012) | 1984 - 2010 | 2.41 | 0.121 |
| Rotemberg and Woodford (1997) | 1980 - 1995 | 1.27 | 0.08 |
| Lubik and Schorfheide (2004) | 1982 - 1997 | 2.19 | 0.3 |
| Boivin and Giannoni | 1979 - 2002 | 2.03 | 0 |
| Calibrated models | | | |
| Galí (2015) | - | 1.50 | 0.125 |
| Ascari and Sbordone (2014) | - | 2.00 | 0.125 |
| Guerrieri and Iacoviello (2015) | - | 2.50 | 0.50 |
| Basu and Bundick (2017) | - | 1.5 | 0.2 |
| Levin et.al. (2000) | - | 1.27 | 0.08 |