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Export Quality, Exports, and Domestic Trade: A Theoretical and an Empirical Analysis

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Abstract

We explore the impact of export quality on the levels of exports and domestic trade. First, we developed a theoretical framework, using the two-country oligopolistic model with quality differentials in product. We find that a consumer-preference driven increase in export quality improves export performance of a country but it reduces domestic trade, in the absence of any constraint on production capacity. We then test the theoretical findings empirically, using annual bilateral inter-country and intracountry trade data for 142 countries from 1963 – 2014 and by applying the gravity model of trade. We use the IMF's Export-Quality Index to estimate a two-stage gravity model and to examine the effects of exports quality on both exports and domestic trade. Our empirical findings are consistent with our theoretical predictions. The empirical findings suggest that an one percent increase in quality leads to an increase in total exports by 1.08% and a fall in intra-national trade by 2.69%. We also find that the effect of export quality is more pronounced for OECD member countries than the non-OECD member countries.

JEL Classifications: F10, F14, H20
Keywords: Export Quality, International Trade, Domestic Trade, OECD

1 Introduction

The importance of quality as a non-price dimension of exports for fostering growth in developing countries is gaining significant attention (Acharyya and Ganguly, 2023). Improving export quality is not only said to enhance a country's export performance but is also pivotal for economic growth, particularly in developing nations (Hausmann et al., 2007, Hallak and Schott, 2011, Henn et al., 2017, Papageorgiou et.al., 2019). Henn et al. (2017) provide evidence of a positive correlation between high-quality exports and economic development. They argue that a higher quality of the varieties of existing goods contribute to improving comparative advantages, increasing productivity, and boosting revenues. Sutton and Trefler (2011) find that between 1980 and 2005 low-income countries (LICs) have moved into more "sophisticated" products that were predominantly produced by rich countries. Furthermore, according to Linder, 1961 and Hallak, 2006) rich countries export and consume high quality products. Thus, the increase in the quality of exports is possibly a reflection, or a result, of the demands of the consumers in the developed countries for high-quality products. However, within industries, LICs are producing low-quality products relative to the developed countries and in spite of the diversification, the LICs have not yet led to a big boost in GDP per capita (Henn et al., 2017). This could be because of an adverse effect of improved export quality on domestic or intra-country trade. The analysis of this differential effect of an increase in export quality is at the heart of our contributions.

In order to examine the effect product quality on exports and growth, one needs to measure product quality. This is not an easy task as quality has many elements, many of which are intangible in nature. Of course, a better quality product, *ceteris paribus*, would be associated with a higher price. Therefore, early on, researchers used unit values as a proxy for product quality (see, for example, Schott (2004), Hallak (2006), and Hummels and Klenow (2005)). Unit values, however, may not be an accurate predictor of export quality as they can be driven, *inter alia*, by production costs differences. Furthermore, if consumers judge quality by price, producers can start to act strategically in pricing products and prices may to reflect quality to some extent. To deal with these issues, Khandelwal (2010) used price and market share as a measure of quality to account for both horizontal and vertical product differentiation in the United States using the data for HS10 level products. Using bilateral trade data for 166 countries over 1962 – 2014 and modifying Hallak's (2006) approach of measuring quality, Henn et al. (2017) found that quality upgrading is more rapid during the early stages of development. They focused on how liberal trade policies, FDI, institutional quality and human capital promote quality upgrading across various sectors of the economy though their impact varies by sector.

The concept of export quality, as indicated above, encompasses both demand and supply aspects. As incomes in developed countries have increased over the decades, consumers have become more sensitive to quality rather than price variations. They tend to prefer higher quality goods, even at a higher price, over cheaper low-quality goods (Acharyya and Ganguly, 2023). Linder's (1961) hypothesis suggests that wealthier countries allocate a higher proportion of their income to high-quality goods compared to poorer countries, making them significant producers of high-quality products. Hallak (2006) also finds that wealthier nations tend to have a stronger demand for high-unit-value imports, which is considered an indirect measure of export quality, and they tend to import more from countries that produce high-quality goods. Schott (2004) uses cross-sectional data across countries and industries, using price as a proxy for higher quality, to conclude that wealthier and more human- and capital-abundant economies tend to export higher-quality varieties. All these indicate that the magnitude of the effect of quality on exports is likely to vary between rich countries and LICs.

Many studies indicate that exporters who sell higher quality goods at premium prices tend to be more successful and productive than those selling lower-quality goods. Tian et al. (2016) conducted a study on China's agri-food exports to 213 countries and found that exporters with higher product quality can capture more demand in the international market. In the presence of heterogeneous consumers, firms often differentiate their products by offering different quality levels and adjusting prices accordingly (Acharyya and Ganguly, 2023).

Although the literature discussed above is very important and path-breaking in many ways, these studies did distinguish between the effect of product quality on international and domestic or intra-national trade. It is possible that the effect on the two types of trade could be qualitatively different even in the absence of production capacity constraints, because of the ways the production and the demand sides of the markets interact. This is the gap in the literature that our paper tries to fill. We do so both theoretically and empirically.

The theoretical framework developed in this study consists of a two-country oligopolistic model with quality differences arising because of heterogeneities in consumer preferences. Our theoretical analysis shows that an increase in quality of the high-quality good will increase the exports of both the high-quality and the low-quality good, and will reduce domestic sales. We then test the theoretical findings empirically, using a bilateral trade dataset for 142 countries from 1963 – 2014 and by applying the gravity model of trade. The empirical findings of this study also confirm that improving export quality increases international trade but reduces intra-national trade. We also find significant differences in the magnitudes of the effects between OECD and non-OECD countries, the effects being more pronounced for the OECD countries. Our study also adds to the literature on export quality and trade by using a relatively newer dataset on export quality by IMF which includes the export quality of most low-income countries.

The rest of the paper is structured as follows. In Section 2, we present our theoretical framework. In Section 3, we describe the dataset and the estimation strategy that we have used for the empirical analysis. Section 4 presents the results, and Section 5 concludes.

2 The Theoretical Model

There are a few different approaches in modeling product quality in oligopolistic models. Grossman and Helpman (1991) considers a quality-ladder model where endogenous research and development (R&D) deteremines the level of quality in a dynamic framework. In Spence (1985), a consumer can consume any amount of a good of a particular level of quality. In contrast, in Kotowitz and Mathewson (1979), Pliskin, Shepard, and Weinstein (1980), and Shaked and Sutton (1982), the quality choice is a binary one with each consumer consuming only one unit of a good of a particular quality and the total demand of goods of different qualities are obtained by summing over different ranges of heterogeneous consumers (see also Das and Donnenfeld (1989) and Khun et al. (2020) for applications of this approach). In this paper, we shall follow the last approach.

To be more specific, we develop a partial-equilibrium, two-country oligopolistic model. The two countries will be referred to as the home country and the foreign country. In our initial specification, the home country is best seen as a developing or emerging country and the foreign country a developed one. Later on, we shall consider different modifications of the initial set up and the present interpretations might change there.

The home country consumes only the low-quality good. The demand of low-quality good in the home country, D_l^H, is:

$$D_l^H = a - bP_l. \tag{1}$$

Turning to the demand side in the foreign country, there is a continuum of consumers with identical consumption preferences but differing in incomes. Incomes are uniformly distributed over the interval [0, 1]. Each consumer has three choices: consumes one unit of the low-quality good, or consumes one unit of the high-quality good, or does not consumer either of the two. We denote the three scenarios by q, where $q = 0,\ 1,\ 2$, representing no consumption, consumption of the low-quality good, and consumption of the high-quality good, respectively.

We follow Pliskin, Shepard, and Weinstein (1980) and Shaked and Sutton (1982) to specify the utility function of a consumer as

$$U(c, q) = cu_q, \tag{2}$$

where c is the consumption of other goods and services (other than the oligopolistic good), and u_q is the sub-utility received from consuming one unit of the good with attribute q.

We assume that the consumers get a higher marginal utility from the consumption of a high quality good i.e. $u_2 > u_1 > u_0$. P_l represents the price of low quality good and P_h is the price of high quality good in the international market, with $P_l < P_h$.

Let us denote by y_0 the level of income at which the consumer with that income is indifferent between not consuming the good and consuming one unit of the low-quality good, that is,

$$U(y_0, q_0) = U(y_0 - P_l, q_1), \tag{3}$$

since at $q = q_0$, the consumer does not consume the oligopolistic good and hence $c = y_0$, and at $q = q_1$, the consumer consumes one unit of the low-quality good, and hence $c = y_0 - P_l$.

From (2) and (3) we solve for y_0 as:

$$y_0 = \frac{P_l u_1}{u_1 - u_0}. \tag{4}$$

Let y^* be the income level such that a consumer is indifferent between high-quality and low-quality goods. Hence, at income level y^*, we have:

$$U(y^* - P_h, q_2) = U(y^* - P_l, q_1). \tag{5}$$

From (2) and (5) we solve for y^* as:

$$y^* = \frac{P_h u_2 - P_l u_1}{u_2 - u_1}. \tag{6}$$

The denominators of Equations (4) and (6) represent utility premia of the low-quality good over no consumption and that of the high-quality good over the low-quality good respectively, while the numerators reflect additional costs associated with them.

Since consumers in the foreign country are assumed to be uniformly distributed over the income space [0, 1], the demand for the low-quality good, D_l^F, and that of the high-quality good, D_h^F, in the foreign country are respectively $y^* - y^0$ and $1 - y^*$. Then, using (4) and (6), we get:

$$D_l^F = \frac{P_h u_2 - P_l u_1}{u_2 - u_1} - \frac{P_l u_1}{u_1 - u_0}, \tag{7}$$

$$D_h^F = 1 - \frac{P_h u_2 - P_l u_1}{u_2 - u_1}. \tag{8}$$

Since the low-quality good is consumed in both countries, but the high-quality good is consumed only in the foreign country, from (1) and (7), we derive the world demand for the low-quality good, D_l, as

$$D_l = D_l^F + D_l^H = \frac{P_h u_2 - P_l u_1}{u_2 - u_1} - \frac{P_l u_1}{u_1 - u_0} + a - bP_l. \tag{9}$$

The world demand for the high-quality good is D_h^F (equation (8)).

The inverse demand functions for low-quality and high-quality goods are derived from (8) and (9):

$$P_l = e - fD_h^F - gD_l, \tag{10}$$

$$P_h = h - kD_h^F - mD_l, \tag{11}$$

where

$$
\begin{aligned}
f &= \frac{u_1 - u_0}{u_1 + b(u_1 - u_0)} > 0,\ g = \frac{u_1 - u_0}{u_1 + b(u_1 - u_0)} > 0,\ e = \frac{(u_1 - u_0)(1 + a)}{u_1 + b(u_1 - u_0)} > 0, \\
k &= -\frac{u_0 u_1 - u_1 u_2 - u_1 u_2 b + u_0 u_2 b + u_1^2 b - u_0 u_1 b}{u_2(u_1 + b(u_1 - u_0))} > 0,\ m = \frac{u_1(u_1 - u_0)}{u_2(u_1 + b(u_1 - u_0))} > 0, \\
h &= \frac{a u_1(u_1 - u_0) + u_1(u_2 - u_0) + b(u_1 - u_0)(u_2 - u_1)}{u_2(u_1 + b(u_1 - u_0)))} > 0.
\end{aligned}
$$

It is normal to assume that $k > m$ and $g > f$: the direct effects are greater than the indirect effects.

Turning to the supply/production side, there are two firms in the home country and one in the foreign country. The low-quality good is produced only in the home country by one of the two firms there. The other firm in the home country and the firm in the foreign country produce the high-quality good. The quantity of low-quality and high-quality good produced in home country are q_l^H and q_h^H respectively, and q_h^F represents the quantity of high-quality good produced in the foreign country.

The market-clearing conditions give us:

$$
D_l = q_l^H, \text{ and } D_h^F = q_h^H + q_h^F. \tag{12}
$$

Profits of the three firms are given by

$$
\begin{aligned}
\pi_h^H &= P_h q_h^H - C_h^H q_h^H, && (13) \\
\pi_l^H &= P_l q_l^H - C_l^H q_l^H, && (14) \\
\pi_h^F &= P_h q_h^F - C_h^F q_h^F, && (15)
\end{aligned}
$$

where C_h^H, C_l^H and C_h^F are the (constant) unit costs of production of the three firms.

Assuming Cournot-Nash conjectures and using (10), (11) and (12), the three first-order conditions are:

$$\begin{aligned}
\frac{\partial \pi_h^H}{\partial q_h^H} &= P_h - C_h^H + \frac{\partial P_h}{\partial q_h^H} q_h^H = P_h - C_h^H - k q_h^H = 0, && (16) \\
\frac{\partial \pi_l^H}{\partial q_l^H} &= P_l - C_l^H + \frac{\partial P_l}{\partial q_l^H} q_l^H = P_l - C_l^H - m q_l^H = 0, && (17) \\
\frac{\partial \pi_h^F}{\partial q_h^F} &= P_h - C_h^F + \frac{\partial P_h}{\partial q_h^F} q_h^F = P_h - C_h^F - k q_h^F = 0. && (18)
\end{aligned}$$

This completes the description of the model. Equations (10), (11), (12), (16), (17) and (18) have seven equations (2 in (12)), and these can be solved for the seven endogenous variables: three outputs, two prices, and two world demands.

2.1 Effects of Quality

We now examine the effect of a change in u_2 on the equilibrium values. An increase in u_2 increases the value consumers attach to quality and therefore raises the demand for the high-quality good in the foreign country. Thus, we see this exercise as an investigation into the effect of a consumer-preference-driven increase in the quality on the levels of exports and on domestic trade. We should note that there is some evidence to suggest that efforts by the developing countries to increase the quality of exports can, to a large extent, be attributed to their response to an increase in demand for high-quality products in the developed countries (Linder, 1961 and Hallak, 2006)). It should also be noted that even though our theoretical model posits a binary division: a low-quality and a high-quality product, an increase in u_2 represents an increase in the level of quality of the high-quality product. Thus, quality is also a continuous variable in our framework.

Taking total derivatives of equations (10), (11), (12), (16), (17) and (18), we find (details are in the Appendix):

$$\frac{dq_h^H}{du_2} = \frac{2g\left(\frac{\partial P_h}{\partial u_2} - \frac{\partial k}{\partial u_2}\, q_h^H\right) - m\left(\frac{\partial P_l}{\partial u_2} - \frac{\partial g}{\partial u_2}\, q_l^H\right)}{2\,(3kg - mf)},$$

$$\frac{dP_l}{du_2} = \frac{\partial P_l}{\partial u_2} - \frac{\partial g}{\partial u_2}\, q_l^H,$$

$$\frac{dq_l^H}{du_2} = \frac{3k\left(\frac{\partial P_l}{\partial u_2} - \frac{\partial g}{\partial u_2}\, q_l^H\right) - 2f\left(\frac{\partial P_h}{\partial u_2} - \frac{\partial k}{\partial u_2}\, q_h^H\right)}{2\,(3kg - mf)},$$

$$\frac{dD_l^H}{du_2} = -b\frac{dP_l}{du_2}$$

These results are formally stated below.

Proposition 1 *An increase in the quality of the high-quality good increases the exports of both the high-quality and the low-quality goods, but reduces domestic trade in the exporting country.*

The results can be explained intuitively as follows. An increase in u_2 will shift the demand curve for the high-quality to to the right, raising the output of both firms, and the price, of the high-quality good. An increase in u_2 also reduced the demand curve of the low-quality good in the foreign country to the left. However, the increase in the price of the high quality good will shift the demand curve of the low-quality good to the right in the foreign country. The latter effect dominates and the net effect on the demand of the low-quality good is positive. This increases its price and output. This will reduce domestic trade of the low-quality good in the home country and increase the export of that good.

2.2 Possible Extensions

The basic model presented and analyzed above can be modified in a number of ways without altering the qualitative results. Some of these modifications are:

- Production of the low-quality good in the foreign country

- No production of the high-quality good in either the foreign country or the domestic country
- No domestic demand of the low-quality good in the domestic market, but the presence of demand for the high-quality good there.

Considering demand for both goods in both countries will complicate matter. Depending on which extension we consider, either country can be interpreted as a developed or a developing country.

3 Empirical Analysis

3.1 Estimation Strategy

This study uses the gravity model like the one suggested by Anderson and van Wincoop (2003) and Bergstrand et al. (2015). The gravity equation derived by Anderson and van Wincoop (2003) is:

$$X_{ij} = \left(\frac{E_j Y_i}{Y_w}\right)\left(\frac{t_{ij}}{P_j \Pi_i}\right)^{1-\sigma}, \quad \sigma > 1. \tag{19}$$

where X_{ij} represents the value of exports from country i to country j. E_j is the total expenditure in country j. Y_i is the sales of goods by country i at destination prices. Y_w is the world output. The parameter t_{ij} represents trade costs for exports from country i to country j. P_j and Π_i are inward and outward multilateral resistances, respectively, and σ represents the elasticity of substitution between the goods.

To estimate the impact of quality on exports and on domestic trade, we conduct a two-stage regression analysis. In the first stage, the gravity equation (20) is estimated using the Poisson pseudo-maximum likelihood (PPML) method suggested by Silva and Teneyro (2006) as it caters for potential endogeneity and bias produced by heteroscedasticity, multicollinearity and serial correlation (Álvarez et al. 2018). It can also include zero export values that would've been otherwise excluded if ordinary least square (OLS) was used (Francois and Manchin 2013).

$$X_{ijt} = \exp\Big[\beta_0 + \beta_1 \text{RTA}_{ijt} + \beta_2 \text{Linder}_{ijt} + \beta_3 \ln(\text{Quality}_{it}) * \text{INTD}_{ij} + \beta_4 \ln(\text{Price}_{it}) * \text{INTD}_{ij} + \beta_5 \ln(\text{Price}_{it}) * \ln(\text{Quality}_{it}) * \text{INTD}_{ij} + \mu_{it} + \theta_{jt} + \delta_{ij}\Big] + \epsilon_{ijt}, \tag{20}$$

where X_{ijt} is the bilateral trade flows from country i to country j at time t. RTA is a binary dummy variable representing regional trade agreements between country i and country j at time t. It takes the value 1 when a regional trade agreement exists between the two countries; otherwise, it takes the value 0. $Linder_{ijt}$ is estimated by taking the absolute value of the di erence between the log of GDP per capita of country i and j.[1] $\ln Quality_{it}$ represents the natural logarithm of a quality index, while $\ln Price$ is the natural logarithm of unit value of export and is used to control for the unit price in the quality index. INTD_{ij} is a binary dummy variable that takes the value 1 in the presence of international trade and 0 otherwise.

The variables μ_{it}, θ_{jt}, and δ_{ij} represent exporter-time, importer-time, and pair-wise fixed e ects, respectively. The inclusion of exporter-time and importer-time fixed e ects deal with inward and outward multilateral resistances and absorbs the e ects of all variables that are country-time specific, like GDP of importers and of exporters and other macroeconomic variables, as highlighted by Larson et al. (2018). Similarly, pair-wise fixed e ects deal with all time-independent bilateral variables, like the distance between the two countries. The three sets of fixed e ects address, to a large extent, endogeneity issues like spurious correlation, that might arise because of omitted variables. Finally, ϵ_{ijt} is the error term in the equation.

Since, export quality (Quality_{it}) is an exporter-time variable, it cannot be identified in the presence of the exporter-time fixed e ect for reasons mentioned above. However, since

[1] $|Linder_{ijt}| = |\ln GDPPC_{it} - \ln GDPPC_{jt}|$ (Kitenge and Lahiri, 2021)

we include data on intra-state trade, we shall use the identification strategy of Heid et al. (2021) to identify it by interacting the dummy variable INTD_{it} with it. The coefficient of this interaction term (β_3) will give us the effect of quality on exports, relative to that on intra-state trade. However, it is possible that the relative effect is positive even when the two individual effects are negative. We follow Paudel and Lahiri's (2023) in introducing the second-stage to separate out the relative and absolute effects. In the second stage, we estimate

$$\begin{aligned}\hat{\mu}_{it} &= \alpha_0 + \alpha_1 \ln \text{GDP}_{it} + \alpha_2 \ln \text{Quality}_{it} + \alpha_3 \ln \text{Price}_{it} + \alpha_4 \ln \text{Quality}_{it} * \ln \text{Price}_{it} \\ &\quad + {}_i + \eta_t + \varepsilon_{it}, \end{aligned} \tag{21}$$

using ordinary least squares (OLS) method where $\hat{\mu}_{it}$ is the exporter-time fixed effects that we generated from our first-stage regression analysis. $\ln \text{GDP}_{it}$ is the log of real GDP of country i at time t. $\ln \text{Quality}_{it}$ and $\ln \text{Price}_{it}$ are the log of export quality and log of unit price of exports of country i at time t, respectively. $\ln \text{Quality}_{it} * \ln \text{Price}_{it}$ is the interaction term between the log of quality and log of unit price of exports. ${}_i$ are the country fixed effects, and η_t represents time fixed effects. ε_{it} is the error term.

From the above two regressions, the absolute effects of quality of exports and intra-national trade are given by $\beta_3 + \alpha_2$ and α_2 respectively (see Paudel and Lahiri's (2023))

3.2 Data

We use annual, bilateral trade data of 142 countries for 52 years (1963–2014), which was constructed by Fouquin and Hugot (2016). This dataset was complemented with observations for intra-national trade, following the Baier et al. (2016)-approach of taking the difference between total value of domestic production and total value of exports. Data for regional trade agreements (RTA) were constructed by Head et al. (2010) and retrieved from the Centre d'Études Prospectives et d'Informations Internationales (CEPII). We use GDP data from the

World Development Indicators (WDI) has been used to calculate the Linder variable. IMF's Export Quality Index and Unit Value of exports are used as a measure of quality and export prices respectively. Data for both the variables is taken from the IMF's Export Quality and Diversification dataset, constructed by Henn et al. (2018) and covers over 800 export products, 166 countries from 1963 - 2014.[2] The quality index has values between 0 and 1.2, with a higher value of the quality index representa a higher quality level.

The summary statistics of the variable are given in the table below.

Table 1: Descriptive Statistics: Summary Statistics of Variables

	Obs	Mean	SD	Min	Max
X_{ijt}	1,523,462	6.12e+08	3.35e+10	0	9.65e+12
RTA	2,579,198	0.629	0.243	0	1
Linder	1,537,265	1.709	1.254	0	7.361
lnGDP	1,690,501	22.345	2.55	15.169	29.997
lnQuality	1,250,204	-0.250	0.253	-1.999	0.111
lnPrice	2,188,072	4.245	0.521	2.51	6.419
INTD	3,367,647	0.996	0.065	0	1

3.2.1 Export Quality Index

Since export quality is the key variable in our analysis and we take the IMF data on export quality, it may be helpful to discuss briefly their methodology. IMF's data comes from a study by Henn et al. (2018) who estimate a gravity equation for 851 sectors separately and then aggregate them to country-time specific values.

They first posit that trade price (unit value) depends on unobservable quality θ_{ijt}, exporter income per capita y_{it}, and distance between importer and exporter, Dist_{ij} as shown in the equation below:

$$\ln p_{ijt} = \zeta_0 + \zeta_1 \ln \theta_{ijt} + \zeta_2 \ln y_{it} + \zeta_3 \ln \text{Dist}_{ij} + \xi_{ijt}, \tag{22}$$

[2]Henn et al. (2018) used unit values at the SITC 4-digit level and then normalized them into a price index for each 2-digit "sector".

where $\ln y_{it}$ represents cost di erences between countries, and $\ln \text{Dist}_{ij}$ allow for transportation costs.

Then, they estimate a quality-augmented gravity equation, specified separately for each product as trade costs and quality may vary across products:

$$\ln(X_{ijt}) = \alpha \ln(Dist_{ij}) + \beta I_{ijt} + \delta \ln(\theta_{ijt}) \ln(y_{jt}) + \eta_i + \quad_j + \epsilon_{ijt}, \tag{23}$$

Where $\quad_j$ and η_i represent importer-country and exporter-country fixed e ects,respectively, I_{mxt} is a set of standard gravity variables except distance, and $ln(\theta_{mxt})ln(y_{mt})$ is the interaction term between the unobserbale quality variable and the importer's income per capita.[3] The last terms comes from the demand for quality.

Substituting $\ln \theta_{ijt}$ from equation (22) into (23), one gets the estimable equations:

$$\begin{aligned} \ln(X_{ijt}) \quad = \quad & \alpha \ln(Dist_{ij}) + \beta I_{ijt} + \eta_i + \quad_j + \zeta_1' \ln p_{ijt} * \ln(y_{jt}) + \zeta_2' \ln(y_{jt}) * \ln(y_{it}) \\ & + \zeta_3' \ln(Dist_{ij}) * \ln(y_{jt}) + \eta_i + \quad_j + \xi_{ijt}', \end{aligned} \tag{24}$$

where $\zeta_1' = \delta/\zeta_1$, $\zeta_2' = -\delta\zeta_2/\zeta_1$, $\zeta_3' = -\delta\zeta_3/\zeta_1$, and $\xi_{ijt}' = -\delta(\zeta_0 + \xi_{ijt}) \ln(y_{jt})/\zeta_1 + \epsilon_{ijt}$.

Because ξ_{ijt} is a part of ξ_{ijt}', $\ln p_{ijt} * \ln(y_{jt})$ is correlated ξ_{ijt}' in equation (24) and it needs to be estimated, for each sector, using instrumental variable (IV) method, and Henn et al. (2018) use $\ln p_{ijt-1} * \ln(y_{jt})$ as the instrument for $\ln p_{ijt} * \ln(y_{jt})$.

From equations (22)-(24), the estimated values of quality is obtained as:

$$\text{Quality-estimate}_{ijt} = \delta \ln \theta_{ijt} = \hat{\zeta}_1' \ln p_{ijt} + \hat{\zeta}_2' \ln y_{it} + \hat{\zeta}_3' \ln Dist_{ij}.$$

The above estimates of quality are then aggregated over sector as nd importers to obtain estimates of quality in the exporter-time dimension.

[3]Detailed methodology about how Export Quality Index was calculated can be found here: https://www.imf.org/external/np/seminars/eng/2014/trade/pdf/henn.pdf

4 Results

In this section, we will present our results for the empirical analysis and provide some robustness checks. Table 2 presents the results of the PPML regression analysis for equation (20), i.e., the first-stage regression in our two-stage approach. The first column is presented for reference purpose only; it has two of the key variables of gravity analysis, viz., RTA (regional trade agreement) and Linder, but does not include *lnQuality**INTD. The coefficients of RTA and Linder are positive and negative and statistically significant, as one would expect. The second column gives our main regression and it includes *lnQuality**INTD. The coefficient of this variable is highly significant and positive. This imples that an increase in export quality significantly increases exports relative to intra-country trade (domestic trade). The coefficients of RTA and Linder continues to be as before qualitatively, the magnitude of their e ects go down significantly That is, the absence of *lnQuality**INTD possibly introduces an upward omitted-variable bias in the magnitudes of these two variables. We find that a 1% increase in *Quality* increases exports (relative to domestic trade) by 4.45%.

The next two columns introduce variables involving *lnPrice* in order to see if our results that quality increases exports, is robust under the new specifications. We find that it is robust. *Price* has a positive e ect on exports (relative to domestic sales). This is possibly a supply-side e ect. This e ect gets amplified as the the quality of exports increases.

Table 3 presents the results of the second-stage OLS regression analysis, i.e., OLS results for equation (21). As mentioned before, the coefficients of this table give us the value of coefficients for intra-country trade. The sum of the coefficients here plus the corresponding coefficients give the values for the e ect on international trade. Column 1 presents the reference regression with only one important variable, viz., GDP which has the country-time dimension. The next equation is the key equation and it includes the variable *lnQuality*. The next two columns introduces the variable *lnPrice* and its interaction with *lnQuality* to examine the robustness of the result on the e ect of product quality on trade. The coefficient of GDP is positive and significant throughout, as one would expect. However, interestingly, the coefficient of *lnQuality* is negative and significant throughout, implying that an incrase in export quality has a negative e ect on domestic trade. An one percent increase in quality a 3.3%. The total e ects of *lnQuality* and other variables on International and Intra-national trade are presented in Table 6.

Table 2: First Stage PPML Estimates

Dependent Variable (log): Total Exports	(1)	(2)	(3)	(4)
RTA	0.426***	0.271***	0.176***	0.173***
	(0.0586)	(0.0542)	(0.0516)	(0.0517)
Linder	-0.292***	-0.190***	-0.0867***	-0.0861***
	(0.0254)	(0.0250)	(0.0276)	(0.0279)
lnQuality*INTD		4.455***	3.633***	2.114***
		(0.424)	(0.419)	(0.782)
lnPrices*INTD			0.457***	0.489***
			(0.0363)	(0.0444)
lnPrices*lnQuality*INTD				0.390**
				(0.168)
Observations	1,113,248	851,574	845,830	845,830
R-square	0.9979	0.9978	0.9979	0.9979
Exporter*Time FE	Yes	Yes	Yes	Yes
Importer*Time FE	Yes	Yes	Yes	Yes
Pair FE	Yes	Yes	Yes	Yes

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3: Second-Stage OLS regression Estimates

Dependent Variable (log): ETFE	(1)	(2)	(3)	(4)
lnGDP	0.607***	0.626***	0.564***	0.578***
	(0.0077)	(0.0109)	(0.0113)	(0.0115)
lnQuality		-3.289***	-2.620***	-2.3122***
		(0.0324)	(0.0315)	(0.1532)
lnPrice			-0.178***	-0.133***
			(0.0163)	(0.021)
lnPrice*lnQuality				-0.090***
				(0.0353)
Observations	8,549	6,258	6,189	6,189
R-square	0.9886	0.9837	0.9866	0.9866
Exporter FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

4.1 Effects of Quality on OECD and Non-OECD countries

In the preceding analysis, we have seen that export quality increases exports, but reduces domestic or intra-national trade. In this section we want to see if there are heterogeneities in these relationships. In particular, we want to examine if these relationships vary between Organization of Economic Cooperation and Development (OECD) member countries and the non-OECD countries. For this, we define a dummy variable OECD which takes the value 1 if the exporting country belongs to OECD, and 0 otherwise. We then modify equations (20) and (26) to include a term that interacts with the quality variable. That is, we estimate:

$$\begin{aligned} X_{ijt} &= \exp\Big[\beta_0 + \beta_1 \text{RTA}_{ijt} + \beta_2 \text{Linder}_{ijt} + \beta_3 \ln(\text{Quality}_{it}) * \text{INTD}_{ij} \\ &\quad + \beta_4 \ln(\text{Price}_{it}) * \text{INTD}_{ij} + \beta_5 * \ln(\text{Quality}_{it}) * \text{INTD}_{ij} * \text{OECD}_i \\ &\quad + \mu_{it} + \theta_{jt} + \delta_{ij}\Big] + \epsilon_{ijt}, \qquad (25) \\ \hat{\mu}_{it} &= \alpha_0 + \alpha_1 \ln \text{GDP}_{it} + \alpha_2 \ln \text{Quality}_{it} + \alpha_3 \ln \text{Price}_{it} + \alpha_4 \ln \text{Quality}_{it} * \text{OECD}_i \\ &\quad + \ {}_i + \eta_t + \varepsilon_{it}. \qquad (26) \end{aligned}$$

The results of the first-stage and second stage regression are shown in Tables 4 and 5 respectively. The e ect of improving export quality both on exports and on intra-national trade are more pronounced for OECD countries than for the non-OECD countries. That is, the estimated value of β_5 in Table 4 is positive and significant, and that of α_4 is negative and significant. As for the magnitude of the e ects, based on the last columns of tables 4 and 5, we find that an one percent increase in export quality increase exports of OECD countries by 3.18 (=3.023+4.778-2.236 -2.383)% and that of non-OECD countries by 0.79 (=3.023-2.236)%. Quality has a negative impact on the domestic trade of both OECD and non-OECD countries. An one percent increase in export quality reduces domestic trade of OECD countries by 4.61 (=2.236+2.283)% and of non-OECD countries by 2.24%. The total e ects of export quality on exports for both OECD and non-OECD countries are shown in Table 6. The column numbers in Table 6 correspond to the column numbers in Tables 2, 3, 4 and 5.

Table 4: First Stage PPML Estimates

Dependent Variable: Total Exports	(2)	(5)
RTA	0.271***	0.167***
	(0.0542)	(0.5045)
Linder	-0.190***	-0.102***
	(0.0250)	(0.0263)
lnQuality*INTD	4.455***	3.023***
	(0.424)	(0.4595)
lnPrices*INTD		0.446***
		(0.3432)
lnQuality*INTD*OECD		4.778***
		(0.9147)
Observations	851,574	845,830
R-square	0.9978	0.9979
Exporter*Time FE	Yes	Yes
Importer*Time FE	Yes	Yes
Pair FE	Yes	Yes

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 5: Second Stage OLS Estimates

Dependent Variable: ETFE	(2)	(5)
lnGDP	0.626***	0.564***
	(0.0109)	(0.011)
lnQuality	-3.289***	-2.236***
	(0.0324)	(0.0308)
lnPrices		-0.168***
		(0.0159)
lnQuality*OECD		-2.383***
		(0.2239)
Observations	6,258	6,189
R-square	0.9837	0.9979
Exporter FE	Yes	Yes
Time FE	Yes	Yes

Table 6: Total E ects

	Intranational Trade				International Trade			
	(2)	(3)	(4)	(5)	(2)	(3)	(4)	(5)
Ln_Quality	-3.288754***	-2.620068***	-2.69309**		1.165931***	1.013028***	1.076782**	
Ln_Price		-0.1778905***	-0.514209**			0.2791984**	1.02841**	
Ln_Quality (OECD)				- 4.618456***				3.182612***
Ln_Quality (Non-OECD)				-2.235771***				0.787644***

4.2 Results with 3-year interval data

Finally, Chang and Wall (2005) suggest that the adjustment of trade in response to changes in other covariates can takes time, and recommend using intervals data instead of continuous panel data. In this section, we ran the same regressions as in Table 2 and 3 with 3-year interval data and the results are reported in Tables 7 and 8. The results remain qualitatively the same.

Table 7: First Stage PPML estimates with 3-year intervals

Dependent Variable: Total Exports	(1)	(2)	(3)	(4)
RTA	0.342***	0.289***	0.170***	0.167***
	(0.0544)	(0.0558)	(0.0526)	(0.0526)
Linder	-0.220***	-0.1852***	-0.0668***	-0.0657***
	(0.0245)	(0.0250)	(0.0276)	(0.0280)
lnQuality*INTD		4.667***	3.757***	2.042***
		(0.4547)	(0.4554)	(0.7841)
lnPrices*INTD			0.509***	0.544***
			(0.0342)	(0.0416)
lnPrices*lnQuality*INTD				0.453***
				(0.1699)
Observations	679,678	283,372	281,650	281,650
R-square	0.9983	0.9977	0.9979	0.9979
Exporter*Time FE	Yes	Yes	Yes	Yes
Importer*Time FE	Yes	Yes	Yes	Yes
Pair FE	Yes	Yes	Yes	Yes

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 8: Second-Stage OLS regression with 3-year intervals

Dependent Variable: ETFE	(1)	(2)	(3)	(4)
lnGDP	0.628***	0.638***	0.576***	0 .590***
	(0.0098)	(0.0181)	(0.0185)	(0.0190)
lnQuality		-3.502***	-2.762***	-2.344***
		(0.0544)	(0.0519)	(0 .2575)
lnPrice			-0.229***	-0.185***
			(0.0271)	(0.0341)
lnPrice*lnQuality				-0.128**
				(0.0589)
Observations	4,977	2,152	2,129	2,129
R-square	0.9906	0.9851	0.9883	0.9881
Exporter FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

5 Conclusion

This paper provides a comprehensive analysis of the impact of improving the export quality on international and domestic trade. The theoretical framework developed in this study shows that an increase in quality of the high-quality good will increase the exports of both the high-quality and the low-quality good, and will reduce domestic sales in the absence of any overall capacity constraint.

The empirical findings from our gravity analysis supports our theoretical predictions. That is, product quality matters for international trade only, impacting exports positively. However, product quality and domestic trade have a negative relationship. Higher quality and high prices reduce domestic trade. The positive e ect of improving export quality on the exports is more pronounced for OECD countries than the non-OECD countries. On the other hand, the negative e ect of product quality on domestic trade is also more pronounced for the OECD member countries.

Thus, as a policy prescription, countries need to be careful when adopting policies to enhance international trade via quality improvement. They also to adopt policies to stimulate domestic trade and otherwise export promotion policy might compromise domestic trade.

References

Abreo, C., Bustillo, R. and Rodriguez, C. (2021) The role of institutional quality in the international trade of a Latin American country: evidence from Colombian export performance, *Journal of Economic Structures*, 24(10).

Acharyya, R. and Ganguly, S. (2023) *Export Quality and Income Distribution*, Cambridge University Press.

Álvarez, I., Barbero, J., Rodríguez-Pose, A. and Zofío, J. (2018) Does institutional quality matter for trade? Institutional conditions in a sectoral trade framework, *World Development*, 103:72–87.

Anderson, J. E. and Wincoop, E. V. (2003) Gravity with Gravitas: A Solution to the Border Puzzle, *American Economic Review*, 93(1):170-192.

Baier, S. and Standaert, S. (2020) Gravity Models and Empirical Trade, *Economics and Finance*.

Bergstrand, J. H., Larch, M., and Yotov, Y. V. (2015) Economic integration agreements, border e ects, and distance elasticities in the gravity equation, *European Economic Review*, 78: 307-327

Das, S. P., and Donnenfeld, S. (1989) Oligopolistic competition and international trade: Quantity and quality restrictions, *Journal of International Economics*. 27: 299-318.

Fouquin, M. and Hugot, J. (2016) Two Centuries of Bilateral Trade and Gravity Data: 1827-2014, *Centre d'tudes Prospectives et d'Informations Internationales (CEPII)*.

Francois, J. and Manchin, M. (2013) Institutions, Infrastructure, and Trade, *World Development*, 46:165–175.

Grossman, G. M. and Helpman, E. (1991) Quality Ladders in the Theory of Growth *The Review of Economic Studies*, 58(1): 43-61.

Hallak, J. C. (2006) Product quality and the direction of trade, *Journal of International Trade*, 68 (1).

Hausmann, R., Hwang, J. and Rodrik, D. (2007) What you export matters, *Journal of Economic Growth*, 12: 1–25.

Head, K. and Mayer, T. (2013) Gravity Equations: Workhorse, Toolkit, and Cookbook, *Centre d'tudes Prospectives et d'Informations Internationales (CEPII).*

Head, K., and Spencer, B. J. (2017) Oligopoly in International Trade: Rise, Fall and Resurgence, *The Canadian Journal of Economics*, 50: 1414-1444.

Henn, C., Papageorgiou, C., Romero, J. M. and Spatafora, N. (2017) Export Quality in Advanced and Developing Economies: Evidence from a New Data Set, *World Bank Group.*

Hummels, D. and Klenow, P. (2005) The Variety and Quality of a Nation's Exports, *American Economic Review*, 95: 704–723.

IMF (2021) Research and Innovation: Fighting the Pandemic and Boosting Long-Term Growth, Chapter 3, World Economic Outlook, October.

IMF (2023) Navigating Global Divergences, World Economic Outlook, October.

Khandelwal, A. (2010) The Long and Short (of) Quality Ladders, *The Review of Economic Studies*, 77(4): 1450-1476.

Khun, C., Lahiri, S. and Lim, S. (2020) Why do U.S. Parents prefer Private to Foster Care Adoptions? The Role of Adoption Subsidies, Gender, Race, and Special Needs, *Economic Inquiry.*

Kitenge, E. and Lahiri, S. (2021) Is the Internet bringing down language-based barriers to international trade? *Review of International Economics*, 30(2): 566-605.

Krugman, P. R. (1989) Chapter 20 Industrial organization and international trade in Handbook of Industrial Organization. 1179-1223.

Larson, J., Baker, J., Latta, G., Ohrel, S. and Wade, C. (2018) Modeling International Trade of Forest Products: Application of PPML to a Gravity Model of Trade, *Forest Products Journal*, 68 (3): 303–316.

Linder, S. (1961) *An Essay on Trade and Transformation*. Almqvist & Wiksell, Stockholm.

Papageorgiou, C., Perez-Sebastian, F., and Spatafora, N. (2019) Quality Upgrading and Export Performance in the Asian Growth Miracle. *IMF Working Paper*.

Paudel, N. S. and Lahiri. S. (2024) The e ects of state-level foreign manufacturing imports on domestic inter-state and intra-state sales in the U.S.A, *Economic Analysis and Policy*, 81, 297 - 305.

Schott, P. (2004) Across-product versus within-product specialization in international trade. *Quarterly Journal of Economics*, 119, 647–678.

Shaked, A. and Sutton. J. (1982) Relaxing Price Competition Through Product Di erentiation, *The Review of Economic Studies*, 49 (1): 3-13.

Silva, J. M. C. S., and Tenreyro, S. (2006) The Log of Gravity. *The Review of Economics and Statistics*, 88(4), 641–658.

Spencer, B. J., and Brander, J. A. (1983) International R & D Rivalry and Industrial Strategy, *The Review of Economic Studies*, 50:707-722.

Sutton, J. and Trefler, D. (2011) Deductions from the Export Basket: Capabilities Wealth and Trade, *National Bureau of Economic Research*, Working Paper 16834.

Tian, D., Hu, N., Wang, X., and Huang, L. (2016) Trade margins, quality upgrading, and China's agri-food export growth. *China Agricultural Economic Review*, 8(2).

World Development Indicators, The World Bank.

Appendix

Additional details of the estimation strategy

The three First-Order Conditions (FOCs) of the profit function are:

$$\frac{{}^{H}_{h}}{q_h^H} = P_h - C_h^H - kq_h^H = 0 \tag{27}$$

$$\frac{{}^{H}_{l}}{q_l^H} = P_l - C_l^H - gq_l^H = 0 \tag{28}$$

$$\frac{{}^{F}_{h}}{q_h^F} = P_h - C_h^F - kq_h^F = 0 \tag{29}$$

$$P_h - kq_h^H = C_h^H \tag{30}$$

$$P_l - gq_l^H = C_l^H \tag{31}$$

Applying Cramer's Rule:

$$\begin{bmatrix} G_{h1}^H & G_{h2}^H & G_{h3}^H \\ G_{h1}^F & G_{h2}^F & G_{h3}^F \\ G_{l1}^H & G_{l2}^H & G_{l3}^H \end{bmatrix} \begin{bmatrix} dq_h^H \\ dq_h^F \\ dq_l^H \end{bmatrix} = - \begin{bmatrix} G_{h4}^H \\ G_{h4}^F \\ G_{l4}^H \end{bmatrix} du_2$$

$$\frac{{}^{H}_{h}}{q_h^H} = P_h - C_h^H + kq_h^H = 0 \tag{32}$$

$$G_{h1}^{H} = -2k \tag{33}$$

$$G_{h2}^{H} = -k \tag{34}$$

$$G_{h3}^{H} = -m \tag{35}$$

$$G_{h4}^{H} = \frac{P_h}{u_2} - \frac{k}{u_2} q_h^{H} \tag{36}$$

$$\frac{{}_{h}^{F}}{q_h^F} = P_h - C_h^F - k q_h^F = 0 \tag{37}$$

$$G_{h1}^{F} = -k \tag{38}$$

$$G_{h2}^{F} = -2k \tag{39}$$

$$G_{h3}^{F} = -m \tag{40}$$

$$G_{h4}^{F} = \frac{P_h}{u_2} - \frac{k}{u_2} q_h^{F} \tag{41}$$

$$\frac{{}_{l}^{H}}{q_l^H} = P_l - C_l^H - g q_l^H = 0 \tag{42}$$

$$G_{l1}^H = -f \tag{43}$$

$$G_{l2}^H = -f \tag{44}$$

$$G_{l3}^H = -2g \tag{45}$$

$$G_{l4}^H = \frac{P_l}{u_2} - \frac{g}{u_2} q_l^H \tag{46}$$

$$|P| = -G_{h4}^H (G_{h2}^F G_{l3}^H - G_{h3}^F G_{l2}^H) - G_{h2}^H (-G_{h4}^F G_{l3}^H + G_{h3}^F G_{l4}^H) + G_{h3}^H (-G_{h4}^F G_{l2}^H + G_{h2}^F G_{l4}^H) \tag{47}$$

$$\begin{aligned} |P| = &- \left(\frac{P_h}{u_2} - \frac{k}{u_2} q_h^H \right) [4kg - mf] + k \left[2g \left(\frac{P_h}{u_2} - \frac{k}{u_2} q_h^F \right) - m \left(\frac{P_l}{u_2} - \frac{g}{u_2} q_l^H \right) \right] \\ &- m \left[f \left(\frac{P_h}{u_2} - \frac{k}{u_2} q_h^F \right) - 2k \left(\frac{P_l}{u_2} - \frac{g}{u_2} q_l^H \right) \right] \end{aligned} \tag{48}$$

$$\begin{aligned} |P| = &- \left(\frac{P_h}{u_2} - \frac{k}{u_2} q_h^H \right) [4kg - mf] + [2kg \left(\frac{P_h}{u_2} - \frac{k}{u_2} q_h^F \right) - mk \left(\frac{P_l}{u_2} - \frac{g}{u_2} q_l^H \right)] \\ &- [mf \left(\frac{P_h}{u_2} - \frac{k}{u_2} q_h^F \right) - 2mk \left(\frac{P_l}{u_2} - \frac{g}{u_2} q_l^H \right)] \end{aligned} \tag{49}$$

$$|P| = -4kg\left(\frac{P_h}{u_2} - \frac{k}{u_2}q_h^H\right) + mf\left(\frac{P_h}{u_2} - \frac{k}{u_2}q_h^H\right) + 2kg\left(\frac{P_h}{u_2} - \frac{k}{u_2}q_h^F\right) - mk\left(\frac{P_l}{u_2} - \frac{g}{u_2}q_l^H\right) - mf\left(\frac{P_h}{u_2} - \frac{k}{u_2}q_h^F\right) + 2mk\left(\frac{P_l}{u_2} - \frac{g}{u_2}q_l^H\right) \tag{50}$$

$$|P| = -2kg\left(\frac{P_h}{u_2} - \frac{k}{u_2}q_h^H\right) + mk\left(\frac{P_l}{u_2} - \frac{g}{u_2}q_l^H\right) \tag{51}$$

$$\frac{dq_h^H}{du_2} = \frac{|P|}{|A|} = \frac{-2kg\left(\frac{\partial P_h}{\partial u_2} - \frac{\partial k}{\partial u_2}q_h^H\right) + mk\left(\frac{\partial P_l}{\partial u_2} - \frac{\partial g}{\partial u_2}q_l^H\right)}{(-k)[6kg - 2mf]} \tag{52}$$

$$\frac{dq_h^H}{du_2} = \frac{|P|}{|A|} = \frac{2g\left(\frac{\partial P_h}{\partial u_2} - \frac{\partial k}{\partial u_2}q_h^H\right) - m\left(\frac{\partial P_l}{\partial u_2} - \frac{\partial g}{\partial u_2}q_l^H\right)}{[6kg - 2mf]} \tag{53}$$

$$g = \frac{u_1 - u_0}{u_1 + b(u_1 - u_0)} > 0 \tag{54}$$

$$m = \frac{u_1(u_1 - u_0)}{u_2(u_1 + b(u_1 - u_0))} > 0 \tag{55}$$

So $g > m$

$$\frac{P_h}{u_2} - \frac{k}{u_2}q_h^H > \frac{P_l}{u_2} - \frac{g}{u_2}q_l^H \tag{56}$$

$$\frac{dq_h^H}{du_2} = \frac{2g\left(\frac{\partial P_h}{\partial u_2} - \frac{\partial k}{\partial u_2}q_h^H\right) - m\left(\frac{\partial P_l}{\partial u_2} - \frac{\partial g}{\partial u_2}q_l^H\right)}{[6kg - 2mf]} > 0 \tag{57}$$

$$\frac{dq_l^H}{du_2} = \frac{|Q|}{|A|} \tag{58}$$

$$|Q| = G_{h1}^H(-G_{h2}^F G_{l4}^H + G_{h4}^F G_{l2}^H) - G_{h2}^H(-G_{h1}^F G_{l4}^H + G_{h4}^F G_{l1}^H) - G_{h4}^H(G_{h1}^F G_{l2}^H - G_{h2}^F G_{l1}^H) \tag{59}$$

$$\begin{aligned}|Q| = -2k&\left[2k\left(\frac{P_l}{u_2} - \frac{g}{u_2}q_l^H\right) - f\left(\frac{P_h}{u_2} - \frac{k}{u_2}q_h^F\right)\right]\\ &+ k\left[k\left(\frac{P_l}{u_2} - \frac{g}{u_2}q_l^H\right) - f\left(\frac{P_h}{u_2} - \frac{k}{u_2}q_h^F\right) - \left(\frac{P_h}{u_2} - \frac{k}{u_2}q_h^H\right)(kf - 2kf)\right]\end{aligned} \tag{60}$$

$$|Q| = -3k^2\left(\frac{P_l}{u_2} - \frac{g}{u_2}q_l^H\right) + 2kf\left(\frac{P_h}{u_2} - \frac{k}{u_2}q_h^H\right) \tag{61}$$

$$\frac{dq_l^H}{du_2} = \frac{|Q|}{|A|} = \frac{3k\left(\frac{\partial P_l}{\partial u_2} - \frac{\partial g}{\partial u_2}q_l^H\right) - 2f\left(\frac{\partial P_h}{\partial u_2} - \frac{\partial k}{\partial u_2}q_h^H\right)}{[6kg - 2mf]} \tag{62}$$

$$k > f, \frac{k}{u_2} < 0; \left(\frac{P_h}{u_2} - \frac{k}{u_2}q_h^H\right) > 0 \text{ and } \left(\frac{P_l}{u_2} - \frac{g}{u_2}q_l^H\right) < \left(\frac{P_h}{u_2} - \frac{k}{u_2}q_h^H\right) \text{ so } \frac{dq_l^H}{du_2} < 0 \tag{63}$$

$$k = -\frac{(u_0u_1 - u_1u_2 - u_1u_2b + u_0u_2b + u_1^2b - u_0u_1b)}{u_2(u_1 + b(u_1 - u_0))} > 0 \tag{64}$$

$$\frac{k}{u_2} = -\frac{(u_1 + u_1 b - u_0 b)(u_1 + b(u_1 - u_0))}{[u_2(u_1 + b(u_1 - u_0))]^2} \tag{65}$$

$$\frac{k}{u_2} = -\frac{(u_1 - b(u_1 - u_0))(u_1 + b(u_1 - u_0))}{[u_2(u_1 + b(u_1 - u_0))]^2} < 0 \tag{66}$$

$$\begin{aligned} P_h = \frac{1}{u_2(u_1 + b(u_1 - u_0))} \Big[& D_h^F \left(u_1(u_0 - u_2) - u_2 b(u_1 - u_0) + u_1 b(u_1 - u_0)\right) \\ & + D_l \left(-u_1(u_1 - u_0)\right) + a u_1^2 - a u_0 u_1 - u_0 u_1 + u_1 u_2 + u_1 u_2 b - u_0 u_2 b - u_1^2 b + u_0 u_1 b \Big] \end{aligned} \tag{67}$$

$$\frac{P_h}{u_2} = \frac{(u_1 + b(u_1 - u_0)) \left[(u_1 + b(u_1 - u_0)) D_h^F + u_1(u_1 - u_0) D_l - (u_1 + u_1 b - u_0 b)(u_1 + b(u_1 - u_0)) \right]}{[u_2(u_1 + b(u_1 - u_0))]^2} \tag{68}$$

$$\frac{P_h}{u_2} = \frac{(u_1 + b(u_1 - u_0))}{[u_2(u_1 + b(u_1 - u_0))]^2} \left[(u_1 + b(u_1 - u_0)) D_h^F + u_1(u_1 - u_0) D_l - (u_1 + b(u_1 - u_0)) \right] > 0 \tag{69}$$

Total Derivative of prices:

$$dP_h = dk q_h^H + dq_h^H k \tag{70}$$

$$dP_l = dg q_l^H + dq_l^H g \tag{71}$$

$$\frac{dP_h}{du_2} = \frac{P_h}{u_2} - \frac{k}{u_2} q_h^H \tag{72}$$

$$\frac{dP_l}{du_2} = \frac{P_l}{u_2} - \frac{g}{u_2} q_l^H \tag{73}$$

$$\frac{dD_l^H}{du_2} = -b \frac{dP_l}{du_2} \tag{74}$$

$$\frac{dD_l^H}{du_2} = -b \left(\frac{P_l}{u_2} - \frac{g}{u_2} q_l^H \right) \tag{75}$$

Since $\frac{\partial P_l}{\partial u_2} > 0$ and $\frac{\partial P_l}{\partial u_2} > \frac{\partial g}{\partial u_2} q_l^H$, so $\frac{dD_l^H}{du_2} < 0$.

$$D_l = \frac{P_h u_2 - P_l u_1}{u_2 - u_1} - \frac{P_l u_1}{u_1 - u_0} + a - bP_l \tag{76}$$

$$P_l = \frac{(u_1 - u_0)(D_l - a)(u_1 + u_1 b - u_0 b) - P_h u_2 (u_1 - u_0)}{(u_0 u_1 - u_1 u_2 - b u_1 u_2 + b u_1^2 + b u_0 u_2 - b u_0 u_1)} \tag{77}$$

$$\frac{P_l}{u_2} = \frac{(u_1 - u_0)(D_l - a)(u_1 + b(u_1 - u_0))}{(u_0u_1 - u_1u_2 - bu_1u_2 + bu_1^2 + bu_0u_2 - bu_0u_1)^2} + \frac{P_h(u_1 - u_0)(u_1 + b(u_1 - u_0))}{(u_0u_1 - u_1u_2 - bu_1u_2 + bu_1^2 + bu_0u_2 - bu_0u_1)^2} + \frac{u_2(u_1 - u_0)(u_1 + b(u_1 - u_0))\frac{\partial P_h}{\partial u_2}}{(u_0u_1 - u_1u_2 - bu_1u_2 + bu_1^2 + bu_0u_2 - bu_0u_1)^2} \tag{78}$$

$$\frac{P_l}{u_2} > 0 \tag{79}$$