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Parui, Pintu; Prettner, Klaus

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Pintu Parui
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Public provision of healthcare and basic science: What are the effects on economic growth and welfare?

Pintu Parui[1] and **Klaus Prettner**[2]

[1] *School of Economics, XIM University, Bhubaneswar, India*

[2] *Vienna University of Economics and Business, Department of Economics, Welthandelsplatz 1, 1090 Vienna, Austria*

Abstract

We propose a generalized R&D-based economic growth model that incorporates i) endogenous human capital accumulation in terms of education and health, ii) endogenous population growth, and iii) the public provision of healthcare and basic science. The government taxes households to pay for healthcare personnel and basic scientists. Since these employees are not anymore available for applied science and for final goods production, important tradeoffs with respect to government spending emerge for economic growth and welfare. We show that increasing public spending, particularly on basic science, leads to faster economic growth in the medium run and tends to raise welfare when compared to actual levels of spending in Organisation for Economic Co-operation and Development (OECD) countries. Our results highlight the difficult tradeoffs associated with public expenditures for healthcare and basic science and emphasize the important role of policymakers in ensuring adequate overall public funding.

Keywords: R&D-Based Growth, Basic Science, Public Healthcare, Children's Health, Education, Fertility, Intertemporal Tradeoffs.

JEL Code: H41; J24; O31; O32; O41

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Email id: *pintuparui6@gmail.com* (P. Parui) *&* *klaus.prettner@wu.ac.at* (K. Prettner)

1 Introduction

How much should governments invest in different areas such as healthcare and basic science to foster economic growth and to improve welfare? To answer this question, we propose a generalized R&D-based model of economic growth in which the government finances basic science, which is an important input in applied R&D, and healthcare expenditures, which improve individual productivity. We use the model i) to analyze the dependence of economic growth on governmental investments in basic science and in healthcare, ii) to calibrate the model to real-world data and highlight the welfare effects of different governmental policies, and iii) to address the tradeoffs that governments face when deciding to invest in healthcare and in basic science.

Most frameworks that have previously been used to answer questions related to the effects of governmental investments on economic growth and welfare were built around different strands of the literature that disregard at least one important dimension in this context. Health investments and their effects on welfare are often analyzed in models of exogenous economic growth following a baseline model structure of Solow (1956), Ramsey (1928), Cass (1965), Koopmans (1965) or Diamond (1965). Of course, in such models and with diminishing returns to physical capital in the production process, there are only limited repercussions of health on economic growth.

Some contributions rely on R&D-based endogenous and semi-endogenous growth models of the Romer (1990) and Jones (1995) types to analyze the long-run growth and welfare effects of healthcare (see, for example, Kuhn and Prettner, 2016; Baldanzi et al., 2019, 2021). The typical finding in this literature is that healthcare investments can raise economic growth and welfare. However, these frameworks are silent on governmental expenditures on basic science.

Finally, another strand of the literature is concerned with the effects of governmental basic science investments on economic growth and welfare (see, for example, Gersbach et al., 2013; Gersbach and Schneider, 2015; Gersbach et al., 2018, 2023; Prettner and Werner, 2016). These frameworks show the importance of basic science as an input in applied R&D, which is the main engine of long-run economic growth. The typical finding here is that basic science investments in rich countries tend to be much lower than the levels that would maximize welfare. These frameworks are, in turn, silent on the effects of governmental expenditures on health.

The discussion so far implies that the only tradeoff analyzed in the discussed literature is whether or not an economy should raise taxes to increase spending on healthcare *or* on basic science. The finding that these types of spending are welfare increasing is perhaps not surprising when there are no other types of spending to consider for the government. By contrast, in reality, governments usually face tradeoffs when planning expenditures in different areas that may all be growth-promoting and welfare-enhancing.

As a consequence, in this contribution, we are interested how the results of the two separate strands of literature change once we allow for both types of governmental expenditures, on health and on basic science. To this end, we propose a generalized R&D-based growth model that includes the quality-quantity trade-off between fertility and human capital accumulation (cf. Prettner et al., 2013; Strulik et al., 2013; Prettner, 2014; Prettner and Werner, 2016). We augment this model to allow for the fact that parents care not only for the education of their children, but also for their health. In so doing, we follow Baldanzi et al. (2021) who, however, abstract from governmental spending on basic research and governmental spending on health.[1] To close this gap, we assume that the government seeks to improve people's health by providing healthcare personnel. This, in turn, can enhance labor productivity in the sectors that employ workers, e.g., by reducing production losses caused by sick employees. Finally, we include basic research as a necessary input in the production of applied R&D, which, in turn, determines economic growth. In so doing, we follow Prettner and Werner (2016).[2] While governmental investments on healthcare and on basic science both have productivity-enhancing effects, a rise in the number of healthcare workers and in the number of basic scientists implies that fewer workers are available for final goods production and for applied R&D. Overall, our model therefore captures a rich set of tradeoffs in terms of governmental spending, along with the quality-quantity trade-off between fertility on the one hand and human capital accumulation in terms of education and health on the other. To our knowledge, our model is therefore the first to generalize the previous R&D-based growth literature to account

[1]For the importance of health investments in generating human capital and for the long-run consequences of governmental health investments, see Well (2007), Prettner et al. (2013), Kuhn and Prettner (2016), Schneider and Winkler (2021), and Kuhn et al. (2023). For empirical evidence, see Weil (2014), Madsen (2016), Bucci et al. (2021), and Bloom et al. (2024).

[2]Other contributions that analyze the effects of basic research on economic growth include Gersbach et al. (2013), Gersbach and Schneider (2015), Gersbach et al. (2018), Akcigit et al. (2020), Gersbach et al. (2023), and Huang et al. (2023). For empirical evidence, see Czarnitzki and Thorwarth (2012), Toole (2012), Coad et al. (2021), and Mulligan et al. (2022).

for all of the following dimensions: endogenous fertility, endogenous health investments, endogenous education investments, and publicly funded healthcare and basic science.

Using our framework, we find that the welfare-increasing level of governmental expenditures on health and basic science is higher than the actual levels of governmental expenditures in these areas. The policy implications of this finding are that i) raising governmental expenditures on basic science and healthcare would be worthwhile in the long run; ii) however, the presence of more domains on which the government can spend its funds productively implies that more care is needed to design and evaluate the corresponding policies.

Our article is structured as follows. In Section 2, we develop the generalized R&D-based economic growth model with endogenous fertility, endogenous education, endogenous health, and governmental expenditures on healthcare and on basic science. In Section 3, we present our analytical results that hold along a balanced growth path. In Section 4, we calibrate the model and solve it numerically for obtaining the transitional dynamics and the welfare effects of government spending. Section 5 is devoted to sensitivity analyses and robustness checks, while we conclude in Section 6.

2 The model

2.1 Consumption side

We consider an economy with three overlapping generations: children, adults, and retirees. Adults decide upon the consumption level c_t, savings for retirement s_t, the number of children n_t, and investments in each of their children in terms of education e_t, and health m_t. The time adults do not spend on raising their children, educating them, and caring for their health is supplied on the labor market. Retirees consume their entire savings carried over from adulthood. Finally, children do not make any economic decisions; instead, they are fed, educated, and cared for by their parents. Following Prettner and Werner (2016) and Baldanzi et al. (2021), the preferences of a single-parent household are captured by the utility function

$$u_t = \ln c_t + \beta \ln \left[(R_{t+1} - 1)s_t\right] + \xi \ln n_t + \theta \ln e_t + \sigma \ln m_t, \tag{1}$$

where $\beta \in (0,1)$ represents the inter-generational discount factor, R_{t+1} represents the gross interest rate on assets between generation t and $t+1$, and $\xi \in (0,1)$, $\theta \in (0,1)$, and $\sigma \in (0,1)$ are utility weights on the number of children, children's education, and children's health, respectively.[3] We assume that next generation's human capital is a multiplicative function of parental education and health spending. Thus, a part of the parental utility function ($\xi \ln n_t + \theta \ln e_t + \sigma \ln m_t$) captures the trade-offs parents face in deciding the number of children and how much time to invest in children's education and children's health. To simplify the model, we assume an exogenously given mortality rate of parents and to rule out solutions in which parents do not have any children at all, we impose the parameter restriction $\xi > \theta + \sigma$.

Following Prettner and Werner (2016), the cost of raising children, educating them, and providing them with a basic health level requires time of their parents.[4] Therefore, the budget constraint of the household reads

$$(1-\tau)(1-\psi n_t - \eta e_t n_t - \chi m_t n_t) w_t h_t = c_t + s_t, \tag{2}$$

where $\tau \in (0,1)$ represents the income tax rate, $\psi > 0$, $\eta > 0$, and $\chi > 0$ denote opportunity costs in terms of time for child-rearing, per child education, and per child health investment, respectively, w_t is the wage rate, and h_t represents effective labor (the human capital of the household). Optimal choices of consumption, savings, fertility, education, and children's health are (see Appendix A for the derivation)

$$\begin{aligned} c_t &= \frac{(1-\tau)w_t h_t}{1+\beta+\xi}, \qquad s_t = \frac{\beta(1-\tau)w_t h_t}{1+\beta+\xi}, \qquad n_t = \frac{\xi-\theta-\sigma}{\psi(1+\beta+\xi)}, \\ e_t &= \frac{\theta\psi}{\eta(\xi-\theta-\sigma)}, \qquad m_t = \frac{\sigma\psi}{\chi(\xi-\theta-\sigma)}. \end{aligned} \tag{3}$$

The population size at time $t+1$ is determined via the fertility rate n_t as

$$L_{t+1} = n_t L_t = \frac{\xi-\theta-\sigma}{\psi(1+\beta+\xi)} L_t. \tag{4}$$

[3]This type of utility function is often used in the literature (cf. Strulik et al., 2013; Prettner and Werner, 2016; Baldanzi et al., 2021) and is based on the "warm-glow motive of giving" (see Andreoni, 1989). It is a special case of the utility formulation used in Galor and Weil (2000), and Galor (2005, 2011) and leads to the same tradeoffs.

[4]For example, parental involvement in a child's physical development by assigning time for the child to participate in different sports and games, to dance, and to do other physical activities will aid in developing the child's health; the same holds true for devoting time to prepare nutritious meals and to care for preventive health measures such as vaccination, etc.

We assume that the individual human capital level of the next generation depends positively on (i) educational effort by the parents, e_t; (ii) parents' productivity in education, A_E; (iii) healthcare effort by parents for their children, m_t[5]; (iv) parents' productivity in healthcare for their children, A_M; and (v) the level of parents' individual human capital h_t in the following way:

$$h_{t+1} = (A_E e_t h_t)^\nu (A_M m_t h_t)^{1-\nu} = \left(A_E \frac{\theta}{\eta}\right)^\nu \left(A_M \frac{\sigma}{\chi}\right)^{1-\nu} \frac{\psi}{\xi - \theta - \sigma} h_t. \tag{5}$$

Equations (4) and (5) capture the trade-offs between child quantity and quality that is summarised in the following proposition.

Proposition 1. *An increase in the desire for a large family (ξ) raises fertility and population growth but reduces human capital accumulation. Increases in the desire for having better educated or healthier children (increases in θ and σ) raise human capital accumulation but reduce fertility and population growth.*

Proof. See Appendix B. □

H_t, the aggregate human capital stock of the economy, is the product of individual human capital (h_t) and the total population size (L_t). Therefore, the human capital stock available for production, basic and applied research, and healthcare $(\tilde{H}_t)$ is given by the aggregate human capital stock adjusted for the time parents spend raising, educating, and caring for their children's health $(\psi n_t + \eta e_t n_t + \chi m_t n_t)$ as

$$\tilde{H}_t = [1 - \psi n_t - \eta e_t n_t - \chi m_t n_t] H_t = \frac{1+\beta}{1+\beta+\xi} h_t L_t. \tag{6}$$

Note that aggregate human capital accumulation and population growth are inversely related.

2.2 Production side

The final goods sector, intermediate goods sector, applied research sector, basic research sector, and healthcare sector constitute the production side of the economy. The first three sectors are based on the standard Romer (1990) and Jones (1995) R&D-driven

[5] Note that, along with the level of education, a better health condition is also an essential component of individual human capital (cf. Rivera and Currais, 2004; Baldanzi et al., 2021).

growth literature. We modify this structure to account for (i) a tax-financed basic research sector that employs scientists to discover and explain the natural laws and phenomena required for applied research (Prettner and Werner, 2016), (ii) public healthcare, which enhances the productivity of human capital (Kuhn and Prettner, 2016; Baldanzi et al., 2021), and (iii) the endogenous evolution of aggregate human capital in the production process, which depends on fertility, education, and health choices of households (Strulik et al., 2013). The model structure is displayed in Figure 1, where households demand goods and supply labor and capital (through savings), The government taxes household wage income to finance healthcare and basic research, and the different production sectors employ labor or the capital of households to produce their corresponding output. Demand and supply for all goods are equal due to the market clearing price vector that emerges endogenously by the interactions of households and firms on the market.

2.2.1 Final goods sector

The perfectly competitive final goods sector employs workers and machines to produce output Y_t according to

$$Y_t = \left(H_{t,M}^{\varepsilon_0} H_{t,Y}\right)^{1-\alpha} \int_0^{A_t} x_{t,i}^{\alpha}\, di, \tag{7}$$

where $H_{t,Y}$ and $H_{t,M}$ refer to the human capital (embodied in workers) employed in the final good and healthcare sectors, respectively, A_t is the technological frontier, $x_{t,i}$ is the amount of the blueprint-specific machine i used in production, and α is the elasticity of output with respect to machines. Employment in the healthcare sector raises the health of workers $H_{t,Y}$ and, thus, affects their productivity according to $H_{t,M}^{\varepsilon_0}$, where ε_0 measures the strength of the effect.[6] We assume that governmental health investments are non-zero such that $H_{t,M} > 0$. For a given total factor productivity (i.e., $H_{t,M}^{\varepsilon_0(1-\alpha)}$), Equation (7)

[6] Consider the following example. An individual's human capital level at the time of entry into the labor force in period t is h_t. This human capital depends on her parents' decision (in period $t-1$) to devote time to her education and health care when she was young. However, if the individual becomes ill, even though she continues to work, she may not be able to perform to her full potential. Public healthcare will assist her in regaining full productivity as soon as possible. As a result, she will be more productive than if she did not have access to public healthcare. In this context, it should be noted that public healthcare may have an impact on children's health. However, for the sake of simplicity, we are ignoring this pathway here. One worthwhile extension of the current model would be integrating this issue and investigating its long-run implications. Note that the likelihood of a health impact on final output productivity is accounted for as a spillover effect by $H_{t,M}^{\varepsilon_0}$, and the magnitude of this health spillover effect is reflected by the parameter ε_0. A similar type of argument can be found in Schneider and Winkler (2017, 2021).

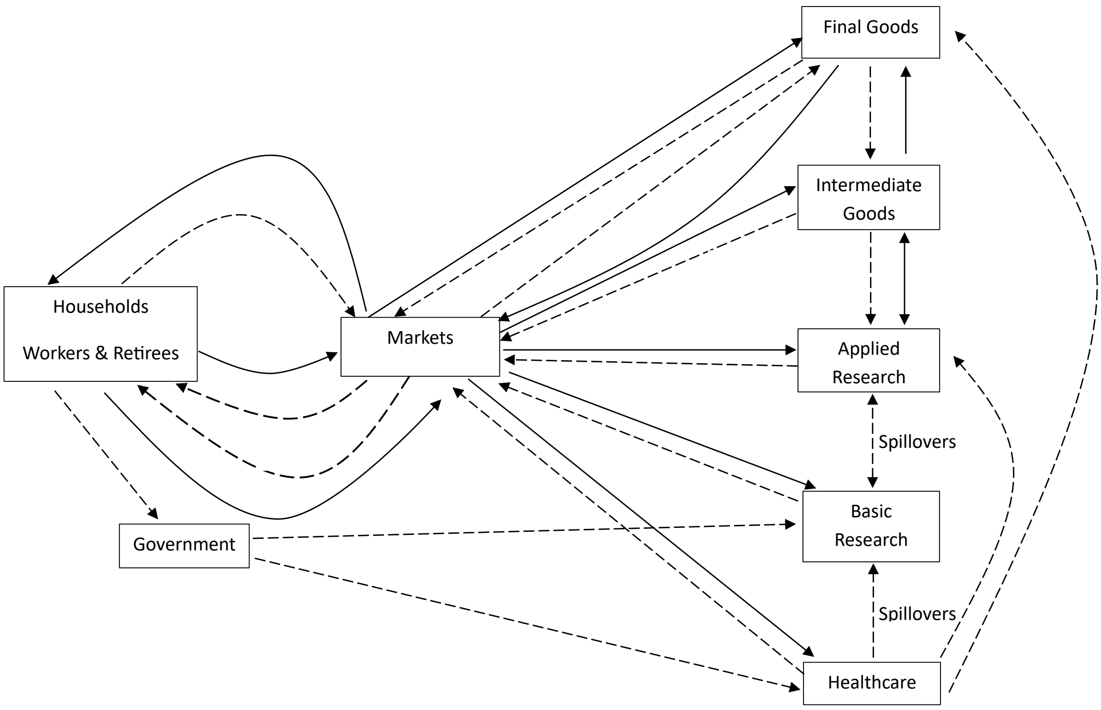


Figure 1: Overview of the structure of the model

exhibits constant returns to scale in $H_{t,Y}$ and $x_{t,i}$. Perfect competition implies the wage rate $w_{t,Y}$ and the prices of machines $p_{t,i}$ are given by the marginal products of workers and machines of type i as

$$w_{t,Y} = (1-\alpha)\left(H_{t,M}^{\varepsilon_0} H_{t,Y}\right)^{-\alpha} H_{t,M}^{\varepsilon_0} \int_0^{A_t} x_{t,i}^{\alpha}\, di = (1-\alpha)\frac{Y_t}{H_{t,Y}}, \tag{8}$$

$$p_{t,i} = \alpha x_{t,i}^{\alpha-1}\left(H_{t,M}^{\varepsilon_0} H_{t,Y}\right)^{1-\alpha}. \tag{9}$$

2.2.2 Intermediate goods sector

Raw physical capital $k_{t,i}$ serves as variable input and one machine-specific blueprint serves as fixed input in the production of the monopolistically competitive intermediate goods sector, which produces the machines for the production of the final good. We assume full depreciation of physical capital over the course of one generation. Thus, operating profits in intermediate goods production are $\pi_{t,i} = p_{t,i}k_{t,i} - R_t k_{t,i}$. Profit maximization then leads to the monopolistic pricing rule for each firm as

$$p_{t,i} = \frac{R_t}{\alpha}. \tag{10}$$

Due to symmetry across firms, each firm employs $k_t = K_t/A_t$ units of physical capital, where K_t represents the aggregate physical capital stock. Thus, the aggregate production function can be re-written as

$$Y_t = \left(A_t H_{t,M}^{\varepsilon_0} H_{t,Y}\right)^{1-\alpha} K_t^{\alpha}, \tag{11}$$

where technological progress appears as labor augmenting.

2.2.3 Applied research sector

The applied research sector employs scientists with a human capital stock of $H_{t,A}$ to design new blueprints that can be patented and sold to the intermediate goods sector. In applied research, the representative firm's production function is given by

$$A_{t+1} - A_t = \delta_1 H_{t,M}^{\varepsilon_1} A_t^{\phi_1} B_t^{\mu_1} H_{t,A}, \tag{12}$$

where $\delta_1 H_{t,M}^{\varepsilon_1}$ is the productivity of inputs in the applied research sector, B_t represents society's stock of basic knowledge, which is discovered by basic researchers and forms the epistemic base for the stock of patented knowledge A_t (Prettner and Werner, 2016), and $\phi_1 \in [0,1]$ and $\mu_1 \in [0,1]$ measure the extent of intertemporal knowledge spillovers in the applied research sector and intersectoral knowledge spillovers from basic to applied research.[7] For a given stock of basic and applied knowledge, ε_1 assesses how strongly public healthcare enhances the productivity of applied researchers. Similar to Prettner and Werner (2016), no blueprints can be developed without any basic knowledge B_t, so we assume that $B_0 > 0$ and $A_0 > 0$. Our framework nests both the endogenous and semi-endogenous growth models of Romer (1990) and Jones (1995) as special cases, which we summarize in Remark 1.

Remark 1. *For $\tau = 0$, $\theta = 0$, $\sigma = 0$, $\xi > \psi(1+\beta+\xi)$, $\mu_1 = 0$, $\varepsilon_0 = \varepsilon_1 = 0$, and $\phi_1 \in (0,1)$, our model nests the Jones (1995) framework, while for $\tau = 0$, $\theta = 0$, $\sigma = 0$, $\xi = \psi(1+\beta+\xi)$, $\mu_1 = 0$, $\varepsilon_0 = \varepsilon_1 = 0$, and $\phi_1 = 1$, our model nests the Romer (1990) framework.*

Firms in the applied research sector hire human capital $H_{t,A}$ so as to maximize their profits

$$\pi_{t,A} = p_{t,A}\delta_1 H_{t,M}^{\varepsilon_1} A_t^{\phi_1} B_t^{\mu_1} H_{t,A} - w_{t,A} H_{t,A} \tag{13}$$

with $p_{t,A}$ being the price of a blueprint and $w_{t,A}$ referring to the applied researchers' wage rate. This leads to the optimality condition

$$w_{t,A} = p_{t,A}\delta_1 H_{t,M}^{\varepsilon_1} A_t^{\phi_1} B_t^{\mu_1}. \tag{14}$$

Following Strulik et al. (2013) and Prettner and Werner (2016), we assume that patent protection lasts for one generation. Once the patent expires, the right to sell the blueprint is handed over to the government, which can either consume or invest the associated proceeds. For a blueprint, firms in the applied research sector charge the entire operating profit of an intermediate goods producer, that is,

$$p_{t,A} = \pi_{t,i} = \alpha(1-\alpha)\frac{Y_t}{A_t}. \tag{15}$$

[7] As in Prettner and Werner (2016), given that patents are partially excludable, whereas the laws of nature, once discovered, can be exploited by scientists freely, one can expect that the spillovers from basic research to applied research are greater than the spillovers in the opposite direction.

The reason is that free entry prevails in intermediate goods production. If a firm would not be willing to pay its entire operating profit for a blueprint, another firm would always be willing to do so.

2.2.4 Basic research sector

Following Prettner and Werner (2016), the production function of basic research is given by

$$B_{t+1} - B_t = \delta_2 H_{t,M}^{\varepsilon_2} A_t^{\phi_2} B_t^{\mu_2} H_{t,B}, \tag{16}$$

where B_t is the stock of basic knowledge, δ_2 is the productivity of basic researchers, $H_{t,B}$ is the human capital stock employed in the basic research sector, and $\mu_2 \in [0,1]$ and $\phi_2 \in [0,1]$ are intertemporal spillover effects within basic research and intersectoral spillover effects from applied research to basic research. As in final goods production and applied research, public healthcare raises the productivity of basic researchers. For a given stock of applied and basic knowledge, ε_2 indicates the strength of this effect.

A part (τ_0) of the government's revenue is spent on employing scientists to discover basic knowledge. Considering aggregate labor supply as given by Equation (6), the tax revenue used for funding basic research is given by the left-hand side of

$$\frac{\tau_0 \tau (1+\beta)}{1+\beta+\xi} w_t h_t L_t = w_t h_t L_{t,B}. \tag{17}$$

This revenue is used to cover the wage bill of scientists in the basic research sector, which is given by the right-hand side of Equation (17). It follows that the amount of human capital employed in the basic research sector is

$$H_{t,B} = L_{t,B} h_t = \frac{\tau_0 \tau (1+\beta)}{1+\beta+\xi} H_t, \quad \left(\equiv \tau_0 \tau \tilde{H}_t\right). \tag{18}$$

Using the production function (16), basic knowledge evolves according to

$$B_{t+1} - B_t = \frac{\delta_2 \tau_0 \tau (1+\beta)}{1+\beta+\xi} H_{t,M}^{\varepsilon_2} A_t^{\phi_2} B_t^{\mu_2} H_t. \tag{19}$$

Note that the accumulation of basic knowledge rises with government spending on it as captured by τ and τ_0.

2.2.5 Healthcare sector

We assume that the government's budget is balanced so that $(1-\tau_0)$ of the tax revenue is spent on employing the healthcare workers in the healthcare sector, i.e.,

$$\frac{(1-\tau_0)\tau(1+\beta)}{1+\beta+\xi} w_t h_t L_t = w_t h_t L_{t,M}.$$

Therefore, the amount of human capital employed in the healthcare sector is

$$H_{t,M} = h_t L_{t,M} = \frac{(1-\tau_0)\tau(1+\beta)}{1+\beta+\xi} H_t, \quad \left(\equiv (1-\tau_0)\tau\tilde{H}_t\right). \tag{20}$$

The government aims to improve people's health by providing healthcare to them and, in doing so, affects the productivity of human capital in final goods production, applied research, and basic research as described above (cf. Kuhn and Prettner, 2016).

2.3 Market clearing and balanced growth path

The labor market clearing conditions are $\tilde{H}_t = h_t[L_{t,Y} + L_{t,A} + L_{t,B} + L_{t,M}] = H_{t,Y} + H_{t,A} + H_{t,B} + H_{t,M}$ and $w_{t,Y} = w_{t,A} = w_{t,B} = w_{t,M} = w_t$. Equations (8), (14), (15), (18), and (20) yield the demand for human capital in the final goods and applied research sectors as

$$H_{t,Y} = \frac{A_t^{1-\phi_1} B_t^{-\mu_1} H_{t,M}^{-\varepsilon_1}}{\alpha\delta_1}, \tag{21}$$

$$H_{t,A} = \tilde{H}_t - H_{t,B} - H_{t,M} - H_{t,Y}$$

$$\implies H_{t,A} = \frac{(1-\tau)(1+\beta)}{(1+\beta+\xi)} h_t L_t - \frac{A_t^{1-\phi_1} B_t^{-\mu_1}}{\alpha\delta_1} \left[\frac{(1-\tau_0)\tau(1+\beta)}{(1+\beta+\xi)} h_t L_t\right]^{-\varepsilon_1}. \tag{22}$$

The development of new blueprints is then given by

$$A_{t+1} = \left(\frac{1+\beta}{1+\beta+\xi}\right)^{1+\varepsilon_1} [(1-\tau_0)\tau]^{\varepsilon_1} (1-\tau)\delta_1 A_t^{\phi_1} B_t^{\mu_1} (h_t Lt)^{1+\varepsilon_1} - \left(\frac{1-\alpha}{\alpha}\right) A_t. \tag{23}$$

As there is full depreciation of physical capital, capital market clearing implies that aggregate savings are used for physical capital accumulation and purchasing new blueprints for intermediate goods production, i.e., $K_{t+1} = s_t L_t - p_{t,A}(A_{t+1} - A_t) = \frac{\beta(1-\tau)}{1+\beta+\xi} w_t h_t L_t - p_{t,A}(A_{t+1} - A_t)$. Equations (8), (11), (15), (20), (21), and (23) yield the aggregate physical capital stock of the next period as

$$K_{t+1} = \left[\frac{\beta(1-\tau)(1-\alpha)\left[(1-\tau_0)\tau(1+\beta)\right]^{\alpha\varepsilon_1+(1-\alpha)\varepsilon_0}}{(1+\beta+\xi)^{1+\alpha\varepsilon_1+(1-\alpha)\varepsilon_0}} K_t^{\alpha} \right] \times$$
$$\left[\left(\frac{A_t^{2-\phi_1} B_t^{-\mu_1}}{\alpha\delta_1} \right)^{-\alpha} A_t \left(h_t L_t\right)^{1+\alpha\varepsilon_1+(1-\alpha)\varepsilon_0} \right]$$
$$- \alpha(1-\alpha)\frac{Y_t}{A_t} \left[\left(\frac{1+\beta}{1+\beta+\xi} \right)^{1+\varepsilon_1} \left[(1-\tau_0)\tau\right]^{\varepsilon_1} (1-\tau)\delta_1 A_t^{\phi_1} B_t^{\mu_1} (h_t Lt)^{1+\varepsilon_1} - \frac{A_t}{\alpha} \right]. \tag{24}$$

Finally, with respect to the evolution of the stock of basic knowledge, Equations (19) and (20) yield

$$B_{t+1} = \frac{\delta_2\tau_0(1-\tau_0)^{\varepsilon_2}\left[\tau(1+\beta)\right]^{1+\varepsilon_2}}{(1+\beta+\xi)^{1+\varepsilon_2}} A_t^{\phi_2} B_t^{\mu_2} (h_t L_t)^{1+\varepsilon_2} + B_t. \tag{25}$$

Now we have all dynamic equations that characterize the evolution of the economy such that we can proceed to our analytical results for the balanced growth path and then to the numerical assessment of the transitional dynamics and the welfare implications.

3 Analytical results for the long-run balanced growth path

From now on, we restrict our attention to the following parameter assumptions to ensure the existence of a balanced growth path and to rule out the empirically implausible scenario of hyper-exponential growth.

Assumption 1. *The intertemporal and intersectoral knowledge spillovers are given by* $\phi_1 \in [0,1)$, $\phi_2 \in [0,1)$, $\mu_1 \in [0,1)$, *and* $\mu_2 \in [0,1)$. *Moreover, it holds that* $\phi_1 + \mu_1 < 1$ *and* $\phi_2 + \mu_2 < 1$.

The growth rates of the stocks of blueprints and of basic knowledge are then given by

$$g_{t,A} \equiv \frac{A_{t+1}-A_t}{A_t} = \frac{(1-\tau)(1+\beta)^{1+\varepsilon_1}\left[(1-\tau_0)\tau\right]^{\varepsilon_1}}{(1+\beta+\xi)^{1+\varepsilon_1}} \delta_1 A_t^{\phi_1-1} B_t^{\mu_1} (h_t Lt)^{1+\varepsilon_1} - \frac{1}{\alpha}, \tag{26}$$

$$g_{t,B} \equiv \frac{B_{t+1}-B_t}{B_t} = \frac{\delta_2\tau_0(1-\tau_0)^{\varepsilon_2}\left[\tau(1+\beta)\right]^{1+\varepsilon_2}}{(1+\beta+\xi)^{1+\varepsilon_2}} B_t^{\mu_2-1} A_t^{\phi_2} (h_t L_t)^{1+\varepsilon_2}. \tag{27}$$

The balanced growth factors (henceforth BGFs) of individual human capital, population size, and aggregate human capital are given by[8]

$$\tilde{h} \equiv \frac{h_{t+1}}{h_t} = \left(A_E \frac{\theta}{\eta}\right)^{\nu} \left(A_M \frac{\sigma}{\chi}\right)^{1-\nu} \frac{\psi}{\xi - \theta - \sigma}, \tag{28}$$

$$\tilde{L} \equiv \frac{L_{t+1}}{L_t} = n_t = \frac{\xi - \theta - \sigma}{\psi(1 + \beta + \xi)}, \tag{29}$$

$$\Omega \equiv \tilde{h}\tilde{L} = \frac{\left(A_E \frac{\theta}{\eta}\right)^{\nu} \left(A_M \frac{\sigma}{\chi}\right)^{1-\nu}}{1 + \beta + \xi}. \tag{30}$$

From now on, we assume that

$$\psi \in \left(\frac{\xi - \theta - \sigma}{\left(A_E \frac{\theta}{\eta}\right)^{\nu} \left(A_M \frac{\sigma}{\chi}\right)^{1-\nu}}, \frac{\xi - \theta - \sigma}{1 + \beta + \xi} \right),$$

which ensures that individual human capital and the population will both grow over time. As a result, $\Omega = \tilde{h}\tilde{L} > 1$ holds unambiguously. The following proposition introduces the main results of this paper for the long-run balanced growth path.

Proposition 2.

(i) The BGFs of A, B, K, and Y are given by

$$\tilde{A} \equiv \frac{A_{t+1}}{A_t} = \Omega^{\frac{(1+\varepsilon_1)(1-\mu_2)+(1+\varepsilon_2)\mu_1}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1}}; \qquad \tilde{B} \equiv \left(\frac{B_{t+1}}{B_t}\right) = \Omega^{\frac{(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1}};$$
$$\tilde{K} \equiv \left(\frac{K_{t+1}}{K_t}\right) = \Omega^{\left[\frac{(1-\mu_2)(2-\phi_1+\varepsilon_1)+\mu_1(1+\varepsilon_2-\phi_2)}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1}\right]+\varepsilon_0} = \tilde{Y} \equiv \left(\frac{Y_{t+1}}{Y_t}\right).$$

(ii) The BGFs increase with aggregate human capital accumulation (Ω), with the knowledge spillovers μ_1, μ_2, ϕ_1, ϕ_2, and with the strength of the effect that healthcare has in enhancing the productivity of workers employed in the applied research sector (ε_1) and in the basic research sector (ε_2). The BGF of GDP also increases in the strength of the effect that healthcare has in enhancing the productivity of workers employed in the final goods sector (ε_0).

(iii) The BGFs are independent of the tax rates, τ, τ_0, and $(1 - \tau_0)$.

(iv) The BGF of individual human capital ($\tilde{h}$) increases with the utility weight of children's education (θ) and health (σ), and decreases with the utility weight of the

[8]Note that $R_t = \alpha p_t = \alpha^2 Y_t/K_t$. Along the balanced growth path, Y_t and K_t are growing at the same rate such that R_t must be constant, i.e., $R_{t+1} = R_t = R$, for all t.

number of children (ξ).

(v) *The BGF of the population* ($\tilde{L}$) *decreases with the utility weight of children's education* (θ) *and health* (σ), *and rises with the utility weight of the number of children* (ξ).

(vi) *The BGF of aggregate human capital* (Ω) *increases with the utility weight of children's education* (θ) *and health* (σ), *and decreases with the utility weight of the number of children* (ξ).

(vii) *The BGF of per capita GDP is given by*

$$\tilde{y} = \frac{\tilde{Y}}{\tilde{L}} = \frac{\left(\frac{\left(A_E \frac{\theta}{\eta}\right)^{\nu} \left(A_M \frac{\sigma}{\chi}\right)^{1-\nu}}{1+\beta+\xi}\right)^{\left(\frac{(1-\mu_2)(2-\phi_1+\varepsilon_1)+\mu_1(1+\varepsilon_2-\phi_2)}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1}\right)+\varepsilon_0}}{\frac{\xi-\theta-\sigma}{\psi(1+\beta+\xi)}}.$$

The per capita GDP growth factor increases with the utility weight of children's education (θ) *and health* (σ), *and decreases with the utility weight of the number of children* (ξ). *It also increases with the knowledge spillovers* μ_1, μ_2, ϕ_1, ϕ_2, *and the strength of the effect that healthcare has on the productivity of workers employed in the final goods sector* (ε_0), *applied research sector* (ε_1), *and the basic research sector* (ε_2).

Proof. See Appendix C. □

Furthermore, the growth models of Prettner and Werner (2016) and Baldanzi et al. (2021) are nested as special cases within our generalized R&D-based growth model, which we summarize in Remark 2.

Remark 2. *For* $\varepsilon_0 = \varepsilon_1 = \varepsilon_2 = 0$, *and* $\nu = 1$, *our model nests the Prettner and Werner (2016) framework, while for* $\varepsilon_0 = \varepsilon_1 = \varepsilon_2 = \mu_1 = \mu_2 = \phi_2 = 0$ *our model nests the Baldanzi et al. (2021) framework as special cases.*

One implication of Proposition 2 is that human capital accumulation is a primary factor for long-run economic growth. A second implication of this proposition is that, although aggregate human capital accumulation is increasing with the desire for educated and healthy children, it is decreasing in population growth. Furthermore, higher intertemporal and intersectoral knowledge spillovers and the strength of the effect of healthcare in enhancing the productivity of workers employed in the applied research, basic research, and final goods sectors lead to a rise in balanced growth rates.

The effects of θ and σ on per capita GDP growth that emerge from our model are higher than in Baldanzi et al. (2021). In Baldanzi et al.'s model, the per capita GDP growth factor is influenced only by ϕ_1. By contrast, along with ϕ_1, in our model, the per capita GDP growth factor is influenced by intertemporal and intersectoral knowledge spillovers such as ϕ_2, μ_1, and μ_2. Another reason for the difference between Baldanzi et al.'s findings and ours is that we incorporate the impact of healthcare in enhancing the productivity of workers in various sectors (through the spillover parameters ε_0, ε_1, ε_2). The impact of θ on per capita GDP growth in our model is, in turn, larger than that of Prettner and Werner (2016), particularly when (i) $\nu = 1$ and (ii) $A_E\theta = A_M\sigma$. Again, the inclusion of the effect that healthcare raises the productivity of workers and the corresponding spillover terms ε_0, ε_1, and ε_2 play a crucial role in this difference. We would also like to highlight that, unlike in Prettner and Werner (2016), who do not consider health and healthcare investments, a rise in parental health investments for children (through a rise in σ) increases the per capita GDP growth factor in our model.

4 The transitional dynamics

We simulate the dynamic system represented by Equations (4), (5), (23), (24), and (25) to illustrate our analytical results numerically and to examine the economy's behavior during the transition. We use 20 years as the length of one generation, which corresponds to the duration of patent protection. We consider data of 30 Organisation for Economic Co-operation and Development (OECD) countries (Austria, Belgium, Switzerland, Chile, Czechia, Denmark, Spain, Estonia, Ethiopia, France, Greece, Hungary, Iceland, Israel, Italy, Japan, Korea Republic, Lithuania, Luxembourg, Latvia, Mexico, Netherlands, Norway, New Zealand, Poland, Portugal, Slovak Republic, Slovenia, Sweden, and United States). This is because basic research data for the rest of the OECD are not available. Table 1 summarises the parameter values that are either taken from the literature (cf. Auerbach and Kotlikoff, 1987; Jones, 1995; Acemoglu, 2008; Prettner and Werner, 2014, 2016; Baldanzi et al., 2021) or otherwise adjusted so that the model's predictions are consistent with the population growth rate and the economic growth rate of OECD

Parameters	Values	Sources
β	0.6892	Auerbach and Kotlikoff (1987), Grossmann et al. (2013a,b), Prettner and Werner (2016)
α	1/3	Jones (1995), Acemoglu (2008), Prettner and Werner (2016)
ψ	0.052	Baldanzi et al. (2021)
τ	0.1552845	OECD(2023a,b)
τ_0	0.044	OECD(2023a,b)
η	0.139	Prettner and Werner (2016)
χ	0.139	Author (similar to Prettner and Werner (2016))
ξ	0.8491	Baldanzi et al. (2021)
θ	0.4	Baldanzi et al. (2021)
σ	0.3	Baldanzi et al. (2021)
A_E	1.18	Author
A_M	1.128	Author
ϕ_1	0.4	Prettner and Werner (2016), Baldanzi et al. (2021)
ϕ_2	0.05	Prettner and Werner (2016)
μ_1	0.3	Author (close to Prettner and Werner (2016))
μ_2	0.3	Prettner and Werner (2016)
ε_0	0.001	Author
ε_1	0.001	Author
ε_2	0.001	Author
δ_1	1	Prettner and Werner (2016)
δ_2	1	Prettner and Werner (2014)
ν	0.5	Baldanzi et al. (2021)

Table 1: Parameter values for the numerical analysis

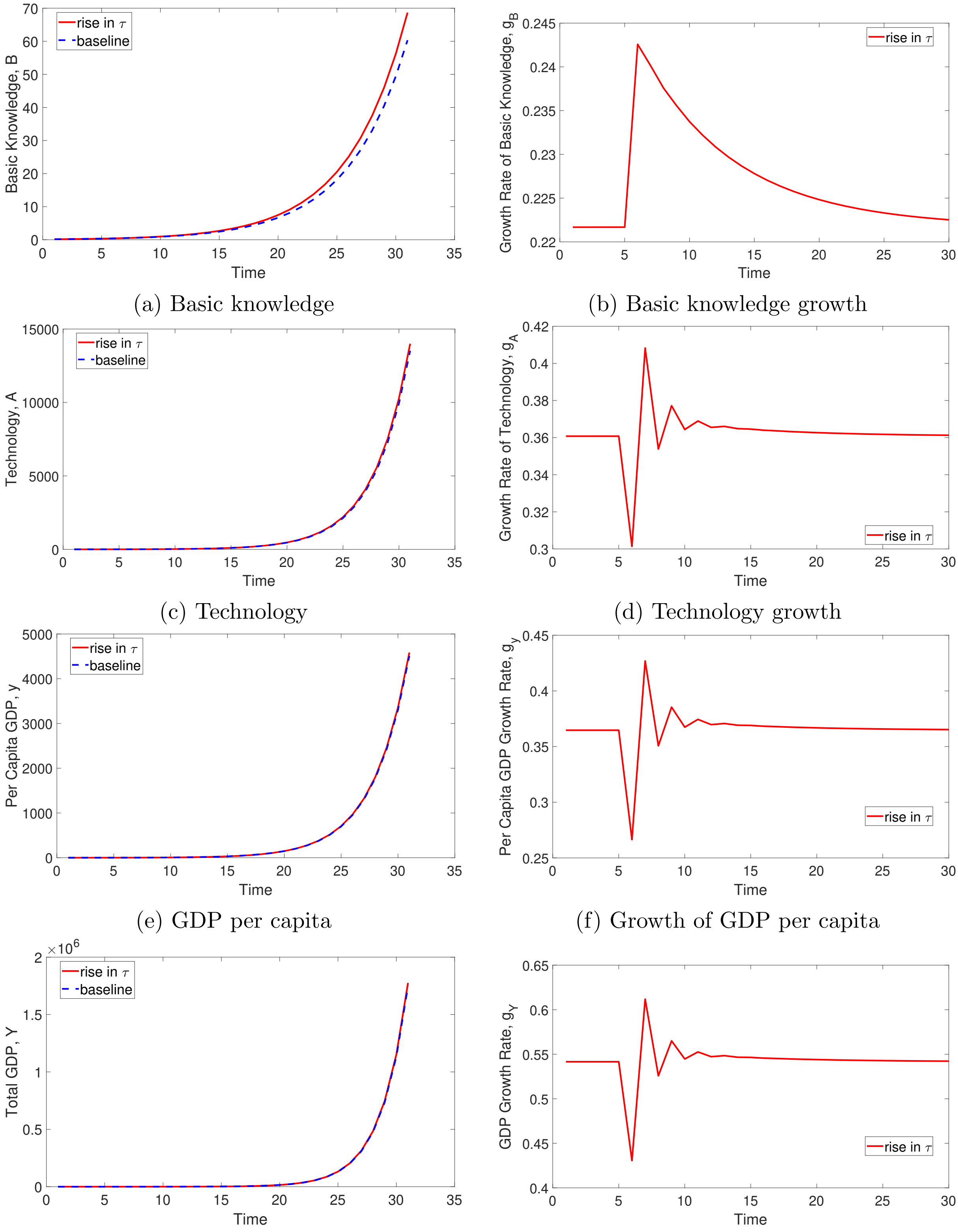
countries from 2000 to 2019, with the data taken from OECD (2023). We are considering data until 2019 because the COVID-19 pandemic disrupted economic activity in the years afterwards. Data on the fraction of GDP that OECD countries spent on basic research and the fraction of GDP spent on publicly-financed healthcare between 2000 and 2019 are taken directly from OECD (2023) and we also get the population growth rate and per capita GDP from this source. The simulated value of the population growth rate is 12.96% over 20 years, which is a reasonable approximation of the inter-generational population growth rate of 12.94% for the countries considered. Similarly, the simulated GDP growth rate is 54.16% over 20 years. This, too, is reasonably close to the inter-generational GDP growth rate of 53.44% for the countries considered.

In line with Prettner and Werner (2016), the discount factor β is computed based on a yearly discount rate of 1.9% and we choose the parameter value $\alpha = 1/3$ based on the literature (cf. Jones, 1995; Acemoglu, 2008; Prettner and Werner, 2016). In addition, we calculate basic research as a percentage of GDP in 2019 for the mentioned OECD countries as 0.4523% and public health expenditure as a percentage of GDP as 9.9%. Therefore, the required tax rate to fund basic research (i.e., $\tau\tau_0$) is 0.004523×(3/2)=0.0067845 (because $\alpha = 1/3$) and the required tax rate for funding public health expenditure (i.e., $\tau(1-\tau_0)$) is 0.0990×(3/2)=0.1485. As a consequence, the values of τ and τ_0 are 0.1552845 and 0.044, respectively.

The effect of an increase in total government expenditure (basic research and health expenditures together) on the growth rates of basic knowledge, technology, per capita GDP, and the physical capital stock (or aggregate GDP) are shown in Figures 2b, 2d, 2f, and 2h. We assume that the economy initially moves along the balanced growth path. After the fifth period in the numerical analysis, we increase government expenditures by 1 percentage point in terms of GDP. Thus, τ increases from 0.1552845 to 0.1702845 (0.01×(3/2)+0.1552845=0.015+0.1552845=0.1702845).

The rise in public spending has the effect of drawing labor away from applied research towards basic research. This slows down the expansion of applied research (see Figure 2d), while accelerating the evolution of basic knowledge (see Figure 2b). Because basic knowledge is a necessary input for applied research, the accumulation of new blueprints accelerates in the medium run despite a short-run slowdown. In the short and medium run, the physical capital stock grows faster than if no policy changes were put into effect (see Figure 2h). As a result of the temporary slowdown in the accumulation of applied research, economic growth in terms of per capita GDP slows down in the short run, whereas growth in per capita GDP picks up in the medium run (see Figure 2f). However, there is no growth effect in the long run because the beneficial growth effects of increased investment in basic research gradually fade away. The effects of an increase in τ on the levels of various variables are shown in Figures 2a, 2c, 2e, and 2g. The solid (red) line illustrates an economy that experienced a rise in τ, whereas the dashed (blue) line depicts an economy that did not experience such an increase. The level of basic knowledge (Figure 2a), the number of patents (Figure 2c), aggregate output (Figure 2g), and per capita GDP (Figure 2e) all are higher in the economy with a rise in τ.

The effect of a change in the composition of government expenditure in favor of basic research vis-a-vis health expenditure (i.e., a rise in τ_0) on the growth rates of basic knowledge, technology, per capita GDP, and the physical capital stock (or aggregate GDP) are shown in Figures 3b, 3d, 3f, and 3h. Again, we assume that the economy



(a) Basic knowledge (b) Basic knowledge growth

(c) Technology (d) Technology growth

(e) GDP per capita (f) Growth of GDP per capita

(g) GDP (h) GDP growth

Figure 2: Effects of a rise in τ by 1 percentage point in terms of GDP

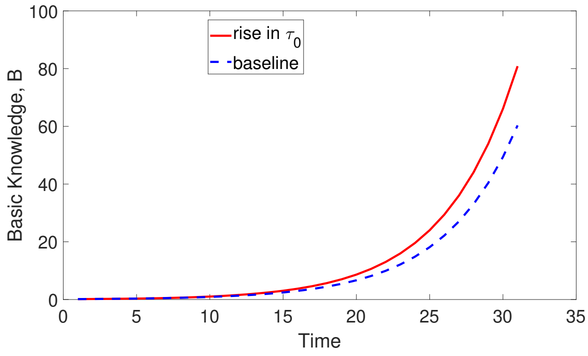
initially moves along the balanced growth path and that an increase in the composition of government expenditure in favour of basic research by 1 percentage point occurs after the fifth period. Therefore, τ_0 increases from 0.044 to 0.054.

The rise of τ_0 has the effect of drawing labor away from the final goods sector towards basic research. This slows down the expansion of final goods (see Figure 3h), while accelerating the evolution of basic knowledge (see Figure 3b). Because basic knowledge is a necessary input for applied research, the accumulation of new blueprints accelerates in the short as well as in the medium run. In the short and medium run, the physical capital stock grows faster than if no policy changes were enacted (see Figure 3h). Economic growth in terms of per capita GDP slows down in the short run, whereas growth in per capita GDP picks up in the medium run (see Figure 3f). Again, there is no growth effect in the long run.

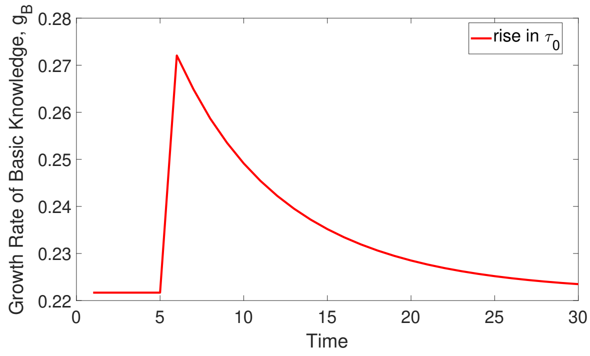
The effects of an increase in τ_0 on the levels of the different variables are shown in Figures 3a, 3c, 3e, and 3g. The solid (red) line illustrates an economy that experienced a rise in τ_0, whereas the dashed (blue) line depicts an economy that did not experience such an increase. The level of basic knowledge (see Figure 3a), the number of patents (Figure 3c), aggregate output and the capital stock (Figure 3g), and per capita GDP (Figure 3e) are all higher in the economy that witnesses a change in the composition of public expenditure in favour of basic research.

Figures 4b and 4a display the impact of an increase in the value of the weight of children's education in the parental utility function (θ) by 1 % after the fifth period on the per capita GDP growth rate and on the level of per capita GDP. We observe that, after the increase in θ, the economy exhibits a higher growth rate as compared to no change. Similarly, an increase in the value of the weight of children's health in the parental utility function (σ) leads to a rise in the per capita GDP growth rate (see Figure 4d). By contrast, an increase in the desire for a large family (ξ) leads to a fall in the growth rate and the level of per capita GDP as shown in Figures 4f and 4e. These are exactly the effects we stated in Proposition 2*(vii)* and they are also consistent with the empirical evidence (Brander and Dowrick, 1994; Ahituv, 2001; Li and Zhang, 2007; Cohen and Soto, 2007; Hanushek and Woessmann, 2012; Herzer et al., 2012).

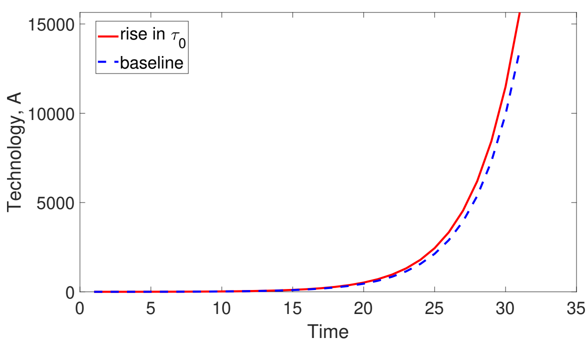
Figure 5 illustrates the impact of increases in the value of intertemporal and inter-sectoral knowledge spillovers ϕ_1, ϕ_2, μ_1, μ_2, and the strength of the effect of healthcare



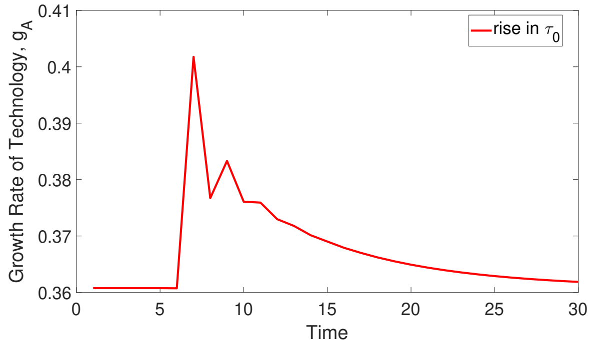
(a) Basic knowledge



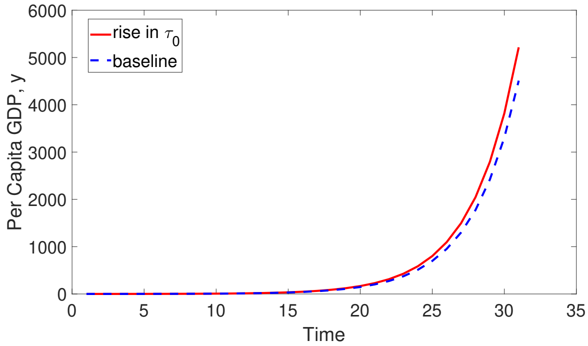
(b) Basic knowledge growth



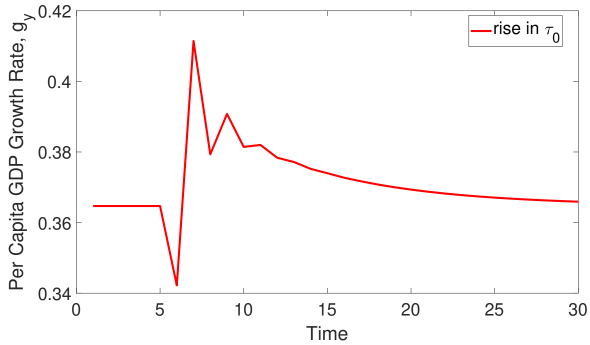
(c) Technology



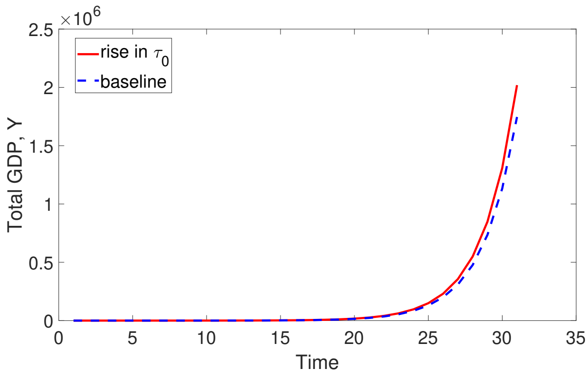
(d) Technology growth



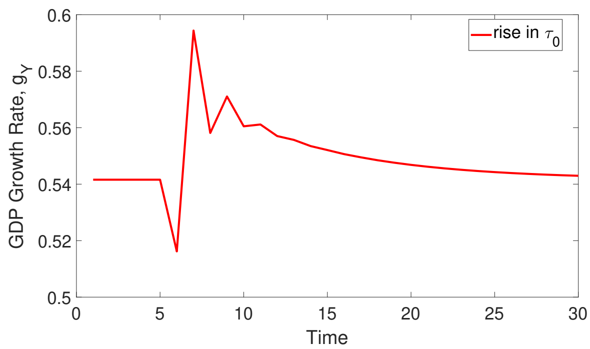
(e) GDP per capita



(f) Growth of GDP per capita

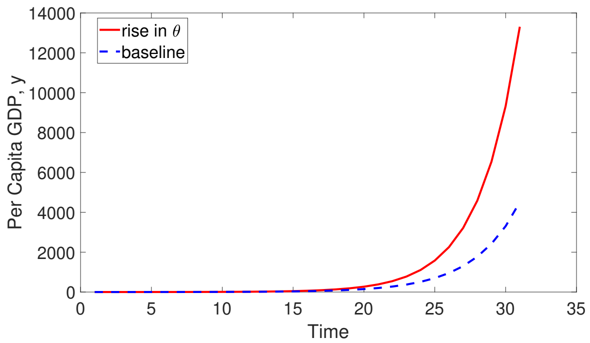


(g) GDP

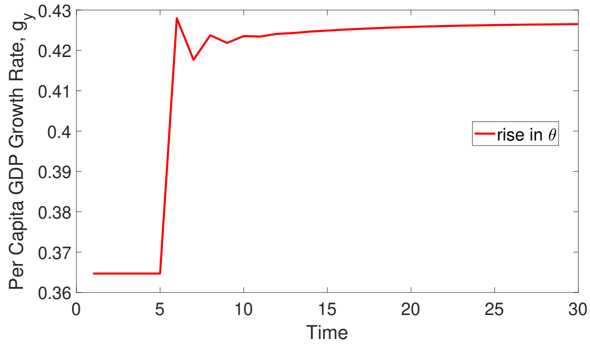


(h) GDP growth

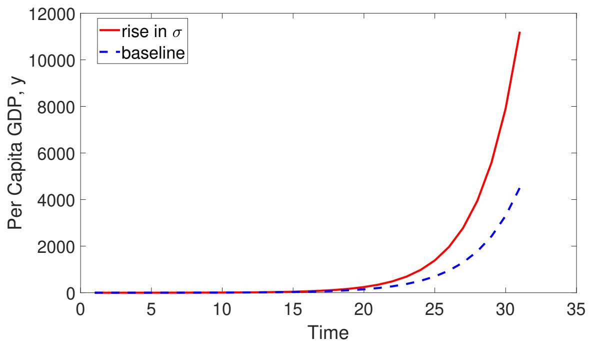
Figure 3: Effects of a rise in τ_0 by 1 percentage point



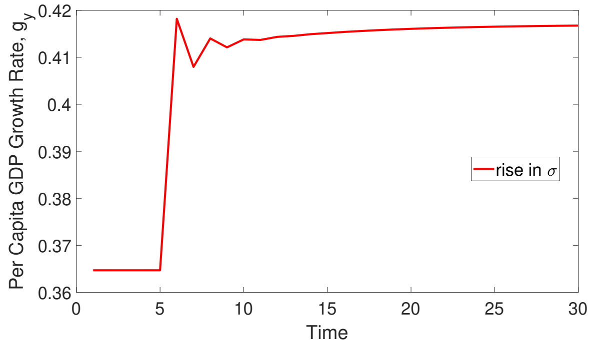
(a) A rise in θ on per capita GDP



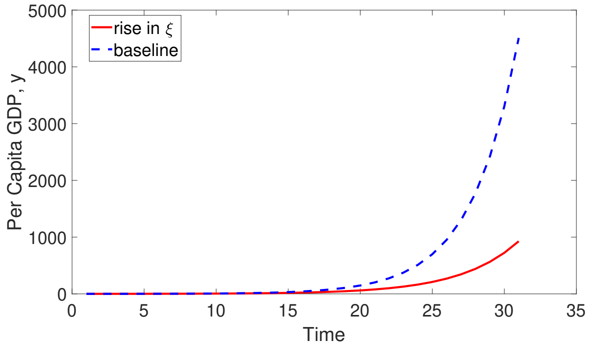
(b) A rise in θ on per capita GDP growth



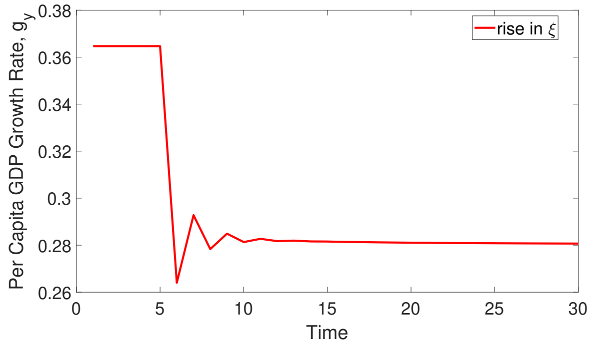
(c) A rise in σ on per capita GDP



(d) A rise in σ on per capita GDP growth

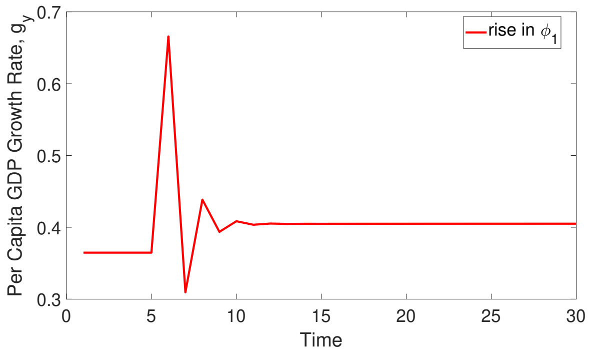


(e) A rise in ξ on per capita GDP

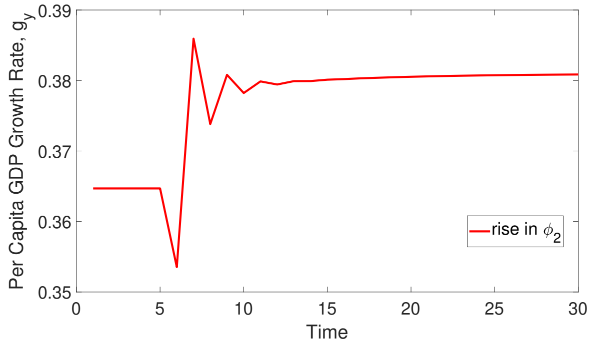


(f) A rise in ξ on per capita GDP growth

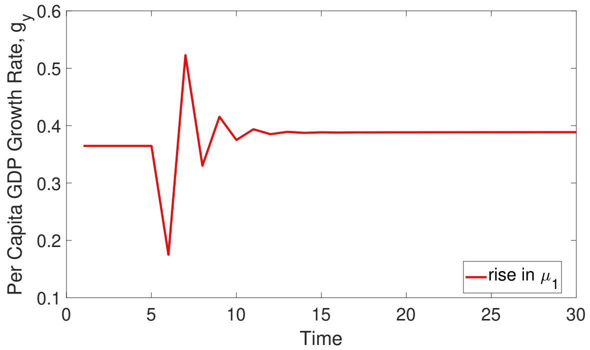
Figure 4: Effect of rise in σ, θ and ξ by 1 percent on per capita GDP and its growth



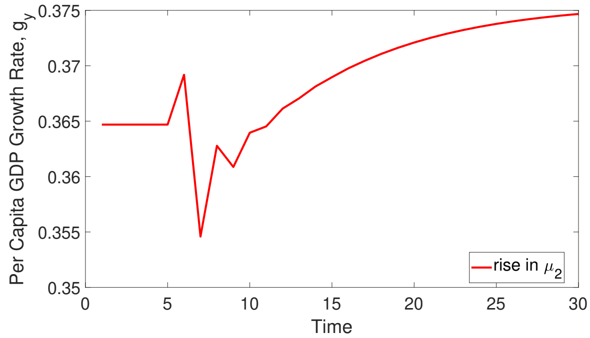
(a) A rise in ϕ_1 on per capita GDP growth



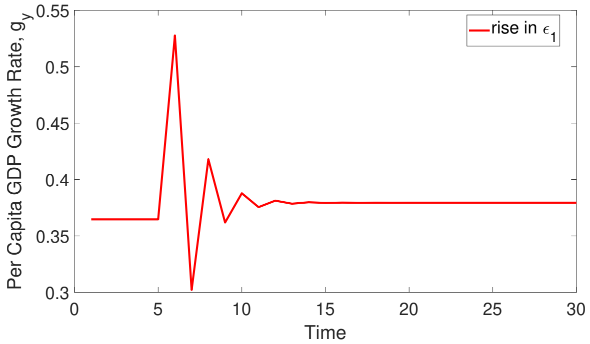
(b) A rise in ϕ_2 on per capita GDP growth



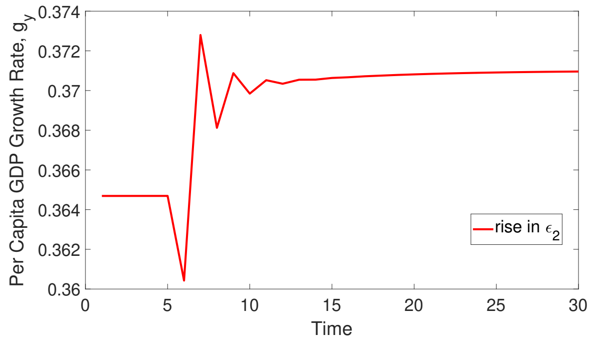
(c) A rise in μ_1 on per capita GDP growth



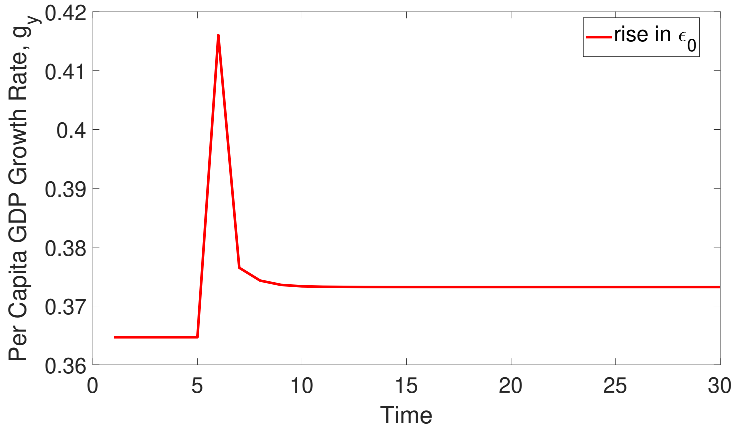
(d) A rise in μ_2 on per capita GDP growth



(e) A rise in ε_1 on per capita GDP



(f) A rise in ε_2 on per capita GDP growth



(g) A rise in ε_0 on per capita GDP growth

Figure 5: Effect of rises in ϕ_1, ϕ_2, μ_1, μ_2, ε_1, ε_2 and ε_0 by 5 percentage points on per capita GDP growth

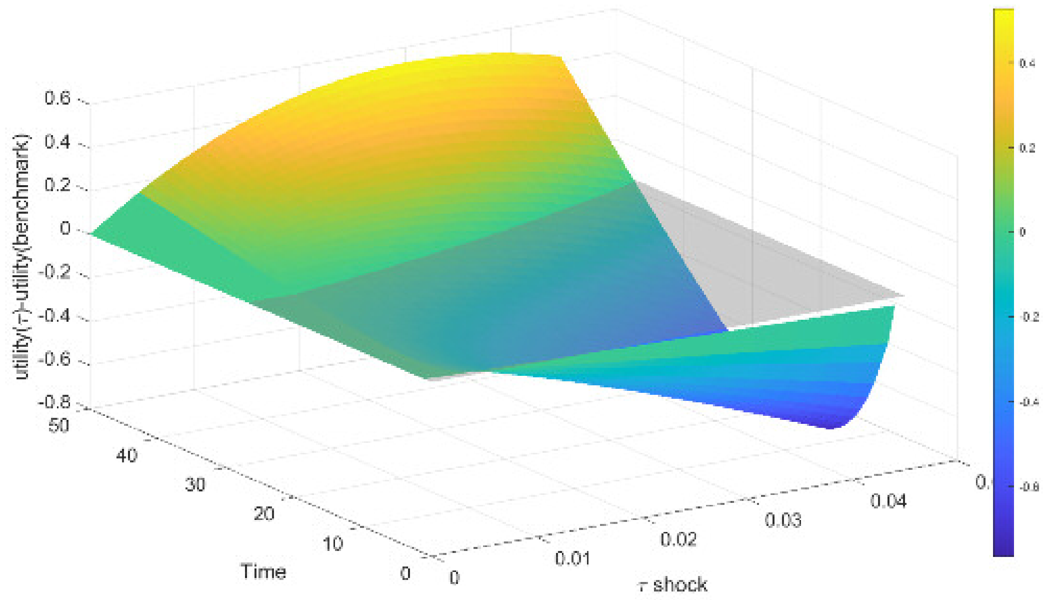


Figure 6: Changes in lifetime utility for changes in τ over different time horizons (x-axis) and different changes in τ (y-axis)

Note: The figure displays the difference in aggregate utility levels between inhabitants of an economy that changes its public policies related to basic research and healthcare and inhabitants of an economy without such a change. The time horizon is displayed on the x-axis, while the change in τ is displayed on the y-axis. If the difference is positive, the inhabitants of the economy with the corresponding change in τ are better off in the relevant time period (because the social welfare is higher). The shaded plane corresponds to the case in which inhabitants of both economies are equally well off, that is, the difference equals zero.

in enhancing the productivity of workers employed in the final goods sector (ε_0), applied research sector (ε_1), and the basic research sector (ε_2) by 5 percentage points on the per capita GDP growth rate. The results are identical to those stated in Proposition 2*(vii)*.

Next, we focus on the welfare effect of the policy change. Our objective is to determine whether the policy leads to an improvement in welfare. If it does, we also aim to identify the optimal tax rate that maximizes welfare over a specific time period. Individuals in our analysis value both present and future consumption, the number of children they have, as well as the education level and the health status of each child (see Equation (1)). As a result, we assess the effects of changes in basic research and healthcare expenditures on utility levels over different time horizons, and various changes in these public expenditures (τ). The difference in the overall utility level between the inhabitants of an economy that implements changes in its basic research and healthcare policies and the inhabitants of an economy that does not make such changes (i.e., $\Delta\tau = 0$) is depicted in Figure 6 (and also in Figure 7 from a different angle). Aggregate utility is calculated by summing up

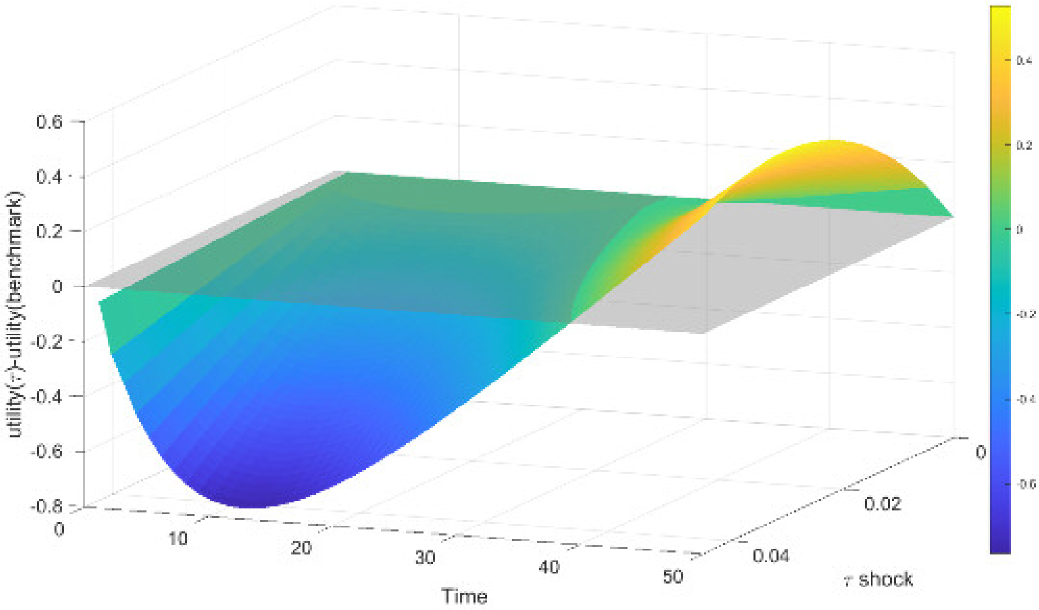


Figure 7: Figure 6 from a different angle

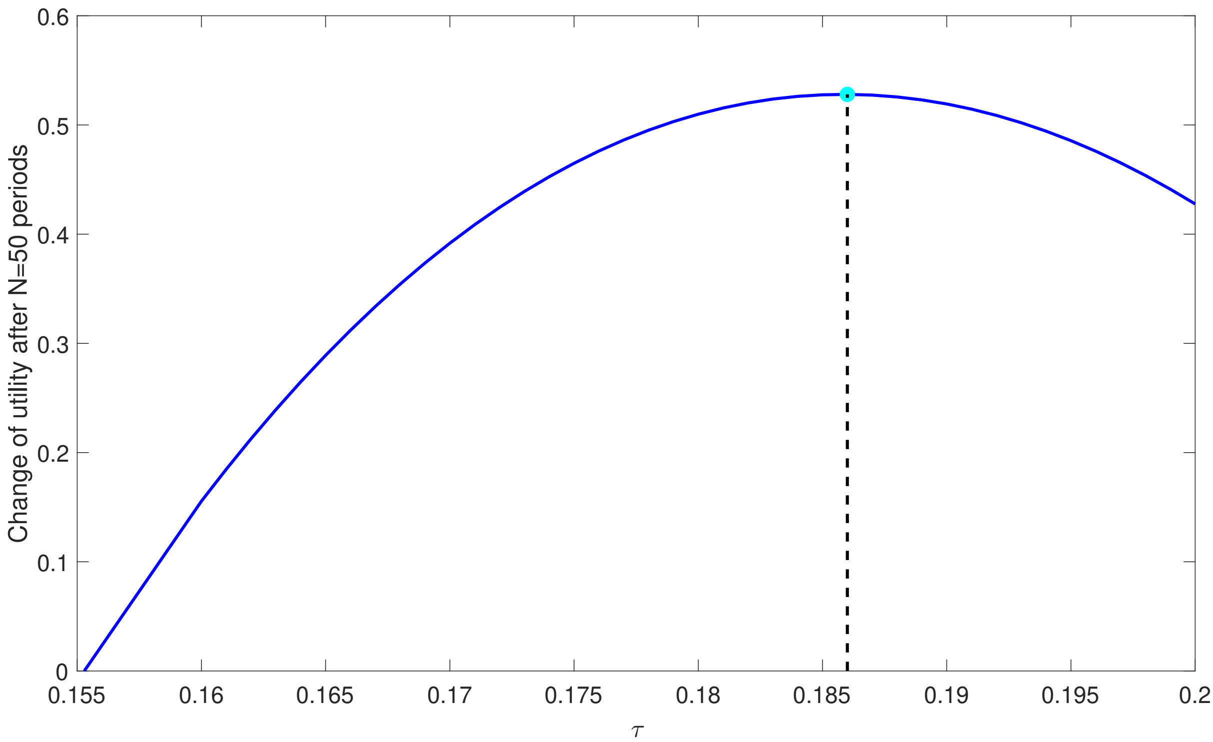


Figure 8: Changes in lifetime utility for changes in τ for the time horizon $N = 50$

the average lifetime utilities until time horizon N according to

$$U_N = \sum_{j=1}^{N} \lambda^{j-1} u_j(c_j, c_{j+1}, e, n, m), \tag{31}$$

where λ represents the social discount factor.

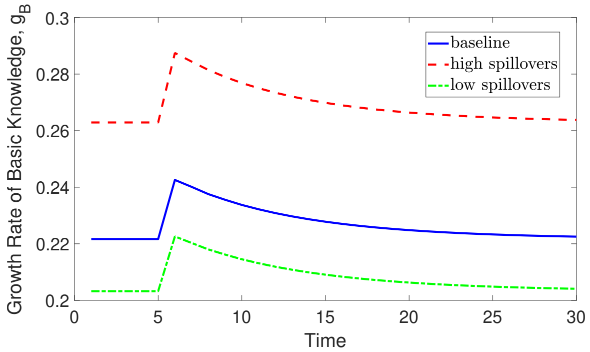
We follow the standard Millian type of welfare measure approach, where the average individual determines social well-being. In Figure 6, the time horizon is represented on the X-axis. Initially (prior to $N = 0$), the economy progresses along a balanced growth path, and at $N = 0$, it encounters a policy change (τ), the magnitude of which is depicted on the Y-axis. The corresponding change in overall utility, compared to the benchmark scenario (without a policy change), is shown on the Z-axis. The graph illustrates that an increase in government expenditure on basic research and public healthcare initially leads to a decline in welfare in the immediate years following the impact. However, it subsequently enhances welfare over longer time horizons because of an increase in economic growth. After 50 generations, the welfare levels in the long run show a positive response to a slight increase in public healthcare and basic research expenditures. However, this positive response turns negative once a certain level of public spending is reached. This suggests that for each increase in τ and for each time horizon, an optimal rate of public spending (i.e., τ_{optimal}) exists that maximizes welfare. According to our results, maximum welfare will be achieved by raising τ to approximately 18.6%, which corresponds to 12.4% of GDP (see Figure 8). It is worth noting that this level of public spending on both basic research and healthcare surpasses to a substantial degree the current expenditure levels in the specified OECD countries corresponding to approximately 10.35% of GDP in 2019.

5 Sensitivity analysis

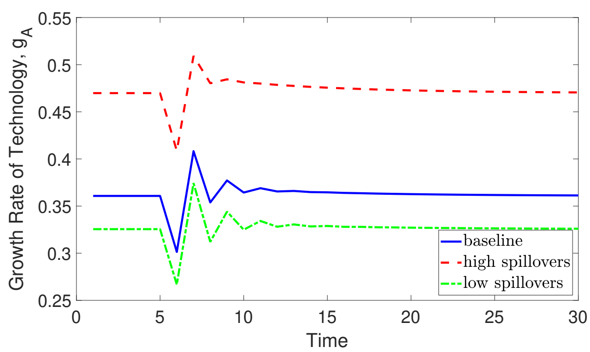
As previously indicated, the parameter settings for the illustrative simulation were chosen to represent either observed data from OECD countries from 2000 to 2019 or common sense derived from empirical findings, as they are frequently used in other research. However, as stated in Prettner and Werner (2016), measuring intertemporal and intersectoral knowledge spillovers is exceptionally difficult. Most research calibrates the relevant parameters so that the model's projected growth series matches the observed ones, which is the approach that we followed as well. In this section, our purpose is to demonstrate that our qualitative findings are fairly robust to changes in the spillover parameters ϕ_1, ϕ_2, μ_1, and μ_2. In doing so, we simulate growth in basic knowledge, applied knowledge, GDP, and per capita GDP in three alternative scenarios: (i) a low spillover scenario with $\phi_1 = 0.38$, $\phi_2 = 0.03$, $\mu_1 = 0.27$, and $\mu_2 = 0.28$; (ii) a high spillover scenario with $\phi_1 = 0.47$, $\phi_2 = 0.07$, $\mu_1 = 0.34$, and $\mu_2 = 0.35$; and (iii) an intermediate spillover scenario corresponding to our baseline specification with $\phi_1 = 0.40$, $\phi_2 = 0.05$, $\mu_1 = 0.30$, and $\mu_2 = 0.30$. Figure 9 depicts the growth effects, with the red dashed line indicating the high spillover scenario, the blue solid line representing the intermediate spillover scenario, and the green dash-dotted line reflecting the low spillover scenario. In summary, boosting public spending on basic research and healthcare (an increase in τ) results in a medium-run rise in the growth rates of basic knowledge, applied knowledge, per capita GDP, and aggregate GDP (or physical capital), a short-run slowdown in the growth rates of these variables, and no long-run growth effects. Thus, our findings related to growth rates remain valid across all of these specifications.

In addition, we examine how changes in spillovers affect the sensitivity of welfare. We observe an interior optimal level of public expenditure on basic research and healthcare (τ) for each generation irrespective of whether the intersectoral and intertemporal knowledge spillovers are low, high, or at the intermediate level. Nevertheless, the optimal public expenditures on basic research and healthcare (τ) are sensitive to changes in the intersectoral and intertemporal knowledge spillovers. Figure 10 exhibits the effects of increases in aggregate utility for low spillovers (green dash-dotted line), intermediate spillovers (blue solid line), and high spillovers (red dashed line) up to generation $N = 50$. The peak of the change in utility concerning a change in τ increases with an increase in spillovers. In particular, for the low spillover scenario, the peak occurs at $\tau = 0.162$, which corresponds to 10.8% of GDP, and for the high spillover case, the maximum occurs at $\tau = 0.231$, which corresponds to 15.4% of GDP. Nonetheless, the optimal public expenditure on basic research and healthcare (τ_{optimal}) from the point of view of future generations is still higher in all three scenarios than the levels presently observed in the above-specified OECD countries.

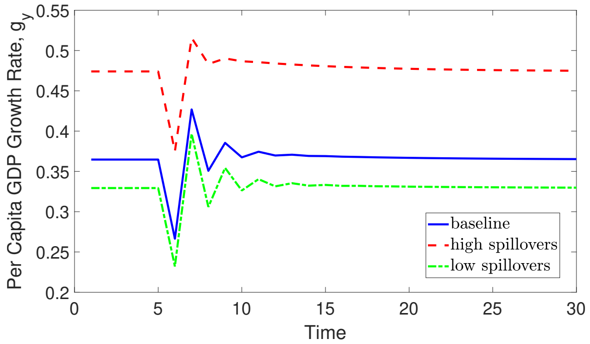
Similarly, we examine the sensitivity of welfare to changes in the weight of children's education (θ) and health (σ) in the parental utility function. We set higher and lower



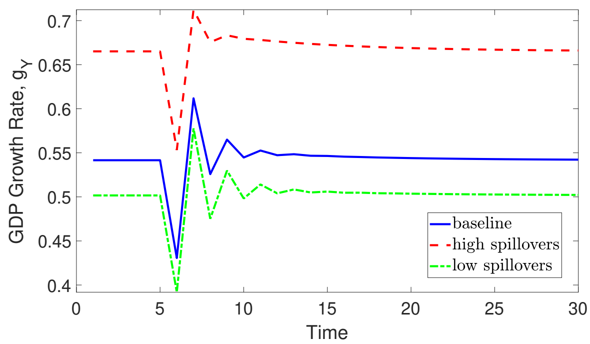
(a) Basic knowledge growth



(b) Technology growth



(c) Growth of GDP per capita



(d) GDP growth

Figure 9: Sensitivity check with respect to intertemporal and intersectoral knowledge spillovers

Note: The solid (blue) line refers to the intermediate spillover scenario, the dashed (red) line refers to the high spillover scenario, and the dash-dotted (green) line refers to the low spillover scenario.

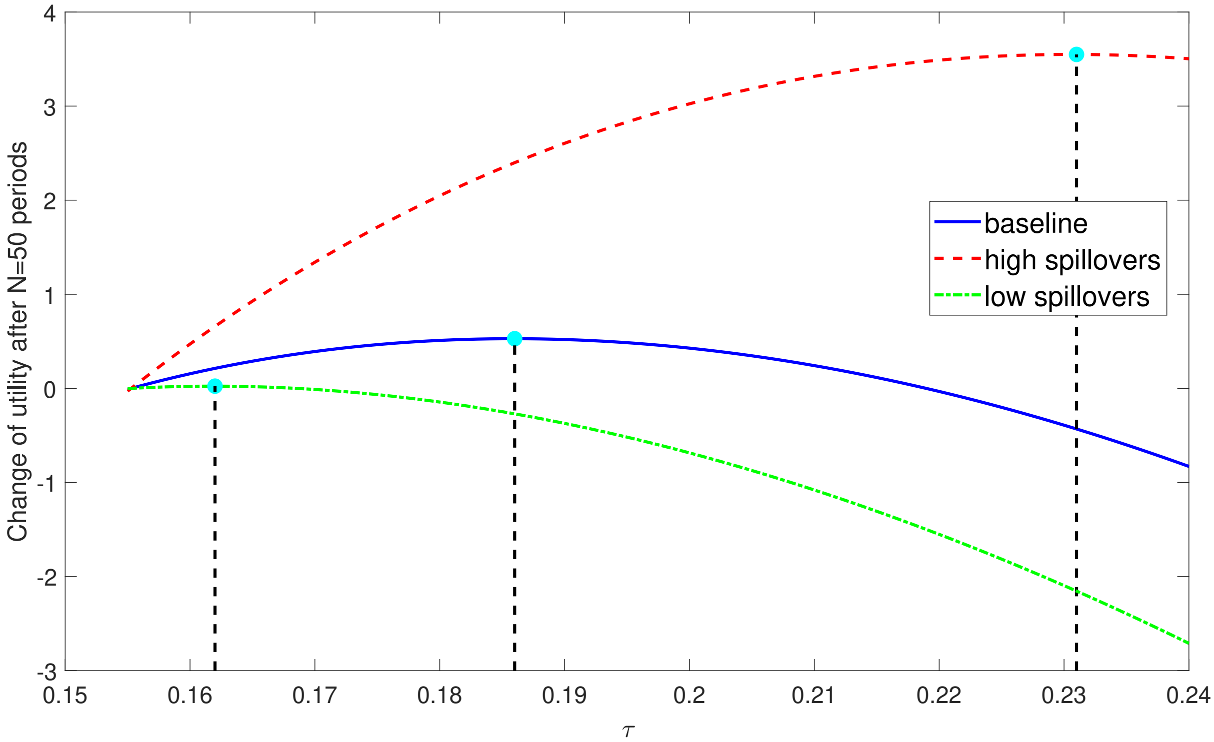


Figure 10: Changes in lifetime utility for changes in τ for the time horizon $N = 50$ and changing ϕ_1, ϕ_2, μ_1, and μ_2

Note: The solid (blue) line refers to the intermediate spillover scenario, the dashed (red) line refers to the high spillover scenario, and the dash-dotted (green) line refers to the low spillover scenario.

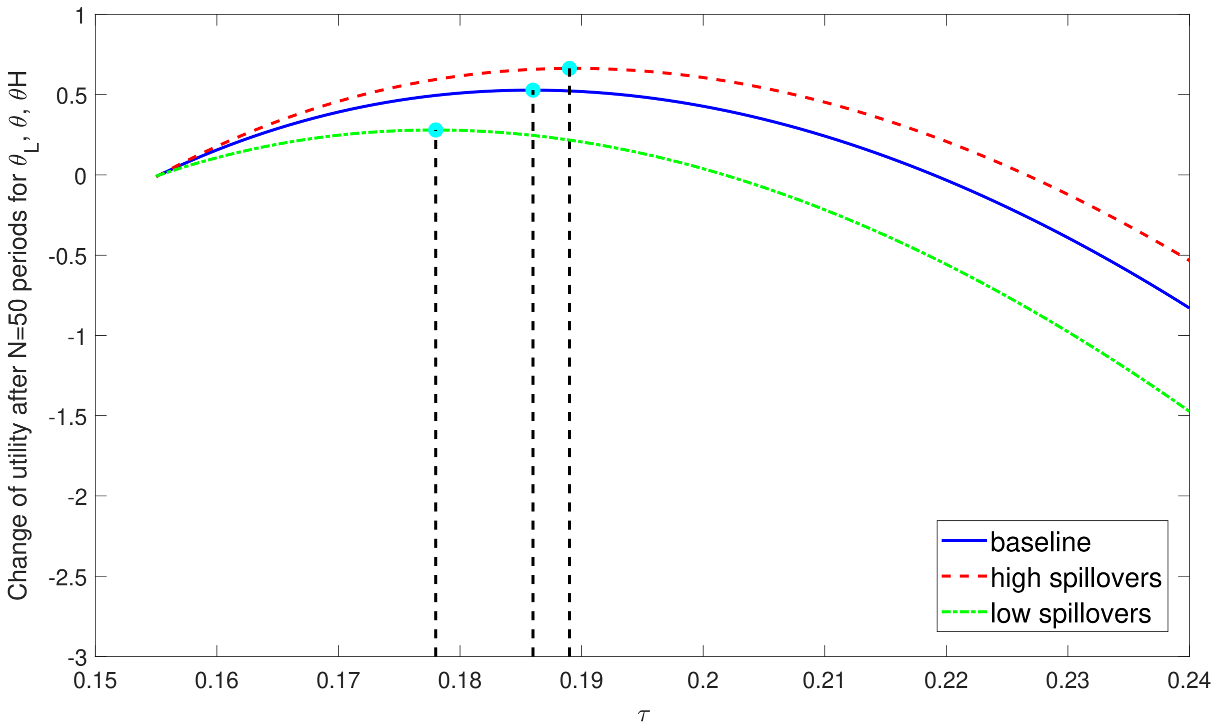


Figure 11: Changes in lifetime utility for changes in τ for the time horizon $N = 50$ and changing θ

Note: The solid (blue) line refers to the intermediate spillover scenario, the dashed (red) line refers to the high spillover scenario, and the dash-dotted (green) line refers to the low spillover scenario.

	θ_{Low} 0.38	$\theta_{\text{Intermediate}}$ 0.4	θ_{High} 0.417	σ_{Low} 0.28	$\sigma_{\text{Intermediate}}$ 0.3	σ_{High} 0.37
τ_{optimal}	0.178	0.186	0.189	0.174	0.186	0.190

Table 2: Sensitivity of the long run optimal public expenditure rate on basic research and healthcare (i.e., τ) with respect to the preferences of households

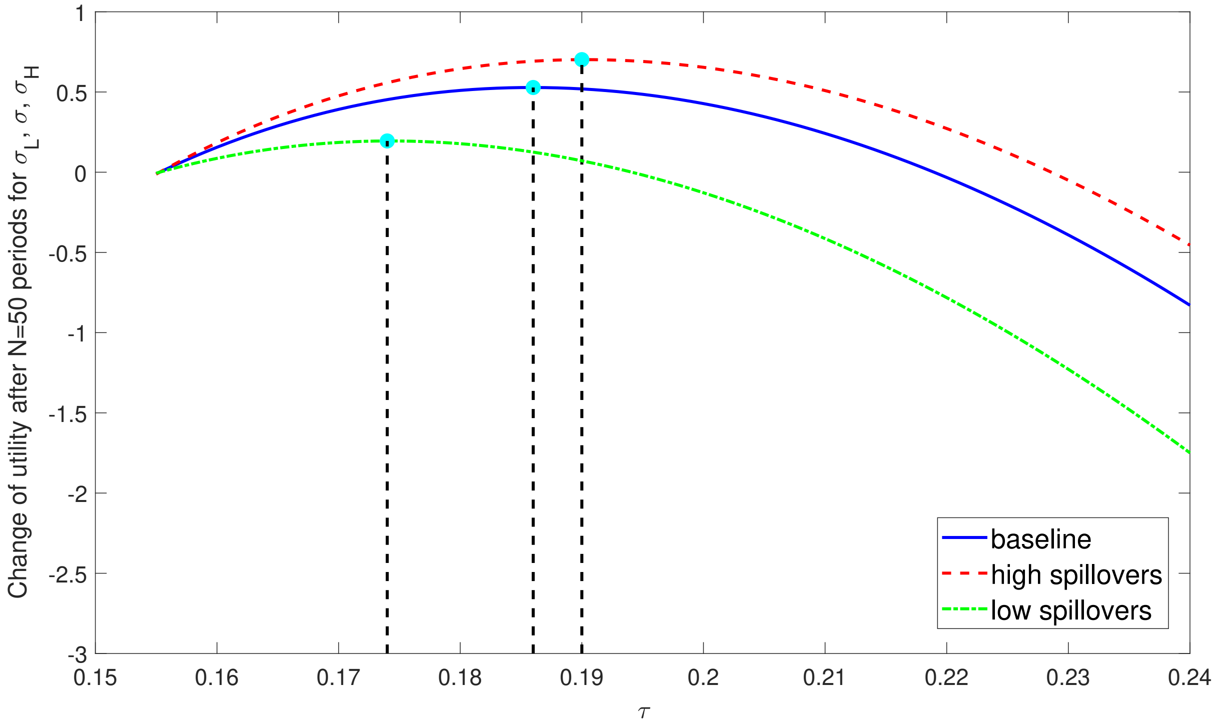


Figure 12: Changes in lifetime utility for changes in τ for the time horizon $N = 50$ and changing σ

Note: The solid (blue) line refers to the intermediate spillover scenario, the dashed (red) line refers to the high spillover scenario, and the dash-dotted (green) line refers to the low spillover scenario.

bounds of these values according to Table 2 in which intermediate values would correspond to our benchmark specification. We observe an interior welfare-maximizing level of τ for each generation irrespective of whether θ and σ are low, high, or at the intermediate level. However, the optimal value of τ is again sensitive to changes in the high and low values of θ and σ. Figure 11 exhibits the effects of increases in aggregate utility for a low value of θ (green dash-dotted line), an intermediate value of θ (blue solid line), and a high value of θ (red dashed line) up to generation $N = 50$.

As the value of θ increases, so does the peak of the change in utility due to a change in τ. Specifically, for the low value of θ ($\theta_{\text{Low}} = 0.38$), the peak occurs at $\tau = 0.178$, which corresponds to 11.87% of GDP, and for the high value of θ ($\theta_{\text{High}} = 0.417$), the peak occurs at $\tau = 0.189$, which corresponds to 12.6% of GDP. Figure 12 exhibits the effects of increases in aggregate utility for a low value of σ (green dash-dotted line), an intermediate value of σ (blue solid line), and a high value of σ (red dashed line) up to generation $N = 50$. As evident from Figure 12, the parameter settings for the peak of the change in utility due to a change in τ rises as the value of σ grows. For instance, for the low value of σ ($\sigma_{\text{Low}} = 0.28$), the peak occurs at $\tau = 0.174$, which corresponds to 11.6% of GDP, while for the high value of σ ($\sigma_{\text{High}} = 0.317$), the peak occurs at $\tau = 0.190$, which corresponds to 12.67% of GDP.

6 Conclusion

We present a highly general R&D-based endogenous growth model emphasizing the role of patentable applied research, publicly-funded basic research, and publicly-funded healthcare. In so doing, we nest two recent contributions by Prettner and Werner (2016) and Baldanzi et al. (2021) as special cases. We use the model to assess the growth and welfare effects of public basic science and public healthcare investments. Our second contribution is to illustrate the role of healthcare for enhancing productivity in various sectors of the economy.

Overall, we show that the basic insights of the previous literature remain intact from a qualitative perspective even in a situation in which different growth and welfare promoting areas are in competition regarding public funding. We show that an increase in publicly funded basic research and health expenditures is still welfare improving as compared with the observable spending levels in the OECD. However, as compared with the results of Prettner and Werner (2016), the quantitative findings are altered to the extent that the welfare-maximizing level of expenditures does not anymore differ from the actual expenditure levels by such a wide margin.

The two main economic policy implications that emanate from our research are that i) governmental expenditures on basic science and healthcare are worthwhile and still these areas are under-funded from a welfare-maximizing perspective; ii) in the presence of more domains on which the government can spend its funds productively, it is even more important to consider thorough cost-benefit analyses to make sound policy decisions and to understand the various tradeoffs involved.

Interesting avenues for future research include i) extending the proposed model to the context of a developing country that is far from the technological frontier and imitates innovations that were made in developed countries. Presumably, investments in basic research would not be similarly important in such a setting and health expenditures would become more prominent; ii) designing a model in which governments have even more scope for enacting different policies such as childcare subsidies, education subsidies, research subsidies (see, e.g., Minniti and Venturini, 2017a,b), etc., and using the model to analyze the extent to which different policies contribute to increase welfare; iii) modeling more explicitly the interactions between private and public healthcare and the complementarities and tradeoffs that they imply.

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Appendix

A Derivation of the optimal values for c_t, s_t, n_t, e_t, and m_t

Using (1) and (2), we set up the Lagrangian as

$$\begin{aligned}\mathcal{L} = &\ln c_t + \beta \ln\left[(R_{t+1} - 1)s_t\right] + \xi \ln n_t + \theta \ln e_t + \sigma \ln m_t \\ &+ \lambda[(1-\tau)(1 - \psi n_t - \eta e_t n_t - \chi m_t n_t) w_t h_t - c_t - s_t].\end{aligned}$$

The first-order conditions are given by

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \implies \frac{1}{c_t} = \lambda, \tag{A.1}$$

$$\frac{\partial \mathcal{L}}{\partial s_t} = 0 \implies \frac{\beta}{s_t} = \lambda, \tag{A.2}$$

$$\frac{\partial \mathcal{L}}{\partial n_t} = 0 \implies \frac{\xi}{n_t} = \lambda(1-\tau)(\psi + \eta e_t + \chi m_t) w_t h_t, \tag{A.3}$$

$$\frac{\partial \mathcal{L}}{\partial e_t} = 0 \implies \frac{\theta}{e_t} = \lambda(1-\tau)\eta n_t w_t h_t, \tag{A.4}$$

$$\frac{\partial \mathcal{L}}{\partial m_t} = 0 \implies \frac{\sigma}{m_t} = \lambda(1-\tau)\chi n_t w_t h_t, \tag{A.5}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \implies (1-\tau)(1-\psi n_t - \eta e_t n_t - \chi m_t n_t) w_t h_t - c_t - s_t = 0. \tag{A.6}$$

Dividing (A.4) by (A.5), we obtain

$$\chi m_t = \frac{\eta\sigma}{\theta} e_t. \tag{A.7}$$

Dividing (A.1) by (A.2), we obtain

$$s_t = \beta c_t. \tag{A.8}$$

Dividing (A.3) by (A.4) and using (A.7) yields

$$e_t = \frac{\theta\psi}{\eta(\xi-\theta-\sigma)}. \tag{A.9}$$

Inserting the value of e_t into (A.7), we get

$$m_t = \frac{\sigma\psi}{\chi(\xi-\theta-\sigma)}. \tag{A.10}$$

Dividing (A.3) by (A.1) and rearranging yields

$$(1-\tau)(\psi + \eta e_t + \chi m_t) n_t w_t h_t = \xi c_t. \tag{A.11}$$

Inserting (A.11) into (A.6), we get

$$(1-\tau) w_t h_t - (1-\tau)(\psi + \eta e_t + \chi m_t) n_t w_t h_t - c_t - s_t = 0$$

$$\implies c_t = \frac{(1-\tau) w_t h_t}{1+\beta+\xi}. \tag{A.12}$$

Therefore,

$$s_t = \frac{\beta(1-\tau) w_t h_t}{1+\beta+\xi}. \tag{A.13}$$

Inserting the values of c_t, s_t, e_t, and m_t into (A.6) and rearranging, we obtain

$$n_t = \frac{\xi-\theta-\sigma}{\psi(1+\beta+\xi)}. \tag{A.14}$$

B Proof of Proposition 1

The partial derivatives of fertility n_t with respect to ξ, θ, and σ are

$$\frac{\partial n_t}{\partial \xi} = \frac{1+\beta+\theta+\sigma}{\psi\,(1+\beta+\xi)^2} > 0, \qquad \frac{\partial n_t}{\partial \theta} = -\frac{1}{\psi\,(1+\beta+\xi)} < 0,$$
$$\frac{\partial n_t}{\partial \sigma} = -\frac{1}{\psi\,(1+\beta+\xi)} < 0. \tag{B.1}$$

The partial derivatives of individual human capital h_{t+1} with respect to ξ, θ, and σ are

$$\begin{aligned} \frac{\partial h_{t+1}}{\partial \xi} &= -\left(A_E\frac{\theta}{\eta}\right)^{\nu}\left(A_M\frac{\sigma}{\chi}\right)^{1-\nu}\frac{\psi}{(\xi-\theta-\sigma)^2}h_t < 0, \\ \frac{\partial h_{t+1}}{\partial \theta} &= \frac{\left(A_E\frac{\theta}{\eta}\right)^{\nu}\left(A_M\frac{\sigma}{\chi}\right)^{1-\nu}\psi h_t}{(\xi-\theta-\sigma)}\left[\frac{\nu A_E}{\eta}\left(A_E\frac{\theta}{\eta}\right)^{-1}+\frac{1}{(\xi-\theta-\sigma)}\right] > 0, \\ \frac{\partial h_{t+1}}{\partial \sigma} &= \frac{\left(A_E\frac{\theta}{\eta}\right)^{\nu}\left(A_M\frac{\sigma}{\chi}\right)^{1-\nu}\psi h_t}{(\xi-\theta-\sigma)}\left[\frac{(1-\nu)A_M}{\chi}\left(A_M\frac{\sigma}{\chi}\right)^{-1}+\frac{1}{(\xi-\theta-\sigma)}\right] > 0. \end{aligned} \tag{B.2}$$

C Proof of Proposition 2

(i) Growth rates of the endogenous variables have to be constant along the balanced growth path. Thus,

$$\frac{g_{t+1,A}-g_{t,A}}{g_{t,A}} = 0 \implies g_{t+1,A} = g_{t,A}$$

$$\implies \frac{(1-\tau)(1+\beta)^{1+\varepsilon_1}\left[(1-\tau_0)\tau\right]^{\varepsilon_1}}{(1+\beta+\xi)^{1+\varepsilon_1}}\delta_1 A_{t+1}^{\phi_1-1}B_{t+1}^{\mu_1}(h_{t+1}L_{t+1})^{1+\varepsilon_1}-\frac{1}{\alpha}$$

$$=\frac{(1-\tau)(1+\beta)^{1+\varepsilon_1}\left[(1-\tau_0)\tau\right]^{\varepsilon_1}}{(1+\beta+\xi)^{1+\varepsilon_1}}\delta_1 A_t^{\phi_1-1}B_t^{\mu_1}(h_tL_t)^{1+\varepsilon_1}-\frac{1}{\alpha}$$

$$\implies \left(\frac{h_{t+1}}{h_t}\frac{L_{t+1}}{L_t}\right)^{1+\varepsilon_1}\left(\frac{A_{t+1}}{A_t}\right)^{\phi_1-1}\left(\frac{B_{t+1}}{B_t}\right)^{\mu_1} = 1$$

$$\Omega^{1+\varepsilon_1}\left(\frac{A_{t+1}}{A_t}\right)^{\phi_1-1}\left(\frac{B_{t+1}}{B_t}\right)^{\mu_1} = 1. \tag{C.1}$$

$$\frac{g_{t+1,B}-g_{t,B}}{g_{t,B}} = 0 \implies g_{t,B} = g_{t-1,B}$$

$$\implies \frac{\delta_2\tau_0(1-\tau_0)^{\varepsilon_2}\left[\tau(1+\beta)\right]^{1+\varepsilon_2}}{(1+\beta+\xi)^{1+\varepsilon_2}}B_{t+1}^{\mu_2-1}A_{t+1}^{\phi_2}(h_{t+1}L_{t+1})^{1+\varepsilon_2}$$

$$=\frac{\delta_2\tau_0(1-\tau_0)^{\varepsilon_2}\left[\tau(1+\beta)\right]^{1+\varepsilon_2}}{(1+\beta+\xi)^{1+\varepsilon_2}}B_t^{\mu_2-1}A_t^{\phi_2}(h_tL_t)^{1+\varepsilon_2}$$

$$\implies \left(\frac{h_{t+1}}{h_t}\frac{L_{t+1}}{L_t}\right)^{1+\varepsilon_2}\left(\frac{A_{t+1}}{A_t}\right)^{\phi_2}\left(\frac{B_{t+1}}{B_t}\right)^{\mu_2-1} = 1$$

$$\left(\frac{B_{t+1}}{B_t}\right) = \Omega^{\frac{1+\varepsilon_2}{1-\mu_2}}\left(\frac{A_{t+1}}{A_t}\right)^{\frac{\phi_2}{1-\mu_2}}. \tag{C.2}$$

Inserting (C.2) into (C.1), we obtain

$$\tilde{A} \equiv \left(\frac{A_{t+1}}{A_t}\right) = \Omega^{\frac{(1+\varepsilon_1)(1-\mu_2)+(1+\varepsilon_2)\mu_1}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1}}. \tag{C.3}$$

Therefore,

$$\tilde{B} \equiv \left(\frac{B_{t+1}}{B_t}\right) = \Omega^{\frac{(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1}}. \tag{C.4}$$

Along the balanced growth path, Y_t and K_t must grow at the same rate. Thus, equation (11) suggests

$$\tilde{Y} \equiv \left(\frac{Y_{t+1}}{Y_t}\right) = \left(\frac{A_{t+1}}{A_t}\right)^{2-\phi_1} \left(\frac{B_{t+1}}{B_t}\right)^{-\mu_1} \left(\frac{H_{t+1,M}}{H_{t,M}}\right)^{\varepsilon_0-\varepsilon_1} = \left(\frac{K_{t+1}}{K_t}\right) \equiv \tilde{K} \tag{C.5}$$

$$\implies \left(\frac{Y_{t+1}}{Y_t}\right) \equiv \tilde{Y} = \tilde{K} \equiv \left(\frac{K_{t+1}}{K_t}\right) = \Omega^{\left[\frac{(1-\mu_2)(2-\phi_1+\varepsilon_1)+\mu_1(1+\varepsilon_2-\phi_2)}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1}\right]+\varepsilon_0}. \tag{C.6}$$

(ii)

$$\frac{\partial \tilde{A}}{\partial \mu_1} = \tilde{A}\ln(\Omega)\frac{(1-\mu_2)\left[(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} > 0,$$

$$\frac{\partial \tilde{B}}{\partial \mu_1} = \tilde{B}\ln(\Omega)\frac{\phi_2\left[(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} \geq 0,$$

$$\frac{\partial \tilde{K}}{\partial \mu_1} = \tilde{K}\ln(\Omega)\frac{(1-\mu_2)\left[(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} > 0,$$

$$\frac{\partial \tilde{A}}{\partial \mu_2} = \tilde{A}\ln(\Omega)\frac{\mu_1\left[(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} \geq 0,$$

$$\frac{\partial \tilde{B}}{\partial \mu_2} = \tilde{B}\ln(\Omega)\frac{(1-\phi_1)\left[(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} > 0,$$

$$\frac{\partial \tilde{K}}{\partial \mu_2} = \tilde{K}\ln(\Omega)\frac{\mu_1\left[(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} \geq 0,$$

$$\frac{\partial \tilde{A}}{\partial \phi_1} = \tilde{A}\ln(\Omega)\frac{(1-\mu_2)\left[(1+\varepsilon_2)\mu_1+(1+\varepsilon_1)(1-\mu_2)\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} > 0,$$

$$\frac{\partial \tilde{B}}{\partial \phi_1} = \tilde{B}\ln(\Omega)\frac{\phi_2\left[(1+\varepsilon_2)\mu_1+(1+\varepsilon_1)(1-\mu_2)\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} \geq 0,$$

$$\frac{\partial \tilde{K}}{\partial \phi_1} = \tilde{K}\ln(\Omega)\frac{(1-\mu_2)\left[(1+\varepsilon_2)\mu_1+(1+\varepsilon_1)(1-\mu_2)\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} > 0,$$

$$\frac{\partial \tilde{A}}{\partial \phi_2} = \tilde{A}\ln(\Omega)\frac{\mu_1\left[(1+\varepsilon_2)\mu_1+(1+\varepsilon_1)(1-\mu_2)\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} \geq 0,$$

$$\frac{\partial \tilde{B}}{\partial \phi_2} = \tilde{B}\ln(\Omega)\frac{(1-\phi_1)\left[(1+\varepsilon_2)\mu_1+(1+\varepsilon_1)(1-\mu_2)\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} > 0,$$

$$\frac{\partial \tilde{K}}{\partial \phi_2} = \tilde{K}\ln(\Omega)\frac{\mu_1\left[(1+\varepsilon_2)\mu_1+(1+\varepsilon_1)(1-\mu_2)\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} \geq 0,$$

$$\frac{\partial \tilde{A}}{\partial \varepsilon_0} = 0 \qquad \frac{\partial \tilde{B}}{\partial \varepsilon_0} = 0 \qquad \frac{\partial \tilde{K}}{\partial \varepsilon_0} = \tilde{K}\ln(\Omega) > 0,$$

$$\frac{\partial \tilde{A}}{\partial \varepsilon_1} = \tilde{A}\ln(\Omega)\frac{(1-\mu_2)}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]} > 0,$$

$$\frac{\partial \tilde{B}}{\partial \varepsilon_1} = \tilde{B}\ln(\Omega)\frac{\phi_2}{[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1]} \geq 0,$$

$$\frac{\partial \tilde{K}}{\partial \varepsilon_1} = \tilde{K}\ln(\Omega)\frac{(1-\mu_2)}{[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1]} > 0,$$

$$\frac{\partial \tilde{A}}{\partial \varepsilon_2} = \tilde{A}\ln(\Omega)\frac{\mu_1}{[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1]} \geq 0,$$

$$\frac{\partial \tilde{B}}{\partial \varepsilon_2} = \tilde{B}\ln(\Omega)\frac{(1-\phi_1)}{[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1]} > 0,$$

$$\frac{\partial \tilde{K}}{\partial \varepsilon_2} = \tilde{K}\ln(\Omega)\frac{\mu_1}{[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1]} \geq 0.$$

(iii)

$$\frac{\partial \tilde{A}}{\partial \tau\tau_0} = \frac{\partial \tilde{B}}{\partial \tau\tau_0} = \frac{\partial \tilde{K}}{\partial \tau\tau_0} = 0,$$

$$\frac{\partial \tilde{A}}{\partial \tau(1-\tau_0)} = \frac{\partial \tilde{B}}{\partial \tau(1-\tau_0)} = \frac{\partial \tilde{K}}{\partial \tau(1-\tau_0)} = 0.$$

(iv)

$$\frac{\partial \tilde{h}}{\partial \xi} = -\left(A_E\frac{\theta}{\eta}\right)^{\nu}\left(A_M\frac{\sigma}{\chi}\right)^{1-\nu}\frac{\psi}{(\xi-\theta-\sigma)^2} < 0,$$

$$\frac{\partial \tilde{h}}{\partial \theta} = \frac{\left(A_E\frac{\theta}{\eta}\right)^{\nu}\left(A_M\frac{\sigma}{\chi}\right)^{1-\nu}\psi}{(\xi-\theta-\sigma)}\left[\frac{\nu A_E}{\eta}\left(A_E\frac{\theta}{\eta}\right)^{-1} + \frac{1}{(\xi-\theta-\sigma)}\right] > 0,$$

$$\frac{\partial \tilde{h}}{\partial \sigma} = \frac{\left(A_E\frac{\theta}{\eta}\right)^{\nu}\left(A_M\frac{\sigma}{\chi}\right)^{1-\nu}\psi}{(\xi-\theta-\sigma)}\left[\frac{(1-\nu)A_M}{\chi}\left(A_M\frac{\sigma}{\chi}\right)^{-1} + \frac{1}{(\xi-\theta-\sigma)}\right] > 0.$$

(v)

$$\frac{\partial \tilde{L}}{\partial \xi} = \frac{1+\beta+\theta+\sigma}{\psi(1+\beta+\xi)^2} > 0; \quad \frac{\partial \tilde{L}}{\partial \theta} = \frac{-1}{\psi(1+\beta+\xi)} < 0; \quad \frac{\partial \tilde{L}}{\partial \sigma} = \frac{-1}{\psi(1+\beta+\xi)} < 0,$$

(vi)

$$\frac{\partial \Omega}{\partial \xi} = -\frac{\left(A_E\frac{\theta}{\eta}\right)^{\nu}\left(A_M\frac{\sigma}{\chi}\right)^{1-\nu}}{(1+\beta+\xi)^2} < 0,$$

$$\frac{\partial \Omega}{\partial \theta} = \frac{\frac{\nu A_E}{\eta}\left(A_E\frac{\theta}{\eta}\right)^{\nu-1}\left(A_M\frac{\sigma}{\chi}\right)^{1-\nu}}{(1+\beta+\xi)} > 0,$$

$$\frac{\partial \Omega}{\partial \sigma} = \frac{\frac{(1-\nu)A_M}{\chi}\left(A_E\frac{\theta}{\eta}\right)^{\nu}\left(A_M\frac{\sigma}{\chi}\right)^{-\nu}}{(1+\beta+\xi)} > 0.$$

(vii)

$$\frac{\partial \tilde{y}}{\partial \xi} = \tilde{y}\left[\frac{-1}{(1+\beta+\xi)}\right]\left[\frac{(1-\mu_2)(2-\phi_1+\varepsilon_1)+\mu_1(1+\varepsilon_2-\phi_2)}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1} + \varepsilon_0 + \frac{1+\beta+\theta+\sigma}{\xi-\theta-\sigma}\right] < 0,$$

$$\frac{\partial \tilde{y}}{\partial \theta} = \tilde{y}\left[\left(\frac{(1-\mu_2)(2-\phi_1+\varepsilon_1)+\mu_1(1+\varepsilon_2-\phi_2)}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1}+\varepsilon_0\right)\left(\frac{\nu}{\theta}\right)+\left(\frac{1}{\xi-\theta-\sigma}\right)\right] > 0,$$

$$\frac{\partial \tilde{y}}{\partial \sigma} = \tilde{y}\left[\left(\frac{(1-\mu_2)(2-\phi_1+\varepsilon_1)+\mu_1(1+\varepsilon_2-\phi_2)}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1}+\varepsilon_0\right)\left(\frac{1-\nu}{\sigma}\right)+\left(\frac{1}{\xi-\theta-\sigma}\right)\right] > 0,$$

$$\frac{\partial \tilde{y}}{\partial \mu_1} = \tilde{y}\ln(\Omega)\frac{(1-\mu_2)\left[(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} > 0,$$

$$\frac{\partial \tilde{y}}{\partial \mu_2} = \tilde{y}\ln(\Omega)\frac{\mu_1\left[(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} \geq 0,$$

$$\frac{\partial \tilde{y}}{\partial \phi_1} = \tilde{y}\ln(\Omega)\frac{(1-\mu_2)\left[(1+\varepsilon_2)\mu_1+(1+\varepsilon_1)(1-\mu_2)\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} > 0,$$

$$\frac{\partial \tilde{y}}{\partial \phi_2} = \tilde{y}\ln(\Omega)\frac{\mu_1\left[(1+\varepsilon_2)\mu_1+(1+\varepsilon_1)(1-\mu_2)\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} \geq 0,$$

$$\frac{\partial \tilde{y}}{\partial \varepsilon_0} = \tilde{y}\ln(\Omega) > 0,$$

$$\frac{\partial \tilde{y}}{\partial \varepsilon_1} = \tilde{y}\ln(\Omega)\frac{(1-\mu_2)}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]} > 0,$$

$$\frac{\partial \tilde{y}}{\partial \varepsilon_2} = \tilde{y}\ln(\Omega)\frac{\mu_1}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]} \geq 0.$$